Estimating Productivity Changes with Flexible Coefficients

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Technical progress in U.S. agriculture is evaluated using a new measure of productivity growth, flexible technical change. This measure allows for nonconstant returns to scale, market structures other than perfect competition, and time-varying coefficients. An integral part of the procedure is the estimation of the production function by Flexible Least Squares. Flexible technical change results are compared with two traditional measures of productivity growth and found to be more stable and more precise in a statistical sense. The results suggest that previous studies which employed total factor productivity measures may have overstated the impact of technology in agriculture.

Key words: flexible least squares, productivity, technical change, time-varying coefficients.

The substantial growth in post-World War II U.S. agricultural production is often attributed to significant technological advances. However, it is difficult to disentangle the effects of two simultaneous processes: changes in technology and increases in input levels. Further, technological change can be of two types. Antle and Capalbo define disembodied technical change as the utilization of existing resources in a manner which achieves higher rates of output per unit of input and embodied technical change as improvements in input quality which lead to higher rates of output per unit of input. These factors are difficult to measure and certainly both have contributed to the increase in U.S. agricultural productivity.

The estimation of the rate of productivity growth has a long history in the literature, with interest in the topic accelerating in the 1960s. Interest has peaked again recently due to claims that U.S. agriculture is losing its comparative advantage in world markets. In an economy-wide setting, Wright rejects the notion of lagging U.S. productivity and suggests that the appearance of a U.S. productivity slowdown relative to other countries has been created by structural changes in the global nonrenewable resource base.

Perhaps the most commonly applied measure of productivity growth has been the notion of total factor productivity (Solow; Jorgenson and Griliches). The assumptions about the production process that are implicit in the measure of total factor productivity have been examined in a number of articles in the last decade. The assumption of constant returns to scale is relaxed by Capalbo and by Baltagi and Griffin. Denny, Fuss, and Waverman allow for nonconstant returns to scale and for market structures that are not perfectly competitive. Cas, Diewert, and Ostensoe relax the assumption of constant coefficients. Swamy, Conway, and LeBlanc provide a theoretical justification for the use of a production function with time-varying parameters when employing aggregate data. They show that the aggregation of microlevel, constant parameter production functions leads to an aggregate function with time-varying parameters. Swamy, Lupo, and Sneed develop a theoretical model which relaxes all three assumptions.

This article also develops a general approach, avoiding all three assumptions, and introduces a new measure, named here flexible technical change. The method is applied to the

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measurement of U.S. agricultural productivity and compared to two alternative methods. By employing a production function approach, no assumptions are needed concerning the market structure of the industry, and the problem of the possible endogeneity of input prices is avoided. Constant returns to scale are simply not imposed on the estimated production function. Time-varying coefficients are introduced by employing a new technique developed by Kalaba and Tesfatsion called Flexible Least Squares (FLS).

The use of a production function approach, which avoids the use of input prices,[1] mitigates some of the problems associated with productivity measurement in the face of disequilibria, but not all. The most common disequilibrium mentioned in the area of productivity measurement has been underutilization of capital (Berndt and Fuss; Hulten; Morrison 1985, 1986; and Slade).

If the capital input to the production function is the entire capital stock and only part of the total capital stock is currently being employed in the production process, estimated productivity will be biased downward (understated). For the U.S. manufacturing sector, Berndt and Fuss suggest that the downward bias from the failure to correct for capacity utilization is approximately 33% when gross output is the dependent variable (as in our production function). Morrison (1986) shows that while adjusting for capacity utilization can be important, the bias introduced by imposing constant returns to scale is even larger. Capalbo also found that imposing constant returns to scale led to a downward bias in the estimate of productivity growth in U.S. agriculture of approximately .5% per year over the period 1950–82. The traditional total factor productivity measure suffers from both types of bias.

The methodology of FLS is ideally suited for the new measure. If productivity is changing through time, it is logical to assume that the entire production function is "evolving" through time.[2] The FLS estimators of the production function coefficients are allowed to follow a gradual dynamic process, thereby allowing for the time element to enter the production function in a new manner far more general than simply as another right-hand-side variable. This article uses aggregate data on U.S. agriculture compiled by Capalbo, Vo, and Wade to compute three measures of productivity growth or technical change including total factor productivity and the new flexible technical change measure.

Theoretical Development of Productivity Measures

Beginning as generally as possible, let

$$Q = f(x_1, \ldots, x_k, t) \tag{1}$$

be the production function in question where Q is the output and $\{x_i, i = 1, k\}$ represents the k inputs to the production process. The variable t represents time and enters the production function in several ways. First, production is assumed to be stochastic in nature due to the weather and physiological phenomena characteristic of agriculture. Thus, at the simplest level the time element of $f(\bullet)$ can be thought of as representing a stochastic error process inherent in production (e.g., an additive error term when estimating $f(\bullet)$ econometrically). Second, it is possible that the actual parameters of the function change with time (e.g., random coefficients models or structural change hypotheses). This second case involves a much more complex time element in the production function and is examined in this article through the application of the FLS methodology.

