Texas Field Crops: Estimation with Curvature

C. Richard Shumway, William P. Alexander, and Hovav Talpaz

Some implications of theory are easily maintained in econometric estimation, but computational costs of maintaining curvature properties (sufficient for existence of an optimal solution) have often proved prohibitive. They also have been violated frequently by unrestricted econometric estimates. A computationally manageable procedure for maintaining and testing curvature is used here to obtain estimates of product supplies and input demands for Texas field crops consistent with the theory of the competitive industry. The curvature properties are tested along with several technology restrictions.

Key words: crops, curvature, demand, duality, supply, technology.

Product supply and input demand functions are reduced-form equations which are based on an underlying behavioral model. In addition to providing economic information, they permit tests about the economically relevant boundary of the technology to be conducted subject to the behavioral assumptions. However, for the estimated reduced-form equations to be consistent, even locally, with the behavioral model from which they were explicitly or implicitly derived, several restrictions implied by that model must be satisfied.

For industries facing perfectly elastic product demands and input supplies and which are comprised of profit-maximizing, price-taking firms, the dual profit function is monotonic, linear homogeneous, and convex in prices. For a twice-continuously differentiable aggregate production function with weak regularity properties (Lau 1978a), an optimal economic solution exists, and the Hessian of the dual profit function is positive semidefinite in prices and symmetric. The product supply and input demand equations are the first derivatives of the profit function. They must be homogeneous of degree zero in prices and have symmetric partial derivatives across equations that form a positive semidefinite Hessian.

Whether the parameters of the profit function are estimated directly or are derived from estimated systems of product supply and input demand equations, the properties of homogeneity and symmetry are often relatively easy to impose or to test. This can be accomplished with linear restrictions and/or normalization for many flexible functional forms. Monotonicity over the data period typically is satisfied by empirical estimates without additional restrictions. It is also possible to determine whether the estimated parameters yield a positive semidefinite Hessian, but unconstrained estimates often do not satisfy this property.

To test the statistical significance of a theory by means of likelihood ratios, all implications of the theory must be imposed in one set of estimates. Since curvature properties frequently are not automatically satisfied, they must be imposed in estimation.[1] But, this requires nonlinear inequality restrictions and substantially increases the computational burden.

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[1] Curvature properties also must be satisfied for estimated cost, production, utility, or indirect utility functions to imply that a solution exists which achieves the presumed behavioral objective.

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Even for relatively small systems of equations, computational costs of maintaining curvature properties in least squares estimation have been prohibitive for many research budgets. As a result, few analysts (e.g., Behrman and Murty; Berndt and Wood; Jorgenson and Fraumeni) have maintained curvature. Consequently, many econometric models with specifications motivated by profit-maximizing or cost-minimizing behavioral assumptions have yielded empirical estimates inconsistent with the specification assumptions. See, for example, recent studies by Antle; Babin, Willis, and Allen; Dixon, Garcia, and Mjelde; Lopez; McKay, Lawrence, and Vlastuin; Ray; Rossi; Shumway. The problem has been especially acute for multiple-product production problems.

The objectives of this article are (*a*) to use a computationally manageable procedure for curvature maintenance in order to obtain estimates of Texas field crop supply and input demand equations fully consistent with the theory of the competitive industry, (*b*) to compare policy-relevant implications of these estimates to those without curvature maintenance, and (*c*) to test for satisfaction of two important technology hypotheses that are frequently maintained in empirical application—short-run nonjoint production and Hicks-neutral technical change (Lau 1978a).

Empirical Problem

In pursuing these objectives, curvature is maintained on a dual model which was previously estimated by Shumway without curvature being maintained or satisfied. The same data are used. As before, the state is modeled as a competitive industry represented by an aggregate, twice-continuously differentiable, multiple-product production function and facing perfectly elastic product demands for six outputs (hay, rice, wheat, sorghum, cotton, and corn); perfectly elastic input supplies for three inputs (machinery operating inputs, fertilizer, and hired labor); and perfectly inelastic input supplies for two additional inputs (family labor and land).[2]

The dual model was specified by Shumway as a normalized quadratic restricted profit function, which may be written compactly as:

$$\tilde{\pi}^* = b_o + C\tilde{P} + .5\tilde{P}'D\tilde{P}, \qquad (1)$$

where $\tilde{\pi}^*$ is profit divided by price of netput 1; $\tilde{P} = [\tilde{p}_2, \ldots, \tilde{p}_m, x_{m+1}, \ldots, x_n]$ is the vector of normalized prices ($\tilde{p}_i = p_i/p_1$) and quantities of fixed inputs and other exogenous variables ($x_{m+1}, \ldots, x_n$); p_i is the price of netput i; the scalar b_o, the vector C, and the symmetric matrix D represent parameters. Following the netput convention (Varian, p. 8), output quantities are positively measured and variable input quantities are negatively measured.

