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An Economic Interpretation of Impact of Phenologically Timed Irrigation on Corn Yield

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An economic production function was derived in an analysis of water application to corn by phenological time periods. The quantities of use of water by growth periods as delineated by physical stages were tested for influence on final output of dry matter and grain. Several years' data for three locations were utilized. The results tend to confirm the conceptual models and previous work in this type of analysis. Certain periods of growth were more critical than others.

Using methodology developed for predicting crop yield, economists can investigate ways to optimize crop production through water control. Optimal yields require that adequate water be applied to crops at the crucial time during a phenological stage.

Following the water balance budget approach, can one determine the phenologically important stages of plant growth and the impact of irrigation on these stages? Can one say anything about management practices that will improve yield? Answers to these and related questions will be discussed in this paper.

Robins and Domingo have reported that soil moisture depletion of one to two days during tasseling resulted in as much as a 22 percent yield reduction, while six to eight days stress reduced yield by 50 percent. They concluded that "yield reductions due to absence of available water after the fertilization period appeared to be related to the maturity of the grain when the available moisture was removed."

Denmead and Shaw found grain yields were reduced by all moisture treatments. Plants subjected to water stress at tasseling were the most affected. The reductions in grain yield were 25 percent when stress was imposed at vegetative stage, 50 percent by stress at tasseling and 21 percent by stress at ear stages. They also found a tendency for stress imposed in one stage to harden the plant against damage (further yield reduction) from stress at a later stage.

In other studies, Charles V. Moore showed that it is possible to impute a value to the irrigation cycle. He further developed a model to determine an optimum water price and changing commodity price during growing season. Arlo W. Biere, et. al. demonstrated the sensitivity of a model to the time of water application. They concluded that the higher the available soil moisture around silking the higher the yield because corn is most sensitive to soil moisture stress at that time. Dan Yaron showed that while production functions with fixed intra-seasonal distribution are estimable by regression methods, difficulties are involved in the regression approach in the estimation of dated production functions. Stewart, et. al. tried to identify the most important stage. They ran separate regressions for four experimental corn-growing sites in four different states (Logan, Utah; Fort Collins, Colorado; Yuma, Arizona; and Davis, California). They found

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that stress at the pollination stages produced the most drastic effects on grain yield.

In this paper we consider special problems with the basic equation in use, the type of econometric analysis needed for tackling the actual regression result, results, management recommendations and conclusions. Our primary objective has been to develop a time sequence production function through applying Hanks [Stewart, et. al.] equation, which uses the water budget approach. The analysis identifies management options that might improve crop yield.

Experimental Data Acquisition

This paper is based on four-hundred and ninety-three observations collected in 1974 and 1975 at Davis, California; Fort Collins, Colorado; and Logan, Utah. Both grain and dry matter yield of corn were collected. The yield equation used enables data pooling because of a uniform approach to measuring crop water requirements and actual evapotranspiration. Stewart, et. al. reported that "It is common knowledge that methods now in use for making these estimates are far from perfect and that the use of different methods often produces different results. Accordingly, the Davis research team has developed what are thought to be improved methods of ET estimation, and these were adopted for use at all experimental sites."

Potential evapotranspiration (ET_p) is closely correlated with pan evaporation and crop growth stage. Accurate measurement of short term ET_p is required when determining the ratios of ET_p for the crop to E_G, where E_G is evapotranspiration for each growth stage. Both measurements depend on the use of sophisticated lysimeter equipment. Such equipment is available in Davis and was utilized in this study. Daily measurements were made of ET_p and of Class A pan evaporation (E_G).

For clarity, and to facilitate measurements among growth stages, the data were summed for short periods (mostly five days each) and ET_p/E_G ratios were computed for each period. This process gave us the actual evapotranspiration (ET_A) used in our regression analyses. For ET_A, each application of water to the soil (including rainfall) starts a new water period, which requires separate consideration. An accounting was kept of water applications as they affect evaporation. ET_p refers to the evapotranspiration possible when water is not limited. Thus, ET_p limits ET_A in any given water period. When $ET_A > ET_p$, it is assumed that drainage down the soil profile was responsible. The data for the short periods within growth stages were pooled for a composite test to show the stage of growth effects.

