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# Identifying the Set of SSD-Efficient Mixtures of Risky Alternatives

**Francis McCamley and James B. Kliebenstein**

Target MOTAD and other direct utility-maximization models provide one way of computing SSD-efficient mixtures. These models are appropriate when the utility function is known and can also be used to identify part of the set of SSD-efficient mixtures even when the utility function is not known. However, they do not always identify all SSD-efficient mixtures. A grid method was proposed by Bawa, Lindenberg, and Rafsky. A third approach, which extends the work of Dybvig and Ross, is presented here. It is illustrated by applying it to data from Anderson, Dillon, and Hardaker.

*Key words:* diversification, SSD efficient set, stochastic dominance.

Historically, mean-variance and mean-absolute deviations criteria have been used to choose appropriate mixtures of risky production and/or marketing activities. These criteria were used in spite of the fact that they are not always consistent with expected utility theory. More recently, methods which are consistent with expected utility theory have been presented.

Some of these methods are, or can be regarded as, direct utility-maximization approaches. Examples include the Target MOTAD model presented by Tauer and by Watts, Held, and Helmers; the safety-first model presented by Atwood, Watts, and Helmers (1985a, b); Porter's mean-target semivariance model; and the direct utility-maximization techniques discussed by Kroll, Levy, and Markowitz and by Lambert and McCarl. Direct utility-maximization methods are most appropriate when the utility function and its parameters are known or can be approximated reasonably well. They have the advantage of providing unique solutions or small sets of solutions.

When the utility function and its parameters are not well known, it may be appropriate to identify all solutions associated with a larger class of utility functions. This may mean applying stochastic dominance criteria. Unfortunately, as Cochran has noted, stochastic dominance techniques are not well developed for problems involving mixtures of alternatives.

A few theoretical results have been published. Hadar and Russell (1971, 1974) and Russell and Seo presented several sets of conditions under which diversification is optimal for risk averters. It has also been shown that there are conditions under which specialization is optimal (Hadar and Russell 1971; Hadar and Seo; McCarl et al.). Dybvig and Ross discussed properties of the portfolio efficient set but did not present a method for identifying it.

Direct utility-maximization techniques can be used to identify subsets of the first (FSD) and second (SSD) degree stochastic dominance-efficient mixtures. For example, Tauer has shown that unique Target MOTAD solutions are SSD efficient and has suggested that a large portion of the SSD-efficient set can often be found by Target MOTAD. Although Target MOTAD can identify a large portion, and sometimes all, SSD-efficient mixtures, it cannot always identify all SSD-efficient mixtures. Other direct utility-maximization techniques seem to share this limitation.

An alternative approach has been proposed

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by Bawa, Lindenberg, and Rafsky. They suggest that their stochastic dominance algorithm could be used to approximate stochastic dominance-efficient sets of portfolios by using a fine grid on the space of feasible portfolios. Their approach could be extended to deal with the more general mixture problems considered by agricultural economists, but it might not be cost effective. Even when the number of observations (states of nature) is not very large, a rather small grid size and, therefore, a large number of lattice points would be required to control sampling errors. However, for many mixture problems in agricultural economics, only a small proportion of the lattice points would belong to the stochastic dominance-efficient set. This suggests using something other than a uniform grid system.

A third approach is discussed in this article. By extending the work of Dybvig and Ross, necessary and sufficient conditions for SSD efficiency are obtained. The relationship between these conditions and an extension of the Target MOTAD model is mentioned. Then, a simple search procedure for identifying all SSD-efficient mixtures is presented and demonstrated.

## Previous Work

### *Assumptions*

Three of the assumptions adopted here are similar to assumptions of the Target MOTAD model. First, linear resource constraints are assumed. Second, it is assumed that there are $s$ states of nature and therefore only $s$ alternative levels of net return associated with a given enterprise mixture. Third, for any state of nature, the net return is a homogenous linear function of the $n$ element activity levels vector, $x$.