The system of $m - 1$ linear product supply and input demand equations obtained as partial derivatives of (1) in $\tilde{p}_2, \ldots, \tilde{p}_m$,

$$x_{it} = c_i + \sum_{j=2}^{m} d_{ij}\tilde{p}_{jt} + \sum_{j=m+1}^{n} d_{ij}x_{jt} + e_{it}, \qquad (2)$$

$$i = 2, \ldots, m, \quad t = 1, \ldots, T,$$

(where e is the error term and t is time), were estimated by Shumway as a seemingly unrelated regression system by Zellner's generalized least squares (GLS).[3] Eight equations were estimated, one each for the six crop supplies and for fertilizer and hired labor demands. Other exogenous variables were the fixed inputs (positively measured), time as a proxy for technical change, an index of actual divided by expected crop yield to represent the *ex post*

[2] The assumed existence of an aggregate state-level production function bypasses (or begs) the issue of what specific assumptions justify the aggregation of firm-level production functions to an industry-level production function. A sufficient aggregation condition for competitive firms is that each has the same production function. However, all that is assumed here is that producers in the state act collectively as though they were a single firm using total input quantities to produce the total quantities of outputs reflected in the state data. Further, because the higher of lagged annual market price or current effective support rate was used to represent expected commodity price, no simultaneity was considered in the estimation of output demand and supply relationships. Some simultaneity likely exists in state-level variable input supply and demand relationships, but it is expected to be relatively small. The markets for all three variable inputs are clearly national in scope. Hired labor is the most likely input to be subject to unique local or state markets. Even at the national level, the effect on implied output supply elasticities of upward-sloping, variable-input supply curves has been found to be modest when land and family labor are treated as fixed inputs (Shumway, Saez, and Gottret). Thus, the empirical results reported in this article are not expected to be affected seriously by this assumption.

[3] A system of m derivative equations (including the profit function or the quadratic demand equation for the numeraire input, x_1) could have been estimated as long as sufficient data were available. Unlike the translog, the covariance matrix of the full set of derivative equations is not identically singular for the normalized-quadratic profit function. Neither the profit function nor the numeraire equation was included in this estimation system because each greatly exacerbated collinearity problems among the independent variables.

effect of weather (Stallings), and diversion payments to represent government commodity programs. The last two variables were crop specific and appeared only in the supply equation of the respective crop. The data consisted of an annual time series for the years 1957–79. Homogeneity in prices was maintained by normalization by the price of machinery operating inputs, and symmetry was maintained by linear restrictions.[4] GLS estimates obtained for this system of equations without imposing the convexity restrictions are reported in Shumway (p. 755).

Maintaining Convexity

Several procedures for maintaining curvature properties in econometric estimation have been proposed (Lau 1978b; Hartley, Hocking, and Cooke; Hazilla and Kopp; Gallant and Golub; Talpaz, Shumway, and Alexander). The problem has been handled most often by using the Cholesky factorization to reparameterize the D matrix (Lau 1978b). This procedure and the eigenvalue decomposition procedure of Talpaz, Shumway, and Alexander were applied to our empirical problem using several optimization algorithms.

Using the most efficient of three nonlinear optimization algorithms, the Cholesky method dominated the eigenvalue procedure both in sum of squared error (SSE) and CPU time required for convergence with each of three data sets.[5] The Cholesky method has since been applied to several additional output supply–input demand systems (all with at least eight equations and often including a quadratic numeraire equation). It has converged in all cases and was the method used in this study. Utilization of this or other methods with suitable software now makes maintenance of curvature properties a manageable task in much empirical research.[6] Details of the Cholesky procedure used here are reported in Talpaz, Alexander, and Shumway.

Implementation—System Covariance Matrix

Coordinated computer programs were written in PROC MATRIX of SAS and in FORTRAN to permit estimation via the Cholesky factorization of complete systems of equations. Seemingly unrelated regression estimates of the system of equations (2) subject to the cross-equation symmetry restrictions were estimated using SAS. These estimates were then used as a starting point in a FORTRAN program. The purpose of the latter program was to perform the Cholesky factorization on the matrix of price parameters and, by utilizing MINOS (version 5.0), to obtain nonlinear least squares estimates of the system subject to the symmetry and convexity restrictions. Alternatively, the system could have included the linear supply and demand equations (2) along with either (1) or the quadratic equation for the numeraire netput (x_1) derived from (1) (e.g., Shumway and Alexander). Or, it could have included the translog profit function along with $m - 1$ share equations with curvature maintained at selected points.[7] In either case, with time-series data, there are often too few ob-

[4] The normalizing price is for the numeraire input, x_1; its quadratic demand equation was omitted from the system of estimation equations.

