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A Cautionary Note on Polynomial Distributed Lag Formulations of Supply Response

Azzeddine Azzam and John F. Yanagida

This paper uses the Pagano-Hartley procedure to estimate the lag length and polynomial degree for the case of a quarterly hog supply equation. The results show that the nicely humped shapes which materialize when using the Almon lag may be caused by the failure in accounting for autocorrelation in determining lag length and polynominal degree.

Key words: Almon lag, autocorrelation, supply response.

Supply response in agricultural commodity markets, especially livestock, has proven difficult to estimate statistically (Chavas and Johnson). Often, the temporal modeling framework does not correspond to the production cycle as in the case of crops. The polynomial distributed lag method developed by Almon has been widely used by agricultural economists to explain livestock supply response (Chen, Courtney, and Schmitz; Meilke, Zwart, and Martin; Kulshreshtha; Meilke). The general form of the Almon model with finite lag length λ can be expressed as

$$(1)\quad Y_t = \sum_{j=0}^{\lambda} \beta_j X_{t-j} + \epsilon_t \qquad t = 1, 2, \ldots, T,$$

where the pattern of the β_j's is described by the p^{th} order polynomial

$$(2)\quad \beta_j = \sum_{k=0}^{p} \alpha_k (j)^k \qquad j = 0, 1, 2, \ldots, \lambda,$$

X_t is nonstochastic and $\epsilon_t \sim (0, \sigma^2)$ for all t.

Rarely are the length of lag and polynomial degree known parameters. Consequently, the previously mentioned studies using the Almon technique relied on ad hoc methods of parameter selection. Although these ad hoc methods can introduce specification error (Harper; Hendry, Pagan, and Sargan) in the distributed lag model by omission (inclusion) of relevant (irrelevant) explanatory variables, it is possible for researchers to obtain desirable statistical results and fail to detect possible misspecifications.

Recently, Pagano and Hartley (PH) proposed a two-step procedure for determining the lag length, λ, and polynomial degree, p in the presence and absence of autoregressive errors.[1] Like alternative parameter selection procedures, the PH method uses the sample data to obtain information about λ and p.[2] Unlike these alternative procedures, the PH method is computationally more efficient by circumventing the need for nested sequential hypothesis testing through the use of orthogonal reparameterization of the basic model.[3] This avoids repeated calculations for different choices of λ and p.

The primary objective of this paper is to reexamine Meilke's study on the supply re-

The authors are, respectively, an assistant professor and an associate professor, Department of Agricultural Economics, University of Nebraska.

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[1] This testing procedure was originally developed by Mizon.

[2] There are numerous procedures and criteria for determing the appropriate lag length and polynomial degree. Useful summaries of these procedures are found in Trivedi and Pagan; Judge et al.; and Hendry, Pagan, and Sargan.

[3] See Batten and Thorton (1983, 1984) for an application of the PH method and comparisons with alternative procedures.

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Table 1. Standardized *t*-Values for Lag Length Selection for Variable (*PBG/PC*) with Lag Length Equal to 20, 19, 17, 15, 13, 11, and 9 Quarters

Lagged Quarters	Lag Length						
	20	19	17	15	13	11	9
0	4.904	5.039	5.221	5.558	5.770	5.914	5.988
1	.589	.600	.641	.652	.649	.834	.865
2	.635	.488	.522	.422	.493	.391	.397
3	1.085	.987	1.011	.863	.885	.882	.925
4	1.389	1.327	1.377	1.356	1.350	1.391	1.460
5	1.309	1.214	1.221	1.367	1.416	1.482	1.544
6	.832	.756	.829	.708	.732	.831	.879
7	1.438	1.234	1.236	1.159	1.265	1.301	1.195
8	1.179	1.281	1.291	1.263	1.315	1.393	1.521
9	1.568[a]	1.474[a]	1.520[a]	1.518[a]	1.567[a]	1.469[a]	1.408[a]
10	.553	.408	.412	.492	.524	.538	
11	.906	.885	.920	.990	.952	.935	
12	.557	.530	.563	.636	.694		
13	.618	.674	.702	.637	.611		
14	.578	.615	.653	.557			
15	.604	.610	.561	.602			
16	1.096	1.099	1.162				
17	.895	.620	.592				
18	.467	.276					
19	.561	.774					
20	1.110						

Notes: Lag length selection process is to find the first statistically significant period starting with the maximum lag length and working backwards to the current period (i.e., lag = 0). Batten and Thorton utilize a significance level of 15% to minimize effects from Type II error. The critical t-values for 15% significance used in this study range from 1.40 to 1.47 for L = 20, 19, 17, 15, 13, 11, and 9.
[a] First significant t-statistic.

sponse of hogs using the PH procedure to select statistically the lag length and polynomial degree. This investigation also examines the effects of nonautoregressive and autoregressive errors in determining λ and p.