Three measures of productivity are examined: total factor productivity, simple technical change, and a new measure, flexible technical change. Total factor productivity (*TFP*) has a long history in the literature as a residual growth of output not explained by the growth of inputs (thus encompassing both embodied and disembodied technical change). A review of related articles is provided by Nadiri, and a recent application to agriculture can be found in Ball. A narrower productivity measure, termed simple technical change (*STC*) here, is the elasticity of the production function with

[1] Agricultural production is carried out with the levels of variable inputs selected by solving a suitable optimization problem employing the expected level of output as a function of input levels. Hoch demonstrates that under these conditions simultaneity between input and output levels disappears and a single-equation estimator of the production function parameters is consistent.

[2] Several justifications for such an assumption are offered in the Results section of this article. The major justification is based on the theory of technological adoption in agriculture (cf., Griliches; Jarvis).

respect to a unit of time. The third measure, flexible technical change (*FTC*), will be defined simply as the equivalent to the *STC* when the coefficients of the production function are allowed to vary over time. Thus, *FTC* allows for a fuller interaction of the time variable with the other arguments of the production function, the inputs.

None of the measures corrects for capacity utilization, but both *STC* and *FTC* allow for nonconstant returns to scale, thereby avoiding one potential source of bias (Morrison 1986; Capalbo). Capacity utilization is a difficult issue in agriculture. In the manufacturing sector, for example, it is easy to see how many hours a factory is used and to measure when workers are laid off. In agriculture many capital items such as harvesters are purchased for use in a specific task and long idle periods are anticipated when the purchase price was set. Therefore, it may be difficult to determine what the producer's expected utilization rate is, let alone the percent of that expected utilization which occurs. Moreover, many of these items have no alternative uses during idle periods. Land is another such input, where mandatory set asides incorporated into the commodity programs force underutilization. Many farmers fulfill their set-aside requirements with marginal land which, again, may have had low or zero expected utilization rates. Because of these difficulties, we have decided to correct the problems caused by the restrictions of constant coefficients and constant returns to scale. While underutilization of capital (including land) may cause our results to be downward biased, they still represent a significant improvement over the traditional *TFP* measure.

In defining these three measures, a variable with a tilde represents the proportional rate of change of that variable with respect to time. For example,

$$\tilde{Q}_t = \left(\frac{\partial Q_t}{\partial t}\right)\left(\frac{1}{Q_t}\right). \tag{2}$$

Total factor productivity is defined (in terms of proportional changes) very simply as

$$\widetilde{TFP}_t = \tilde{Q}_t - \sum_{i=1}^{k} s_{it}\tilde{x}_{it}, \tag{3}$$

where s_{it} is the cost share of the ith input at time t.

A simple application on equation (2) of the chain rule of differentiation leads to an implicit definition of simple technical change, $\widetilde{STC}_t = (\partial f/\partial t)/Q_t$:

$$\tilde{Q}_t = \sum_{i=1}^{k}\left(\frac{\partial f}{\partial x_{it}}\right)\left(\frac{\partial x_{it}}{\partial t}\right)\left(\frac{1}{Q_t}\right) + \left(\frac{\partial f}{\partial t}\right)\left(\frac{1}{Q_t}\right). \tag{4}$$

Solving the expression for $\widetilde{STC}$ and simplifying gives:

$$\widetilde{STC} = \tilde{Q}_t - \sum_{i=1}^{k}\left(\frac{\partial f}{\partial x_{it}}\right)\left(\frac{x_{it}}{Q_t}\right)\tilde{x}_{it}. \tag{4'}$$

The tilde is employed because $\widetilde{STC}$ is also a proportional change since it can be written as $\widetilde{STC} = (\partial f(\bullet)/\partial t)/f$. By comparing (3) and (4'), it is obvious that the only difference between the total factor productivity measure and the simple technical change measure is the weights employed in the aggregation of the changes in input use. In fact, Capalbo shows in a few simple steps that the two measures are identical assuming cost minimization if the elasticity of total cost with respect to output is unitary. In this production function world, the equivalent condition is that the production function be homogeneous of degree one. Note that we are not imposing such an assumption (of constant returns to scale) here, so that the two measures will differ.