[5] Since both procedures are reparameterizations of the problem, the same estimates for the parameters in the original parameter space are theoretically attainable. The differences experienced are due to the necessity of using numerical solution methods. The two procedures were first programmed in the SAS programming language, PROC MATRIX, using the Davidon-Fletcher-Powell variable metric optimization algorithm (Fletcher and Powell) modified with the golden section line search (Talpaz). The eigenvalue decomposition procedure required fewer iterations for convergence on a three-equation sample problem and gave convex parameter estimates with lower SSE than the Cholesky factorization procedure (Talpaz, Shumway, and Alexander, p. 663). Using the reduced-gradient nonlinear programming procedure in the algorithm code, MINOS version 4.0 (Murtagh and Saunders 1978, 1980), the Cholesky procedure converged more rapidly with lower SSE on two orthogonal sample data sets (five and eight equations) than the eigenvalue procedure. However, it failed to converge on the Texas field crop data. MINOS version 5.0 (Murtagh and Saunders 1983) was the algorithm code used here. It converged on the Texas field crop data and required 21–97% less CPU time than the eigenvalue procedure for each of the three data sets.

[6] On the Texas A&M University Amdahl V-8 mainframe computer, CPU time required to maintain curvature on this system of equations was 88 seconds. Since its Hessian matrix is a matrix of constants, the normalized quadratic is the simplest functional form to which the Cholesky decomposition can be applied. The reparameterization required by this method for other common forms of the profit function, such as the translog, is also straightforward although somewhat more involved and renders curvature restrictions only as local properties. With other more complicated functional forms, the method may be impossible to apply.

[7] Diewert and Wales note that a number of recent curvature-imposing applications maintain sufficient conditions for curvature, sometimes reduce the flexibility of the functional form, and generally maintain curvature only locally. The restrictions maintained in this study are both necessary and sufficient, do not reduce flexibility, and maintain curvature globally for the normalized quadratic functional form. They would maintain curvature locally for the translog or for any of several other functional forms.

servations to estimate the quadratic numeraire or translog profit equation by OLS.

Given the validity of the restrictions, two procedures give GLS estimators with the same asymptotic properties for our system of seemingly unrelated regression equations: (*a*) estimate each equation by OLS, compute the covariance matrix across equations, use it to transform the observation matrix, and reestimate with symmetry restrictions maintained by GLS; or (*b*) stack the system of equations and estimate the entire system by OLS with symmetry restrictions maintained, compute this covariance matrix across equations, transform the observation matrix, and obtain GLS estimates.[8] The first procedure was used by Shumway. The second was used here both to derive the covariance matrix and to obtain a starting point for the nonlinear least squares (NLS) estimation which maintains curvature properties. Although asymptotic properties are the same (assuming the symmetry restrictions are valid), the two sets of GLS estimates differ because different system covariance matrices are used to transform the observation matrix. Neither gives results for our data set that are consistent with a convex restricted profit function.

The NLS estimation requires iterative search to obtain parameter estimates. Iterating also on the system covariance matrix, as recommended when obtaining GLS estimates of a translog system, is not advisable here for several reasons. First, it would greatly increase computational burden. Second, it would not yield estimates for the normalized quadratic profit function (unlike the translog) invariant to choice of numeraire; this is because (*a*) the covariance matrix of the full system of the normalized quadratic derivative equations is not singular, and (*b*) changing the numeraire changes the entire specification since the numeraire equation is quadratic while the rest of the derivative equations are linear.[9] Third, since noniterative GLS gives the same asymptotic distribution as the maximum likelihood estimator, which is obtained by iterating, the only benefit from iterating would be a (possible) gain in efficiency. Hence, the system covariance matrix computed from the OLS estimates of the stacked system was used to transform the observation matrix throughout both the GLS and NLS estimations.[10]

With regard to properties of the GLS and NLS estimators, it should be noted that the dimension of the parameter space is not reduced by maintaining the inequality convexity constraints. When the null hypothesis that the profit function is convex is true, the unconstrained GLS estimator is consistent and asymptotically equivalent to the constrained NLS estimator. That is, the probability that the convexity constraint is binding tends toward zero with increasing sample size.

