Forecasting Price Movements: an Application of Discriminant Analysis

D. J. Menkhaus and R. M. Adams

The purpose of this paper is to demonstrate a technique, discriminant analysis, which may be useful in predicting the direction of movement between fall feeder calf prices and spring yearling prices. The results of the discriminant analysis model are then compared with a conventional regression approach in terms of relative accuracy of predictions. The usefulness of incorporating the direction of price movement as a variable in a price prediction model is also evaluated. Generally, the results suggest that the discriminant analysis approach provides useful information, and the directional variable improves forecasts when incorporated into a traditional forecasting model.

The increasing complexity of decisionmaking in agriculture enhances the value of information. An integral component of the decision process is accurate price forecasts. A few recent efforts have been directed toward comparing the price forecasting ability of the futures market and econometrically-based models [Just and Rausser, Martin and Garcia]. In general, however, the development and testing of new forecasting approaches has generated little attention among economists in recent years.

Three general forecasting approaches used by agricultural economists are (1) the models presented by Box and Jenkins, which include the autoregressive-integrated moving-average (ARIMA) forecasting models (noncausal); (2) economic models (causal); and (3) composite forecasting models. The ARIMA models were employed by [Oliveira, *et al.*] to forecast beef prices. Economic models have recently been used by [Agriculture Canada 1978a, 1978b] in forecasting and have traditionally received the most attention from agricultural economists. The composite model approach, which combines the ARIMA and economic models, has been applied to livestock price forecasting [Bessler and Brandt].

There appears to be no clear consensus as to which method, causal or noncausal, is the best forecasting technique in terms of ease of generating forecasts and accuracy of forecasts. A number of researchers have compared the Box — Jenkins type and econometric models [Naylor, *et al.*; Nelson; Leuthold, *et al.*; and Bechter and Rutner]; conclusions relative to the above criteria are mixed. The structural or causal models do have the advantage of being closely linked to economic theory and may more readily account for structural changes in the system under investigation.

Zellner, Bates and Granger and others suggest that a synthesis of the conventional econometric techniques and time series analysis techniques is a promising approach. While promising, this approach too would benefit by improving forecasts generated from the causal or eonomic model approach. [Zellner p. 641] concludes that while considerable progress has been made in the work with structural economic models, "an economic model as satisfactory as the Ford Model T has not as yet appeared." The intent of this paper is to suggest a modification of

---

D. J. Menkhaus and R. M. Adams are Associate Professors in the Division of Agricultural Economics at the University of Wyoming.

Wyoming Agricultural Experiment Station Journal Article Number 991. This research was partially funded by Western Regional Cooperative Reseach Project, W-145, Impacts of Relative Price Changes of Feed and Cattle on Marketing of U.S. Beef. The authors wish to acknowledge the constructive suggestions of the anonymous *Journal* reviewers on earlier versions of this paper.

traditional econometric models which could improve forecasts generated from these types of models.

Objective

The purpose of this paper is to demonstrate the usefulness of modifying the economic model approach to include a forecast of the direction of movement of an economic variable, namely price. From the standpoint of producer decisionmaking the direction of price movement alone can be useful information. Given that most economic data exhibit positive serial correlation, it is relatively easy to predict a continuation of time series movements. However, the development of a forecasting model which has the ability to identify turning points offers a greater challenge.

Specifically, this paper demonstrates a technique, discriminant analysis, which might be useful in predicting the direction of movement between fall feeder calf prices and spring yearling prices. The discriminant analysis approach is then compared to regression in terms of relative accuracy in predicting the direction of price movement. Finally, the usefulness of incorporating the direction of price movement as a variable in a model to predict the magnitude of price is evaluated. Thus, the forecasting approach suggested here explicitly recognizes that price is made up of two components, direction and magnitude. The belief is that forecasting models and their results can be improved by considering each of these components separately.

Problem and Approach

One decision facing feeder calf producers is whether to sell spring calves in the fall or to hold these animals until the following spring. A rancher wishing to determine the optimal time for marketing cattle needs to have knowledge about the physical production response of livestock as well as price outlook [Kearl, p. 11].

