A Debt Selection Model for Banks of the Cooperative Farm Credit System

Loren Tauer and Michael Boehlje

This article discusses the application of a quadratic programming model to the bond and note participation decision of a Cooperative Farm Credit Bank. The model generates an efficient frontier of bond and note portfolios from which a bank can choose. The composition of these portfolios depends upon the expected cost and variance-covariance of cost for the bond and note activities, debt needs, and the debt policy constraints of a bank. The results indicate that the interest rate risk of various bond and note issues should be considered when making debt participation decisions.

The Cooperative Farm Credit System has become a major supplier of credit to agriculture. On January 1, 1980, the 37 banks of the System collectively extended in excess of $48 billion to agriculture. This entailed 36 percent of all farm real estate debt and 25 percent of all farm nonreal estate debt. The System obtains funds for loans from the national money market by issuing system-wide consolidated bonds and discount notes. The interest cost on bonds and notes depends upon market interest rates which have been very volatile in recent years and are not controllable by the System [Bildersee]. In order to obtain sufficient funds, the System must be willing to offer investors competitive rates. Although cost control is imperative in all segments of the System's operations, it is especially important in the funds acquisition process since interest paid on System debt securities typically accounts for over 90 percent of the interest rate charged to the member-borrowers.

Each bank has flexibility in determining its participation in the various note and bond issues of the System. If interest rates are expected to decrease, then it is advantageous for a bank to select shorter maturity bonds. This would allow the bank to refinance sooner at a lower interest rate. If interest rates are expected to increase, then it would be beneficial to participate heavily in longer-term bonds in order to lock in a low interest cost. Also of consideration in these decisions is the amount of funds needed immediately and in the future for new lending and replacement of maturing bonds and notes.

However, future interest rates and debt needs are not known with certainty. Interest rates have been extremely volatile in recent years, and fluctuating rates can significantly increase the costs of the bank and ultimately the costs and financial risks of the borrowers from the System. For example, participating in shorter-term maturity issues with the expectation of decreasing interest rates could result in significantly higher costs and rates for borrowers if interest rates increase instead. Thus, the risk of interest rate changes should be considered along with expected interest rate levels in debt issue participation decisions. This article discusses the use of a quadratic programming model to derive expected cost - variance of cost debt structures

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(an EC-VC efficient frontier) for banks of the Cooperative Farm Credit System. From this efficient frontier, bank management can select a specific debt strategy depending upon their risk preference.

Previous researchers have analyzed the debt selection activities and policies of the Farm Credit System. Hollenhorst analyzed the Federal Land Bank's debt management policies for the period 1947 to 1961. He tested some of the debt policy rules that were used by the bank during that period and found them to be ineffective in obtaining low cost debt structures. Swortzel and Jensen completed a study that projected the funding needs of the various banks of the system. A limitation of their model is that it only forecasts two months into the future. Bildersee, Percival, and Morris and Smith evaluated the financing needs of the Farm Credit System in a 1973 study and proposed the use of debt simulation models. They concluded that the lowest cost debt structure changes frequently and argued for flexibility in the timing and terms of debt participation. Crane, Knoop, and Pettigrew constructed a linear programming model to select bond maturities for Federal Land Banks, but the linear model has limited usefulness when interest rates are extremely variable. Numerous researchers have studied and modeled the asset and liability selection process of other financial intermediaries including Chambers and Charnes, and Cohen and Hammer. Almost all of these studies have dealt with commercial banks.