Application of the Pagano-Hartley Procedure to Hog Supply Response

The technical details of the PH procedure are developed in the appendix. In this section, we apply the PH procedure to estimating lag length and polynomial degree for the following quarterly hog supply equation (Meilke):

$$Q_t^s = \alpha_0 + \sum_{i=1}^{3} \alpha_i D_i + \alpha_4 T + \sum_{j=0}^{\lambda} \beta_j \left(\frac{PBG}{PC}\right)_{t-j} + \epsilon_t, \tag{3}$$

where

$$\beta_j = \sum_{k=0}^{p} \alpha_k (j)^k \tag{4}$$

and Q^s is commercial pork production (million pounds, carcass weight), *PBG* is the average price of barrows and gilts, seven markets ($/hundredweight), *PC* is the average price received for corn, No. 2 yellow ($/bushel), T is a time trend, and D_i represent seasonal dummy variables; λ and p are, respectively, the lag

Table 2. Standardized *t*-Values for Polynomial Degree Selection for L^* = 9 Quarters

Polynomial Degree[a]	Absolute Value of t-Statistics[b]
0	2.445
1	5.357
2	2.719
3	2.065[c]
4	.259
5	.924
6	.549
7	.073
8	.217
9	.352

[a] Polynomial degree selection process is to find the first statistically significant polynomial degree starting with the maximum degree length and working backwards to degree equal to zero.
[b] The critical t-value for 15% significance is 1.40.
[c] First significant t-statistic.

Table 3. Standardized *t*-Values for Lag Length Selection with Correction for Serial Correlation for Variable (*PBG/PC*) with Lag Length Equal to Twenty Quarters

Lagged Quarters	Absolute Value of *t*-Statistics
0	7.021
1	.653
2	.041
3	.704
4	1.084
5	1.020
6	.311
7	1.164
8	.830
9	1.470[a]
10	.305
11	.85
12	.599
13	.957
14	.888
15	.968
16	1.385
17	1.396
18	.003
19	.084
20	.483

Notes: See notes in table 1.
[a] First significant *t*-statistic.

Table 4. Standardized *t*-Values for Polynomial Degree Selection with Correction for Serial Correlation ($L^* = 9$ Quarters)

Polynomial Degree[a]	Absolute Value of *t*-Statistics
0	.919
1	4.566
2	4.526
3	3.977
4	.550
5	2.506
6	1.389
7	1.141
8	1.126
9	1.745[b]

[a] See footnotes a and b, table 2.
[b] First significant *t*-statistic.

length and degree of the polynomial to be determined. Equation (3) is estimated in double-logarithmic form for the time period 1960–83 (96 quarters).

Under the assumption of serially uncorrelated errors, we chose a maximum lag length $L = 20$ and reparameterized equation (A.2) as shown in the appendix. The resulting t_{L-j} statistics are listed in table 1. The PH procedure chose a lag length of nine periods. Alternative values of L (lag lengths of 19, 17, 15, 13, 11, and 9 periods) were specified to test the sensitivity of the lag length selection to the initial choice of L. As shown in table 1, the results were unchanged in all cases. The standardized t values for polynomial degree selection with $L^* = 9$ are listed in table 2. The procedure yields a polynomial degree equal to 3.

A test for serial correlation with $L^* = 9$ and $P^* = 3$ produced a Durbin-Watson statistic equal to .328, indicating strong positive autocorrelation of the residuals. This suggests that the disturbances should be modeled as an autoregressive process.

Using the Gallant and Goebel method and Harvey (p. 204), we model the autoregressive effects by

$$(1 - \rho_1 B)(1 - \rho_4 B^4)\epsilon_t = v_t,$$

where B is the lag operator and ρ_4 captures the seasonal effects, while ρ_1 accounts for the serial correlation from quarter to quarter. The estimates, $\hat{\rho}_1$ and $\hat{\rho}_4$, are $-.861$ and .010, respectively. The t-values for lag length selection with correction for serial correlation are in table 3. It is evident that lag length selection, in our particular example, is invariant when accounting for serial correlation. However, the degree of the polynomial changed from 3 to 9 (see table 4) when adjusting for serial correlation. The Durbin-Watson statistic after correcting for serial correlation is 1.936. This result provides further support to PH's conclusion that estimation of polynomial distributed lags is extremely sensitive to the adjustment of the autoregressive process.

Summary

In this paper, we apply the PH procedure to estimate the lag length and polynomial degree for the case of a quarterly hog supply process. Our study closely resembles the work done by Meilke but utilizes a different time period for analysis and revised data. One implication from our results is that with $L^* = P^* = 9$, the Almon procedure is the same as unrestricted least squares.

These results also serve as a warning to future users of the Almon technique in supply analyses. Namely, the nicely humped shapes which materialize when applying the Almon distributed lag procedure may well be caused by the failure in accounting for serial correlation in determining lag length and polynomial degree. Furthermore, the economic implication from our results is that short-run and long-run elasticities derived from the estimated coefficients of the Almon model should be interpreted with caution.