Estimation Results

Although the own-price supply and demand parameters estimated by Shumway all had the expected signs, the estimated profit function was not convex since the matrix of price parameters was not positive definite. Neither was the matrix of GLS price parameters estimated by stacking the equations in the first (OLS) step. See table 1 for price elasticities from both sets of estimates. Because the covariance matrices used to transform the observation matrix differ between these two methods, the GLS estimates and corresponding elasticities also differ. The extent to which parameter estimates can differ for asymptotically equivalent estimators is readily apparent from these elasticities. For example, eight of our own-price elasticities have the expected sign as compared to all nine in Shumway's estimates. Own-price elasticities for machinery operating inputs, rice, and corn are, respectively, .63, 1.27, and 1.24 from our estimates and −.37, .72, and .07 from Shumway's. Some cross-price elasticities also had different signs and others differed greatly in magnitude. Parameters at least 1.96 times as large as standard errors include 39% of the price parameters and 36% of all parameters in

[8] With as few observations as we have, small sample properties also are important. Unfortunately, only the asymptotic properties of these estimators are known.

[9] The machinery operating inputs category was chosen as the numeraire because quantity data for this input were less reliable than for other inputs or outputs. The empirical results are not independent of this choice. Even if the numeraire equation had been included in the system of estimation equations, the results still would have been dependent on the choice of numeraire. Gottret's examination of the impact of numeraire choice on U.S. and regional agricultural production estimates revealed no sensitivity of several theoretical and technical hypothesis test conclusions but considerable sensitivity of own-price output supply and input demand elasticities.

[10] Theil and Clements (p. 116) recently have cautioned that, relative to a known true covariance matrix, symmetry-constrained estimation of the covariance matrix can yield final parameter estimates with "impaired efficiency" and approximate standard errors that "give an overly optimistic picture of their precision."

Table 1. Product Supply and Input Demand Elasticities, Two Generalized Least Squares (GLS) Estimates, 1979[a]

	Elasticity with Respect to the Price of								
Output or Input	Machinery Operating Inputs	Fertilizer	Hired Labor	Hay	Rice	Wheat	Sorghum	Cotton	Corn
Machinery Operating Inputs	−.37	−.15	−.01	.07	−.17	.75	−.09	.26	−.30
	.63	−.14	.24	.07	−.32	.04	−.58	−.09	.15
Fertilizer	−.15	−.70	.20	.13	.38	.05	.05	.03	.01
	−.14	−.83	.14	.23	.51	.01	.09	.03	−.03
Hired Labor	−.01	.39	−.43	.11	.21	.03	−.84	.19	.34
	.44	.26	−.38	−.22	−.06	.41	−.75	.11	.19
Hay	−.07	−.13	−.06	.10	.26	.01	−.01	.06	−.16
	−.06	−.22	.11	.28	.55	−.05	−.10	−.002	−.52
Rice	.24	−.55	−.16	.39	.72	.16	.36	−.28	−.88
	.45	−.74	.05	.84	1.27	−.05	.31	−.82	−1.32
Wheat	−.57	−.04	−.01	.01	.09	.43	−.18	.01	.26
	−.03	−.01	−.17	−.04	−.03	.30	−.05	−.14	.15
Sorghum	.05	−.03	.27	−.01	.15	−.14	.62	−.74	−.17
	.34	−.05	.24	−.06	.13	−.04	.74	−.90	−.39
Cotton	−.05	−.01	−.02	.01	−.04	.003	−.26	.25	.11
	.02	−.01	−.01	−.0004	−.12	−.04	−.31	.29	.17
Corn	.29	−.01	−.18	−.16	−.59	.33	−.27	.52	.07
	−.14	.03	−.10	−.52	−.88	.19	−.63	.80	1.24

[a] Top elasticities are GLS with first-stage covariance matrix computed from unrestricted OLS estimates of eight equations (Shumway, p. 756, corrected for column- and row-label error). First-stage covariance matrix of bottom GLS elasticities was computed from OLS estimates of the stacked system of equations with symmetry maintained.

our estimates compared to 50% and 53%, respectively, of Shumway's.

Collinearity among the independent variables in this eight-equation system is fairly strong. A condition index of 347 was computed for the centered and scaled stacked matrix of independent variables.[11]

The hypothesis of no first-order serial correlation in our GLS estimates was tested and not rejected at the 5% level for any equation. Durbin-Watson statistics ranged from 1.72 for rice supply to 2.61 for sorghum supply, all within the inconclusive range.