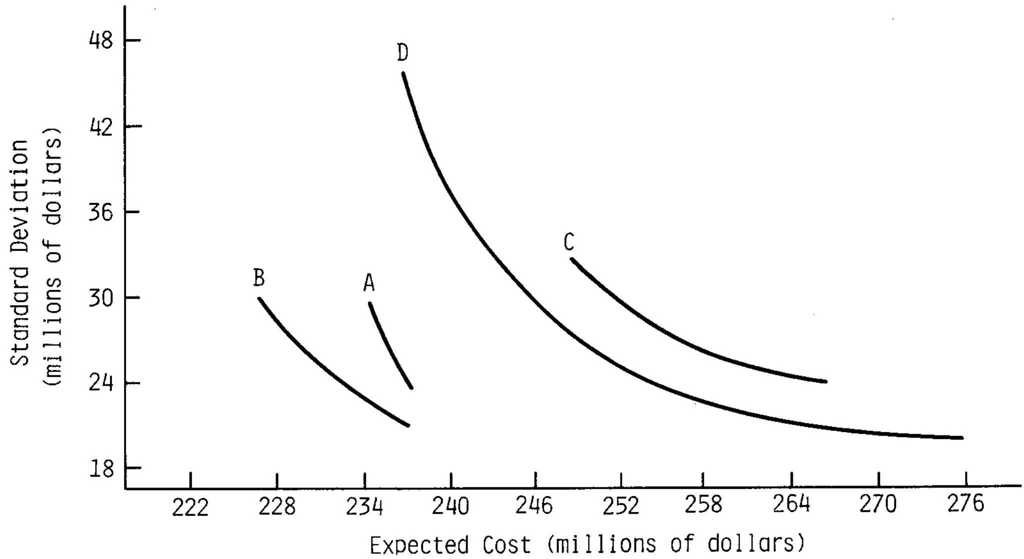
The Conceptual Framework

Quadratic programming as originally formulated by Markowitz has had extensive applications to asset selection. In agriculture the procedure has been used to analyze investment [Sadan], production [Burt and Johnson, Scott and Baker], and marketing [Heifner] decisions. A comprehensive application of quadratic programming to a commercial bank was completed by Robison. He illustrated how the procedure could be used to measure empirical adjustments to portfolios from changes in policy variables, as well as changes in costs and demands.

Although the quadratic model is intuitively appealing, it has some theoretical and empirical limitations. These limitations include using only the expected return and variance of return, the first two moments, in the analysis when these two moments do not completely specify the probability distribution for an activity [Borch]. This problem is mitigated if the utility function of the decision-maker can be accurately specified as quadratic. Erroneous results can also occur when the coefficients of the model are not accurately estimated [Frankforter, Phillips, and Seagle]. These limitations and others are discussed in Hakansson, Robison and Brake, and elsewhere. Even with limitations, however, quadratic programming allows the introduction of risk into a decision model which is not readily possible with linear programming and other deterministic models.

In this study quadratic programming is applied to the liability rather than the asset side of the balance sheet. At any level of expected debt cost, one unique debt structure is determined that minimizes the variance of debt cost. As expected cost is reduced by selecting another debt structure, the minimum cost variance at that expected cost increases. Aversion to risk, which produces a direct relationship between return and variance of return on an E-V frontier, produces an indirect relationship between cost and variance of cost on a EC-VC frontier (Figure 1). The shape of the frontier results from applying quadratic programming to the feasible space of debt portfolios. The feasible space is a convex set in expected cost - variance of cost space. Because the quadratic program minimizes variance at a fixed level of expected cost (varied parametrically), the solution set or efficient frontier that is traced out lies along the lower left border of the feasible space.

The activities of the quadratic programming model used here consist of the various present and future note and bond issues. Expected cost and variance-



A = Most probable forecast of expected debt cost and debt requirements from PCA loan projections, with debt policy constraints
B = Most probable forecast of expected debt cost and debt requirements from PCA loan projections, no debt policy constraints
C = Recession forecast of expected debt cost and debt requirements from time series forecast, with debt policy constraints
D = Recession forecast of expected debt cost and debt requirements from time series forecast, no debt policy constraints

Figure 1. Expected Cost and Standard Deviation Efficiency Frontiers (in Millions of Dollars).

covariance cost matrices are constructed for these activities. Estimation of these coefficients is discussed later. The right-hand side of the model contains the funding needs of the bank. Each bank is required by law to meet the sound credit needs of its members, but these funding needs are not known with certainty, so it is necessary to convert probabilistic estimates of funding needs into deterministic values. One possibility is to use the expected values of those stochastic funding needs. However, in some instances it may be optimal to plan for note and bond debt outstanding to be an amount greater or less than expected debt needs; an excess can be invested and a deficit could be obtained from short-term debt sources.

Using an inventory model it is possible to calculate the least expected cost quantity of bonds and notes to have outstanding for any time period given the probability distribution of funding needs, expected cost of bonds and notes, expected cost of other short-term funds, and expected return from short-term investments. After the optimal bond and note quantities to be outstanding are determined for each time period, these values can be inserted as part of the right-hand side of the quadratic program, and the optimal maturity structure of bond and note debt can be determined.

The funding need for each period is defined as the amount of funds necessary to service the bank's loans outstanding for that period. The vast majority of funding need occurs instantaneously at the start of a period as outstanding loans are carried into the new period. Therefore, probabilistic funding needs are estimated for the beginning of the period. After the start of each period the excess or shortage of funds for the remainder of the period will be invested or borrowed, respectively.

Mathematically, determination of the optimal quantity of bonds and notes entails minimizing the expected cost equation:

$$(1)\quad E[c(y)] = c \cdot y + p \int_{y}^{\infty}(v-y)\, f(v)\, dv - h \int_{-\infty}^{y} (y-v)\, f(v)\, dv$$

where:

v = amount of funds needed for a given time period, f(v) = probability density function for the possible values of v, c = expected cost of new bonds and notes, p = expected short-term debt cost, h = expected short-term excess funds return, y = amount of bonds and notes outstanding.

