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Incorporating Labor-Leisure Choice

into a Static General Equilibrium Model

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Alan K. Fox*

Abstract

This paper illustrates how a simple labor-leisure choice is incorporated into the U.S. International Trade Commission's CGE model of the U.S. economy. Following the work of Ballard (1999), special attention is paid to the parameterization of the labor-leisure choice so that the relative responsiveness of the model is consistent with econometrically observed behavior. The model is extended using a simple top CES nest between leisure and the consumption good composite.

1. Introduction

For a number of years, the U.S. International Trade Commission (USITC) has used a CGE model to analyze various policy questions. One of the major applications for the USITC's U.S. model has been in conducting the analyses for the periodic study, *The Economic Effects of Significant Import Restraints***.** The next edition of the study, the third update, is expected to be released in the summer of 2002. One of the innovations in the model used for the latest study is the incorporation of a labor-leisure choice for the representative household. A full description of the model employed in each study can be found in its respective technical appendix.

Over time, additional features have been added to the model to increase its usefulness for this type of analysis. This paper lays out one of the most recent features of the model, the adoption of a new specification for labor supply. The U.S. model has, until now, had a simple behavioral description of the labor market: the supply of labor is expressed in terms of efficiency units, and the quantity supplied does not vary with the wage. This paper illustrates how the model has been adopted to include a simple labor-leisure choice for the representative household, thus allowing labor to respond to a change in the real wage. While the econometric evidence suggests that the long-run responsiveness of labor supply is relatively small, including some response is nonetheless useful in illustrating how the economy responds to a liberalization of tariffs or other import restraints.

In order to incorporate a labor-leisure choice in the U.S. model, it is necessary for us to construct a current level of leisure that is consistent with the output of the U.S. economy and

*Office of Economics, U.S. International Trade Commission, Washington, DC 20436. The author may be contacted by email at Alan.Fox@usitc.gov. This paper is the result of ongoing professional research of USITC Staff and solely represents the opinions and professional research of the individual author. This paper is not meant to represent in any way the views of the U.S. International Trade Commission or any of its individual Commissioners.

the known characteristics of the U.S. supply of labor. The method presented here relies on the work of Ballard (1999). Ballard emphasizes that the parameterization of the labor-leisure choice is critical to the outcome of simulation. Specifically, the choice of the time endowment parameter, or how many hours are in a day, determines the total-income elasticity of labor supply. This can have a material impact on the relative responsiveness of changes in tax policy. For example, if the total time endowment is too large relative to the benchmark level of hours worked, the responsiveness of labor supply to a policy change can be implausibly large, despite the fact that the other parameters describing the labor market are well within the range of generally accepted values.

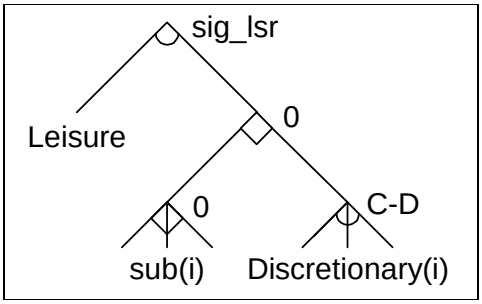
Below we provide a basic overview of the household in the U.S. model, followed by the derivation of the necessary parameters. In order to parameterize the model, it is necessary to derive the compensated and uncompensated elasticities of labor supply and leisure demand, as well as the total-income elasticity of labor supply.

2. Overview of Consumer Model

The original formulation of consumer's utility in the U.S. model used a Linear Expenditure System[†] (LES) to model behavior of the household. In figure 1 below, the original formulation consists of the right-hand nest of subsistence consumption (sub(i)) and discretionary consumption (Discretionary(i)).

The household utility function is then modified to take into account the consumer's choice between labor and leisure. The new model structure adds a top CES nest allowing substitution between leisure and the consumption of goods and services. The top nest of Figure 1 illustrates this structure, with Leisure and the consumption composite combined in a CES function with an elasticity of sig_lsr.

Figure 1: Household utility function



1.1 Derivation of Behavioral Parameters

The consumer utility function is extended by allowing the household to substitute consumption of leisure for consumption of goods and services. The top nest between leisure and other consumption is aggregated as a CES function. While the household in fact consumes a wide variety of goods and services, for the purposes of this exposition they will

[†] For example, see Deaton and Mulbauer (1980, pp. 64-7) for details on the LES specification.

be treated as a composite good X. The consumer faces the following utility maximization problem (expressed in share terms):

$$U = \left[\theta \left(\frac{L}{L_0} \right)^{\rho} + (1-\theta) \left(\frac{X}{X_0} \right)^{\rho} \right]^{1/\rho}, \text{ such that } M + W\bar{L} = WL + P_X X . \quad (1)$$