Other symbols are defined as follows: $p$ denotes a row vector of probabilities associated with $s$ states of nature; $C$ is a matrix of net returns associated with the activities for the various states of nature; $C_{ij}$ is the net return per unit of activity $j$ when the $i$th state of nature occurs; and $y$ is a vector of (total) net returns for the various states of nature. Thus,

(1) $$y - Cx = 0.$$

Here, $A$ is a matrix of resource or technical requirements, and $b$ is a vector of resource levels. The constraints on activity levels are

(2) $$Ax \leqq b, \text{ and}$$

(3) $$x \geqq 0.$$

Although the assumptions about the joint probability distribution and about the relationship of net returns (for various states of nature) to activity levels are somewhat specialized, they can be extended to approximate more general situations. For example, Lambert and McCarl show that constraints much like (1) can approximate the joint density function of continuous random variables. As stated, the equations in (1) require net returns for each state of nature to be a linear homogenous function of enterprise activity levels, but this requirement could be relaxed to deal with alternative assumptions such as complementary enterprises. It would also be relatively easy to deal with the "additional penalty" case discussed by Robison and Lev or with increasing marginal income tax rates. None of these extensions would require drastic changes in the approach proposed in this article.

### *Conditions for DR Efficiency*

One set of conditions for what Dybvig and Ross call portfolio efficiency is relevant for the class of problems discussed above. The efficiency concept associated with this set of conditions is called DR efficiency here to avoid the implication that it is relevant only for portfolio problems.

Dybvig and Ross' theorem 1 implies that a net returns vector, $y^o$, is DR efficient if, and only if, there exists a vector, $z^o$, which satisfies the following conditions:

(4) $z^{o\prime}y^o \geqq z^{o\prime}y$ for all $y$ vectors which satisfy (1), (2), and (3);

(5) $z_i^o p_i \geqq z_j^o/p_j \quad \text{if } y_i^o < y_j^o \quad$ for all $i, j$;

(6) $z^o > 0.$

Dybvig and Ross developed these conditions by considering the problem of maximizing the expected value of a (weakly) concave utility function. Within that context, $z^o$ can be interpreted as a support vector; $z^o$ can also be regarded as a vector of relative shadow prices for the net returns associated with various states of nature or as a generalized marginal expected utilities vector. Thus, each $z_i^o/p_i$ can be regarded as a relative marginal utility. Conditions (4) and (6) are necessary for vector maxima and,

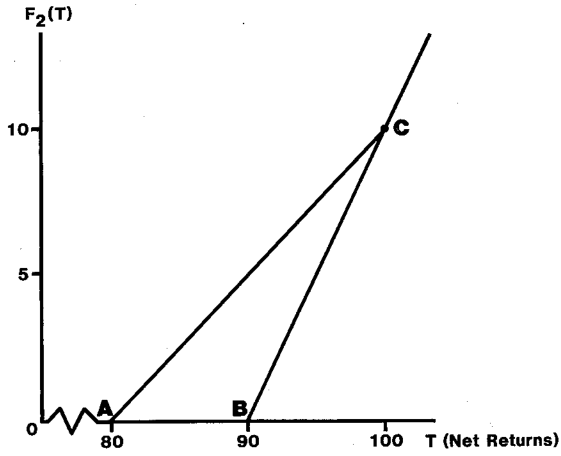


**Figure 1. $F_2$ functions for selected mixtures**

therefore, FSD efficiency. For DR-efficiency, condition (5) must also be satisfied. It ensures that marginal utility is a nonincreasing function of net returns.

### *Properties of the Set of DR-Efficient Vectors*

Dybvig and Ross discussed the properties of the efficient set for the perfect market case. Even though the problem considered here is different, two of their properties are relevant. The DR-efficient set is connected and is the union of a finite number of closed convex subsets. This union need not be convex.

Each of the DR-efficient subsets is an intersection of the plane representing the feasible set and the subset of the $s$-dimensional Euclidian space, $R^s$, for which all $y$ vectors share the same rank order. The term rank order is used in a weak sense since the boundaries (where one or more "ties" exist in the elements of the $y$ vectors) of the subsets are included in the "same rank order" subsets rather than separate subsets.