Solving for the minimum net expected cost produces:[1]

$$(2)\qquad \int_{-\infty}^{y^*} f(v)\, dv = \frac{p-c}{p-h}$$

If f(v) is estimated as a normal equation, then y^* can be determined from standard normal tables. But y^* is only defined if $0 \leq \frac{p-c}{p-h} \leq 1$. This can occur only under either of two conditions: (a) $p \geq c > h$, (b) $p \leq c < h$. For a finite solution to exist for y^*, condition (a) must hold.

The use of an inventory model to estimate future bond and note utilization, although an improvement over using expected values, is not without problems. First, although the model treats debt needs as probabilistic it assumes that costs are known with certainty or that bank management is risk neutral. It is possible to integrate the inventory model directly into the quadratic programming model so that expected cost, cost risk, optimal bond and note debt quantity to be outstanding, and the maturity structure of that debt are determined simultaneously, but such an approach requires a nonsequential stochastic quadratic programming model. Solutions can be obtained using separable programming techniques with a quadratic algorithm, but the size of the optimization model is greatly increased, and modeling and solution costs become exorbitant. In addition, demand for debt (loans) might be a function of expected debt cost and cost risk, but that relationship is not part of the current approach. The simultaneous determination of expected debt cost, cost risk, and loan demand would again require much more sophisticated and costly solution procedures than those used here. A partial remedy using the model proposed here is to revise debt needs after selection of a debt portfolio and continue to resolve the quadratic model and revise the debt needs until debt costs converge.

The Numerical Model

A numerical debt selection model was constructed for a Federal Intermediate Credit Bank in the Midwest. Although the structure of the model and coefficients were derived for that specific bank, with modifications, the same numerical model can be used for any of the other 36 banks of the Cooperative Farm Credit System.

Model Structure

The planning horizon of the model is three years. Three years enables the analysis of the impact of sequential funding with the system's discount notes, six-month bonds, and nine-month bonds but limits the model to a size that can be solved at a reasonable computational cost.

The model is multiperiod; the first 18 periods are monthly periods, and the last six

[1]The first derivative of equation (1) with respect to the control variable y set equal to zero is:

$$\frac{dE[c(y)]}{dy} = c - p \int_{y}^{\infty} f(v)\, dv - h \int_{-\infty}^{y} f(v)dv = 0.$$

By definition,

$$\int_{y}^{\infty} f(v)\, dv = 1 - \int_{-\infty}^{y} f(v)\, dv.$$

Inserting this identity into the derivative of equation (1) and simplifying results in equation (2).

periods are quarterly periods. Monthly periods were selected because the six-month and nine-month bonds are issued at the beginning of each month. The last half of the planning horizon was aggregated into quarters to reduce the number of activities in the model and still provide some detail for the last half of the planning horizon.

Eighteen nine-month bond activities were defined for the 18 monthly periods, and six nine-month bond activities were defined for the last six quarters. Six-month bond activities were defined in a similar manner. Long-term bond activities were defined as bonds issued the first month of each quarter for the first 18 months and then the beginning of each quarter for the last half of the planning horizon. Long-term bonds issued by the System have terms of at least 18 months, but most are three years or more, so in the model they provide funds for the duration of the planning horizon regardless of when they are issued. Discount notes can be issued daily in maturities of five to 270 days but were defined in the model as notes issued at the beginning of each monthly or quarterly period with a maturity of one month or a quarter. In essence, the model will determine the average participation in discount notes between bond issue dates but will not determine the participation in specific discount note offerings.

Although some bonds are outstanding at the end of the planning horizon, no salvage or liquidation activities were defined in the model. If it is expected that certain bonds will have salvage costs or values at the end of the planning horizon because they are more or less expensive than bonds which will be issued beyond the three-year planning horizon, those terminal costs or values should be included in the net cost of the bond activities. Because the price of securities could not be reasonably estimated beyond the three-year planning horizon or in essence are uncertain, no termination values or costs were included.

The model contains 24 rows for the 24 periods of funding needs of the bank. The first 18 rows are for the 18 monthly periods. The last six rows are for the last six-quarters. Transfer rows and columns were used to force into solution the debt structure outstanding at the beginning of the planning horizon since initial outstanding debt obligations will provide for some of the funding needs of the bank during the three-year horizon.

Additional constraints were included to determine the effects of various debt management policies that the bank imposed during the test period dates. These policies were used by the bank primarily to reduce potential fluctuations in interest costs. One policy restriction is that no more than 10 percent of debt outstanding at any time can be acquired by a single bond issue. Another restriction is that no more than ten percent of all debt can be held in discount notes. A third restriction is that at least 30 percent of debt must be held in term bonds.