With respect to price, knowledge of the direction of its movement between fall and spring (calves to yearlings) may be useful information for the decisionmaker. In fact, when considering the effects of supply or demand shifts, economists are often initially more concerned about the direction of price movement than about magnitude. The approach suggested here is to develop a model which can predict the direction of price movement between fall and spring feeder animals and then use this information in a model to predict magnitude.

The method presented in this paper could be used to determine the direction of movement in any economic variable. Furthermore, depending on the nature of the problem under investigation, quarterly or monthly data could be used instead of annual data. Thus, the problem outlined above, aside from its practical appeal, can also serve to demonstrate the usefulness of the suggested technique.

The Model

The intent of the model is to classify price movements into one of two mutually exclusive classes: increase (up) or decrease (down). The discriminant analysis approach can provide such delineation, given its objective of classifying individuals, objects or phenomena into one of two or more mutually exclusive categories or classes on the basis of a set of independent variables. This discrimination is accomplished by combining the set of independent variables into a linear function or index in such a manner that the difference between the means of the index for the mutually exclusive categories per unit of dispersion about the means is maximized [Duncan and Leistritz, p. 5].[1]

The discriminant model can be used as a type of economic model. Thus, the selection of discriminating variables is analogous to the selection of independent variables for a multiple regression model. An identification of

[1]For more complete discussions of discriminant analysis and/or its application to economic problems, refer to Araji and Finley, Bauer and Jordan, Blood and Baker, Bromley, Cooley and Lohnes, Duncan and Leistritz, Fisher, Morrison, Press, and Reinsel.

variables consistent with economic theory would appear to be one strength of this procedure relative to autoregressive techniques, spectral analysis and harmonic motion analysis. Economic variables (which should more accurately reflect the movement of a specific economic variable) are used as the independent variables in discriminant analysis rather than past observations of the dependent variable, trigonometric functions or moving averages.

The following model is used to classify the direction of price movement between October feeder calf prices and the succeeding March yearling prices. Since the objective of the model is forecasting, it is specified in an *ex post* manner, and predicts the direction of movement in March prices in October or as soon as selected September price series are available. The model is: $Z_t = B_0 + B_1X_{1t} + B_2X_{2t} + B_3X_{3t} + B_4X_{4t} + B_5X_{5t}$; where Z_t = the discriminant score used in categorizing the direction of movement (up or down) in the price of October feeder calf prices in year t as compared to the March price of yearlings in year t + 1 and B_i = the discriminant or weighting coefficient for the i^{th} variable. The X_i's are specifically defined as: X_1 = yearling price in September (deflated); X_2 = direction of movement in yearling prices between March and September in year t (1 if up and 0 if down); X_3 = price of September slaughter steers (deflated); X_4 = price of September corn (deflated); and X_5 = percent of January 1 inventory slaughtered.

Prices of feeder calves (cattle) are derived from the prices of slaughter cattle (X_3) and the costs of feeding these animals to slaughter weight, which is primarily a function of the cost of grain (X_4). Subsequent prices may also depend on the direction of price movement (Kearl, p. 8), as represented by X_2 in the above model. The September yearling price (X_1) acts as a positioning variable and X_5 depicts liquidation and expansion periods in cattle inventories. This latter variable may also serve as a proxy for changes in the quantity of feeder calves where the quantity of feeder calves is affected by the proportion of cows and potential herd replacement animals which are slaughtered.

For comparative purposes, a regression model containing the same independent variables as the discriminant model was also estimated. The dependent variable was redefined as the March yearling price in year t + 1 (deflated) minus the October feeder calf price in year t (deflated). Finally, a variable denoting the direction of price movement was included in the regression model to test its value in improving the accuracy of price magnitude prediction.

Data

Sources of data included selected issues of the U.S.D.A. publications *Livestock and Meat Situation, Livestock and Meat Statistics and Agricultural Statistics*. Corn prices were obtained from the *Commodity Year Book*. Data for the period 1925-1969 were used to estimate the model. Data for the years 1970-1980 were reserved to test the forecasting ability of the model. During the period 1925-1969 there were 24 observations in which the October price of good and choice feeder steer calves at Kansas City was greater than the next year's weighted average of all weights and grades of March feeder steer (yearling) price. The movement in yearling prices (X_2) was calculated using March and September yearling prices of the same year. Corn price is the September 15 average price received by U.S. farmers. The slaughter price is represented by the average cost per 100 lbs. of sales out of first hands for choice slaughter steers at Chicago, 1925-1949. For the period 1950-1980, this price is represented by the price of choice slaughter steers at Omaha, average cost per 100 lbs. live weight of sales out of first hands and more recently 900-1100 lbs. Percent of January 1 cattle and calf inventory slaughtered was calculated using the sum of cattle and calf commercial slaughter.

