Cotton Production Under Risk: An Analysis of Input Effects on Yield Variability and Factor Demand

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The risk flexible production model developed by Just and Pope is estimated for the case of cotton in California's San Joaquin Valley and the implications of the model for factor demand are examined. Results indicate risk-reducing roles for farm machinery, labor, and fertilizer in contrast to restrictions imposed by traditional stochastic production specifications. Qualitative assessment of estimated risk effects on factor employment under risk aversion are evaluated by comparison to the risk-neutral solution.

The lead U.S. agriculture maintains in worldwide production of food and fiber can be attributed largely to the use of innovative, energy-intensive farm machinery and contemporary farming practices, such as irrigation, fertilization, and pest management. However, the public has become more concerned in recent years over the possible adverse effects of agricultural inputs on the environment and society as a whole; e.g., both pesticides and farm mechanization are now controversial issues. For intelligent regulation of the use of productive factors, government agencies must be aware of the roles various inputs play in production and the implications of their roles on factor employment.

For example, farm machinery may be employed to reduce uncertainty associated with labor supply during harvest. Analyses of policies designed to mitigate farm labor displacement should account for this effect. Additionally, pest control input usage is supposedly influenced by production risk considerations [Feder]. Since these inputs may be used both to increase output and to decrease output variability, changes in the availability of pesticides have implications regarding variability of output. Production function analyses of regulation should account for this potential impact. Such models of the productive process should be sufficiently flexible to assess the impacts of inputs on both absolute output and output variance. Unfortunately, a review of previous empirical and theoretical production studies indicates little or no attention has been directed toward the issue of input effects on output variability. Traditional models have implicitly introduced assumptions that prevent the opportunity to investigate this issue.

Just and Pope recently showed the unduly restrictive nature of traditional stochastic models of agricultural production processes. Their results demonstrate that for the class of traditionally specified log-linear stochastic production functions — including the Cobb-Douglas, translog, generalized power, CES, and transcendental — the marginal impact of an input on yield variability and marginal product are always positive and always negative, respectively. To facilitate more flexibility regarding risk in both empirical and theoretical investigations, they proposed a technical relationship incorporating separate effects of inputs on the mean and variance of output. Our purpose here is to provide an empirical test of this production framework to determine if *a priori* assumptions regard-

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ing the impact of input use on yield variability are supportable and to consider implications of the empirical results for derived demand.

Production Model and Derived Demand

The general form of the production function employed for this study is

$$(1) \qquad y = f(x;\alpha) + h^{1/2}(x;\beta)\varepsilon$$

where $E[\varepsilon]=0$; $E[\varepsilon^2]=1$; and y is output, x is a vector of inputs, α and β are vectors of unknown parameters. Just and Pope developed this production model and its properties with emphasis on its flexibility with respect to an input's impact on the variance of output.

The risk-flexible technology depicted in equation (1) may be incorporated into a firm decision model and implications for derived factor demand can be examined if very specific assertions regarding firm behavior and technology are made in the conceptual development of the demand model. First, it is assumed that the firm produces output subject to the technical conditions expressed in (1) and that the error term, ε, is distributed according to the standard normal density. Second, product price, p, and a k-vector of input prices, γ, are assumed known and given. Third, firm behavior is characterized by selection of a k-vector of inputs, x, to maximize expected utility, $E[U(\cdot)]$, a monotonic function of profit, π.

Under these assumptions, a firm's objective function is given by

$$E[U(\cdot)] = \int_{-\infty}^{\infty} U(\xi)g_{\pi}(\xi)d(\xi)$$

where

$$\pi = p[f(x;\alpha) + h^{1/2}(x;\beta)\varepsilon] - \gamma'x$$

and

$$g_{\pi}(\xi) = [2\pi p^2h(x;\beta)]^{-1/2}$$

$$\exp\left[-1/2\left(\frac{(\xi - pf(x;\alpha) + \gamma'x)}{ph^{1/2}(x;\beta)}\right)^2\right]$$