Variance-Covariance Matrix

The variance-covariance matrix was derived from secondary market yields of federal government securities from the 13-year period 1965 to 1977.[2] This period spanned several economic cycles. Monthly observations were available for one, three, six, and 12-month, as well as two, three, four, five, and ten-year term-to-maturity securities. A linear segmented yield curve for each month was constructed from these observations to obtain yields for any other term-to-maturity.

A three-year monthly moving observation from the 13 years of data was used to generate 120 three-year observations of actual rates. The expectations theory of the term structure of interest rates was then used to obtain expected values for the interest rates

[2]Secondary market yields were used because initial placement rates were not available for all currently used Farm Credit securities. However, a regression of secondary nine-month yields on available initial FICB nine-month rates produced a statistically significant (at the .01 percent level) slope coefficient of one and an intercept coefficient of zero, which indicates that secondary rates are good proxies for initial rates.

in each of the 120 three-year observation periods. The expectations theory states that the current long-term spot rate is the geometric mean of the current short-term spot rate and future short-term rates that are expected to occur during the duration of the long-term security [Meiselman][3]. The rate thus derived is the expected rate determined by the market as market participants price various terms-to-maturity.

To compute the expected rate of a future bond, current short-term and long-term bond rates are used. The term-to-maturity of the current short-term bond is the number of months into the planning horizon that the future bond will be issued. The term-to-maturity of the current long-term bond is then that number of months into the planning horizon that the future bond will be issued plus the term-to-maturity in months of the future bond whose rate is being estimated. For example, to compute the expected interest rate values for the nine-month bonds into the three-year future for one of the 120 observations, the following formula was used:

$$(3) \qquad E(r_{0,k,k+9}) = \left[(1 + r_{0,0,k+9})^{\frac{k+9}{12}} / (1 + r_{0,0,k})^{\frac{k}{12}} \right]^{\frac{12}{9}} - 1,$$

where:

r_{t_1, t_2, t_3} = an interest rate, t_1 = the date on which the rate is computed, t_2 = the date on which the bond is issued, t_3 = the date on which the bond matures, k = the month into the future that the bond will be issued (varies from 1 to 35).

Similar formulas were used to compute the expected rates for the one-month, six-month, and three-year securities. The actual interest rates that occurred and the estimated expected rates were converted into actual and expected costs per $1,000 of debt. The deviations of the actual costs from the expected costs were then squared and divided by 120 to obtain the coefficients of the variance-covariance matrix.

The variance-covariance matrix thus calculated is 84 by 84, corresponding to the 84 activities of the model. Interest rates for the first month are considered known with certainty, so any variance or covariance term involving any of the first month's debt activities is zero. As debt activities occur further into the three-year future, the variance of those activities becomes larger. This increase occurs because there is more risk concerning the expected interest rates that will occur. Covariances of activities in different periods approach zero as the time between periods becomes greater. This occurs because any economic condition that affects interest rates in one period will have a smaller impact on interest rates in a different time period as the time between periods becomes greater. Debt activities early in the first year have almost zero covariance with debt activities late in the third year which implies that an activity early in the planning horizon will not affect the selection of an activity later in the planning horizon via the variance-covariance matrix. Thus, debt diversification can be accomplished by selecting any first and third year debt activity, and it is unnecessary to extend the planning horizon beyond three years to achieve additional debt diversification potential.

Expected Costs

Although the variance-covariance matrix is assumed to be invariant for all applications of the model, revised expected costs and debt needs are necessary for each application of the model. The model results that follow are for the planning horizon of January 1, 1979, to December 31, 1981. Two projections of interest rates for this time period were obtained from the Bank and used as the expect-

[3]Calculated forward rates were used as expected rates. A liquidity premium was not included in the calculation because its existence, value and stability is debatable (however, see MuCulloch). In addition, the forward rates calculated did not have a positive (or negative) bias when compared with the actual rates. A positive bias should occur with the existence of a liquidity premium.

ed costs in two separate applications.[4] A "most probable" forecast projected interest rates to decrease during 1979, increase during 1980, and then fall again during 1981. A "recession" forecast projected interest rates to increase the first two quarters of 1979, to fall drastically during the third and fourth quarter of 1979 as a recession develops, and then to decrease moderately during 1980 and 1981. The projection aternatives do not encompass the drastic increases in interest rates that actually occurred in late 1979 and the first half of 1980. The risk of that occurring, however, is the reason for the variance-covariance matrix and the quadratic model. Costs of issuing securities were added to expected interest costs to obtain expected debt costs. Because the model is multiperiod, the expected costs and variance-covariance coefficients were discounted to the present.