One problem which must be dealt with in a forecasting model is the range of the sample data relative to the range of the validation and post sample data. Variables which are subject to inflation may move rapidly beyond the range of the data over which the model is estimated. To adjust for inflationary trends, price data were deflated using the consumer price index.

Results

The results of the estimated discriminant and regression models are presented in Table 1. In the discriminant model, the standardized coefficients identify the relative importance of the independent or discriminating variables in explaining the dependent variable. Of the five variables in the model, percent inventory slaughtered (X_5) is the most powerful discriminating variable, followed by the September yearling price (X_1), September corn price (X_4), direction of movement in yearling prices between March and September (X_2) and September slaughter steer price. The regression model predicting the difference between deflated March (year t + 1) and deflated October (year t) feeder prices is consistent relative to signs with the discriminant model, with the exception of the slaughter price variable. In terms of relative magnitudes of the standardized coefficients, the two models are generally comparable.

In the discriminant model, the unstandardized discriminant coefficients can be used to predict the direction of price movement, i.e., up or down. The classification procedure is as follows [Johnston, p. 339]: if $Z_t > Z_{cv}$, classify observation t as belonging to an upward movement. If $Z_t < Z_{cv}$, classify observation t as belonging to a downward movement. Z_{cv} is defined as the critical value of Z_t and is calculated as follows: $Z_{cv} = b_0 + b_1x_1 + b_2x_x + b_3x_3 + b_4x_4 + b_5x_5 = 0.042$; where: x_i = the average of X_i group (uptrend or downtrend) means, and b_i = the estimated discriminant coefficients.

Evaluation and Comparison of the Forecasting Accuracy of the Models

The primary test of a forecasting model should be based on how well it predicts. However, consistency of estimated signs with theoretical expectations and significance of the coefficients, in the case of regression, are also important. The t statistics indicate that all coefficients in the regression model are significantly different from zero with the

TABLE 1. Estimated Results for the Discriminant and Regression Models.

	Discriminant Coefficients		Regression Coefficients	
Variable	Standardized[a]	Unstandardized	Standardized	Unstandardized
Constant	----	−8.269	----	−13.060 (−3.404)[b]
X_1	−0.431	−0.0624	−0.508	−0.265 (−2.124)
X_2	0.169	0.388	0.257	2.152 (2.489)
X_3	0.0324	0.00459	−0.0980	−0.0502 (−0.357)
X_4	0.290	0.519	0.387	2.500 (2.640)
X_5	0.826	0.242	0.415	0.438 (3.860)
			R^2 =	0.62
			F =	12.772

[a]Standardized by the measurement scales and variability in the original data.
[b]Numbers in parenthesis are t statistics.

exception of slaughter cattle price, which also exhibits an incorrect sign. The sign of B_1 is difficult to discern *a priori*. However, the remainder of the coefficient signs appear to be correct.[2] The signs associated with the discriminant coefficients also conform with theoretical expectations. The sign of B_2 suggests that if the price of yearlings is increasing (decreasing) it will continue to increase (decrease) to March, at least relative to October feeder calf prices.

The forecasting ability of the above models is evaluated according to how well price movements were predicted for the period 1925-69 (the estimation period) and validated over the period 1970-1980 (post-estimation period). The results for the estimation period are reported in Table 2. For the years 1925-1969, the discriminant model properly classified 16 of the 21 upward price movements and 20 of the 24 downward price movements, or 80 percent of the cases. A naive no-change model would have properly classified approximately 70 percent of the cases. In terms of picking up the turning points, i.e., when the direction of price movement changes, the model correctly predicted 11 of the 18 directional changes.

Using the same criteria as presented above, the regression model performed slightly better than the naive no-change model, correctly classifying 73 percent of the price movements and properly predicting 10 of the 18 turning points. Theil's U_2 coefficient (0.505) indicated that the regression model is better than the naive no-change extrapolation. In summary, the discriminant model is slightly better than the regression framework in forecasting the direction of price movement between fall feeder calves and spring yearlings for the period of estimation, and also slightly better in terms of predicting turning points.