Model Specification

Research to evaluate the influence of irrigation management on corn production where water and salinity limited production has been carried out in Arizona, California, Colorado and Utah [Hanks, et. al., Stewart, et. al.]. We use Hanks' model because it provides for a direct relationship between evapotranspiration by growth stage and yield. Additionally, it can predict transpiration, while other models require measured data. Another advantage is that the Hanks' model is readily transferable; all it requires to predict yields are basic soil, climate, crop and irrigation data. Furthermore, the Hanks' model recognizes that yield is related to transpiration. According to Hanks, the yield-transpiration relationship is important but the plant, soil and climatic factors are difficult to separate.

The Hanks' water budget model shows yield as a function of evapotranspiration. It is represented in equation form as

1)

$$\frac{Y}{Y_p} = C\left(\frac{ET_A}{ET_p}\right)_v^{\lambda_1}\left(\frac{ET_A}{ET_p}\right)_p^{\lambda_2}\left(\frac{ET_A}{ET_p}\right)_m^{\lambda_3}$$

where Y = Tons per hectare of harvested grain or dry matter. Y_p = Potential yield is the highest measured value of Y. C = Regression constant. ET_A = Measured evapotranspiration. It is the amount of applied water depleted by plants, taking into account losses from drainage and run-off. ET_A is measured in centimeters. ET_p = Potential evapotranspiration: The highest measured value of ET_A. The subscripts v, p, m represent a phenological time period. Where: v = vegetative stage. p = pollination stage. m = maturation stage.

Vegetative stage is defined as extending from planting to first tassle. This stage varies with location, but for Logan, Utah, it averaged sixty-three days based on a two-year (1974-1975) experiment. Pollination stage includes from first tassle to blister kernel. For the two year experiment in Logan, this stage averaged twenty-six days. Maturation stage, from blister kernel to physiological maturity, averaged forty-three and a half days for the 1974-1975 Logan experiment.

The exponents λ_1, λ_2, λ_3, in the Hanks' model represent the relative importance of water for the three different stages. The λ_1 values represent the elasticity of crop production to an increase in actual evapotranspiration (ET_A) during its vegetative stage. Similarly, λ_2 and λ_3 represent the elasticity of crop production to an increase in actual evapotranspiration (ET_A) during pollination and maturation stages, respectively.

Yield is measured either as grain or as dry matter. Grain yield is the actual amount of corn kernels harvested, weighed dry in tons per hectare. Dry matter yield is the actual weight of everything on the corn plant from a few inches up from the roots where the stalk was cut. It was weighed dry in tons per hectare.

Difficult Issues Associated With Model Specification

The functional form of our model is

$$2)\quad Y = \begin{bmatrix} \{f^1 (ET_A)_v, (ET_A)_p, (ET_A)_m\} & (ET_A)_v \leq (ET_p)_v; (ET_A)_p \leq (ET_p)_p; (ET_A)_m \leq (ET_p)_m \\ \{f^2 (ET_A)_v, (ET_A)_p, (ET_A)_m\} & (ET_A)_v > (ET_p)_v; (ET_A)_p > (ET_p)_p; (ET_A)_m > (ET_p)_m \end{bmatrix}$$

$$ET_A, ET_p > 0$$

This specification means that actual yield is a function of actual evapotranspiration. The functions are designated f^1, f^2; the integers 1 and 2 are not powers. The first function, f^1, implies that, to get yield, actual evapotranspiration has to be less than or equal to potential evapotranspiration. In practice, potential always exceeds actual. The other function, f^2, has strict inequality. Here there is no question that moisture is required for production. But this is the case where $ET_A > ET_p$. The definite waste involved may be due to drainage down the soil profile and excess runoff. This is termed waste because water is not used by the crop.

From equation (2) we develop

$$3)\quad Y = C_a (ET_A)_v^{\lambda_1} \; (ET_A)_p^{\lambda_2} \; (ET_A)_m^{\lambda_3}$$

where C_a is actual regression constant. To minimize the impact of combining data from three different locations, a ratio of actual to potential observations (ET_A/ET_p) is required. Actual observations represent data collected from the field and maximum values represent potential observation. Such a ratio also helps minimize climatic effect from one year to the next, it minimizes disease effect etc. So for the potential counterpart of equation (3) we will have

$$4)\quad Y_p = C_p (ET_p)_v^{\propto_1} \; (ET_p)_p^{\propto_2} \; (ET_p)_m^{\propto_3}$$

where C_p is a potential regression constant.