## Necessary and Sufficient Conditions for SSD Efficiency

Ordinarily, the description of a probability distribution involves (at least implicitly) a set of possible outcomes and the associated probabilities, $p$. In this article, only the $y$ vectors differ among alternative probability distributions; the probabilities associated with the various states of nature are fixed. Therefore, when describing any net returns distribution, only its $y$ vector is mentioned. We regard an enterprise mixture, $x^o$, as being SSD efficient if the net returns vector, $y^o$, associated with it is SSD efficient.

A net returns vector, $y^o$, is SSD efficient only if it is DR-efficient. A simple example shows that the converse is not true. Consider the net returns matrix,

$$C = \begin{bmatrix} 80 & 100 \\ 100 & 80 \end{bmatrix}. \tag{7}$$

Assume equiprobable states of nature, a single resource constraint such as

$$x_1 + x_2 \leq 1 \tag{8}$$

and a nonnegativity constraint such as (3).

Clearly, all feasible mixtures for which the sum of $x_1$ and $x_2$ equals one yield net returns vectors which are DR-efficient. Each of these net return vectors is "supported" by the vector, $z^o = (1 \ 1)'$, which satisfies conditions (4) through (6). However, only one of the DR-efficient net return vectors is SSD efficient. It is the one for which both $x_1$ and $x_2$ equal one-half.

This is illustrated graphically in figure 1. There, $AC$ is the relevant portion of the graph of the $F_2$ function when $x_1$ equals one and $x_2$ equals zero (or vice versa);[1] $BC$ is the analogous portion of the $F_2$ function's graph when both $x_1$ and $x_2$ equal one-half. The $F_2$ graphs associated with other DR-efficient mixtures are strictly betwen the two $F_2$ graphs shown when $T$ is between 80 and 100.

### *Stronger Conditions*

A $y$ vector can be DR efficient without being SSD efficient because condition (5) permits marginal utility to be a nonincreasing function of net returns. That is, $z_i^o/p_i$ can equal $z_j^o/p_j$ even when $y_i^o$ is less than $y_j^o$. A stronger condition can be obtained by requiring marginal utility to be a strictly decreasing function of net returns and replacing condition (5) with

$$z_i^o/p_i > z_j^o/p_j \quad \text{if} \quad y_i^o < y_j^o \qquad \text{for all } i, j. \tag{5'}$$

[1] $F_2$ ($T$) is the area to the left of $T$ under the cumulative distribution function. The assumptions in this article imply that the $CDF$s are step functions, and the $F_2$ functions are piecewise linear functions of $T$.

Conditions (4), (5′), and (6) are necessary and sufficient for SSD efficiency.[2]

## Linear Programming Formulations

Determining whether conditions (4), (5′), and (6) are satisfied involves solving any one of several similar saddlepoint problems. The lagrangians for these saddlepoint problems are the same as those associated with appropriately formulated pairs of linear programming problems. One pair of these linear programming problems is described in detail. Useful variations are also discussed briefly.

### *The Dual*

Our statement of the dual assumes that the states of nature have been permuted so that the elements of $y^o$ are in ascending order and that there are no ties among these elements. The first of these assumptions is trivial and merely simplifies the notation. The second is somewhat less trivial; it is relaxed later.

The dual is

$$\text{(9)} \qquad \text{minimize } v'b - \sum_{j=1}^{s} w_j p_j y_j^o$$

subject to

$$\text{(10)} \quad A'v - C'z \geqq 0$$

$$\text{(11)} \quad z_j - p_j w_j = 0 \qquad \text{for } j = 1, 2, \ldots, s$$

$$\text{(12)} \quad w_j - w_{j+1} \geqq 1 \qquad \text{for } j = 1, 2, \ldots, s-1$$

$$\text{(13)} \quad w_s \geqq 1$$

$$\text{(14)} \quad v, w \geqq 0.$$

In this formulation, $v$ is the shadow price vector for the resource constraints; $z$ has an interpretation similar to that for $z^o$ and $w$ is a vector of marginal utilities whose elements are related to $z$ and $p$ as shown in (11).