To examine the potential usefulness of incorporating the direction of movement in price as a variable in a price prediction model, the actual direction of price movement is added to the regression model as a dummy variable.[3] From a forecasting standpoint, such a model would require an auxiliary equation to predict the movement in price [Montgomery and Johnson, p. 27]. The discriminant function set forth above could serve as a method of obtaining these ancillary forecasts. The addition of the direction of price movement variable to the regression equation decreases Theil's U_2 coefficient to 0.363, and the R^2 increases to 0.80. The estimated regression equation and t statistics follow:

$$Y = \underset{(-1.427)}{-4.485} - \underset{(2.029)}{0.186X_1} + \underset{(2.618)}{1.664X_2}$$

$$- \underset{(-0.546)}{0.056X_3} + \underset{(2.643)}{1846X_4} + \underset{(1.366)}{0.133X_5} + \underset{(5.955)}{3.940X_6};$$

where: Y = deflated March yearling price in year t + 1 minus the deflated October feeder calf price in year t; X_6 = the direction of movement in fall feeder calf prices to the next spring yearling prices (1 if upward and 0 if downward). Using information about the direction of price movement in the regression framework not only improves the directional forecasts (96 percent correctly classified), as expected, but also the magnitude predictions, relative to a no-change model, as measured by Theil's U_2 coefficient (Table 3).

[2]The expected sign of the coefficient associated with the price of corn in a factor demand relationship is negative. However, in the case presented here, e.g., the difference between yearling and feeder calf prices, the relationship should be positive. Feedlot operators or feeders, in cases of high grain prices, pay premiums for heavier animals relative to lighter animals.

[3]It is recognized that using the actual direction of price movement presents difficulties from a practical or applied standpoint. However, for the purpose of evaluating the usefulness of incorporating this variable into conventional econometric models to improve forecasts, it makes more sense to use the actual directional movement than a forecast of this variable. The use of a predicted price movement, and the effects of an incorrect forecast of this variable, are discussed later in this paper.

TABLE 2. Discriminant and Regression Results, 1925-1969.

	Discriminant[a]		Regression			Discriminant[a]		Regression	
Year	Actual	Predicted	Actual[b]	Predicted	Year	Actual	Predicted	Actual[b]	Predicted
1925	1	1	3.429	3.072	1948	0	1*	−2.746	−1.504
1926	1	1	3.075	2.310	1949	1	1	1.148	−1.852*
1927	1	1	3.231	3.422	1950	1	0*	4.494	−2.239*
1928	0	0	−0.253	0.192*	1951	0	0	−8.239	−7.698
1929	0	0	−1.384	−1.064	1952	0	0	−7.019	−4.250
1930	0	0	−1.420	.2045*	1953	1	1	3.658	1.490
1931	0	0	−1.053	−1.456	1954	1	1	0.497	1.535
1932	0	0	−2.103	−3.192	1955	0	1*	−4.127	0.749*
1933	0	0	−0.387	−1.070	1956	1	1	0.123	2.117
1934	1	1	7.382	5.399	1957	1	1	0.902	2.760
1935	0	0	−0.414	0.427*	1958	0	0	−6.594	−3.286
1936	1	1	3.590	4.194	1959	0	0	−6.277	−4.818
1937	0	1*	−1.047	4.003*	1960	0	0	−1.736	−2.435
1938	1	0*	1.635	−.7261*	1961	0	0	−3.538	−3.206
1939	0	0	−1.370	−1.108	1962	0	0	−6.060	−2.335
1940	0	0	−0.429	−1.411	1963	0	0	−4.213	−2.972
1941	0	0	−1.224	−1.430	1964	1	0*	0.194	−1.439*
1942	1	1	2.275	0.778	1965	1	1	3.185	0.378
1943	1	0*	0.849	−1.039*	1966	0	1*	−4.105	−1.610
1944	1	1	2.808	1.893	1967	0	0	−1.890	.0078*
1945	1	1	2.727	1.862	1968	0	0	−0.240	−2.182
1946	1	1	5.128	3.706	1969	1	0*	0.383	−2.624*
1947	1	1	6.786	6.077					

[a] A 1 indicates an upward movement in the direction of prices of yearlings relative to calves; 0 is a downward movement.

