Evaluating Farmland Investments Considering Dynamic Stochastic Returns and Farmland Prices

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This paper examines farmland investment decisions using a stochastic dynamic programming framework. Consideration is given to the dynamic, stochastic nature of farmland returns, linkages between farmland returns and farmland prices, and the effects of the above dynamic factors on a farm's financial structure. Optimal decisions to purchase or sell farmland are found for a central Illinois farm with high quality farmland. Sizes and debt distributions are then determined, given that the optimal decision rule is followed. Decisions from the dynamic programming model also are compared to a capital budgeting model.

Key words: farmland investments, risk, stochastic dynamic programming.

Land transactions have significant impacts on a farm's profitability and financial structure. Much research has analyzed various aspects of these impacts with emphasis given to financing firm growth. Using debt capital to finance firm growth requires an increase in leverage position which increases the firm's risk position (Barry, Hopkin, and Baker), alters the time pattern of cash flows (Ellinger, Barry, and Lins; Lee), reduces its liquidity (Barry and Baker), and may affect the optimal production organization (Baker). Alternative strategies for managing debt and equity capital also affect a firm's risk position (Held and Helmers; Hinman and Hutton), consumption patterns, and production decisions (Johnson and Boehlje).

These financing issues are important when considering farmland purchase or sale decisions. Other important factors include the stochastic, dynamic nature of farmland returns (Alston; Burt) and linkages between farmland returns and prices (Burt). Most previous studies either assume that future farmland returns and prices are known with certainty or that their distributions are known unconditionally.

A major objective of this paper is to analyze optimal farmland investment decisions considering the above two dynamic factors and their effects on a farm's financial structure. A stochastic dynamic programming (DP) model of a fully owned crop farm is conceptualized and then numerically solved to determine optimal farmland transactions for a central Illinois farm with high quality farmland. The DP results then are compared to those obtained from a traditional, static capital budgeting model (e.g., Barry, Hopkin, and Baker; Lee et al.). This comparison evaluates possible performance gains attributable to fully integrating the dynamic components into the analysis.

The crop farm is conceptualized in the next section. In the second section the conceptual model is used to specify a dynamic programming model. Numerical parameters for a central Illinois farm with high quality land are given in the third section. The final sections present results from the dynamic programming and capital budgeting models, considering their implications for investment analysis.

Crop Farm Model

The crop farm model is developed by dividing time into yearly periods. At the beginning of

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each year, a decision is made to either purchase or sell farmland (i.e., farmland investment decision). During the year, realizations of stochastic, dynamic returns and dynamic farmland prices occur. These realizations, along with the farmland investment decision, affect the farm's debt-to-asset position. Key elements of the model include relationships representing the stochastic, dynamic nature of farmland returns, dynamic farmland price movements, and the crop farm's financial structure.

Farmland returns are represented by direct returns per acre. Direct returns give returns before fixed factor payments and equal gross revenue minus variable costs (i.e., seed, chemical, fertilizer, machinery, and hired labor costs). The stochastic, dynamic nature of direct returns is captured by a first-order Markovian density function:

$$DR_{t+1} = f_1(DR_t, u_t), \tag{1}$$

where DR_t equals direct return in year t, and u_t is a random variable. The $f_1(\cdot)$ function gives the DR_{t+1} density function conditional on the DR_t level.

Farmland prices are determined by a deterministic relationship depending on the lagged direct return and two previous farmland prices:

$$P_{t+1} = f_2(DR_t, P_t, P_{t-1}). \tag{2}$$

This Markovian relationship is based on a modified version of a capitalization formula developed by Burt.

The remaining specifications reflect a fully owned farm's financial components. (The full ownership assumption is used to focus attention on farmland investment decisions.) Financial components are modeled by defining financial stocks and flows. Financial stocks describe the farm's asset and debt balances at the beginning of each year. Financial flows then change these balances during the year.