Forming the ratio

$$5)\quad \frac{Y}{Y_p} = \frac{C_A}{C_p} \; \frac{(ET_A)_v^{\lambda_1}}{(ET_p)_v^{\propto_1}} \; \frac{(ET_A)_p^{\lambda_2}}{(ET_p)_p^{\propto_2}} \; \frac{(ET_A)_m^{\lambda_3}}{(ET_p)_m^{\propto_3}}$$

Simplification and assuming $\propto_i = \lambda_i \, \forall_i$ (Where $\forall_i$ stands for over all i) will yield the model equation

1)

$$\frac{Y}{Y_p} = C\left(\frac{ET_A}{ET_p}\right)_v^{\lambda_1}\left(\frac{ET_A}{ET_p}\right)_p^{\lambda_2}\left(\frac{ET_A}{ET_p}\right)_m^{\lambda_3}$$

Shape of Function

Given empirical evidence, with no water, there will be no product. Introduction of water implies $ET_A > 0$ and thus some product, even if only as measurable dry matter. An increase in water supply implies a cumulative ET_A and that the evapotranspiration is increasing. The evapotranspiration rate goes up because, as the plant develops, so does its transpirational capacity. This would increase the tonnage of dry matter yield. Similarly, as the transpiration capacity of corn increases so would grain yield. Production is increased as corn kernels increase in size and fill up the corn cob. If we keep $(ET_A)_v$, $(ET_A)_m$ at a level where the crop is not stressed and let any increase in total ET_A come only during $(ET_A)_p$, dry matter and grain yields will both increase. The model we are dealing with conforms to the regular Cobb-Douglas shaped curve for both grain and dry matter yield (figure 1).

The question of actual evapotranspiration equaling respective stage potential evapotranspiration $[(ET_A)_{v,p,m} = (ET_p)_{v,p,m}]$ can be problematic. Taking the ratio $\frac{Y}{Y_p}$ we find $Y = Y_p$ because $\frac{Y}{Y_p} = (1)^{\lambda_1} \cdot (1)^{\lambda_2} \cdot (1)^{\lambda_3}$ (assuming $\frac{CA}{C_p} = 1$). Thus $Y = Y_p = 1$.

The production function is subject to increasing returns to scale until $Y = Y_p$. Increasing returns to scale further imply continuing utilitization of factors of production. Hence, with an increase in production factor inputs, actual yield is supposed to approach potential output, Y_p. Where $\lambda_i = 0 \, \forall_i$, potential yield is not obtainable because the elasticity or factor share of each stage is zero. Where then is the economic problem?

Once the actual yield measured equals maximum yield, the function will no longer exhibit increasing returns to scale. If one chooses to increase a factor of production when actual yield $Y = Y_p = 1$, zero returns to scale should be expected, and beyond $Y = Y_p$ negative returns to scale should be expected.

As long as $Y_p > Y$, there is a fraction, and fractions of ET imply that better management (defined as stage-oriented water application) could make actual yield approach potential yield(Y_p). Normally when $\sum_{i=1}^{3} \lambda_i > 1$, it is a case of increasing returns to scale. How Y_p is approached would dictate the rate of increase in Y. Since our function is positively sloped, (see figure 2), then the rate of increase of Y would be increasing for grain and constant for dry matter.

Economic Basis of Analysis

The value of marginal product of water (VMP_w) is defined as price (a constant in a