### *Relationship of Dual to Necessary and Sufficient Conditions*

The objective function (9) is related to (4). The inequalities in (10) require the imputed shadow prices for the resources to be large enough to guarantee that, at the margin, the value (in marginal utility terms) of the resources used by each activity (or enterprise) is at least as large as the expected marginal utility (of net returns) associated with that activity. Inequalities (11) through (13) ensure that (5′) and (6) are satisfied.[3] Since these constraints require $w$ (and $z$) to be positive, $w$ is included in (14) (and $z$ is excluded) merely to make the dual fully compatible with our preferred primal specification.

### *The Primal*

The primal problem can be derived directly from the dual. It is

$$\text{(15)} \qquad \text{maximize } \sum_{j=1}^{s} t_j,$$

subject to

$$\text{(16)} \qquad t_j - t_{j-1} - p_j y_j \leqq -p_j y_j^o \quad \text{for } j = 2, 3, \ldots, s$$

$$\text{(17)} \qquad t_1 - p_1 y_1 \leqq -p_1 y_1^o$$

$$\text{(18)} \qquad y - Cx = 0$$

$$\text{(19)} \qquad Ax \leqq b$$

$$\text{(20)} \qquad x, t \geqq 0.$$

In stating the primal, it is both convenient and appropriate to let $x$ and $y$ be the primal variables (vectors) associated with dual constraints (10) and (11), respectively. This choice makes it obvious that (18) and (19) are the same as (1) and (2).

### *Relation to Usual SSD-Efficiency Test*

The relationship between the primal and the usual test for SSD efficiency becomes somewhat more apparent when (16) and (17) are replaced with equivalent constraints. The variable $t_{j-1}$ can be eliminated from any inequality in (16) by adding the inequalities in (16) for which $j$ is smaller and then adding (17). Doing this and changing the sign of the resulting inequalities (by multiplying by $-1$) shows that (16) and (17) imply

[2] Dybvig and Ross's theorem 1 assumed the class of strictly monotonic (increasing), (weakly) concave utility functions. Their table 1 indicates that condition (5′) is implied by the class of strictly increasing, strictly concave utility functions. This class is slightly more general than the class of functions associated with SSD efficiency in Bawa's article.

[3] The choices of right-hand side values for (12) and (13) affect the optimal value of (9). This is not a problem since the critical question is whether (9) is zero or positive.

(21) $$\sum_{k=1}^{j} p_k y_k - t_j \geqq \sum_{k=1}^{j} p_k y_k^o \quad \text{for } j = 1, 2, \ldots, s.$$

The inequalities in (21) are all equalities when the objective function in (15) is maximized. Thus, (21) also implies (16) and (17).

The difference between the sums in the $j$th constraint of (21) equals the difference between the (usual SSD criterion) $F_2$ functions for $y$ and $y^o$ when the elements of both $y$ and $y^o$ are in ascending order and the $F_2$ functions are evaluated at any income level, $T$, which is no smaller than either $y_j^o$ or $y_j$ and no larger than either $y_{j+1}^o$ or $y_{j+1}$. (Substitute infinity for $y_{j+1}^o$ and $y_{j+1}$ when $j$ equals $s$.)