To derive the third measure, the measure of flexible technical change, all that is necessary is to recognize that with time-varying coefficients interacting with the inputs in the production function the marginal products have another dimension of variation. In the constant coefficient world, $\partial f/\partial x_i$ can vary if $f(\bullet)$ has nonlinear terms in x_i and the x_is vary through time. Most production functions possess this property, and certainly most input levels are time varying. However, with time-varying coefficients even the parameters of $f(\bullet)$ that appear in the derivative will be changing through time. This represents an alternative manner for biased technical change to manifest itself. Swamy, Conway, and LeBlanc show that the aggregation of microlevel production functions leads to an aggregate production function with time-varying parameters. Note that the only difference between the measures of simple technical change and flexible technical change is that the coefficients of the production function are time varying. To emphasize the time-varying coefficients, define the proportional change in *FTC*, $\widetilde{FTC}$, by:

(5) $\widetilde{FTC}_t = \tilde{Q}_t - \sum_{i=1}^{k} \left(\frac{\partial f_F}{\partial x_{it}}\right)\left(\frac{x_{it}}{Q_t}\right)\tilde{x}_{it}.$

In (5), $f_F(\bullet)$ represents the production function estimated using FLS, so that the parameters involved in the calculations of the marginal products change each period. This is a somewhat more general use of time-varying parameters to measure productivity changes than a new approach by Baltagi and Griffin which uses time-period-specific dummy variables to estimate an index of technical change that they embed in a translog cost function.

Up to this point all three measures have been discussed in a continuous time frame, but production in agriculture is discrete in general, and the data on production are definitely discrete in their availability. Therefore, the three productivity measures must be converted to their discrete approximations. This is done using the Tornqvist-Theil discrete approximation to a Divisia index because the production function is modeled as a translog function. See Caves, Christensen, and Diewert; and Cas, Diewert, and Ostensoe for demonstrations of why Tornqvist-Theil indices are theoretically "exact" when used in conjunction with a translog function. Using the same notation for the three measures (since only discrete results will be produced, confusion should be minimal), the discrete approximations to $\widetilde{TFP}$, $\widetilde{STC}$, and $\widetilde{FTC}$ are:

(6) $$\widetilde{TFP}_t = \ln\left(\frac{Q_t}{Q_{t-1}}\right) - \frac{1}{2}\sum_{i=1}^{k}(s_{it} + s_{it-1})\ln\left(\frac{x_{it}}{x_{it-1}}\right),$$

(7) $$\widetilde{STC}_t = \ln\left(\frac{Q_t}{Q_{t-1}}\right) - \frac{1}{2}\sum_{i=1}^{k}\left[\left(\frac{\partial f(t)}{\partial x_{it}}\right)\left(\frac{x_{it}}{Q_t}\right) + \left(\frac{\partial f(t-1)}{\partial x_{it-1}}\right)\left(\frac{x_{it-1}}{Q_{t-1}}\right)\right]\ln\left(\frac{x_{it}}{x_{it-1}}\right),$$

and

(8) $$\widetilde{FTC}_t = \ln\left(\frac{Q_t}{Q_{t-1}}\right) - \frac{1}{2}\sum_{i=1}^{k}\left[\left(\frac{\partial f_F(t)}{\partial x_{it}}\right)\left(\frac{x_{it}}{Q_t}\right) + \left(\frac{\partial f_F(t-1)}{\partial x_{it-1}}\right)\left(\frac{x_{it-1}}{Q_{t-1}}\right)\right]\ln\left(\frac{x_{it}}{x_{it-1}}\right).$$

Again, note that the production function in (8) used to define $\widetilde{FTC}_t$ is subscripted with an F to emphasize that the production function has been estimated by FLS.

Flexible Least Squares

A common assumption in the modeling of economic systems is one of constant coefficients. This assumption can, of course, be easily relaxed through the application of various techniques such as Swamy random coefficients. However, many people want to believe that there is a true model underlying the observed data which has a set of deterministic parameters. In the present application, this would be a belief in a constant coefficient production process generating the stochastic supply outcomes we observe. A compromise position is one which states that perhaps the underlying process is not stable in an ordinary constant parameter sense but is instead an evolving process with parameters that follow some gradual adjustment paths. A technique recently proposed by Kalaba and Tesfatsion, FLS, allows for the simple estimation of the parameters for a linear model of this "slowly dynamic" form and represents a compromise between constant coefficient and fully random coefficient models.

Let the process to be modeled be represented by the linear function

(9) $y_t = x_t' \beta_t + \epsilon_t, \qquad t = 1, 2, \ldots, T.$

Here, $\{y_t\}$ is the process to be modeled, x_t is a (column) vector of explanatory variables, β_t is a vector of the model's parameters, and ϵ_t is the result of a stochastic process which will be referred to here as the observation error. Further, assume that the parameters of the model, β_t, only change from their values in the previous period by some small amount, v_t, called the dynamic error. Making this explicit mathematically, let

(10) $\beta_t = \beta_{t-1} + v_t, \qquad t = 2, 3, \ldots, T.$

Two possible interpretations to the model outlined by (9) and (10) add some intuition to this approach. The first is the obvious: a model with parameters that change slowly through time as described is believed by the researcher to be the true model for a given application. The second interpretation is that the parameters are believed to be constant by the researcher, but to guard to some extent against misspecification error, a stochastic restriction of constant parameters is placed on a random parameter model. This is done by stochastically restricting the set of v_t to be small. A slight variation of this second story is a Bayesian approach. A Bayesian might have a subjective prior belief that the parameters were constant with probability p and time varying with probability $(1 - p)$.