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Appendix

Pagano-Hartley Procedure

The basic PH model, in matrix form, is similar to equation (1) with the addition of a matrix of contemporaneous explanatory variables Z, i.e.,

$$\text{(A.1)} \qquad Y_\lambda = Z_\lambda \delta_\lambda + X_\lambda \beta_\lambda + \epsilon_\lambda,$$

where Z_λ and X_λ are, respectively, $(T \times m)$ and $(T \times (\lambda + 1))$ matrices of observations on the explanatory variables; and δ_λ and β_λ are, respectively, $(m \times 1)$ and $((\lambda + 1) \times 1)$ vectors of parameters to be estimated.

Replacing the unknown λ by the largest lag L, we can write equation (A.1) as

$$\text{(A.2)} \qquad Y_L = [Z_L \vdots X_L]\begin{bmatrix} \delta_L \\ \beta_L \end{bmatrix} + \epsilon_L = W_L \psi_L + \epsilon_L,$$

where W_L and ψ_L are a $((T - L) \times (m + L + 1))$ and $((m + L + 1) \times 1)$ augmented matrix and vector, respectively.

By the Gram-Schmidt decomposition, W_L can be decomposed into $W_L = Q_L R_L$, where Q_L is a $((T - L) \times (m + L + 1))$ matrix with orthonormal columns, and R_L is an upper triangular square matrix of order $(m + L + 1)$. Equation (A.2) now can be rewritten as

$$\text{(A.3)} \qquad Y_L = Q_L a_L + \epsilon_L,$$

where $a_L = R_L \psi_L = [a^\delta : a^\beta]'$. Since Q_L is orthonormal, the least squares estimate of a_L is $\hat{a}_L = [\hat{a}^\delta : \hat{a}^\beta]' = Q_L' Y_L$. To recover the structural parameters $\hat{\psi}_L$, we simply calculate $R_L^{-1}\hat{a}_L$.

By virtue of orthogonal reparameterization of equation (A.1), the hypotheses regarding the β_j's in equation (A.1) and the a_j's in equation (A.3) are equivalent, i.e.,

$$H_{L-j} : \beta_L = \beta_{L-1} = \ldots = \beta_{L-j} = 0 \qquad \text{for } j = 0, 1, 2, \ldots, L, \text{ and}$$

$$H^*_{L-j} : a^\beta_L = a^\beta_{L-1} = \ldots = a^\beta_{L-j} = 0 \qquad \text{for } j = 0, 1, 2, \ldots, L.$$

Pagano and Hartley note that in view of the orthogonality of the procedure defining the a_j's, one can equivalently consider the simple hypothesis:

$$\text{(A.4)} \qquad K_{L-j} : a^\beta_{L-j} = 0 \quad \text{for } j = 0, 1, 2, \ldots, L.$$

The estimate of the lag length is equal to the index $L-j$, where j is the smallest number for which K_{L-j} is rejected. The appropriate t-statistic for the hypothesis K_{L-j} is

$$t_{L-j} = a^{\beta}_{L-j}/s,$$

where s is the standard error such that $s^2 = \dfrac{Y'_L Y_L - \hat{a}_L \hat{a}_L}{T - m - L - 1}$.

The procedure for selecting the polynomial degree is analogous to that of the lag length. Denoting L^* as the optimal lag, we rewrite equation (A.1) as

$$\text{(A.5)} \qquad Y^*_L = Z_{L^*}\delta_{L^*} + X_{L^*}\beta_{L^*} + \epsilon_{L^*}.$$

Substituting for the β_j's from equation (2) in the text, and setting $p = L^*$, we obtain

$$\text{(A.6)} \qquad Y_{L^*} = Z_{L^*}\delta_{L^*} + X^*_{L^*}\alpha + \epsilon_{L^*}$$

where $X_{L^*} = X_{L^*}H$, and

$$H = \begin{bmatrix} 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 1 & 1 & \cdot & \cdot & \cdot & 1 \\ 1 & 2 & \cdot & \cdot & \cdot & 2^{L^*} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & L^* & \cdot & \cdot & \cdot & L^{*L^*} \end{bmatrix}.$$

By orthogonal reparameterization of equation (A.6), we arrive at the analogous hypothesis in equation (A.4) regarding the polynomial degree.

Dropping the assumption of serially uncorrelated errors, we slightly modify the PH procedure for determining the lag length L^* and the degree of the polynomial P^*. This adjustment is an extension of the work done by Gallant and Goebel on nonlinear regression with autocorrelated errors in the linear case. The essence of the Gallant-Goebel method is (*a*) to construct a Γ matrix from the estimates of the autocovariances up to a lag r of the autoregressive process,

$$\epsilon_t + \sum_{j=1}^{r} \rho_j \epsilon_{t-j} = v_t,$$

and (*b*) to find a matrix U such that $U'U = \Gamma_r^{-1}$. The matrix U is used to transform the original observations of the regression model. The generalized least squares (GLS) estimate is then obtained by using ordinary least squares (OLS).