Debt Requirements

Normal probability density functions of FICB debt were estimated for each month for 18 months into the future and then for each quarter for an additional six quarters into the future. The two parameters of the normal distribution, the mean and the standard deviation, were obtained from a linear regression of FICB debt on selected regressors. The forecasted values from the estimated regression equations were used as the means for the future periods. The variances of the error of forecast were used as the measure of variances for the distributions.

Two separate linear regressions were estimated to obtain two different forecasts of FICB debt for the 1979 to 1981 period. The first equation was estimated by a time series regression of FICB debt using dummy seasonal variables. The second equation was estimated by a regression of FICB debt upon PCA loans outstanding. Projections of future PCA loans were obtained from the Bank. Each equation generated a slightly different type of projection. The times series equation provided a projection that increased every month, but with the greatest increase occurring the first quarter of each year as farmers prepared for the crop season. The second equation projected debt to generally increase over the three-year horizon with larger increases occurring during the first quarter and decreases occurring during the fourth quarter of each year as farmers reduce their debt at the end of the crop season. The probability density functions of FICB debt along with the average cost of bond debt, short-term debt cost, and excess funds return were used to derive the optimal level of bond and note debt for each period with the application of the inventory model.

Numerical Results

"Most Probable" Rate Forecast

Two applications of the numerical model for the January 1979 decision date will be reported. One application involved the use of the most probable forecast of interest rates to derive expected costs and debt forecasts from PCA loan projections to obtain optimal debt requirements. This application resulted in 116 individual efficient debt portfolios for the three-year planning horizon. This efficient frontier is traced out in Figure 1 as curve A. Curve A illustrates the range and tradeoff between expected cost and variance (standard deviation). If a lower expected cost portfolio is desired, a larger standard deviation of cost or risk must be assumed. The specific composition of any portfolio generated by the model depends upon the expected cost and variance-covariance of cost of the activities, the change in debt needs, and the debt policy constraints.

Three debt portfolios on this efficient frontier — the highest expected cost, an intermediate cost, and the lowest expected cost alternatives — are shown in Table 1. The lowest standard deviation portfolio involves extensive use of term bonds. A movement to higher standard deviation but lower expected

[4]Specific interest rate projections and other input data are available from the authors.

TABLE 1. Debt Issuance for the Planning Horizon: Most Probable Forecast of Expected Debt Cost and Debt Requirements from PCA Loan Projections, With Debt Policy Constraints (in Millions of Dollars).

	Lowest Standard Deviation Portfolio				Intermediate Standard Deviation Portfolio				Highest Standard Deviation Portfolio			
	Term Bonds	Nine-Month Bonds	Six-Month Bonds	One-Month Discount Notes	Term Bonds	Nine-Month Bonds	Six-Month Bond	One-Month Discount Notes	Term Bonds	Nine-Month Bonds	Six-Month Bonds	One-Month Discount Notes
1979												
January	126.073	0.0	0.0	0.0	93.479	0.0	32.594	0.0	93.479	0.0	0.0	32.594
February		105.853	0.0	0.0		0.0	88.789	17.064		0.0	19.772	118.675
March		51.109	0.0	61.901		0.0	0.0	130.074		0.0	101.611	130.074
April	130.615	0.0	17.992	0.0	130.615	0.0	86.094	0.0	5.353	0.0	130.615	80.741
May		111.750	0.0	0.0		0.0	93.393	18.357		0.0	59.803	132.689
June		110.402	0.0	5.945		0.0	0.0	134.704		0.0	114.331	134.704
July	136.697	0.0	0.0	0.0	136.697	136.697	0.0	24.656	20.275	0.0	108.485	136.697
August		58.010	0.0	39.768		0.0	77.749	133.475		0.0	120.773	133.475
September		0.0	0.0	131.664		7.243	86.463	131.664		63.653	131.664	131.664
October	129.673	0.0	0.0	0.0	129.673	68.172	0.0	0.0	129.673	0.0	0.0	112.693
November		126.972	0.0	0.0		40.943	0.0	73.569		61.830	0.0	131.785
December		1.453	0.0	70.775		0.0	0.0	94.688		133.338	0.0	133.897
1980												
January	136.009	0.0	0.0	0.0	136.009	0.0	0.0	23.913	136.009	73.263	0.0	98.344
February		139.825	0.0	10.088		139.825	0.0	0.0		139.825	0.0	117.455
March		49.359	0.0	109.295		122.413	0.0	2.214		143.642	0.0	143.642
April	147.458	0.0	0.0	0.0	147.458	29.616	0.0	0.0	147.458	34.347	0.0	0.0
May		50.325	1.122	17.576		11.012	0.0	0.0		11.012	0.0	0.0
June		0.0	0.0	28.588		0.0	0.0	18.255		74.665	0.0	0.0
Third Quarter	150.761	0.0	0.0	0.0	150.761	0.0	0.0	0.0	0.0	0.0	122.714	0.0
Fourth Quarter	65.832	0.0	0.0	0.0	0.0	59.798	0.0	0.0	0.0	235.983	0.0	0.0
1981												
First Quarter	0.0	53.931	0.0	440.979	0.0	252.640	0.0	0.0	0.0	419.129	0.0	0.0
Second Quarter	0.0	0.0	200.047	202.919	0.0	0.0	0.0	322.827	0.0	0.0	0.0	472.158
Third Quarter	0.0	0.0	0.0	502.070	99.764	0.0	0.0	502.070	167.336	158.365	0.0	502.070
Fourth Quarter	0.0	0.0	94.162	448.798	0.0	0.0	92.824	448.798	149.598	109.715	0.0	448.798
	Expected Discounted Cost = 237.184				Expected Discounted Cost = 235.590				Expected Discounted Cost = 234.310			
	Standard Deviation = 22.021				Standard Deviation = 23.755				Standard Deviation = 28.578			