[b] Deflated March yearling price in year t + 1 minus the deflated October feeder calf price in year t.

*Denotes an incorrect classification of price movement.

TABLE 3. Actual and Predicted Price Differences Using a Regression Model Incorporating the Actual Direction of Price Movement as a Variable, 1925-1969.

Year	Actual[a]	Predicted	Year	Actual[a]	Predicted	Year	Actual[a]	Predicted
1925	3.429	4.108	1940	−0.429	−2.548	1955	−4.127	−2.123
1926	3.075	3.416	1941	−1.224	−2.671	1956	0.123	2.423
1927	3.231	4.633	1942	2.275	2.765	1957	0.902	3.064
1928	−0.253	−1.171	1943	0.849	1.774	1958	−6.594	−4.433
1929	−1.384	−1.917	1944	2.808	2.848	1959	−6.277	−4.969
1930	−1.420	−0.805	1945	2.727	2.613	1960	−1.736	−3.558
1931	−1.053	−1.884	1946	5.128	4.509	1961	−3.538	−3.977
1932	−2.103	−2.813	1947	6.786	4.952	1962	−6.060	−3.209
1933	−0.387	−1.257	1948	−2.746	−3.978	1963	−4.213	−3.492
1934	7.382	5.240	1949	1.148	0.083	1964	0.194	0.999
1935	−0.414	−1.210	1950	4.494	0.256	1965	3.185	2.194
1936	3.590	5.061	1951	−8.239	−6.777	1966	−4.105	−3.358
1937	−1.047	0.771*	1952	−7.019	−4.256	1967	−1.890	−2.015
1938	1.635	1.603	1953	3.658	2.651	1968	−0.240	−3.780
1939	−1.370	−2.436	1954	0.497	2.350	1969	0.383	−0.045

[a]Deflated March yearling price in year t + 1 minus the deflated October feeder calf price in year t.
*Denotes an incorrect classification of price movement.

While the results of the models presented above for the estimating period appear to be encouraging with respect to improving forecasts, the real test of these models is how well do they predict? The observations for the post-estimation period, 1970-1980, were used to validate the forecasting models. Three alternatives are reviewed: the discriminant model, the regression or price prediction model using the direction of price movement as estimated by the discriminant function, and the regression model using the actual direction of price movement as an observation for each of the post sample years. The results of this latter alternative are used for comparison to determine the influence of an error in the prediction of the price movement variable on the forecast of price magnitude. Practically, of course, this approach would be of no value for forecasting, in that the value of the actual direction would not be known in an applied case. Thus, the accuracy of the actual forecasts would be overstated for this alternative.

Table 4 presents the validation results for the post-sample period. The discriminant model misclassified two of the eleven cases. The two cases misclassified are turning points. The regression model also incorrectly classified two cases. A comparison of Theil's U_2 coefficient indicates that, as expected, the use of the actual trend or direction variable improves forecasts over those generated from the model without this variable. However, if incorrect predictions of trend are made, the predictions are no better (and may be worse) than if the direction variable were not included. These results support the hypothesis that a direction variable improves forecasts if the trend or turning point variable can be predicted accurately.

It should be noted that the validation period (1970-80) represents a tough test period. During this period, among other factors, fluctuating grain export demand, a price freeze, increased interest rates and rapid inflation greatly affected cattle prices. Most of these factors, and their economic and psychological impacts, are difficult to include in an economic model, particularly when their effects would not have been a factor during the estimation period used in this study. Partially as a result of these factors, several of the observations for the validation period may be considered as outliers or extreme values. Thus, the variance of the forecast

TABLE 4. Predictions Using Discriminant and Regression Equations and Using Information Concerning the Direction of Movement in Price, 1970-1978.