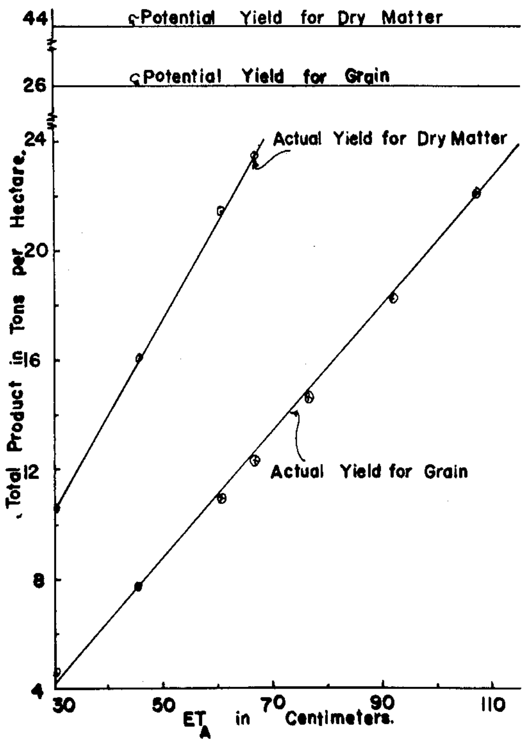

Figure 1. Total product versus input (quantity of water(ET_A)).

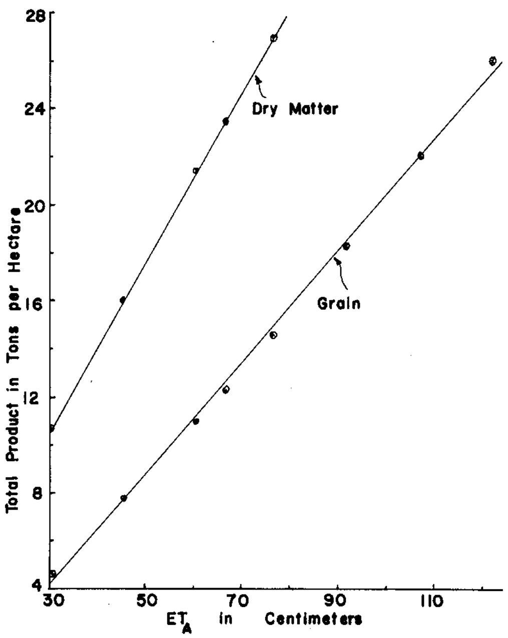

Figure 2. Total production approaching potential production(Y_p) as water quantity(ET_A) increases.

given year) times marginal physical product of water (MPP) at a given stage. MPP represents the marginal contribution of a unit of water to the total product of grain yield. VMP changes because MPP values differ for each stage of growth.

To determine MPP for each stage of growth requires adjusting equation (1), which yields:

$$Y = \frac{Y_p C}{ET_{P_v}^{\lambda_1}\ ET_{P_p}^{\lambda_2}\ ET_{P_m}^{\lambda_3}}\ ET_{A_v}^{\lambda_1}\ ET_{A_p}^{\lambda_2}\ ET_{A_m}^{\lambda_3}$$

Defining $\frac{Y_p C}{ET_{P_v}^{\lambda_1}\ ET_{P_p}^{\lambda_2}\ ET_{P_m}^{\lambda_3}}$ as $\hat{C}$, the equation can be written as

$$7)\quad Y = \hat{C}\ ET_{A_v}^{\lambda_1}\ ET_{A_p}^{\lambda_2}\ ET_{A_m}^{\lambda_3}$$

Differentiating Y with respect to evapotranspiration of each growth stage would yield the marginal physical product of that growth stage.

Using non-optimally allocated water as was done in the experiment, there was a tendency for MPP of water to be highest during pollination stage for grain yield.

$$8)\quad MPP(ET_A)_p > MPP(ET_A)_v > MPP(ET_A)_m$$

Relationships in (8) mean that a change in total yield divided by a corresponding change in ET_A is highest at the pollination stage and the change in total yield divided by a corresponding change in ET_A at the vegetative stage is higher than that of the maturation stage.

To get an optimal solution, a Lagrangian, L, was formed, using relationship (7).