The model described by (9) and (10) allows both this Bayesian interpretation and the stochastic restriction approach to be implemented by a procedure that attempts to minimize the sets of estimated ϵ_t and v_t according to some weighting scheme. The FLS estimator is defined in such a manner. Denote the FLS estimates of the set of β_t as $\{b_t, t = 1, 2, \ldots, T\}$. Then the set of b_t is defined as the solution to the minimization problem:

$$\text{(11)} \quad \min L = \sum_{t=2}^{T} \mu(b_t - b_{t-1})'(b_t - b_{t-1}) + \sum_{t=1}^{T} (y_t - x_t'b_t)'(y_t - x_t'b_t) = \sum_{t=2}^{T} \mu \hat{v}_t' \hat{v}_t + \sum_{t=1}^{T} \hat{\epsilon}_t' \hat{\epsilon}_t.$$

The hats over the errors are used to stress that they are only estimates of the true set of dynamic and observation errors, resulting from the selection of the set of estimates of the parameter vectors and the choice of μ.

The parameter μ which appears in the optimization problem makes explicit the weighting scheme for trading off observation and dynamic errors. While it is taken as a scalar here, there is no particular difficulty in generalizing this to a matrix weight. In fact, Kalaba and Tesfatsion already have developed a FORTRAN program that allows such a generalization. The parameter μ represents the inverse of the prior precision in a manner directly analogous to the case of least-squares estimation subject to a stochastic restriction set. As μ increases, the researcher is demanding a closer adherence to constancy from the b_t parameter vectors. In the limits, $\mu = \infty$ is equivalent to OLS estimation and $\mu = 0$ is equivalent to a random coefficients model.[3] In practice, a researcher may desire to try several values of μ and then assess the various performances of the estimates by plotting what Kalaba and Tesfatsion call the residual efficiency frontier. This frontier shows the tradeoff between the sum of the squared observation errors and the sum of the squared dynamic errors. It is convex and often can give a useful guide to how an amount of "randomness" in the coefficients leads to a drop in the (sum of the squared) observation errors. In many cases a large decrease in the sum of the squared observation errors can be achieved by allowing only a small amount of dynamics to enter into the parameter vectors.

If the v_t are assumed to be nonstochastic (i.e., the β_t evolve in a deterministic but unknown manner) and the ϵ_t are assumed to be normally distributed, it is possible to show that all of the FLS estimates of the β_t are distributed as Student t random variables, each with the usual $(T - k)$ degrees of freedom. This assumption that the true coefficients are nonstochastic simplifies the calculation of the covariances of the estimated parameters but adds the provision that the estimated covariances will be conditional on the choice of μ.

For details of the algebra involved in obtaining FLS coefficients and for a demonstration of the distribution of the estimates, see the appendix.

Data

The data employed to empirically estimate the production function were derived from the data described in Capalbo, Vo, and Wade for agricultural production in the U.S. This data source includes Divisia quantity and implicit price indices for six output and 10 input categories for the period 1948–83.

For the purposes of the present study, the outputs were aggregated to a single total using

[3] Although reference has been made to how well the FLS methodology models processes which are "slowly dynamic," this should not be taken to imply that the procedure cannot allow for rapid structural shifts in the underlying relationship. Kalaba and Tesfatsion include simulation experiments in which the FLS coefficients track true parameters moving in sinusoidal and elliptical patterns. Tesfatsion and Veitch show the ability of FLS to track structural shifts in the underlying parameters. They further show, in an empirical example involving a money demand function, that FLS coefficients can follow processes which might be characterized as "nonsmooth" or "rapidly dynamic."

the Tornqvist-Theil approximation to the Divisia index. The weights in this procedure are the value shares of each commodity. The implicit assumption embedded in such an aggregation is input-output separability. While this may be somewhat unrealistic in principle, it is unavoidable here because the aggregate input quantities cannot be separated according to their employment in the production of the various outputs.

The production function was specified to have three inputs: labor, capital, and materials. Labor includes both family and hired labor. Capital includes land, structures, inventories, equipment, and breeding stock. Included in materials are the inputs energy, fertilizer, pesticides, feed and seed, and a miscellaneous category. The aggregation from 10 to three inputs was also performed using the Tornqvist-Theil approximation to the (chain) Divisia index. The base year for all of the aggregation computations was set at the sample midpoint, 1966.