Fourth, it is further assumed that the firm's decision-maker exhibits constant risk averse behavior characterized by the following exponential function[1]:

$$U(\xi) = a - b\exp[-c\xi] \; ; \; a,b,c>0$$

Hence:

$$E[U(\pi)] = a - b\exp[-c[pf(x;\alpha) - \gamma'x] + (1/2)[cph^{1/2}(x;\beta)]^2]$$

and necessary conditions characterizing the firm's postulated optimal behavior require that

$$\partial E[U(\pi)]/\partial x_i = 0; \; i=1,2,\ldots\ldots,k$$

or that

$$(2) \qquad p\partial f(x;\alpha)/\partial x_i = \gamma_i + (1/2)cp^2 [\partial h(x;\beta)/\partial x_i] \; ; \; i=1,2,\ldots,k .$$

Sufficient conditions indicate that the matrix with elements H_{ij} given by

$$(3) \quad H_{ij} = p[\partial^2 f(x;\alpha)/\partial x_i \partial x_j] - (1/2)cp^2 [\partial^2 h(x;\beta)/\partial x_i \partial x_j] \; ; \; i,j = 1,2,\ldots k$$

is negative definite. Thus, equation (2) suggests that a firm employs an input to the point of equality between the value of marginal product and the input price plus a term involving the marginal effect of the input on risk.

In the case of risk-increasing inputs (derivative of $h(\cdot)$ positive), strict concavity of the mean of output satisfies conditions for a local maximum by equation (3). On the other hand, for risk-reducing inputs (derivative of $h(\cdot)$ negative), optimality requires the value of marginal product to be steeper than the subjective value of the marginal impact on risk. As an illustration, the value of marginal product (VMP_i) and "value of marginal risk" (VMR_i) curves depicted in Figure 1 for the

[1]The exponential utility was chosen to simplify computations.