$$9)\quad L = \hat{C}\,(ET_A)_v^{\lambda_1}\ (ET_A)_p^{\lambda_2}\ (ET_A)_m^{\lambda_3} - \phi\ [(ET_A)_v^{\lambda_1} + (ET_A)_p^{\lambda_2} + (ET_A)_m^{\lambda_3} - \overline{w}]$$

where w is total water used as evapotranspiration and is defined as

$$10)\ \overline{w} = (ET_A)_v + (ET_A)_p + (ET_A)_m$$

Taking first order conditions and solving for ϕ, the optimum condition becomes

$$11)\quad \frac{MPP(ET_A)_v}{P_{w_v}} = \frac{MPP(ET_A)_p}{P_{w_p}} = \frac{MPP(ET_A)_m}{P_{w_m}} =$$

or alternatively

12)

$$\frac{VMP(ET_A)_p}{P_{w_p}} = \frac{VMP(ET_A)_v}{P_{w_v}} = \frac{VMP(ET_A)_m}{P_{w_m}} = K \geq 1$$

where P_{w_v}, P_{w_p}, P_{w_m}, is price of water at a given stage. Since price of water remains the same during the irrigation season, then $P_{w_v} = P_{w_p} = p_{w_m}$.

But equation (8) is suboptimal for a farmer seeking maximum profit because of the strict inequalities. By spending a dollar less (not applying a dollar's worth of water) on water during vegetative and maturation stages, the farmer affects total ET_A and rate of $(ET_A)_{v,m}$. This would cause loss in dry matter but gain in the production of corn grain. Net output will increase by a factor greater than zero for the same total cost. Shifting the amount spent on water from the less productive to the most productive plant growth stage can thus restore profit maximization. Going one more step, the dollars spent on water during maturation stage can be reduced to further enhance profit.

Econometric Analysis of Growth Stage Importance

The exponents of the Hanks' equation are crucial in determining stage importance. Given an equation $Y = C\,A^{\lambda_a} B^{\lambda_b}$ we know $\lambda_a = \left[\frac{dY}{Y}\right] \div \left[\frac{dA}{A}\right]$, $\lambda_b = \left[\frac{dY}{Y}\right] \div \left[\frac{dB}{B}\right]$. An x percent increase in factor A, will yield a $x\lambda_a$ percent increase in output Y. Similarly for B, $x\lambda_b$ percent is the increase in Y. If $\lambda_a > \lambda_b$, the factor A contributes more to the production of Y than factor B. Thus the need exists to test if $\lambda_a > \lambda_b$. We used a null and an alternate hypothesis approach to compare two stages at a time and then to decide if $\lambda_a > \lambda_b > \lambda_c$.

Using the regression result, a simple t-test will be used to test

$$H_O^1: \lambda_2 - \lambda_1 = 0$$

$$H_A^1: \lambda_2 - \lambda_1 > 0$$

To do this test we shall apply the formula

13)

$$\frac{W^1\lambda + W^1\beta}{\sqrt{W^1\,[S^2(X^1X)^{-1}]\,W}} \sim t_{n-k}$$

where: λ = Least Square Estimator; W = Fixed weight vector; β = Parameter vector. $\beta = 0$, therefore negligible, t_{n-k} = t-statistics done with n observations and k degrees of freedom; and $[S^2\,(X^1X)^{-1}]$ = the estimated coefficient of variance co-variance matrix. Since three hypotheses for each stage of growth will be tested, W^1 takes the following values:

$$W^1 = \begin{bmatrix} 0 & -1 & 1 & 0 & = \lambda_2 > \lambda_1 \\ 0 & 1 & 0 & -1 & = \lambda_1 > \lambda_3 \\ 0 & 0 & 1 & -1 & = \lambda_2 > \lambda_3 \end{bmatrix}$$

The solution to equation (13) is

$$\frac{0.227}{\sqrt{0.580 \times 10^{-2}}} = 2.980$$

Similar hypotheses testing of $\lambda_1 > \lambda_3$, $\lambda_2 > \lambda_3$ yielded 0.250 and 4.207 respectively. These results indicate that water affects yield differently at different stages of growth.

For α = 1 percent, one tailed test, the t-statistic is 2.980 and from a standard table, the t value of 1 percent is 2.326. Since 2.980 > 2.326, we accept the alternate hypothesis stated as $H_A^{2,1}: \lambda_2 - \lambda_1 > 0$ which implies that when growing grain, contribution to total product for each additional irrigation is higher during pollination stage than vegetative stage.

The same conclusion is reached when pollination stage is compared to maturation stage. Statistically, categorical statements as for the two cases above cannot be made when comparing vegetative stage with maturation stage. Since $\lambda_1 = 0.347$ while $\lambda_3 = 0.330$, one can conclude that numerically $\lambda_1 > \lambda_3$.