The NLS estimates (with convexity maintained) are reported in table 2. These estimates globally maintain three properties of the restricted profit function for a state industry that behaves like a competitive firm with a twice-continuously differentiable aggregate production function—homogeneity, symmetry, and convexity. The fourth property, monotonicity, was checked at all data points and was not violated at any point for any equation. Thus, these estimates are fully consistent with the competitive theory for a price-taking, state-level industry.

Consistency of the curvature properties with the data was tested by determining whether the estimated nonconvex parameter estimates fell within a 95% confidence ellipsoid around the convex parameter estimates. The logic for the test statistic used here, which is distributed approximately as an F, is developed in the appendix. The observed F was 1.04. Thus, the curvature properties are not rejected by these data at the 5% level of significance (critical value of $F^{.05}_{78,106} = 1.43$).

Nearly as many NLS parameter estimates (33%) as our GLS estimates (36%) were asymptotically significant at the 5% level.[12] However, only a little more than three-fourths as many NLS as GLS parameters on the price

[11] While this condition index suggests fairly strong collinearity (Belsley, Kuh, and Welsch; Hocking and Pendleton), it is lower than frequently observed in such systems (e.g., Shumway and Alexander).

[12] The absolute magnitudes of the estimated parameters reported in this article are not directly comparable to those reported by Shumway because the data have been scaled here to promote convergence in the nonlinear estimation.

Table 2. Nonlinear Least Squares Estimates of Texas Field Crop Supplies and Input Demands

Parameter	Estimate	Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
c_2	−.0744	d_{46}	.0000460	d_{69}	.000398	d_{212}	−.0134
	(.0613)		(.0000513)		(.001509)		(.0025)
c_3	.00879	d_{56}	−.000177	d_{79}	−.00318	d_{312}	−.000103
	(.00938)		(.000223)		(.00397)		(.000335)
c_4	−.00116	d_{66}	.00205	d_{89}	.0126	d_{412}	−.0000044
	(.00181)		(.00077)		(.0090)		(.0000649)
c_5	−.0157	d_{27}	−.0188	d_{99}	.00181	d_{512}	−.0000322
	(.0105)		(.0080)		(.00684)		(.0003454)
c_6	−.0962	d_{37}	.0000537	d_{210}	−1.28	d_{612}	−.000468
	(.0181)		(.0007277)		(1.85)		(.000669)
c_7	−.0459	d_{47}	.0000245	d_{310}	−.791	d_{712}	−.000165
	(.0524)		(.0001139)		(.248)		(.001311)
c_8	−.854	d_{57}	.000468	d_{410}	−.031	d_{812}	.0246
	(.172)		(.000459)		(.049)		(.0068)
c_9	−.170	d_{67}	−.000397	d_{510}	−.063	d_{912}	.00341
	(.074)		(.001190)		(.264)		(.00250)
d_{22}	.577	d_{77}	.0116	d_{610}	−1.24	d_{413}	.0000151
	(.062)		(.0041)		(.52)		(.0000064)
d_{23}	−.00194	d_{28}	.0178	d_{710}	.194	d_{513}	.000116
	(.00207)		(.0151)		(.915)		(.000032)
d_{33}	.0000075	d_{38}	−.0000577	d_{810}	4.10	d_{613}	.000416
	(.0002345)		(.0018742)		(5.20)		(.000044)
d_{24}	−.00112	d_{48}	.0000050	d_{910}	1.36	d_{713}	.00102
	(.00029)		(.0003704)		(1.69)		(.00015)
d_{34}	.0000041	d_{58}	−.000566	d_{211}	1.33	d_{813}	.00756
	(.0000310)		(.001437)		(9.17)		(.00049)
d_{44}	.0000033	d_{68}	.00177	d_{311}	−.0928	d_{913}	.000122
	(.0000073)		(.00321)		(1.4278)		(.000144)
d_{25}	−.0132	d_{78}	−.0548	d_{411}	.195	d_{614}	−.0000045
	(.0013)		(.0064)		(.265)		(.0000020)
d_{35}	.0000443	d_{88}	.271	d_{511}	2.09	d_{714}	−.0000155
	(.0001230)		(.037)		(1.38)		(.0000128)
d_{45}	.0000216	d_{29}	.0255	d_{611}	13.3	d_{814}	−.00157
	(.0000247)		(.0095)		(2.9)		(.00033)
d_{55}	.000319	d_{39}	−.0000847	d_{711}	10.3	d_{914}	.0000315
	(.000126)		(.0008334)		(5.0)		(.0000320)
d_{26}	−.000168	d_{49}	−.0000408	d_{811}	21.1		
	(.003256)		(.0001587)		(24.6)		
d_{36}	.0000023	d_{59}	−.000620	d_{911}	11.1		
	(.0003255)		(.000671)		(7.3)		