At first glance, the conditions under which the inequalities in (21) are related to the differences in the $F_2$ functions may seem too restrictive to be very useful. Fortunately, there are two mitigating considerations. The statement of the conditions could be weakened. Of more importance to an intuitive understanding is that for mixture problems of the sort considered here, the conditions are satisfied for a critical subset of the feasible $y$ vectors. It is possible to show that an income vector, $y^o$, may be dominated by one or more feasible $y$ vectors only if it is dominated by a feasible $y$ vector which is very "close" to $y^o$. If a $y$ vector is sufficiently close to $y^o$, its elements will have the same rank ordering as those of $y^o$, and the difference between the pairs of sums in the various constraints of (21) will accurately represent the difference in the SSD cumulative functions for most (and in a limiting sense, all) relevant $T$ values.[4]

Thus, in effect, the primal program simply answers a question which is appropriate for any SSD efficiency test. That is, is there another feasible $y$ vector whose $F_2$ graph lies on or below that for $y^o$ at all values of $T$ and strictly below the $F_2$ graph for $y^o$ at some value of $T$? If not, then the optimal value of the objective function (for the dual and both versions of the primal) is zero and $y^o$ is SSD efficient. If $y^o$ is dominated, the optimal value of the objective function is positive. A positive objective function value means that $y^o$ is dominated, but it does not always mean that it is dominated by the particular $y$ vector which is part of the primal solution.

*An Alternative Test Criterion*

Although the obvious test criterion is the value of the objective functions in (9) or (15), the optimal $y$ vector is a more sensitive indicator. Because it may be necessary (due to the fact that linear programming algorithms produce very precise rather than exact solutions) to perturb the right-hand sides of (16) and (17) to obtain feasible solutions, a small positive objective function value may be obtained even when $y^o$ is SSD efficient. It is possible to determine the effect of perturbations on the objective function. However, it is usually simpler to look at the optimal $y$ vector. It tends to be very different from $y^o$ when $y^o$ is not SSD efficient.

*Modifications When There Are Ties in the Elements of $y^o$*

The linear programming formulations presented above assumed that no two elements of $y^o$ are the same. Relaxing this assumption requires minor changes. When there are one or more ties among the elements of $y^o$, then the $j$th inequality in dual constraint (12) would be replaced by either

(12′) $$w_j - w_k \geqq 1$$

or

(12″) $$w_j \geqq 1.$$

In (12′), $k$ is the smallest integer for which $y_k^o$ is greater than $y_j^o$. If no integer, $k$, satisfies this requirement, then (12″) is used. Note that when there are no ties (12′) is the same as the $j$th inequality in (12).

Corresponding changes are required in the primal. Replace (16) with either

(16′) $$t_j - t_k - p_j y_j \leqq -p_j y_j^o$$

or

(16″) $$t_j - p_j y_j \leqq -p_j y_j^o$$

In (16′), $k$ is the largest integer for which $y_k^o$ is smaller than $y_j^o$. If no positive integer, $k$, satisfies this requirement, use (16″) instead of (16′).

*Relationship to Target MOTAD*

Since every Target MOTAD solution which is unique in the sense defined by Tauer is SSD

[4] The intuitive argument presented here provides the basis for a more rigorous proof of the proposition that $y^o$ is SSD efficient if and only if the optimal value of (15) is zero. This proof as well as proofs of certain statements about characteristics of the SSD efficient set are sketched in McCamley and Kliebenstein (1987).

efficient, it is apparent that these solutions also satisfy the necessary and sufficient conditions for SSD efficiency presented in this article. The converse is not true. However, if the Target MOTAD model were extended to include $s-1$ targets, then the set of Target MOTAD solutions associated with unique $y$ vectors would be identical to the set of SSD-efficient solutions. Despite the relationship between the multiple Target MOTAD model and SSD efficiency, use of a multiple Target MOTAD formulation does not appear to be a cost effective way of identifying the set of SSD-efficient $y$ vectors.

## Properties of the SSD-Efficient Set

Conditions (4), (5′), and (6) and the equivalent linear programming formulations permit determining the SSD-efficiency status of a specific $y$ vector and/or its associated mixture(s) without explicitly knowing or considering any other feasible $y$ or $x$ vector. This can be useful. Of more significance is the fact that two properties of the SSD-efficient set make it possible to identify the entire SSD-efficient set by determining the SSD-efficiency status of a finite number of vectors.