cost portfolios entails a general shift to nine-month bonds, then six-month bonds, and finally to discount notes. However, if the expected cost of any debt activity is low enough, it may remain as part of a debt portfolio regardless of the variance level of the portfolio. An example of this is the low expected cost of the November 1979 nine-month bond. Discount notes and six-month bonds are also low expected cost activities during the last half of 1979, yet they are not a dominant part of any low cost debt portfolio because their use entails a higher risk that interest rates might increase and lead to a high cost debt portfolio later.

The highest standard deviation portfolio is the bond and note potfolio that would be utilized if risk was ignored; it is the portfolio that would be obtained with a traditional linear programming analysis. The dramatic difference between this portfolio and other portfolios on the efficient frontier suggests that risk attitudes and the riskiness of various bond and note issues cannot be ignored in making debt portfolio decisions.

The changes in debt needs between periods also affect the debt activities included in a portfolio. If debt needs increase each period, then any debt activity can be used, and the selection will depend upon expected cost and variance. If debt needs decrease, there must be some debt maturing that period. This effect is difficult to notice in the portfolios of Table 1 because much of the debt maturing to satisfy the decreased debt needs in the last half of 1979 was issued before January, 1979. However, some of the six-month and nine-month bonds issued early in 1979 assist in the adjustment to decreased debt needs later in 1979.

Also affecting the composition of the portfolios are the debt policy constraints. At least $93 million of term bonds must be issued in January, 1979, in order to maintain 30 percent of the debt in term issues. That must be done even when it is expected that interest rates will be lower for the rest of the three-year horizon. The constraints restricting any debt selection activity to no more than 10 percent of debt outstanding also affect the activities in a debt portfolio by ensuring that a large amount of any specific activity is not selected. For example, the October 1979 term bond is the lowest expected cost activity of the entire three-year planning horizon and has relatively low variance, yet participation in it is limited to a maxmum of $129.673 million because that is exactly 10 percent of the debt needs at that time.

To determine the effects of the constraints, they were removed and another efficient frontier of 25 portfolios was generated (Curve B in Figure 1). Releasing all constraints shifted the frontier to the left — at any level of variance the expected cost of a portfolio is less without the constraints. In this application the policy constraints do not eliminate the portfolios with high variances. The highest standard deviation portfolio on the frontier derived with debt policy constraints is actually one percent greater than the $28.195 million standard deviation derived without the policy constraints. In addition to failing to limit potentially high cost portfolios, the constraints increase the minimum standard deviation portfolio obtainable (Figure 1).

Eliminating the constraints alters the composition of the debt portfolios. The low standard deviation portfolios have more term bonds while the high standard deviation portfolios have more discount notes when the constraints are not imposed; nine-month and six-month bonds are also used more extensively in the unconstrained intermediate standard deviation portfolios, but there is still debt diversification. Without constraints, any low expected cost debt activity, such as the October 1979 term bond, will dominate the portfolios, especially in high standard deviation portfolios where debt diversification is not prevalent.

The shift from frontier A to frontier B, which occurs when the constraints are eliminated, is not a parallel shift; the reduction in expected cost is larger at high standard deviation levels. This occurs because the term bond in October of the first year is used at a volume as large as $960.182 million in the unconstrained frontier but is restricted to a maximum of $129.673 million in the constrained portfolio.

"Recession" Rate Forecast

A second application of the model for January, 1979, used the recession forecast of interest rates to derive expected debt cost and the time series regression to derive optimal debt requirements. This application generated 141 unique debt portfolios which are plotted as curve C in Figure 1. Since the time series procedure projected more debt needs during the planning horizon compared to projections using PCA loans, all of the portfolios on frontier C reflect a higher expected cost than those on frontier A. Frontier C also spans a greater area of expected cost because expected interest rates change drastically during the three-year horizon.