Note: Standard errors are in parentheses. See equation (2) for parameter identification. Parameter subscripts identify variables: the first identifies variable i and the second and third identify variable j. Variable numbers: 1 is machinery operating inputs, 2 is fertilizer, 3 is hired labor, 4 is hay, 5 is rice, 6 is wheat, 7 is sorghum, 8 is cotton, 9 is corn, 10 is family labor, 11 is total acres planted to the six crops, 12 is time, 13 is the crop-specific weather proxy, and 14 is crop-specific effective diversion payments. Prices p_2 through p_9 are normalized by p_1.

variables were significant at this level. Weighted diversion payments were significant in the NLS supply equations for wheat and cotton but only for cotton in the GLS equations. Quantity of family labor was a significant variable in the NLS equations for hired labor demand and wheat supply but only for hired labor demand in the GLS equations. In both sets of estimates, total acreage planted was significant for wheat and sorghum supplies, time was significant for fertilizer demand and cotton supply, and the weather proxy variable was significant for all output supplies except corn. Five own-price parameters were significant in the NLS equations and four in the GLS equations. Only five cross-price parameters were significant in the NLS equations compared to 10 in the GLS equations.

Elasticities

Although convexity was not rejected, maintenance of the curvature restrictions can substantially alter policy-relevant implications. To

Table 3. Product Supply and Input Demand Elasticities, Convexity Maintained, 1979

	Elasticity with Respect to the Price of								
Output or Input	Machinery Operating Inputs	Fertilizer	Hired Labor	Hay	Rice	Wheat	Sorghum	Cotton	Corn
Machinery Operating Inputs	−.93	−.003	.001	.19	−.12	.46	−.60	.79	.22
Fertilizer	−.003	−.85	.06	.32	.65	.003	.22	−.05	−.34
Hired Labor	.001	.11	−.01	−.04	−.08	−.001	−.02	.01	.04
Hay	−.18	−.31	.02	.18	.20	.13	.05	.003	−.10
Rice	.17	−.95	.06	.30	.76	−.13	.26	−.08	−.39
Wheat	−.35	−.002	.001	.11	−.07	.27	−.04	.04	.04
Sorghum	.36	−.13	.01	.03	.11	−.03	.62	−.77	−.20
Cotton	−.16	.01	−.001	.001	−.01	.01	−.27	.35	.07
Corn	−.21	.33	−.02	−.10	−.26	.05	−.31	.33	.20

illustrate, price elasticities from the NLS estimates are reported in table 3. All own-price elasticities have the expected sign (since convexity requires it), but most are lower in absolute value than the elasticities from the GLS estimates. This latter finding is opposite to the result obtained by Diewert and Wales for U.S. manufacturing using the translog functional form; this difference partly may be due to our imposition of curvature restrictions which are both necessary and sufficient while Diewert and Wales' restrictions are only sufficient. The only own-price elasticity that changed sign is for the numeraire input, machinery operating inputs. The parameters of the numeraire equation were not estimated separately but were derived under the homogeneity and symmetry restrictions.

One-fourth of the cross-price elasticities changed sign. They also tended to be lower in absolute magnitude with convexity maintained—50 were not greater than .2 (compared to 44 in the GLS estimates), 16 were between .2 and .4 (11 GLS), and only six were above .4 (17 GLS).

Technology Tests

Shumway conducted several indirect tests (some approximate) on the structure of the multiple-product production technology, including nonjointness, homotheticity, and separability. The test for short-run nonjointness in all outputs was repeated here along with tests for Hicks-neutral technical change.

The asymptotic distributions of the unconstrained test statistics are unaffected by the inequality nature of the curvature restrictions, when valid, since they reduce the allowable region for the estimates but not the dimensions of the region. They do not alter the minimum variance bounds of the parameter estimates. Thus, it is unnecessary with large samples to repeat the structural tests with convexity maintained (Jorgenson and Lau, pp. 71–72; Rothenberg, pp. 49–58). However, since the parameter estimates do change for our small sample when convexity is imposed, the tests were conducted with both sets of estimates in order to provide a comparison of actual test statistics with a limited number of observations (table 4).

Table 4. Nonjointness and Technical Change Tests

Test	Degrees of Freedom	Chi-Square Statistic[a] GLS	NLS
Nonjointness in all outputs	15	162.8	129.0
Global Hicks-Neutral Technical Change:			
In Fertilizer and Hired Labor	2	50.4	31.0
In all outputs	6	17.2	20.6
Local Hicks-Neutral Technical Change:			
In Fertilizer and Hired Labor	1	21.3	26.3
In all outputs	5	18.1	16.9

[a] For nonjointness and global technical change tests, critical values of the Wald Chi-Square statistic at the .01 level are 9.2, 16.8, and 30.6 with two, six, and 15 degrees of freedom, respectively. For local technical change tests, critical values at the .01/23 level are 12.0 and 22.0 with one and five degrees of freedom, respectively.