### *Subsets*

As is the case for the set of DR-efficient $y$ vectors, the SSD-efficient set is the union of a finite number of closed convex subsets. However, the characteristics of these subsets differ in two ways from those discussed by Dybvig and Ross. One difference reflects the fact that the set of feasible $y$ vectors is not usually a hyperplane but a more general convex polyhedron.[5] Each DR-efficient $y$ vector lies on the surface of this polyhedron. Each DR-efficient subset is the intersection of a proper face of the polyhedron and a "same rank order" subset of $R^s$. Proper faces include vertices and edges as well as those portions of the surface which might intuitively be thought of as being faces.[6]

Although the set of SSD-efficient $y$ vectors is a subset (sometimes improper) of the set of DR-efficient $y$ vectors, the set of subset candidates is larger for SSD efficiency than for DR efficiency. The example associated with (7) and (8) illustrates the need to include additional subsets. The reader can verify that the only SSD-efficient $y$ vector belongs to two DR-efficient subsets but is not, by itself, a DR-efficient subset.

To differentiate the additional subset candidates associated with SSD efficiency from those also associated with DR efficiency, they will be called "tie" subsets. Although there will typically be several "tie" subset candidates for most problems, their role is ordinarily very limited. An example presented later will demonstrate that the SSD efficiency status of "tie" subsets is often so obvious that they do not even need to be explicitly evaluated.

Subset candidates satisfy a type of "all or nothing" relationship. If a strictly "interior" $y$ vector of a subset candidate is SSD efficient, then the entire subset is SSD efficient. That is, all "interior" and all "boundary" vectors are SSD efficient. As demonstrated by the example above, it is possible for one or more of the boundaries of a subset candidate to be SSD efficient even though the balance of the subset is not SSD efficient. Note that the collection of subset candidates is defined so that these boundaries are, in turn, separate subset candidates. This permits us to adopt the convention of regarding a subset candidate as being SSD efficient when all of its vectors are SSD efficient and SSD inefficient (or not SSD efficient) when at least one of its $y$ vectors is not SSD efficient.

### *Connectedness*

The SSD-efficient set is connected. Connectedness simplifies identifying the SSD efficient set. If $y^1$ and $y^2$ belong to a conected set, then there exists a continuous path within the set which "connects" them (Murty, p. 466).

### *The Set of SSD-Efficient Mixtures*

The properties of the set of SSD-efficient mixtures ($x$ vectors) are similar, but not identical, to the set of SSD-efficient $y$ vectors. Connectedness of the set of SSD-efficient $y$ vectors implies connectedness of the set of SSD-efficient enterprise mixtures. The definition of the set of subset candidates is similar to that given earlier. That is, each subset candidate in mix-

[5] We assume that the polyhedron is bounded. (The phrase, convex polytope is sometimes used to denote a bounded polyhedron.) It appears that this assumption could easily be relaxed.

[6] Stoer and Witzgall's definitions of a face and a proper face are assumed here.

ture space is the intersection of a face of the polyhedron of feasible mixtures with either a subset of mixtures for which the elements of $Cx$ have the same (weak) rank ordering or a subset of mixtures for which the elements of $Cx$ not only have the same rank ordering but for which there is at least one tie among the elements of $Cx$. One extension must be made. For some "degenerate" problems, interior mixtures which are not on any proper face of the polyhedron can be SSD efficient. Therefore, it may be necessary to consider subset candidates lying in the interior of the polyhedron.

## Search Strategies

The best strategy for identifying the SSD-efficient set is likely to vary from problem to problem. The considerations discussed above suggest some guidelines.

### *Identifying All SSD-Efficient y Vectors*

First, it is appropriate to start by examining the set of $y$ vectors which maximize expected net returns. At least one of these vectors must be SSD efficient.

Second, the connectedness property means that at each stage in the search procedure it is appropriate to consider only those candidate subsets which are "adjacent" to one or more subsets already known to be SSD efficient.

Third, when there are several "adjacent" subset candidates, it is appropriate to give highest priority to examining those which lie on higher-order intersections of the polyhedron and its boundary planes. This exploits the fact that the set of alternative marginal utility vectors consistent with a higher-order intersection is ordinarily larger than the set associated with a lower-order intersection. Thus, "adjacent" candidate subsets which lie on an edge of the polyhedron are more promising than those lying on more general faces.