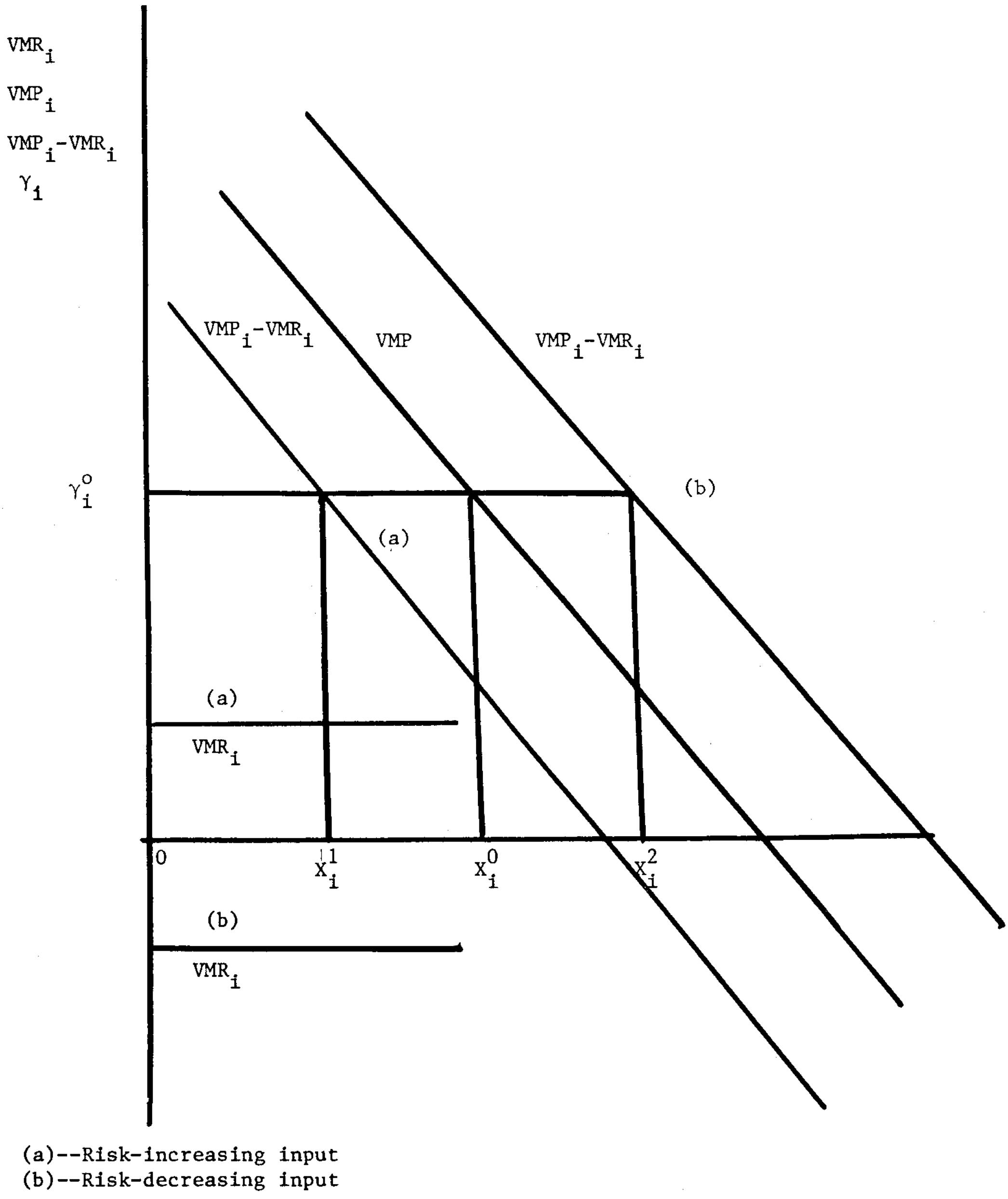

Figure 1. Value of Marginal Product and Derived Input Demand Under Risk Aversion.

case of a risk-increasing and risk-reducing input satisfy these conditions. Likewise, a corresponding derived input demand function for a risk-reducing and a risk-increasing input is also shown in Figure 1. Note that in this figure the value of marginal product curve is shifted according to the marginal effect of the input on risk. Specifically, at input price γ_i, employment of input x_i is displaced from its level under risk neutrality, x_i^0, to either x_i^1 or x_i^2 when risk aversion is introduced. Only a downward shift is permitted when the traditional stochastic production model is combined with risk-averse behavior. Figure 1 depicts the intuitive hypothesis that risk averse behavior implies more (less) of an input is employed if it has a decreasing (increasing) impact on risk than would be employed in the absence of risk aversion.

Estimation

Coefficient estimates based on (2) are difficult to obtain directly due to problems of identifiability. However the production model in equation (1) may be independently estimated, and resulting estimates can be used with other data to make inferences about input demand. The latter procedure is used in this empirical analysis. Consistent and efficient estimation of the parameters in equation (1) is provided by the following four-step procedure:

(Stage 1) Application of nonlinear ordinary least squares to:
$y_t = f(x_t;\alpha) + u_t$
to obtain $\hat{\alpha}$ and $\hat{u}_t = y_t - f(x_t;\hat{\alpha})$.

(Stage 2) Application of ordinary least squares to:
$\ln|u_t| = \ln[h^{1/2}(x_t;\beta)] + v_t = (1/2)\ln(u_t^2)$
to obtain $\hat{\beta}$.

(Stage 3) Application of nonlinear generalized least squares to:
$y_t = f(x_t;\alpha) + u_t$
with consistent covariance matrix estimate developed from the results of stage 2. Results provide, α^*, an efficient estimate of α.

(Stage 4) Application of linearized maximum likelihood to obtain an efficient estimate of β given by:

$$\beta^* = \hat{\beta} + .6352e_1 - 1/2\left[\sum_{t=1}^{T} \ln x_t \ln x_t'\right]^{-1} \cdot$$

$$\sum_{t=1}^{T}\left[1 - \hat{u}_t^2 \cdot \exp(-2\ln x_t'(\hat{\beta} + .6352e_1))\right]\ln x_t$$

where $\hat{\alpha}$ and $\hat{\beta}$ are defined in Stages 1 and 2 respectively, and e_1 is a k-vector with first element 1 and remaining elements zero.

In the following estimation, the four-step method is applied to a Cobb-Douglas production function which incorporates the stochastic aspects of (1), and is given by

$$(4)\; y_t = \alpha_0 \prod_{i=1}^{k} x_{it}^{\alpha_i} + \beta_0 \prod_{i=1}^{k} x_{it}^{\beta_i} \varepsilon_t\,;$$

$$t = 1,2\ldots,T$$

where

$$E[\varepsilon_t] = 0 \text{ and } E[\varepsilon_t\varepsilon_{t'}] = \begin{cases} 1, & \text{if } t = t' \\ 0, & \text{otherwise} \end{cases}$$