Consistent with the findings of Shumway, short-run nonjointness in all outputs was soundly rejected at the 1% level by both tests. We cannot conclude that the supply of each output is independent of the price of all other outputs (i.e., $d_{ij} = 0$, for all $i \neq j$, i and j outputs) (Lau 1978a, p. 183).

Technical change is indirectly Hicks neutral in the variable inputs of fertilizer and hired labor if all demand ratios are independent of time (Lau 1978a, p. 202), i.e., if

$$(3) \qquad d_{212}x_3 - d_{312}x_2 = 0.$$

By the same condition on output supplies, technical change is indirectly Hicks neutral in outputs if

$$(4) \quad d_{i12}x_j - d_{j12}x_i = 0, \qquad \text{for all } i, j = 4, \ldots, 9.$$

Technical change is globally indirectly Hicks neutral in variable inputs (outputs) if all parameters in (3) (or (4)) are zero. Global indirect Hicks neutrality was rejected in the variable inputs and in outputs at the 1% level by both the GLS and NLS estimates.

Since global neutrality was rejected, local indirect Hicks neutrality was tested by (4) for outputs and by (3) for variable inputs at the data points. Chi-square statistics were computed at each observation, and the largest for each form of neutral technical change constitutes the test statistic. Using a test size of .01/23, where 23 is the number of observations, the probability of a joint Type I error was at most 1% by Bonferroni's inequality (Bickel and Doksum, p. 288). By both the GLS and NLS estimates, local Hicks-neutral technical change was rejected in variable inputs but not in outputs. The failure to reject local Hicks neutrality in outputs (which would not have been rejected even at the 5% level) contrasts sharply with the recent tests by Shumway and Alexander for the ten U.S. Department of Agriculture farm production regions. In that case, local Hicks neutrality in outputs was rejected at the 5% level in all regions for the data period 1951–82.

Although the actual χ^2 statistics for these technology tests differ between the GLS and NLS estimates, the conclusions rendered are consistent.

Conclusions

Product supply and input demand equations for Texas field crop production are reported in this article and are constrained so that their estimates are fully consistent with the theory of the competitive industry with a twice-continuously differentiable aggregate production function. Symmetry, homogeneity, and convexity of the profit function in prices were maintained in the constrained estimation. Monotonicity was satisfied by all equations at all observations.

Curvature (convexity) properties were maintained using a Cholesky factorization. Utilization of this method with suitable software has made this previously burdensome problem computationally manageable. Solution took 88 seconds on an Amdahl V-8 mainframe computer and cost less than \$10 at educational rates. With such feasibility, convexity now warrants widespread testing in empirical research along with other implications of theory.

Maintaining curvature properties substantially increased mean-squared error but did not impose major restrictions on our data. When tested, convexity was not rejected by these data at the 1% level.

Product supply and input demand elasticities were altered substantially when curvature properties were maintained. They also were highly sensitive to choice among asymptotically equivalent procedures for obtaining the covariance matrix in the first step of GLS estimation.

Technology test results were consistent across estimation approaches. Short-run nonjointness in all outputs was again soundly rejected, implying that it would not be legitimate to model the production of each of these six crops as single-product decision problems. Local indirect Hicks-neutral technical change in outputs was not rejected by either the GLS or NLS estimates. Consequently, it would appear justifiable to model output supply ratios over the data period without major concern about bias introduced by disembodied technical change. Since Shumway failed to reject homotheticity of the production technology in these six outputs and in the variable inputs, this would also imply that changes in marginal rates of substitution among the outputs and among the variable inputs have been largely independent of disembodied technical change (as represented by the time variable).[13]

[13] Indirect Hicks neutrality implies and is implied by direct Hicks neutrality only if either the production function is homothetic or it is additive in time (Lau 1978a, p. 202).

This article addressed two issues: (*a*) the feasibility of imposing curvature properties in econometric estimation by the Cholesky factorization, and (*b*) their impact on a specific set of empirical estimates. Tests of hypotheses using the curvature-constrained parameter estimates have been performed, but no statistical interpretation of the method for maintaining curvature has been provided. For the latter purpose, the sampling-theoretic approach to inequality constraints of Kodde and Palm and the Bayesian approach of Chalfant and Gray might be considered.