Similarly, for dry matter the hypothesis is

$$H_O: \lambda_1 - \lambda_3 = 0$$
$$H_A: \lambda_1 - \lambda_3 > 0$$

With a t-statistic result of 2.779.

Vegetative stage contributes to production more than maturation stage because 2.779 > 2.235 for a 1 percent level of significance. Thus we reject the null hypothesis and accept the alternate hypothesis which states $H_A: \lambda_1 - \lambda_3 > 0$. Similarly, the pollination stage contributes more than the maturation stage but at a lower level of significance. From standard statistical tables at $\alpha = 5$ percent, one tailed test, the value is 1.645. Since the computed value for $H_O: \lambda_2 - \lambda_3 = 0$ test is 1.706 which is greater than 1.645, we reject the null hypothesis and accept the alternative hypothesis which states $H_A: \lambda_2 - \lambda_3 > 0$. Statistically, vegetative and pollination stages differ very little. Thus we accept the null hypothesis $H_O: \lambda_1 - \lambda_2 = 0$.

In general, the statistical analysis indicates that pollination is the most important stage at which to apply irrigation if grain yield is to be optimized while for dry matter, vegetative stage is the most important.

Production Function Equations

The result of the regression analysis is the base from which stage importance will be determined using a composite hypothesis. A composite test implies testing within a range as opposed to the simple test which tests only a point.

The composite hypotheses are stated for grain as

$$H_O^{G2,1,3}: \lambda_2 > \lambda_1 > \lambda_3$$
$$H_A^{G2,1,3}: \lambda_2 < \lambda_1 < \lambda_3$$

and for dry matter as

$$H_O^{D1,2,3}: \lambda_1 > \lambda_2 > \lambda_3$$
$$H_O^{D1,2,3}: \lambda_1 < \lambda_2 < \lambda_3$$

The $H_O^{G2,1,3}$ implies that stage two is more important than stage one, and stage one is more important than stage three for grain yield. The regression results for grain show statistically that pollination is the most important stage when compared to vegetative and maturation stages. For dry matter, vegetative stage is shown to be the most important stage when compared to maturation and pollination stages.

For the test, comparing all three growth stages, all 493 observations were used for the regressions. Using the following simplification

$$\log Y - \log Y_p \equiv \log R_y;$$

$$\left[\frac{ET_A}{ET_p}\right]_v \equiv R_v;$$

$$\left[\frac{ET_A}{ET_p}\right]_p \equiv R_p;$$

$$\left[\frac{ET_A}{ET_p}\right]_m \equiv R_m\ ,$$

equation (1) was respecified in log linear form as

$$14)\quad \log R_y = \log C + \lambda_1 \log R_v + \lambda_2 \log R_p + \lambda_3 \log R_m$$

The concern is with

$$H_O: \lambda_2 > \lambda_1 > \lambda_3$$
$$H_A: \lambda_2 < \lambda_1 < \lambda_3$$ for grain, and for dry matter

$$H_O: \lambda_1 > \lambda_2 > \lambda_3$$
$$H_A: \lambda_1 < \lambda_2 < \lambda_3$$

Using the following substitution

$$\lambda_1 = \lambda_3 + d$$

and $\lambda_2 = \lambda_1 + e$

or $\lambda_2 = \lambda_3 + d + e$ solving the basic equation, and making relevant substitutions, the result is

$$15)\quad \log R_y = \log C + d(\log R_v + \log R_p) + e \log R_p + \lambda_3 (\log R_v + \log R_p + \log R_m)$$

The regression result confirms the higher contribution λ_2 makes. The result of the composite test, which tests a hypothesis within a range as contrasted with a simple test which tests only a point, is d = 0.017, e = 0.227 and λ_3 = 0.330 for grain while for dry matter d = 0.124, e = −0.052, and λ_3 = 0.269. In equation form