Financial Stocks of a Crop Farm

Definition of a crop farm's financial stocks begins with the standard accounting identity:

$$W_t = Assets_t - Debts_t, \tag{3}$$

where W_t is wealth (i.e., equity capital) at the beginning of year t. Assets and debts are divided into three categories leading to a standard accounting identity of:

$$W_t = P_t{*}OA_t + OFA(OA_t) + HFI_t, \tag{4}$$

where OA_t equals the number of owned acres, $OFA(\cdot)$ is a function giving the value of other farm assets, and HFI_t equals the holdings of financial instruments. The first two terms to the right of equation (4)'s equality sign give the value of farm assets. The first term, $P_t{*}OA_t$, equals the value of farmland while the second term, $OFA(OA_t)$, gives the value of other farm assets. Other farm assets are all assets other than farmland including production inventories, supplies, investment in growing crops, machinery, and buildings. Negative amounts of HFI_t represent debt used to finance farm assets. In addition, positive amounts of HFI_t represent holdings of financial assets when no debt is needed to finance financial asset holdings. This representation implies that an individual can hold portions of wealth in both farm and financial assets or expand farm asset holdings by reducing farm asset holdings and increasing debt holdings. Debt holdings modeled by HFI_t do not include operating debt. Operating debt is handled separately as discussed below. In addition, interest rates on debt and financial asset holdings vary according to relative holdings of financial instrument and farm assets.

These relative holdings are measured by a debt-to-farm-asset ratio (DFA_t):

$$DFA_t = -HFI_t/[P_t{*}OA_t + OFA(OA_t)]. \tag{5}$$

This ratio differs from commonly used debt-to-asset ratios. Positive amounts of DFA_t equal debt relative to total assets, the typical debt-to-asset ratio. Negative amounts represent financial asset holdings relative to farm asset holdings. Suppose, for example, that DFA_t equals .25. By rearranging equation (5)'s terms, the HFI_t value can be found to equal $-.25[P_t{*}OA_t + OFA(OA_t)]$. The negative value indicates that debt capital is used to finance farm assets. If, instead, DFA_t equals $-.25$, then HFI_t is a positive $.25[P_t{*}OA_t + OFA(OA_t)]$, indicating financial asset holding.

Using equation (5), the accounting identity in (4) is stated as:

$$W_t = [P_t{*}OA_t + OFA(OA_t)][1 - DFA_t]. \tag{6}$$

Wealth is summarized by three variables: the price of farmland (P_t), the number of owned acres (OA_t), and the debt-to-farm-asset ratio (DFA_t). During the year, the farmland price changes due to the Markovian farmland price relationship (equation (2)). Financial flows change the latter two variables.

Financial Flows of a Crop Farm

Farmland purchases or sales can occur at the beginning of a year. Variable DOA_t gives the number of acres purchased or sold, with positive and negative amounts respectively representing purchases and sales. Any 80-acre increment of farmland can be purchased or sold at the current farmland price (P_t). Farm size resulting from the decision equals:

$$(7) \qquad OA_{t+1} = OA_t + DOA_t.$$

Farmland purchases (sales) require an investment (disinvestment) in farm assets (INV_t) equaling:

$$(8) \quad INV_t = P_t{*}DOA_t + OFA(OA_{t+1}) - OFA(OA_t) + TC(DOA_t, P_t, DFA_t).$$

The ($P_t{*}DOA_t$) term represents the value of farmland purchased (sold) while $[OFA(OA_{t+1}) - OFA(OA_t)]$ equals the change in other farm asset holdings. The function $TC(\cdot)$ gives two types of transaction costs. First, a 1% surcharge is placed on the amount of newly acquired debt. Second, a 5% charge representing brokerage fees is placed on the value of farmland sales. Both of these values represent typical service fees and other costs associated with land transactions. Investment (disinvestment) yields new financial asset holdings ($CHFI_t$) equaling:

$$(9) \qquad CHFI_t = HFI_t - INV_t,$$

where INV_t is investment given by equation (8)

After farmland investment decisions are made, realization of before-tax income occurs. Before-tax income is defined as:

$$(10) \quad I_{t+1} = DR_{t+1}{*}OA_{t+1} - FC(OA_{t+1}) + i(DFA_t, INV_t, OA_{t+1})CHFI_t - OD(DFA_t, INV_t, OA_{t+1}),$$

where

(*a*) ($DR_{t+1}{*}OA_{t+1}$) gives the gross margin from owned acres. DR_{t+1} is a random variable whose distribution is given by equation (1). Thus, before-tax income also is a random variable.

(*b*) $FC(OA_{t+1})$ is a function giving nonland fixed costs. Fixed costs depend on farm size and include depreciation, hired labor, farm supplies, buildings and fence repair, utilities, and insurance. All fixed costs, including depreciation, are assumed to be cash costs.