Implicit in the input aggregation is the assumption that the inputs are additively separable. In light of this, care was taken to aggregate in a manner consistent with previous research in productivity and according to the authors' prior beliefs about agricultural input separability. In any case, a certain amount of input aggregation becomes necessary to accommodate degrees-of-freedom considerations and to avoid excessive multicollinearity.

Results

The production function was estimated as a translog with the single output and three inputs described above. The translog was chosen, as mentioned previously, to match the choice of indexing method employed in constructing both the data set and the three productivity measures (the Divisia index). As the coefficients are of little interest here, they are not reported. It is worth noting, though, that the FLS estimates of the production function are related to the OLS estimates. Kalaba and Tesfatsion prove that the OLS estimates are a (matrix) weighted average of the set of the FLS estimates of a given model.

To confirm the necessity of time-varying coefficients, an empirical test for constancy of the coefficients of the translog production function was performed. The procedure is one recently developed by Ploberger, Krämer, and Kontrus. The test is based on fluctuations in the parameter estimates as successively larger subsamples of the data set are employed to produce OLS estimates. The basis of the test is that if the OLS estimate of any coefficient varies by more than a specified amount as the number of observations used to estimate it changes from k to T (number of parameters to entire sample), then the null hypothesis of constant parameters is rejected for that model and data set. The critical value of the test given by Ploberger, Krämer, and Kontrus for $k = 10$ at the 99% confidence level is 1.95 and the calculated test statistic was 1,179. This strong rejection of the null hypothesis of constant coefficients provides a clear justification for the use of the FLS algorithm in the estimation of the translog production function for this aggregate U.S. agricultural data set and agrees with the theoretical results of Swamy, Conway, and LeBlanc.

FLS estimates were produced for 14 different values of μ. These sets of FLS estimates were used to produce an estimated residual efficiency frontier (as discussed previously) which was employed in determining the desired value of μ for this application. The residual efficiency frontier that was produced is shown in figure 1. The parameter μ was set at 1.5 after examining the sets of estimates and the residual efficiency frontier (denoted with an * on figure 1). It was felt that this provided the optimal tradeoff between observation error and dynamic error. Allowing for this amount of dynamics in the production function coefficients produced a 71% decrease in the sum of the squared measurement errors. Yet the sum of squared dynamic errors is only 16% of what it would be for a random coefficients model. Thus, as the figure shows, allowing a little dynamics in the parameters can greatly reduce the observation errors of the model. While the choice of μ is somewhat arbitrary, it is directly analogous to the prior precision employed in stochastic restriction estimation. An alternative interpretation is that of a loss function which reflects the loss felt by the model's users from squared dynamic errors relative to squared observation errors. In this context, μ is the relative loss parameter. Essentially, the loss function approach is the method used by the authors to select the value of μ.

After estimating the production function by

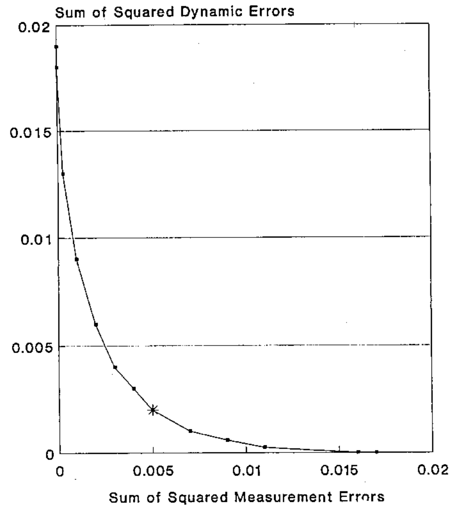

Figure 1. Residual efficiency frontier

both FLS and OLS, the proportional changes in the three productivity measures could be calculated using equations (6)–(8). The average values by decade for the three measures are presented in table 1. Note that these are the percent changes in the measures themselves, not the proportional changes generated by equations (6)–(8). The measure of total factor productivity clearly paints the rosiest picture of technological growth. Unfortunately, it is also the least appealing theoretically. Capalbo shows that U.S. agriculture did not display the constant returns to scale that would make *TFP* a reasonable measure of technological gains. The measures of *STC* and *FTC* produced considerably lower estimates of the productivity gains in U.S. agriculture, with the *FTC* measure being the higher. Particularly interesting is the fact that the *FTC* measure diverges sharply from the *STC* during the 1970s, showing much higher technological growth. This matches the findings in Morrison (1986) that the relaxation of assumptions made in productivity measures (such as constant returns to scale or constant coefficients) tended to raise estimated productivity growth during the seventies. Note that the *TFP* growth rate also increased in the 1970s, although by a smaller percentage than the *FTC* growth rate. The year-to-year changes are highly variable and are listed in table 2 and graphed against time in figure 2.