			Regression[a]			
Year	Discriminant[a] Actual	Discriminant[a] Predicted	Actual[b]	Predicted W/O Direction Variable	Predicted W/Actual Direction Variable	Predicted W/Predicted Direction Variable
1970	0	0	−3.79	−2.77	−3.84	−3.84
1971	0	0	−0.52	−3.96	−4.69	−4.69
1972	1	0*	0.78	−4.19*	−0.50*	−4.44*
1973	0	0	−11.56	−7.91	−6.58	−6.58
1974	0	0	−0.70	−0.99	−1.75	−1.75
1975	1	0*	4.00	0.75	2.82	−1.12*
1976	1	1	0.28	0.80	2.21	2.21
1977	1	1	4.83	1.36	2.49	2.49
1978	1	1	6.04	−1.13*	0.76	0.76
1979	0	0	−8.16	−6.57	−6.31	−6.31
1980	0	0	−4.80	−4.86	−4.92	−4.92
				U_2 = 0.48	U_2 = 0.40	U_2 = 0.50

[a]Results obtained by using an average of cattle and calf slaughter for the first nine months of each year and multiplying by twelve. The September consumer price index was used as a proxy for the annual index to deflate prices.

[b]Deflated March yearling price in year t + 1 minus the deflated October feeder calf price in year t.

*Denotes an incorrect classification of price movement.

error should be expected to be large as compared to cases where the magnitude between the independent variable used in a post-estimation period and the estimation period sample mean is small. Such circumstances have been a main source of error in recent forecasting attempts. It, therefore, may not be surprising that the discriminant model was not able to correctly identify the direction of price movement in all cases during the 1970-1980 period. Further, as expected, the use of incorrect ancillary forecasts of the trend variable resulted in forecasts less accurate than if correct values were used.

Concluding Remarks

The purpose of this paper was to demonstrate a method which may be useful in improving forecasting models and thus forecasts of economic variables. This technique is based on predicting the direction of price movement and incorporating this information into a model to predict magnitude.

Since price is made up of direction and magnitude, as is explicitly recognized in forecast evaluation methods [Tomek and Robinson, pp. 362-363], the incorporation of a trend variable has intuitive appeal. It would seem logical that a model specifically designed to predict turning points should be more accurate than a conventional model which predicts magnitude and, implicitly, direction. This is verified in the results presented above for the period 1925-1969. That is, the discriminant model performed slightly better than a comparable regression model in predicting the direction of movement in price. In addition, when the actual direction of movement in price was used as a variable, magnitude forecasts were improved (based on Theil's U_2 coefficient), indicating the potential of using the direction variable in a price forecasting model. Results for the validation period, 1970-1980, indicate that if a correct value for the directional variable is used, the forecasts of price magnitude are improved. However, if an incorrect value for the same variable is used, the forecasts of price magnitude may be worse than if this variable were not used at all.

While the results do not fully resolve the difficult issue of correctly forecasting turning points, they do suggest the potentially high payoff from the inclusion of a correct prediction of the directional variable in forecasting models. Further refinement of the discriminant model may be warranted in order to more accurately predict the direction of movement in fall calf prices and spring yearling prices. Another alternative is that an approach other than discriminant analysis may be required to predict the direction of price movement, for example, a logit model.

From a practical standpoint, information concerning the direction of price movement would be useful to producers of agricultural commodities in their efforts to ease the effects of cycles. For instance, the model under investigation in this paper should provide input for decisionmakers in determining whether to market feeder animals in the fall or to hold those animals for future sale. A prediction of future price direction and magnitude could serve as one input to improve marketing decisions.

References

Araji, A. A., and R. M. Findley. "Managerial Socioeconomic Characteristics and the Size of Operation in Beef Cattle Feeding — An Application of Discriminant Analysis." *American Journal of Agricultural Economics*. 53 (1971): 647-50.

Bates, J. M. and C. W. J. Granger. "The Combination of Forecasts." *Operations Research Quarterly*. 20 (1969): 451-68.

Bauer, L. L., and J. P. Jordan. *A Statistical Technique for Classifying Loan Applications*. Bul. No. 476, Ag. Expt. Sta., Univ. of Tennessee, Knoxville, 1971.

Bechter, D. M. and J. L. Rutner. "Forecasting with Statistical Models and A Case Study of Retail Sales." *Economic Review*, Fed. Res. Bank of K.C. 63(1978): 3-11.

Bessler, D. A. and J. A. Brandt. *Composite Forecasting of Livestock Prices: An Analysis of Combining Alternative Forecasting Methods*. Sta. Bul. 265. Ag. Expt. Sta. Purdue Univ., Dec. 1979.