The lowest, an intermediate, and the highest standard deviation portfolios for this application are shown in Table 2. The bond and note composition of the portfolios on frontier C are different than that obtained with the most probable interest rate projections (Curve A in Figure 1), but there are many similarities. Again the lower standard deviation portfolios consist primarily of term bonds and nine-month bonds; the higher standard deviation portfolios consist of six-month bonds and discount notes. The intermediate standard deviation portfolios consist of six-month and nine-month bonds. The response to the movement in expected interest rates is very pronounced in some of these portfolios — in general nine-month or six-month bonds are used before interest rates climb in early 1979, and then six-month or discount notes are used as interest rates decline later in 1979.

In this application without the debt policy constraints, the efficient frontier shifts to the left (Curve D of Figure 1) so that at any standard deviation level, expected cost is lower. Eliminating the constraints in this case exposed the bank to higher variance portfolios; however, it also made available lower standard deviation portfolios. The high variance, low expected cost portfolio in this case contained almost all discount notes to roll over debt as expected interest rates decreased.

First Period Activities

When a debt portfolio is selected, the first period activity is the most imminent and irrevocable. Before the second and later periods arrive, conditions and expectations will change, and the model can be used to derive new debt portfolios for the three-year planning horizon. Many of the portfolios have the same activities in the first period. In fact, in the recession application with debt policy constraints, there is only one first-period option. This is shown in Table 3 along with the first-period options for the other applications.

In most instances the low risk but high expected cost first-period option is to use long-term bonds. The risk is low because a long-term bond will lock in a fixed interest cost for an extended time. An increased risk option is to use nine-month or six-month bonds. Because these two securities are so similar in maturity, cost, and variance, they are good substitutes for each other. The highest risk, but lower expected cost, first-period option in many instances is to use discount notes.

Summary and Conclusions

This article discusses the development and application of a quadratic program to aid banks of the Cooperative Farm Credit System in determining participation in the various bonds and notes that the System issues. The quadratic programming model generates an efficient frontier of debt portfolios from which a bank can choose the optimal portfolio. The composition of the portfolios depends upon the expected cost and variance-covariance of cost of the debt activities, the change in debt needs, and the debt policy constraints of the bank. In general, with all applications of the model, a movement along the efficient frontier from low expected-cost

TABLE 2. Debt Issuance for the Planning Horizon: Recession Forecast of Expected Debt Cost and Debt Requirements from Time Series Forecast, With Debt Policy Constraints

	Lowest Standard Deviation Portfolio				Intermediate Standard Deviation Portfolio				Highest Standard Deviation Portfolio			
	Term Bonds	Nine-Month Bonds	Six-Month Bonds	One-Month Discount Notes	Term Bonds	Nine-Month Bonds	Six-Month Bond	One-Month Discount Notes	Term Bonds	Nine-Month Bonds	Six-Month Bonds	One-Month Discount Notes
1979												
January	134.288	44.579	0.0	0.0	134.288	44.579	0.0	0.0	134.288	44.579	0.0	0.0
February		114.883	0.0	0.0		114.883	0.0	0.0		114.883	0.0	0.0
March		78.089	0.0	44.564		112.654	0.0	0.0		122.654	0.0	0.0
April	138.490	0.0	0.0	0.0	0.0	93.926	0.0	0.0	0.0	93.926	0.0	0.0
May		100.062	0.0	0.0		100.062	0.0	0.0		9.205	90.857	0.0
June		83.912	0.0	21.329		105.241	0.0	0.0		0.0	50.204	55.037
July	141.206	0.0	0.0	0.0	0.0	119.877	0.0	0.0	0.0	0.0	33.708	141.206
August		109.269	27.958	0.0		0.0	0.0	137.227		0.0	136.504	141.929
September		0.0	20.952	95.474		111.082	0.0	142.571		0.0	115.783	142.571
October	143.005	0.0	1.386	0.045	5.845	143.005	19.187	23.450	5.845	0.0	143.005	42.638
November		108.463	0.0	11.611		0.0	0.0	143.524		0.0	143.524	110.004
December		0.0	0.0	94.884		127.319	0.0	144.042		0.0	144.042	144.042
1980												
January	144.603	0.0	0.0	0.0	143.085	0.0	0.0	144.603	49.441	0.0	127.352	144.603
February		86.887	0.0	39.750		98.817	0.0	144.465		0.0	144.465	144.465
March		0.0	0.0	144.448		105.092	0.0	144.448		0.0	115.634	144.448
April	144.294	0.0	0.0	0.0	137.678	144.294	0.0	0.0	0.0	142.678	0.0	143.235
May		81.662	0.0	29.893		2.286	0.0	0.0		0.0	144.523	144.523
June		0.0	0.0	38.944		120.133	0.0	0.0		6.761	145.428	145.428
Third Quarter	146.333	0.0	0.0	0.0	0.0	0.0	220.530	0.0	31.792	0.0	410.932	0.0
Fourth Quarter	93.988	0.0	0.0	65.354	0.0	0.0	48.993	445.511	7.231	0.0	136.043	445.511
1981												
First Quarter	0.0	11.288	0.0	451.276	0.0	451.276	112.705	451.276	43.701	304.234	407.632	451.276
Second Quarter	0.0	0.0	81.541	387.911	0.0	0.0	306.864	49.761	11.068	0.0	159.366	461.237
Third Quarter	0.0	0.0	0.0	469.384	0.0	0.0	0.0	469.384	9.052	0.0	423.020	469.384
Fourth Quarter	0.0	0.0	112.353	475.892	0.0	301.772	475.892	475.892	7.231	0.0	475.892	475.892
	Expected Discounted Cost = 264.536				Expected Discounted Cost = 250.678				Expected Discounted Cost = 247.961			
	Standard Deviation = 21.590				Standard Deviation = 27.989				Standard Deviation = 31.738			