An interesting question to answer concerns the effect of allowing dynamic parameters in

Table 1. Average Values (%) of the Changes in *TFP, STC,* and *FTC*

	1950s	1960s	1970s	1980–83	1949–83
TFP	1.077	1.344	1.731	−0.299	1.078
STC	0.280	0.142	0.177	0.309	0.0778
FTC	0.363	−0.287	0.740	0.373	0.214

Note: *TFP* is total factor productivity, *STC* is simple technical change, and *FTC* is the measure of flexible technical change. These measures are defined in equations (6)–(8), respectively.

the production function on the productivity measures. The results show that, in fact, the *FTC* measure is the least variable of the three measures. Allowing for dynamic parameters in the production process results in a more stable estimate of technological growth. The measures *TFP, STC,* and *FTC,* when expressed in percent changes, had sample variances of 10.905, 11.012, and 6.946, respectively. A possible, and intuitive, explanation of this stability in the *FTC* is that the dynamic coefficient specification is a better model of the true production process than a constant coefficient one (at least for the specification tried here).

Research has provided a basis for anticipating, a priori, a fairly smooth process of technical change. Certainly new developments occur in discrete increments, but implementation and adaptation to real-world conditions occur continuously. Studies by Griliches and by Jarvis suggest that adoption of new technology follows a logistic pattern. If one graphed the percent of adopters versus time for some new technological improvement, the result would likely be an S-shaped curve similar to the cumulative distribution function of a normal random variable. This would lead the coefficients of the production function to time vary, but in a "smooth," albeit nonlinear, manner. Restricting the coefficients from being completely random protects against truly stochastic changes in the marginal products of the inputs, changes which are not due to technical change. The greater stability of this measure may well come from the effects of a key missing input: weather. Annual weather variations certainly affect the marginal products of other inputs for many agricultural products. The FLS methodology allows the production function parameters to adjust to this phenomenon, while the usual OLS (or GLS) methodology cannot. Thus adverse (favorable) weather conditions would tend to bias the *TFP* and *STC* measures

Table 2. Annual Changes (%) in Productivity

Year	%Δ*TFP*	%Δ*STC*	%Δ*FTC*
1949	−2.603	−3.910	−2.169
1950	−2.288	3.732	−2.279
1951	2.790	0.436	1.346
1952	4.430	−1.486	−0.991
1953	3.524	0.843	0.776
1954	−0.312	−1.300	−0.997
1955	−0.866	−0.517	0.250
1956	7.591	4.562	4.707
1957	−2.478	−2.911	−4.010
1958	1.884	0.937	1.841
1959	−3.505	−1.493	−1.574
1960	2.854	0.800	1.268
1961	0.869	0.656	0.175
1962	1.894	0.017	−0.062
1963	1.709	−0.697	−0.731
1964	2.316	2.802	0.231
1965	−1.025	−6.254	−3.332
1966	1.022	0.057	−1.747
1967	2.472	2.524	1.255
1968	0.152	−0.055	0.090
1969	1.174	1.573	−0.015
1970	−2.605	−4.285	−1.915
1971	5.161	4.995	3.778
1972	1.100	1.064	1.123
1973	1.602	−4.028	−0.800
1974	3.515	1.156	1.036
1975	2.134	−1.797	0.362
1976	−0.011	2.899	1.355
1977	3.810	3.933	3.025
1978	−2.411	−7.194	−4.595
1979	5.017	4.429	4.030
1980	−0.582	−5.047	−3.195
1981	7.947	6.733	7.770
1982	0.669	2.874	1.341
1983	−9.2317	−3.323	−4.424

Note: *TFP* is total factor productivity, *STC* is simple technical change, and *FTC* is flexible technical change.

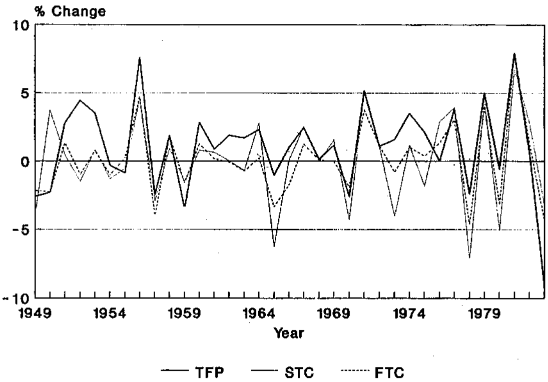

Figure 2. Annual productivity growth

downward (upward). A model with random coefficients would be affected by weather conditions in a reverse manner, with good weather resulting in an overestimate of productivity growth. For this reason, FLS seems especially well suited to agricultural applications.