Data

The data used to estimate the model in equation (4) consist of records from a random sample of 41 cotton growers in California's San Joaquin Valley. The data pertain to yield and input levels in 1974.[2] Variables included in the model are:

y_t = cotton lint (pounds per acre)
x_{1t} = irrigation (acre feet per acre)
x_{2t} = labor (dollars per acre)
x_{3t} = machinery (dollars per acre)
x_{4t} = fertilizer[3] (pounds per acre)
x_{5t} = insecticides[4] (pounds per acre)

Table 1 presents summary statistics, the matrix of simple correlation coefficients of the data, and F-statistics which were used on an

[2]The use of cross-section data raises the possibility of estimation bias which can result from omitted interfirm factors. The extent of such bias is an empirical question dependent on the data set under consideration. The absence of suitable managerial variables prevents investigation of the extent of this bias.

[3]Fertilizer use included manure as well as the popular chemical fertilizers.

[4]Includes miticides.

TABLE 1. Data Summary Statistics[a].

Item	Cotton Lint	Irrigation	Labor	Machinery	Fertilizer	Insecticides[b]
Mean	976.424	3.071	79.564	127.352	2641.550	14.857
Standard Deviation	211.263	1.182	48.739	73.738	5129.560	16.377
F-statistic[c]	—	2.083	1.489	.840	2.074	1.331
		Matrix of Simple Correlation Coefficients				
Cotton lint	1.000					
Irrigation	.197	1.000				
Labor	.198	−.065	1.000			
Machinery	.278	−.184	.260	1.000		
Fertilizer	.047	−.194	−.205	.000	1.000	
Insecticides	.066	.373	.107	−.203	−.109	1.000

[a]Based on 1974 data, sample size = 41.
[b]Includes miticides.
[c]F-test for evaluation of collinearity in variables (see Farrar and Glauber).

F-test suggested by Farrar and Glauber to test for multicollinearity. In no case did the reported F-statistics exceed the critical value, $F_{0.95}(5,36) \approx 2.48$, and thus serious collinearity among the various predetermined variables was not indicated.[5]

Production Estimates

Just and Pope show that the traditionally specified Cobb-Douglas production function is a special case of the risk-flexible production function (1) when, among other conditions, $\alpha_i = \beta_i$; $i=1,2,\ldots,k$. Hence for comparison purposes, Table 2 reports both the usual stochastic production model, estimated in log-linear form by ordinary least squares, and the risk flexible model of equation (4).[6] Least squares estimates (shown in the first column of numbers) indicate positive output elasticities for all inputs; however, none of the coefficient estimates are significantly different from zero. The corresponding efficient output elasticity estimates of the risk-flexible formulation (reported in the fifth column of Table 2) differ considerably from the ordinary least squares results; moreover, all input coefficients are significant, with the notable exception of insecticides. The latter coefficient possesses a negative sign and is statistically insignificant. Furthermore, efficient estimates of risk impacts (presented in column 6) reveal statistically significant risk-increasing and reducing roles for irrigation and farm machinery, respectively. For the remaining inputs, parameter estimates of labor and fertilizer suggest risk-reducing impacts whereas the parameter estimate for insecticides suggests a risk-increasing impact on output. Considering the different parameter estimates resulting from the alternative model specifications, the hypothesis, $\alpha_i = \beta_i$, was tested to aid in model selection. Application of Hotelling's T^2 test led to rejection of the hypothesis $\alpha_i = \beta_i$, which suggests that the risk-flexible form is more appropriate than the traditionally specified Cobb-Douglas production function.

Input Productivities and Risk Impacts

In empirical work, the usefulness of the risk-flexible model lies in its ability to sepa-

[5]Note that the assumed firm behavior does not generally imply collinearity among input variables as Doll suggests under cost minimization.

[6]The TROLL/1 System Dogleg Macro facility was employed for nonlinear estimation.