16)

$$Y_{grain} = 0.970 \quad R_v^{0.017} \quad R_p^{0.227} \quad R_m^{0.330}$$

$$(-1.646) \; (0.235) \; (2.616) \; (9.926)^*$$

$$R^2 = 54\%$$

where Y_{grain} = Grain Yield

17)

$$Y_{dry\ matter} =$$

$$0.950 \quad R_v^{0.124} \quad R_p^{-0.052} \quad R_m^{0.269}$$

$$(-3.879) \; (2.559) \; (-0.903) \; (0.123)^*$$

$$R^2 = 62\%$$

$Y_{dry\ matter}$ = dry matter yield

For grain yield, d, e > 0 while for dry matter e < 0. Considering grain yield, for $\propto$ = 1 percent for one tailed test, the importance of the pollination stage is further shown when compared to vegetative and maturation stages. Statistically, no statement can be made with regard to the importance of the stage when vegetative and maturation stages are compared. For dry matter, the importance of vegetative stage is shown when compared to other stages of growth, but statistically one cannot say that pollination stage is more important than vegetative stage for e < 0.

The stronger hypothesis only statistically confirms with respect to grain yield the importance of pollination stage as compared to vegetative and maturation stages. And with respect to dry matter yield, it comfirms the importance of vegetative stage as compared to the other two stages.

For a composite test the substitution made is

$$\lambda_1 = \lambda_3 + d$$

and

$$\lambda_2 = \lambda_1 + e$$

Using the composite test transformation, exponents of our production function can be derived. The d and e values are found from composite test regression results to be 0.017 and 0.227, respectively. Hence λ_1 = 0.330 + 0.017 = 0.347 and λ_2 = 0.347 + 0.227 = 0.574.

Substituting λ_1, λ_2, and λ_3 we have a production function as shown in equations (18) and (19). We construct the following two equations using regression results.

18)

$$Y_{grain} =$$

$$0.970 \quad R_v^{0.347} \quad R_p^{0.574} \quad R_m^{0.330}$$

$$(-1.646) \; (5.840) \; (12.038) \; (9.926)^*$$

$$R^2 = 54\%$$

19)

$$Y_{dry\ matter} =$$

$$0.950 \quad R_v^{0.394} \quad R_p^{0.343} \quad R_m^{0.269}$$

$$(-3.897) \; (10.034) \; (10.870) \; (12.268)^*$$

$$R^2 = 62\%$$

Discussion of Results

All results confirm the importance of pollination stage for grain yield and vegetative stage for dry matter yield. This result indicates where irrigation management emphasis should be placed. Physical conditions are such that producers can only approach potential yield. Thus, rate of production due to increase in water applied is a product of a proportionality variable and potential yield.

$$R_i = k\,Y_p \qquad (20)$$

where R_i = Increase in water applied; k = Proportionality variable depending on ET, a fraction; and Y_p = Potential yield.

For optimal solution during the growing season, amount of ET_A was highest during pollination stage. Thus emphasis should be on pollination stage.

Next we show the returns to scale associated with our production functions. To ascertain if $\sum_{i=1}^{3} \lambda_i \gtreqless 1$ is increasing, decreasing or constant returns to scale we need to find $\sum_{i=1}^{3} \lambda_i$ significant or not significant t-statistically. To do this, the following hypothesis is required — H_O: $\sum\lambda_i = 1$ and H_A: $\sum\lambda_i > 1$.

Testing this linear combination of the λ coefficients would follow

$$\frac{W^1\lambda - W_O}{\sqrt{W^1S^2(X^1X)^{-1}W}} \qquad (21)$$

where W = fixed weight (unit) vector; λ = Least Square Estimator; W_O = Unity (1); and $S_2(X^1X)^{-1}$ = Estimated coefficient of variance-covariance matrix.

Solving, one gets

$$\frac{1.451 - 1}{\sqrt{3.472\ X10^{-3}}} = 7.598$$

Following the t-statistic form of analysis employed earlier, we find $7.598 > 2.326$, which is the book value for t at 1 percent level of significance. This result calls for a rejection of the null hypothesis stated as H_O: $\sum\lambda_i = 1$. Consequently for grain yield, we accept the alternate hypothesis stated as H_A: $\sum\lambda_i > 1$. Therefore we conclude that the production function stated exhibits increasing returns to scale. Furthermore, the major contributor to increasing returns is water applied during the pollination stage when considering grain yield. Since $\sum_{i=1}^{3} \lambda_i > 1$, as the amount of water applied is increased, its utilization also increases. The yield starts by increasing at an increasing rate. With further increases in water application, the rate of increase declines. From our analysis, grain yield increase will come through pollination stage relatively more when compared to the other two stages.