Fourth, it may be appropriate to determine whether a face satisfies the vector-maximum conditions, (4) and (6), before attempting to determine whether any of the subset candidates associated with that face are SSD efficient.[7] Faces satisfy an all-or-nothing property with respect to conditions (4) and (6). That is, if any interior vector of a face satisfies (4) and (6), then the entire face satisfies conditions (4) and (6). Of greater significance is the fact that if any interior vector fails to satisfy conditions (4) and (6), then none of the interior vectors on that face satisfy conditions (4) and (6). This, in turn, means that none of the interior vectors on the face can be SSD efficient. Because a face may include several subsets which are candidates for SSD efficiency, finding that the interior of the face fails to satisfy (4) and (6) may preclude several tests for SSD efficiency.

Fifth, a candidate subset can be ignored if any of the candidate subsets which comprise its "boundaries" are known to be SSD inefficient.

### *Identifying All SSD-Efficient Mixtures*

There are at least two ways of identifying the set of SSD-efficient enterprise mixtures. One way is to first identify all SSD-efficient $y$ vectors. When only one enterprise mixture is associated with each $y$ vector, some of the intermediate calculations may, if preserved, provide sufficient information to identify the set of SSD-efficient enterprise mixtures. When more than one enterprise mixture is associated with some $y$ vectors, additional calculations may be required.

An alternative, but very closely related, approach finds the SSD-efficient mixtures more directly. There seems to be little reason to prefer one of these approaches to the other. The second approach is chosen for the following example simply because the graph of the feasible set of enterprise mixtures can be more easily presented in a way which can be understood.[8]

## An Example

The search strategy is illustrated by applying it to an example from Anderson, Dillon, and Hardaker (pp. 209–10). This example was chosen because it has the smallest number of activities (three) which allows some general properties (e.g., nonconvexity) to be exhibited

[7] Our dual and primal formulations become tests for a vector maximum if $w_{j+1}$ is omitted from (11) of the dual and $t_{j-1}$ is omitted from (16) in the primal.

[8] Both feasible sets are three dimensional. However, the set of feasible mixtures lies in an easily recognizable three-dimensional space, while the set of feasible $y$ vectors lies in a three-dimensional subspace of a five-dimensional space.

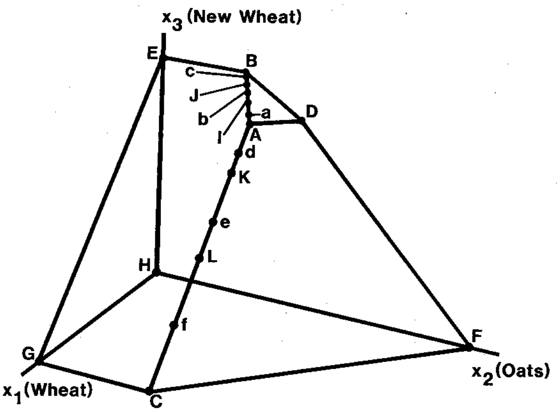


**Figure 2. Feasible crop mixes**

and the largest number of activities which permits graphical presentation of the feasible set. For this article, each of the five states of nature (observations) is assumed to be equally likely. Figure 2 provides a perspective view of the feasible set and some relevant subsets. Table 1 presents selected enterprise mixtures. Upper case letters are used to identify vertices (corners) of the feasible set and/or its subsets. Lower case letters identify selected "interior" mixtures of some of the subsets.

Since only one enterprise mixture, mixture *A*, maximizes net returns, it must be SSD efficient. It provides a logical starting point for the process of identifying the SSD-efficient mixtures.

Several subset candidates are adjacent to (include) *A*. Those associated with edges are most likely to be SSD efficient. The mixtures on the interior of edge *AD* fail to satisfy conditions (4) and (6). Thus, apart from *A*, none of the mixtures on *AD* are SSD efficient. Edges *AB* and *AC* satisfy conditions (4) and (6). Thus, the subsets on these edges will be examined further.