TABLE 2. Estimates of Risk Flexible Production Function for Cotton, San Joaquin Valley, California[a]

Estimator	Ordinary Least Squares[b]	4-Step					
		Stage 1	Stage 2	Stage 3	Stage 4	Direction of Impact	
Parameter	$\hat{\alpha}_i$	$\hat{\alpha}_i$	$\hat{\beta}_i$	α_i^*	β_i^*	Mean	Variance
Constant	5.9549	5.3639	8.1681	4.5907	7.1502		
Irrigation	.1964	.2409	.5153	.2121	.4902	+	+
	(.1144)[d]	(.0980)	(.4007)	(.0606)	(.2869)		
Labor	.0005	.0167	−.0516	.1052	−.0608	+	−[e]
	(.0282)	(.0352)	(.0989)	(.0429)	(.0708)		
Machinery	.1101	.1899	−.4349	.2666	−.3452	+	−[e]
	(.0713)	(.0678)	(.2496)	(.0576)	(.1788)		
Fertilizer	.0245	.0415	−.1824	.0492	−.0867	+	−[e]
	(.0304)	(.0243)	(.1065)	(.0183)	(.0763)		
Insecticides[c]	.0086	.0052	.0228	−.0139	.0058	−[e]	+
	(.0202)	(.0193)	(.0707)	(.0158)	(.0506)		

[a]Based on 1974 data, sample size = 41.
[b]Assumes multiplicative, homoscedastic disturbances.
[c]Includes miticides.
[d]Numbers in parentheses are estimated asymptotic standard errors.
[e]Indicates a change in sign from traditional stochastic specification.

rate the effect each input has on mean output and variability of output. Since traditional production models have confounded these effects, it is important to assess each input's separate effects qualitatively and to compare with commonly held beliefs in the agricultural community.

Irrigation positively affects mean yield and yield variability (columns 7 and 8 of Table 2). As expected, this input has a positive marginal product, yet it also increases output variability. The risk-increasing aspect of irrigation may appear counterintuitive since establishment of an irrigation schedule supposedly mitigates the role played by nature in the production process. Several illustrations complicate this supposedly simple hypothesis. For example, irrigation may interact positively with random variations in weather variables such as sunlight and temperature, to promote above average yields. On the other hand, growth problems associated with an insufficient number of degree days are exacerbated by an irrigation schedule that could further lower soil temperature and contribute to plant disease. Moreover, the ability of growers to respond to other farm management problems is somewhat restricted during periods when irrigation limits mobility in the field. Finally, California-based agricultural consultants have noted the major role of irrigation in aggravating farm problems, such as crop protection and crop nourishment.

Each of the next three inputs in Table 2 — labor, machinery, and fertilizer — has the same sign configurations, suggesting a positive marginal product and a risk-reducing role in production. In the cases of labor and machinery, increasing these inputs should permit growers to respond more rapidly to problems, particularly during harvest when a rapid response may be crucial in reducing crop losses. Finally, fertilizers appear to reduce yield variability perhaps by helping maintain plant vitality despite adverse weather conditions or agricultural pests that find the fertilizer-induced overgrowth a prime breeding ground and unlimited food source.

Most surprising and interesting in Table 2 are the apparently anomalous results for insecticides. Although both estimates of marginal product and the impact on output variability are insignificant, the signs are the opposite of what one might ordinarily expect. Several prior production studies [Headley, Fischer, Campbell] indicate large and significant mean returns to insecticide treatments and thereby foreshadow potentially large costs for regulating insecticide use. It is also generally accepted that growers apply additional insecticides as a form of self-insurance against losses. Justification for insurance-spraying centers around the idea that insecticides reduce the likelihood of pest populations attaining economically damaging levels.

Nevertheless, closer scrutiny of the most recent pest management economics literature indicates several factors which satisfactorily explain the finding in this study. First, depletion of the effectiveness of the insecticide arsenal (increased degree of resistance by insects), resurgence, and secondary pest outbreaks threaten the viability of insecticides in agricultural production.[7] For example, Carlson's analysis of nationwide cross-section data during three separate years indicates a decining expenditure elasticity for insecticides over time that was not significantly different from zero in the most recent year considered (1969). Second, Feder used decision analysis to show that excessive insecticide use by risk-averse growers may occur. Hence, in the presence of overuse and declining effectiveness, parameter estimates indicating substantial gains from additional insecticide application might not be expected. Third, insecticide elasticity estimates based on cross-section data, which include the previously cited reports, are conditioned on the typically unobserved level of the pest population. Extraneous qualitative evaluation here indicates that relevant infestation levels were quite low during the sample period, thus further limiting the importance of insecticides in the production process. Finally the phytotoxic effects of insecticides on plants cannot be ruled out. In summary, the negative sign attached to insecticides may be attributed to these factors or to sampling variation. Additional research is necessary to verify the trend of declining effectiveness of insecticides and to identify the relevant factors.