TABLE 3. Debt Activities the First Period (in Millions of Dollars).

Portfolio Numbers	Range in Expected Cost	Range in Standard Deviation	Term Bonds	Nine-Month Bonds	Six-Month Bonds	Discount Notes
		"Most Probable" Forecast With Debt Policy Constraints				
1	237.184	22.021				
			126.073	0	0	0
56	235.994	23.084				
			118.135	0	7.938	0
58	235.590	23.755				
			93.479	0	32.594	0
107	234.362	26.581				
			93.479	0	30.458	2.136
110	234.032	27.068				
			93.479	0	0	32.594
116	234.310	28.578				
		"Most Probable" Forecast Without Debt Policy Constraints				
1	237.062	18.803				
			126.073	0	0	0
19	230.781	23.597				
			96.665	0	29.408	0
20	228.733	25.735				
			0	0	126.073	0
24	226.800	27.910				
			0	0	0	126.073
25	226.602	28.195				
		"Recession" Forecast With Debt Policy Constraints				
1	264.536	21.590				
			134.288	44.579	0	0
141	247.961	31.738				
		"Recession" Forecast Without Debt Policy Constraints				
1	275.332	17.775				
			178.867	0	0	0
65	240.153	37.314				
			0	178.867	0	0
74	236.870	44.934				

and high cost-variance portfolios to higher expected cost but lower cost-variance portfolios typically entailed a shift from one-month discount notes to six-month bonds to nine-month bonds to term bonds. A projected decrease in expected interest rates over the planning horizon caused shorter-term bonds and notes to be used to take advantage of the decrease. The specific terms used depended upon the duration of the movement and variance level of the efficient frontier. The long-term activities used at lower variances were term bonds; long-term activities used at higher variances were nine-month bonds. Short-term activities used at lower variances were nine-month and six-month bonds; short-term activities used at higher variances were discount notes. These results imply that the optimal debt maturity structure depends to a significant degree on risk attitudes as well as expected cost.

Debt policy constraints used by the bank to reduce risk were also included in the model. When these policy constraints were imposed, expected debt cost was higher at each level of variance. This resulted in a shift to the right of the efficient frontier. With the most probable forecast of interest rates, expected cost was $5 to $8 million (2-3 percent) higher when constraints were included; with the recession forecast, expected cost was $2 to $3 million (approximately 1 percent) higher. The increase in expected cost was greater for the most probable forecast of rates because the debt policy constraints limited the extensive use of a low-cost term bond. With the recession forecast, the policy constraints generally limited the high levels of variance (cost risk) to which the bank could be exposed by truncating the upper section of the efficient frontier. But, the policy constraints also truncated the lower section of the efficient frontier and limited the potential to use low-variance solutions as well, consequently increasing risk potential. Thus, the model results suggest that such policy constraints may actually be increasing rather than reducing risk.

The dramatic difference between the portfolio which would be chosen if risk is ignored and the other portfolios on the efficiency frontier suggests that risk attitudes and the riskiness of various bond and note issues should be considered in making debt participation decisions. With recent volatility in interest rates, lenders such as the Cooperative Farm Credit System that make the majority of their loans on a variable rate basis must be more cognizant of interest rate volatility in their funds acquisition decisions. Unless such risks are explicitly considered, the frequency and amount of rate adjustments and thus the financial risk of the borrower will increase. Risk programming models such as the quadratic program used here can be utilized to formulate debt management policies that consider both expected cost and cost risk. Extensions of the methodology used here to allow simultaneous determination of expected cost, cost risk, loan quantity, and maturity structure may prove useful, but the model used and results presented here clearly indicate that a methodology that ignores risk is not appropriate.

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