The *FTC* measure was also significantly more precise in a statistical sense than the *STC* measure. The *TFP* measure is simply a construction from the data and has no statistical properties. The *FTC* and *STC* measures, because they are based partially on estimated coefficients, can have confidence intervals placed around them and other statistical tests performed given some parametric assumptions. If the ϵ_t are assumed to be independently and identically distributed normals with an unknown variance, the $\widetilde{FTC}_t$ (as discussed previously) and the $\widetilde{STC}_t$ are distributed as Student t random variables. The $\widetilde{STC}_t$ are distributed as t random variables since they are linear functions of the estimated OLS coefficients of the production function. The variances of both the $\widetilde{FTC}_t$ and $\widetilde{STC}_t$ measures were calculated being careful to account for all covariances, especially between b_t and b_{t-1} for the $\widetilde{FTC}_t$ measure. The variances allowed both measures to be tested for statistical significance relative to a null hypothesis of zero productivity growth (e.g., $\widetilde{FTC}_t = 0$). The results were striking: 33 of the 35 estimates of $\widetilde{FTC}_t$ were statistically different from zero while only eight of 35 estimates of $\widetilde{STC}_t$ were significantly nonzero. While there is no reason to believe that this makes the $\widetilde{FTC}_t$ a superior measure (since zero is a completely arbitrary value to test the measures against), the results do indicate the overwhelming increase in precision gained by employing the $\widetilde{FTC}_t$. The variances of the $\widetilde{FTC}_t$ were generally two orders of magnitude smaller than the corresponding variances of the $\widetilde{STC}_t$. This accuracy is a direct result of the decrease in the sum of the squared observation errors achieved by allowing the parameters to time vary. Because the variances are estimated for the $\widetilde{FTC}_t$ and $\widetilde{STC}_t$ measures in their proportional change form (rather than the easier to interpret percent change form), the t-values and the proportional changes in the measures are presented in table 3. Note a zero proportional change is, of course, a zero percent change and that any hypothesis concerning percent changes could be tested by converting the value desired into the equivalent number suitable for testing the proportional changes.

Table 3. *FTC* and *STC* Measures and Their *t*-Values

Year	*FTC*	*STC*	Year	*FTC*	*STC*
1949	−3.157 (−41.70)	−.1372 (−.6135)	1967	.0143 (55.73)	.0237 (.1687)
1950	−.1312 (−26.84)	.3453 (2.554)	1968	−.0107 (−260.9)	−.0402 (−.5292)
1951	−.3414 (−37.76)	.0311 (.1823)	1969	−.0042 (−9.718)	−.0159 (−.1171)
1952	−.2792 (−53.70)	−.3068 (−1.390)	1970	−.0268 (−66.38)	−.0612 (−.4373)
1953	−.0736 (−21.35)	−.0871 (−.8299)	1971	.0207 (44.78)	−.0375 (−.4309)
1954	.0009 (.5267)	.0332 (.2788)	1972	−.0085 (−162.1)	−.0702 (−3.125)
1955	.0474 (20.43)	.4077 (5.995)	1973	.0112 (33.08)	−.0370 (−.3153)
1956	.0476 (11.62)	.0342 (1.333)	1974	.0903 (301.7)	.2619 (6.910)
1957	−.0005 (−.1297)	.1724 (.6093)	1975	.0066 (56.42)	−.0475 (−1.133)
1958	.0177 (6.342)	.3246 (4.352)	1976	−.0037 (−29.65)	−.0220 (−.2288)
1959	−0.559 (−49.44)	.1715 (2.422)	1977	.0399 (77.21)	.0221 (.3275)
1960	−.0043 (−3.664)	.0114 (.1425)	1978	−.0907 (−581.3)	−.3105 (−2.622)
1961	−.0058 (−4.304)	.0458 (.4226)	1979	.0282 (65.35)	−.0662 (−1.544)
1962	−.0025 (−2.803)	−.0198 (−.2495)	1980	.0664 (133.1)	.2387 (5.985)
1963	−.0449 (−25.93)	.0135 (.1897)	1981	.0572 (136.6)	.0431 (.3967)
1964	−.0205 (−16.09)	.0117 (.0662)	1982	.0203 (40.86)	.0152 (.0976)
1965	−.0571 (−90.42)	−.0279 (−.2412)	1983	−.0778 (−41.37)	.1021 (.9942)
1966	−.0248 (−33.17)	.0024 (.0137)			

Note: Numbers in parentheses are the *t*-values of the measures. *STC* and *FTC* are defined in equations (7) and (8), respectively.

Conclusions

In this article a new measure of productivity growth was introduced, flexible technical change. The proportional changes in this new measure were defined to be the time elasticity of a production function that has been estimated employing the new methodology of Flexible Least Squares. A simple formula for calculating these proportional changes based on the marginal products of the inputs and the changes in the output and input levels was presented. The approach is very general and can be applied to any functional form desired. Analogies to cost and profit function approaches are obvious.