That is, the consumer maximizes utility as a weighted average of leisure (L) and a composite consumption good (X), such that the budget constraint is satisfied. The consumer's available resources are the sum of other (non-wage) income M and the endowment of time ($\bar{L}$) evaluated at the wage rate (W). The cost of consumption is simply leisure evaluated at the wage rate plus consumption of the composite good at the consumer's price (P_X). The parameters L_0 and X_0 represent the benchmark demands for leisure and the consumption good. The share parameter θ is defined as the benchmark share of the budget devoted to leisure:

$$\theta = \frac{W_0 L_0}{M_0 + W_0 \bar{L}} \quad (2)$$

Thus, utility in Equation (1) is set to 1 at the benchmark. Solving the consumer's problem, we find the following expressions for demand for leisure and the consumption good:

$$L^* = L_0 \left(\frac{M + W\bar{L}}{M_0 + W_0 \bar{L}} \right) \left(\frac{W}{W_0} \right)^{-\sigma} \left[\theta \left(\frac{W}{W_0} \right)^{1-\sigma} + (1-\theta) \left(\frac{P_X}{P_{X0}} \right)^{1-\sigma} \right]^{-1} \quad (3a)$$

$$X^* = X_0 \left(\frac{M + W\bar{L}}{M_0 + W_0 \bar{L}} \right) \left(\frac{P_X}{P_{X0}} \right)^{-\sigma} \left[\theta \left(\frac{W}{W_0} \right)^{1-\sigma} + (1-\theta) \left(\frac{P_X}{P_{X0}} \right)^{1-\sigma} \right]^{-1} \quad (3b)$$

These can then be used to derive the indirect utility function V:

$$V(P_X, W, M) = \frac{M + W\bar{L}}{M_0 + W_0 \bar{L}} \cdot \frac{1}{e(W, P_X)} \quad (4)$$

where e is the unit expenditure function and is defined as

$$e(W, P_X) = \left[\theta \left(\frac{W}{W_0} \right)^{1-\sigma} + (1-\theta) \left(\frac{P_X}{P_{X0}} \right)^{1-\theta} \right]^{\frac{1}{1-\sigma}} . \quad (5)$$

For the purposes of our model, we need to derive expressions tying together the compensated and uncompensated leisure demand and labor supply elasticities. The compensated leisure demand elasticity $\eta_L|_{\bar{u}}$ is derived from the leisure demand function:

$$\eta_L|_{\bar{u}} = \sigma \left[\theta \left(\frac{W_0 e(P_X, W)}{W} \right)^{\sigma-1} - 1 \right] \quad (6)$$

At the benchmark, $\eta_L|_{\bar{u}} = \sigma[\theta - 1]$.

We also know the following relationship between the compensated leisure demand and labor supply elasticities:

$$\varepsilon_L|_{\bar{u}}\,(\bar{L} - L) + \eta_L|_{\bar{u}}\,L = 0 \tag{7a}$$

where $\varepsilon_L|_{\bar{u}}$ is the compensated elasticity of labor supply. This can be rewritten as

$$\varepsilon_L|_{\bar{u}} = -\eta_L|_{\bar{u}}\,\frac{L}{\bar{L} - L} \tag{7b}$$

Next our attention turns to the derivation of the uncompensated leisure demand and labor supply elasticities. The uncompensated leisure demand elasticity can be derived from Equation (3a) above.

$$\eta_L = \underbrace{\frac{W\bar{L}}{M + W\bar{L}} - \theta\left[\frac{W_0}{W}\cdot e(P_X, W)\right]^{\sigma-1}}_{\eta_I} + \underbrace{\sigma\left[\theta\left(\frac{W_0}{W}\cdot e(P_X, W)\right)^{\sigma-1} - 1\right]}_{\eta_L|_{\bar{u}}} \tag{8}$$

The uncompensated leisure demand elasticity is equal to the sum of the total income elasticity and the compensated leisure demand elasticity from Equation (6) above. Next, we can establish a relationship between the uncompensated elasticity of leisure demand and labor supply in a fashion similar to that for their compensated versions shown in Equation (7).

$$\eta_L L + \varepsilon_L(\bar{L} - L) = 0 \tag{9a}$$

which can be rewritten as

$$\varepsilon_L = -\eta_L\,\frac{L}{\bar{L} - L} \tag{9b}$$

Now we can establish the relationship between the uncompensated and compensated supply elasticities. First, we note that in Equation (8), the benchmark value of η_I is determined as follows:

$$\eta_I = \frac{\bar{L}W_0}{M_0 + \bar{L}W_0} - \theta \tag{10a}$$

If we define $\lambda = L/\bar{L}$, we can rewrite this expression as

$$\eta_I = \theta\left(\frac{1-\lambda}{\lambda}\right) \tag{10b}$$

Substituting Equation (9b) into Equation (8) yields

$$\varepsilon_L = \frac{-\lambda}{1-\lambda}\left[\eta_I + \eta_L\,|_{\bar{u}}\right] \tag{11}$$

We can then substitute Equations (10b) and (7b) into (11) to arrive at the following relationship:

$$\varepsilon_L = \varepsilon_L\big|_{\bar{u}} - \theta \tag{12}$$

Now we need only establish the initial value for leisure. Let us define one additional variable, *H*, to represent the number of hours worked, such that $\bar{L} \equiv L + H$.