Blood, D. M., and C. B. Baker. "Some Problems of Linear Discrimination." *Journal of Farm Economics*., 40(1958): 674-83.

Box, G. P. E. and G. M. Jenkins. *Time Series Analysis: Forecasting and Control*. 2nd edition. San Francisco: Holden - Day, 1976.

Bromley, D. W. "The Use of Discriminant Analysis in Selecting Rural Development Strategies." *American Journal of Agricultural Economics*, 53(1971): 319-22.

Canada Agriculture. *Commodity Forecasting Models for Canada Agriculture Volume I*. Policy, Planning and Economic Branch. Ottawa. 1978a.

Canada Agriculture. *Commodity Forecasting Models for Canada Agriculture Volume II*. Policy, Planning and Economics Branch. Ottawa. 1978b.

Commodity Research Bureau, Inc. *Commodity Year Book*, New York. Selected Years.

Cooley, W. and P. Lohnes. *Multivariate Data Analysis*. New York: John Wiley & Sons, 1971.

Duncan, M. and F. Larry Leistritz. *Multivariate Statistical Analysis: Concepts and Economic Applications*. Ag. Econ. Misc. Report No. 10, Dept. of Ag. Econ., North Dakota State Univ., Fargo, 1972.

Fisher, R. A. "The Use of Multiple Measurements in Taxonomic Problems." *Annals of Eugenics*, 8 (1936): 179-188.

Johnston, J. *Econometric Methods*, 2nd ed. New York: McGraw-Hill Book Company, 1972.

Just, R. E. and G. C. Rausser. "Commodity Price Forecasting with Large-Scale Econometric Models and the Futures Market." *American Journal of Agricultural Economics*. 63(1981): 197-208.

Kearl, W. G. *Cattle — Price Behavior and Rancher's Decision Making*. Agricultural Experiment Station. Bulletin 408. University of Wyoming. October 1963.

Leuthold, R. M., A. J. MacCormick, A. Schmitz, and D. C. Watts. "Forecasting Daily Hog Prices and Quantities: A Study of Alternative Forecasting Techniques." *Journal of the American Statistical Association*. 65(1970): 90-107.

Martin, L. and P. Garcia. "The Price-Forecasting Performance of Futures Markets for Live Cattle and

Hogs: A Disaggregated Analysis." *American Journal Agricultural Economics*. 63(1981): 209-15.

Montgomery, D. C. and L. A. Johnson. *Forecasting and Time Series Analysis*. New York: McGraw-Hill Book Co. 1976.

Morrison, D. G. "Discriminant Analysis." In Robert Ferber, *Handbook of Marketing Research*. New York: McGraw-Hill Book Company, 1974.

Naylor, T. G., T. G. Seaks and D. W. Wichern. "Box-Jenkins Methods: An Alternative to Econometric Models." *Inst. Stat. Rev*. 40(1972): 123-137.

Nelson, C. R. "The Prediction Performance of the FRB-MIT-PENN Model of the U.S. Economy." *American Economic Review*. 62(1972): 902-17.

Oliveira, R. A., C. W. O'Conner and G. W. Smith. "Short-Run Forecasting Models of Beef Prices." *Western Journal of Agricultural Economics*. 4(1979): 45-55.

Press, J. S. *Applied Multivariate Analysis*. New York: Holt, Rinehard & Winston, Inc., 1972.

Reinsel, E. I. *Discrimination of Agricultural Credit Risks from Loan Application Data*. Unpublished Ph.D. dissertation, Michigan State University.

Tomek, W. G. and K. L. Robinson. *Agricultural Product Prices*. Ithaca, Cornell University Press. 1972.

U.S. Department of Agriculture. *Agricultural Prices*. Economics, Statistics and Cooperatives Service, Washington, D.C., selected issues.

U.S. Department of Agriculture. *Livestock and Meat Situation*. Economics, Statistics and Cooperatives Service, Washington, D.C., selected issues.

U.S. Department of Agriculture. *Livestock and Meat Statistics*. Stat. Bul. No. 522, Ag. Marketing Service, Washington, D.C. selected issues.

Zellner, A. "Statistical Analysis of Econometric Models." *Journal of the American Statistical Association*. 74(1979): 628-643.