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Appendix

It is not possible to test the null hypothesis of convexity of the profit function by the standard methods of linear models. Such methods require that the null hypothesis be stated in terms of linear equality restrictions on the parameters. One suggestion is to construct a confidence ellipsoid for the parameters and to reject the null hypothesis if and only if this ellipsoid does not contain parameter values which lead to a convex profit function.[14]

Let G be a subspace of the parameter space which is defined by

$$G = \{\theta \in \mathbb{R}^k : \theta \text{ yields a convex profit function}\}.$$

An approximate $1 - \alpha$ confidence ellipsoid for θ is given by

$$E = \{t : F(t) \leq F^{\alpha}_{k,d}\},$$

where $F(t) = [(\hat{\theta} - t)'(W'\hat{\Omega}^{-1}W)(\hat{\theta} - t)/k]/MSE$; $\hat{\theta} = (W'\hat{\Omega}^{-1}W)^{-1}W'\hat{\Omega}^{-1}Y$ is the GLS estimate of θ; W is the matrix of stacked independent variables for the system of equations; Y is the stacked vector of dependent variables; $\hat{\Omega} = \hat{\Sigma} \otimes I$; $\hat{\Sigma}$ is the contemporaneous covariance matrix estimated from the OLS residuals; k is the number of parameters estimated; $d = (m - 1)T - k$; T is the number of observations; $m - 1$ is the number of equations estimated; MSE is the mean-squared error for the GLS estimate; and $F^{\alpha}_{k,d}$ is the $100(1 - \alpha)$ percentage point of the $F_{k,d}$ distribution. The null hypothesis of convexity is rejected if and only if the intersection of G and E is empty.

This test is conservative in the sense that the probability of a Type I error is no greater than α. To see this, let $\theta_0 \in G$ be the true value of the parameter vector θ and note that

$$\begin{aligned}\alpha = P[\theta_0 \notin E] &= P[E \subseteq \mathbb{R}^k - \theta_0] = P[E \in G^c \cup G_0] \\ &= P[E \subseteq G^c] + P[E \subseteq G_0] \\ &\quad + P[E \not\subseteq G^c, E \not\subseteq G_0, E \subseteq G^c \cup G_0] \\ &\geq P[E \subseteq G^c] = P[\text{Type I error}],\end{aligned}$$

where G^c is the complement of G and $G_0 = G - \theta_0$.

Determining whether the intersection of E and G is empty may seem to be a difficult problem to solve. Given the constrained NLS estimators, it is not. It can be shown that the vector of constrained NLS parameter estimates, $\tilde{\theta}$, is also a solution to

$$\min_{t \in G} F(t).$$

Hence, E and G have a nonempty intersection if and only if

$$F(\tilde{\theta}) \leq F^{\alpha}_{k,d}.$$

The value of $F(\tilde{\theta})$ is interesting in its own right as it is a measure of squared statistical distance from the unconstrained to the constrained parameter estimates.

Under a condition like $W'\Omega^{-1}W/T \rightarrow Q$ as $T \rightarrow \infty$, where Q is positive definite, one expects this test to be consistent. That is, the probability of rejecting the null hypothesis when it is false tends to one as T increases. The size of the test will tend to zero, however, if there is an open neighborhood about θ_0 contained in G. This doesn't seem of great concern since one would not need to conduct the test if the GLS and NLS estimates are the same.

Summarizing, the test of convexity of the profit function is conducted as follows:

(*a*) Find the OLS estimate, $\hat{\Sigma}$, of Σ from the OLS residuals. The estimate of Ω is

$$\hat{\Omega} = \hat{\Sigma} \otimes I.$$

This estimate of Ω is used throughout.

(*b*) Find the GLS estimate, $\hat{\theta}$, of θ as

$$\hat{\theta} = (W'\hat{\Omega}^{-1}W)^{-1}W'\hat{\Omega}^{-1}Y.$$

(*c*) Using the methods discussed, find the constrained NLS estimate, $\tilde{\theta}$, of θ by minimizing the function

$$(Y - W\theta)'\hat{\Omega}^{-1}(Y - W\theta)$$

with respect to θ subject to θ yielding a convex profit function.

(*d*) Calculate

$$F(\tilde{\theta}) = [(\hat{\theta} - \tilde{\theta})'(W'\hat{\Omega}^{-1}W)(\hat{\theta} - \tilde{\theta})/k]/MSE$$

and reject H_o if and only if $F(\tilde{\theta}) > F^{\alpha}_{k,d}$.

[14] Our appreciation is extended to Robert L. Basmann for this suggestion. It is similar to the test recently proposed by Wolak.