[7]A recent study demonstrates that organophosphorus insecticides applied to San Joaquin Valley cotton not only contribute to the destruction of beneficials but also significantly stimulate the reproductive rate of two-spotted spider mites (see Leigh and Wynholds).

Factor Demand

Expected marginal productivities and expected marginal impacts of input use on yield variability were evaluated and are reported in Table 3. Table 4 shows the expected value of marginal product and average input prices paid by farmers in the San Joaquin Valley of California. Earlier behavioral postulates indicate that the value of marginal product exceeds factor price for risk-increasing inputs and is less than factor price when an input functions so as to reduce risk. The following results were obtained for irrigation, labor, and fertilizer. The value of marginal product for irrigation exceeds unit cost, whereas for the risk-reducing inputs, labor and fertilizer, the opposite occurs. Results in the cases of fertilizer and irrigation are highly significant. Implied factor demands under constant risk aversion indicate decreased (increased) employment of risk-increasing (risk-decreasing) inputs compared with the risk neutral solution for a range of risk coefficients. This result is most evident in the case of risk-reducing inputs. Thus, in particular, these inputs tend to support *a priori* expectations regarding derived demand under risk.

Conclusions

This paper investigates the supposedly twofold role inputs have on mean output and output variability and assesses qualitatively what effect risk impacts may have on factor employment. The traditionally specified Cobb-Douglas production function and the risk-flexible production function, developed by Just and Pope, were estimated and compared. Based upon the available cross sectional sample data obtained from 41 Califor-

TABLE 3. Expected Marginal Mean and Variance Impacts on Inputs for Cotton, San Joaquin Valley, California[a].

Input	Expected Marginal Product[b] OLS	Expected Marginal Product[b] 4-Step	Expected Marginal Impact on Yield Variability[b] OLS	Expected Marginal Impact on Yield Variability[b] 4-Step
	(pounds per acre/input unit)			
Irrigation	65.1517	70.3598	11,374.2137	323,311.3782
Labor	.0261	5.5167	4.5774	−5117.8223
Machinery	3.0069	7.2810	524.9442	−22464.4719
Fertilizer	.0311	.0624	5.4223	−437.3457
Insecticides	1.2946	−2.0924	226.0063	6491.3931

[a]Based on estimates reported in Table 2.
[b]Evaluated at the geometric mean.

TABLE 4. Comparison of Expected Value of Marginal Product and Factor Price for Cotton, San Joaquin Valley, California[a].

Input	Unit	Expected Value of Marginal Product	Price
Irrigation	dollars/acre foot	33.35 (9.5287)[c]	4.50
Labor	dollars/hour	2.70 (.3677)	2.90
Machinery	dollars/hour	3.46 (.2486)	3.00
Fertilizer	dollars/pound	0.03 (.0109)	0.21
Insecticides[b]		--	--

[a]Based on the estimates reported in Tables 2 and 3.
[b]Output elasticity of insecticides was insignificant.
[c]Numbers in parentheses are estimated asymptotic standard errors.

nia cottongrowers, the econometric analysis led to rejection of the hypothesis implicitly assumed in the Cobb-Douglas production function; namely, that all inputs increase output variability. Results from estimating the risk-flexible production function for cotton suggest a significant risk-increasing role for irrigation and a significant risk-reducing role for farm machinery. Results also indicate labor and fertilizer reduce yield variability and insecticides increase yield variability.

Given the quantitatively different roles inputs have on output variability, a simple decision model was specified to aid in qualitative assessment of risk effects on factor demand. The assumption of constant risk-averse behavior was incorporated into the decision model, and the intuitive hypothesis that demand for an input is inversely related to its risk impact was examined. Results tend to support this hypothesis for risk-reducing inputs.

Some caution about these conclusions should be exercised since the cross-section data employed are region and crop specific and do not necessarily permit general inference regarding risk impacts of agricultural inputs. However, the importance of correctly identifying the role of inputs on output variability cannot be overestimated. This is especially true for chemical inputs that have potentially large negative externalities associated with their use. Policy-makers should be particularly aware of yield variation attributable to inputs when evaluating costs associated with regulatory controls on input use. The risk-flexible production model presented by Just and Pope provides a practical framework for examining both input-yield variability interaction and government policies designed to reduce risks in agriculture.

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