The same procedure can be used for dry matter. It too has $\sum_{i=1}^{3} \lambda_i > 1$. But, the t value is 0.168. Since $0.168 < 2.326$ we cannot reject the null hypothesis H_O: $\sum_{i=1}^{3} \lambda_1 = 1$. Thus for dry matter, the production function may yield constant returns to scale.

Management Recommendations

Irrigation practices used by farmers generally follow "rule of thumb" decision making for frequency and amount of water applied. Many follow the practice of running the water to the end of the row every two weeks without concern for infiltration rates, lengths of row, or other determinants of the amount of water applied. Such practices could hardly be expected to achieve optimal water application practices in amounts or timing.

For grain production we found that the optimal allocation of water would give the pollination stage highest ET_A value. ET_A was 0.299 of total ET_A for vegetative stage, 0.446 of total ET_A for pollination stage and 0.255 of total ET_A for maturation stage. (This means 30 percent, 45 percent, and 25 percent of the water applied in the respective stages.) Thus, the pollination stage needs 19 percent more ET_A than maturation stage and 16 percent more ET_A than vegetative stage.

The relevant question for management is how to optimally allocate the increase ET_A during pollination stage. A transfer of units of water from one stage to another is an attempt to change the unequal marginal physical product of water during the three stages. By transferring units of ET_A (input) from the less efficient stages to those established as the most efficient, a farmer can approach an optimal allocation of water.

Possible alternatives for a farmer to employ are:

1) Varying irrigation frequency is the key to obtaining optimal yield. For example, a farmer should vary the number of days between irrigations so as to get the 16 and 19 percent differential between pollination and vegetative and maturation stages, respectively.

2) Strict adherence to an irrigation schedule that favors higher water application during vegetation stage will tend to increase dry matter yield. The crop should be irrigated so as to get the 5 and 7 percent differential between vegetative, pollination and maturation stages respectively.

3) The schedule in terms of amount of water and irrigation frequency should allow for important characteristics such as soil, land slope, and so on.

4) On some types of soils, it may be better to vary duration of irrigation while maintaining the same number of days between irrigations.

5) Regardless of irrigation frequency, irrigating above field capacity at any given irrigation would waste water. If the irrigation schedules calls for irrigating when moisture content is down to a desired field capacity fraction, irrigation should not be delayed.

6) Transferring irrigation water to another stage at a particular time can save water and labor cost. Such management would increase yield if the water was shifted from a lower utilization stage to a higher one. Eliminating waste will reduce costs.

Conclusion

If corn is being grown for grain, the crop should not be stressed during pollination stage. To promote silage yield, stress should be avoided during vegetative stage.

We cannot categorically assign stage importance between vegetative and maturation stages for grain; however, the null hypothesis stating H_O: $\lambda_1 > \lambda_2 > \lambda_3$ indicates that regression analysis using $\lambda_1 = \lambda_3 + d$ gave $d > O$ which implies that λ_3 cannot be more important than λ_1 because $\lambda_1 > \lambda_3$.

Similarly for dry matter H_O: $\lambda_1 > \lambda_2 > \lambda_3$. Regression analysis using $\lambda_1 = \lambda_2 + d$ gave $d > O$, which implies that λ_2 cannot be more important than stage 1. To put it another way, analysis using $\lambda_2 = \lambda_1 + d$ would yield $d > O$ which means that λ_3 cannot be more important than λ_1.

The increasing returns indicate that, as the amount of water applied increases, there would be increasing utilization of water up to a point. At the moment, it cannot be said what that point is. For a given soil, field capacity is reached only after a certain quantity of water has been applied at a suitable intake rate. It would be wasteful to exceed field capacity.

One way to enhance yields is to be sure the plant does not go through stress. This can be done by increasing the irrigation frequency, reducing the time period between any two consecutive irrigation, or by increasing the amount per irrigation. This is a practical management option to be decided on the basis of relative costs and physical factors.

Care must be taken during vegetative and pollination stages. The data show that the level of water applied at a particular stage of growth can affect yield. More research is necessary to ascertain precisely which stage of growth follows in importance after pollination stage for grain and vegetative stage for dry matter.

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