The subset of mixtures on *AB* for which the $y$-vector elements have the same rank order as for mixture *A* ($y_2 \geqq y_1 \geqq y_4 \geqq y_3 \geqq y_5$) is considered first. This subset, line segment *AI*, is determined to be SSD efficient by evaluating mixture *a*. Subset (line segment) *IJ* is found to be SSD efficient by evaluating *b*. The "interior" of subset *JB* is not SSD efficient because *c* is not. Mixtures *I* and *J* are "tie" subsets. It was not necessary to test these subsets explicitly for SSD efficiency because *I* and *J* belong to other subsets already found to be SSD efficient.

**Table 1. Selected Enterprise Mixtures**

| Identifier | Wheat $x_1$ | Oats $x_2$ | New Wheat $x_3$ |
|---|---|---|---|
| | | (hectares) | |
| *A* | 1.33 | 4.00 | 6.67 |
| *B* | 0 | 3.20 | 8.00 |
| *C* | 8.00 | 4.00 | 0 |
| *D* | 0 | 5.33 | 6.67 |
| *E* | 0 | 0 | 8.00 |
| *F* | 0 | 12.00 | 0 |
| *G* | 8.00 | 0 | 0 |
| *H* | 0 | 0 | 0 |
| *I* | .67 | 3.60 | 7.33 |
| *J* | .28 | 3.37 | 7.72 |
| *K* | 2.52 | 4.00 | 5.48 |
| *L* | 4.71 | 4.00 | 3.29 |
| *a* | 1.00 | 3.80 | 7.00 |
| *b* | .50 | 3.50 | 7.50 |
| *c* | .20 | 3.32 | 7.80 |
| *d* | 2.00 | 4.00 | 6.00 |
| *e* | 4.00 | 4.00 | 4.00 |
| *f* | 7.00 | 4.00 | 1.00 |

Evaluation (in sequence) of mixtures *d*, *e*, and *f* confirms that all five subset candidates (*AK*, *K*, *KL*, *L*, and *LC*), and thus all mixtures on edge *AC*, are SSD efficient. As was the case for edge *AB*, it was not necessary to test explicitly the "tie" subsets represented by mixtures *K* and *L*.

Because the corner mixture, *C*, is SSD efficient, it is appropriate to consider those edges, *CF* and *CG*, connected to it which have not yet been examined. The interiors of each of these edges fail to satisfy conditions (4) and (6).

It is possible to show that the set of SSD-efficient mixtures has now been identified. The known SSD-efficient set is completely "surrounded" by mixtures which are either infeasible or SSD inefficient. The connectedness property implies that no mixtures other than those in the union of *JA* and *AC* can be SSD efficient.

Even though this example is very simple, it illustrates the advantages of exploiting the properties of the SSD-efficient set. Only five tests for a vector maximum and six tests for SSD efficiency were required to identify the SSD-efficient set.

## Concluding Remarks

The approach proposed in this article extends the work of Dybvig and Ross in three ways.

Their efficiency conditions were revised slightly to obtain conditions for SSD efficiency. Two properties of the efficient set for their perfect market case were modified to be consistent with the conditions for SSD efficiency and with problems which may include inequality constraints. The properties of the SSD-efficient set were exploited to develop a procedure for identifying it.

For the example considered above, the set of SSD-efficient mixtures is identical to the set of (single) Target MOTAD solutions. As noted earlier, this is not a general result. A more general result is the fact that the set of SSD-efficient mixtures includes rather diverse crop mixes. This underscores the importance of identifying the utility functions or risk preferences of relevant decision makers. More precise knowledge of risk preferences may help define an appropriate proper subset of the SSD-efficient mixtures. For example, it has been shown (McCamley and Kliebenstein 1986) that applying a restricted version of the generalized stochastic dominance criterion can reduce the size of the efficient set of mixtures.

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