The new measure, flexible technical change, was shown to be more stable in the sense of having a smaller variance around a constant percentage growth rate. It was also shown to produce considerably lower estimates of productivity growth than the traditional notion of total factor productivity. These values were also lower than those found by Ball and by Capalbo for U.S. agriculture in earlier studies. In contrast to Capalbo's results with a cost function approach, we found that relaxing the incorrect assumptions in the traditional *TFP* measure leads to a lower estimate of productivity growth.

These lower estimates of productivity growth would cause a downward revision in estimates of the returns to agricultural research, since the benefit from research is usually measured as increased productivity. Other policy implications arising from these lower estimates are difficult to identify. This is because all three measures—*TFP, STC,* and *FTC*—seem to show an absence of any trend in the productivity of U.S. agriculture. Unlike the U.S. manufacturing sector where the great productivity decline has sparked much debate, agricultural productivity growth appears to have been reasonably constant (on average) over the last 30 years. Therefore, while the new *FTC* measure suggests a lower rate of productivity growth, it suggests no recent downturn (or upturn) in the rate of progress. This suggests that the shift in the global resource base has not affected the comparative advantage of U.S. agriculture, probably because the U.S. is still well-endowed with the resources necessary for agricultural production. However, the new *FTC* measure does suggest that the development of the resource base and quality improvements in inputs have played a large role in the growth and competitiveness of U.S. agriculture.

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Appendix

To demonstrate that the distribution of the b_t is indeed a t-distribution will require some of the algebra used in calculating the estimate. Kalaba and Tesfatsion produce the FLS coefficients in two steps, first calculating sequential parameter estimates based only on the observations up to the period and then recursively creating smoothed estimates that depend on each period's sequential estimate and the next period's smoothed estimate. Thus, the final smoothed estimates depend on the entire set of observations. These smoothed estimates are the ones employed in this article, and the method for calculating them can be written in matrix form as

$$\text{(A.1)} \qquad b^{FLS} = A^{-1}X'y$$

where b^{FLS} is the $(Tk \times 1)$ vector of the b_t, $(b^{FLS})' = (b_1', \ldots, b_T')'$, y is the $(T \times 1)$ column vector of y_t, and the matrices A $(Tk \times Tk)$ and X $(T \times Tk)$ are defined as

$$\text{(A.2)} \qquad A = \begin{bmatrix} \mu I + x_1x_1' & -\mu I & 0 & \cdots & \cdots & 0 \\ -\mu I & 2\mu I + x_2x_2' & -\mu I & 0 & \ddots & \vdots \\ 0 & -\mu I & 2\mu I + x_3x_3' & -\mu I & 0 & \vdots \\ \vdots & 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & 0 & -\mu I & 2\mu I + x_{T-1}x_{T-1}' & -\mu I \\ 0 & \cdots & \cdots & 0 & -\mu I & \mu I + x_Tx_T' \end{bmatrix}$$

and

$$\text{(A.3)} \qquad X' = \begin{bmatrix} x_1 & 0 & \cdots & \cdots & 0 \\ 0 & x_2 & 0 & \ddots & \vdots \\ \vdots & 0 & \ddots & 0 & \vdots \\ \vdots & \ddots & 0 & x_{T-1} & 0 \\ 0 & \cdots & \cdots & 0 & x_T \end{bmatrix}.$$

Both of the matrices A and X' have full column rank and therefore rank greater than or equal to T. Thus, from (A.1)

$$\text{(A.4)} \qquad \mathrm{cov}(b^{FLS}) = \mathrm{cov}(A^{-1}X'y) = (A^{-1}X')\mathrm{cov}(y)(A^{-1}X')'.$$

Given the assumption that the true β^{FLS} is nonstochastic, if we also assume that the ϵ_t are independently and identically distributed normal random variables with variance σ^2, then the covariance matrix of the b^{FLS} is

$$\text{(A.5)} \qquad \mathrm{cov}(b^{FLS}) = \sigma^2A^{-1}X'XA^{-1}.$$

All the components of A and X are direct transformations of the xs and μ. Thus, for a known σ^2, b^{FLS} would be distributed as a multivariate normal, conditional on μ. In actual applications, with an unknown σ^2 and the usual estimate of the variance of the errors substituted, b^{FLS} is distributed as a multivariate Student t random variable with an estimated covariance matrix equal to the expression in (A.5) with the σ^2 replaced by its estimate.

The FLS estimates of the production function that are employed here to calculate the measure of flexible technical change were produced using a FORTRAN program slightly modified from the one presented in Kalaba and Tesfatsion. The program uses a sequential solution technique to produce the set of estimators that minimize the objective functions presented in (11). For further details, see Kalaba and Tesfatsion. While the matrix notation presented here is useful for understanding the process and analyzing the estimators' statistical properties, the inversion of the $(Tk \times Tk)$ matrix A can be a computationally inefficient solution method. Therefore, the sequential method of Kalaba and Tesfatsion (which avoids taking the inverse of A) is recommended for large data sets.