This allows us to solve Equation 2 for *L* (leisure) in terms of θ, *M*, and *H*. Note that, at the benchmark, *W* is set to 1.

$$\begin{aligned} L &= \theta(M + \bar{L}) \\ &= \theta(M + H + L) \\ L(1-\theta) &= \theta(M + H) \\ L &= \frac{\theta(M+H)}{1-\theta} \end{aligned} \tag{13}$$

The value of the expression $M + H$ is taken to be equal to the benchmark value of final demand. Ballard (1999) suggests that reasonable values for the supply elasticities are $\varepsilon_L = 0.1$ and $\varepsilon_L\big|_{\bar{u}} = 0.3$. These values, combined with Equation 12, allow us to determine a value of $\theta = 0.3 - 0.1 = 0.2$. Combining this with Equation 13 allows us to pin down the benchmark value for *L*, leisure. This particular parameterization suggests that leisure is worth about one-quarter of the final demand for goods and services.

Lastly, we need to establish the benchmark value of σ, the elasticity of substitution between labor and leisure. We turn to the benchmark value of Equation (6):

$$\eta_L\big|_{\bar{u}} = \sigma(\theta - 1) \tag{14}$$

Combining this with Equation (7b), we can solve for σ:

$$\sigma = \frac{\varepsilon_L\big|_{\bar{u}}}{1-\theta} \cdot \frac{\bar{L} - L}{L} \tag{15}$$

Thus, given econometric estimates of ε_L and $\varepsilon_L\big|_{\bar{u}}$, we can combine these with the benchmark level of expenditure on goods and services and Equations (12), (13), and (15) to derive the parameters necessary for the model: σ, the elasticity of substitution between labor and leisure; *L*, the total amount of leisure consumed at the benchmark. These permit us to incorporate the labor-leisure choice into the U.S. model such that the model's response from the baseline is consistent with the econometrically observed responsiveness to changes in tax policy.

2. Implementation in MPSGE/GAMS

Implementation in MPSGE/GAMS is actually quite straightforward. Extracts from the model code are given in Section 4. The first block lists the parameter declarations and the equations implementing Equations (12), (13), and (15) above. Given values for η_L and $\eta_L|_{\bar{u}}$, we can combine these with the benchmark final demand to establish the benchmark level of leisure.

The next block illustrates how leisure is incorporated into the household utility function. We see that the top nest of consumption is set to `s:sig_lsr`. Leisure enters in the final line of the `PROD:U` block as `I:wanet Q:lsr`.

3. Sample Model Results

Because the third update of the study "The Economic Effects of Significant Import Restraints" has not yet been released, we must withhold results relating to that study. Unfortunately, we have not had the time to construct alternative scenarios and results to illustrate how the labor-leisure choice works in the U.S. model. The author does believe, however, that the parameter choices made lead to plausible outcomes in the labor market. For example, as the real wage rises in response to a tariff liberalization, a modest amount of labor is drawn into the workforce, therefore damping the wage response.

An updated version of this paper including alternative scenarios, results, and sensitivity analysis of different parameter choices will be made available on the GTAP website after the conference.

4. GAMS Code

4.1 Parameter Declarations

```
* ---------------------Labor Leisure-------------------------*
*       Inputs    - Uncompensated labor supply elasticity
*                 - Compensated labor supply elasticity
*                   (or income elasticity or leisure)
*       Outputs   - Substitution elasticity between Consumption
*                   and Leisure
*                 - Additional endowment of Labor
* -----------------------------------------------------------*

parameter   sig_lsr     Elast. of subst. b/w leisure and consumption
            lsr         Benchmark value of leisure
            l_bar       Benchmark labor endowment
            theta_l     Leisure's share in overall budget
            eps_l       Uncompensated labor supply elasticity
            eps_l_u     Compensated labor supply elasticity
            ballard     Ballard coefficient;

eps_l   = 0.1;
eps_l_u = 0.3;
theta_l = eps_l_u - eps_l;
lsr     = sum(i,fd(i,"hhld")) * theta_l / (1-theta_l);

display lsr;

l_bar = (ylab0+labsav0-ltaxt0-lytaxt0) + lsr;

display l_bar;

sig_lsr = (eps_l_u/(1- theta_l)) * ((l_bar - lsr)/lsr);

display sig_lsr;

ballard = (l_bar /(l_bar - lsr));

display ballard;
```

4.2 Household Utility

```
$PROD:U s:sig_lsr les:0 cd(les):1
            O:cpi    Q:(sum(I,fd,(I,”hhld”))+lsr)
            I:pq(i)  Q:sub(i) les:
            I:pq(i)  Q:(fd(I,”hhld”)-sub(i)) cd:
            I:wanet Q:lsr

$report:
V:hhd(i) I:pq(i) PROD:u
V:D_lsr  I:wanet PROD:u
```

References

Ballard, Charles L. (October 1999) "How Many Hours Are in a Simulated Day? The Effects of Time Endowment on the Results of Tax-Policy Simulation Models," Working Paper. East Lansing, Michigan: Department of Economics, Michigan State University.

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