(*c*) $i(DFA_t, INV_t, OA_{t+1})$ is a function giving the interest rate on holdings of financial assets. This function allows differing interest rates on positive and negative financial instrument holdings and an increasing interest cost structure for higher positive debt-to-farm-asset ratios. The $CHFI_t$ equals the holdings of financial assets after making farmland decisions. Thus, the $[i(DFA_t, INV_t, OA_{t+1})CHFI_t]$ term gives the returns from financial asset holdings or the interest costs incurred from nonoperating debt.

(*d*) $OD(DFA_t, INV_{t+1}, OA_{t+1})$ is a function giving the interest costs on operating debt. Operating debt equals cash requirements per acre times the number of acres farmed less any positive financial asset holdings. Operating debt is multiplied by the interest rate to determine interest costs. Only interest costs on operating debt are accounted for by $OD(\cdot)$ because actual operating expenses are accounted for in DR_{t+1}.

Tax payments and consumption withdrawals are defined by a flow of funds ($FLOW_{t+1}$) equation:

$$(11) \quad FLOW_{t+1} = I_{t+1} - TAX(I_{t+1}, INV_t, DOA_t) - C_t,$$

where I_{t+1} equals before-tax income given by equation (10), $TAX(\cdot)$ is a function defining federal and state tax liabilities, and C_t is a withdrawal for family living purposes. Note that consumption withdrawals are not treated as a decision variable. This treatment abstracts from economic and financial theory in which investment, financing, and withdrawal decisions are made jointly. Consumption is represented as a fixed withdrawal because this method models farmers' consumption patterns reasonably well (Davis, Mullen, and Bryant; Giraro, Tomek, and Mount).

Given INV_t and $FLOW_{t+1}$, the debt-to-farm-asset ratio at the end of the year (DFA_{t+1}) equals the beginning financial instrument holdings $[(P_t{*}OA_t + OFA(OA_t))DFA_t]$, plus investment ($INV_t$), less flow ($FLOW_{t+1}$). The resulting quantity is divided by the ending total farm asset value, $[P_{t+1}{*}OA_{t+1} + OFA(OA_{t+1})]$, to give:

$$(12) \quad DFA_{t+1} = \frac{[(P_t{*}OA_t + OFA(OA_t))DFA_t] + INV_t - FLOW_{t+1}}{P_{t+1}{*}OA_{t+1} + OFA(OA_{t+1})}.$$

Although equation (12) is a deterministic relationship, DFA_{t+1} is a random variable because $FLOW_{t+1}$ is a random variable.

The Dynamic Programming (DP) Model

Specification of the DP model requires one stochastic state variable—direct return (DR_t)—and four nonstochastic state variables—the previous two farmland prices (P_t and P_{t-1}), the number of owned acres (OA_t), and the debt-to-farm-asset ratio (DFA_t). State transition equations for the direct return, farmland price, owned acres, and debt-to-farm-asset ratio variables are given by equations (1), (2), (7), and (12), respectively. The decision variable is the number of acres to purchase or sell (DOA_t).

The presumed objective is to maximize the expected value of terminal after-tax wealth. Denoting the final year as T, after-tax wealth can be written as a function of the state variables:

$$(13) \quad V_T(DR_T, P_T, P_{T-1}, OA_T, DFA_T) = [OA_T*P_T + OFA(OA_T)](1 - DFA_T) - TC(OA_t, P_t) - ETAX(P_t, OA_t, TC(\cdot)),$$

where $V_T(\cdot)$ is the recursive objective function for year T, $TC(\cdot)$ gives the transaction costs on a total sale of farmland, and $ETAX(\cdot)$ is a function giving taxes on the total sale of farmland. Equation (13) leads to a general recursive equation of:

$$(14) \quad V_t(DR_t, P_t, P_{t-1}, OA_t, DFA_t) = \max_{DOA_t} E[V_{t+1}(DR_{t+1}, P_{t+1}, P_t, OA_{t+1}, DFA_{t+1})],$$

where $E[\cdot]$ is an expectations operator. The $V_{t+1}(\cdot)$ gives the expected value of after-tax wealth for each state variable level, assuming that optimal decisions are made.

Given the recursive equation and the state transition equations in (1), (2), (7), and (12), the maximization problem for an arbitrary year is:

$$(15\text{-a}) \quad V_t(DR_t, P_t, P_{t-1}, OA_t, DFA_t) = \max_{DOA_t} E(V_{t+1}(DR_{t+1}, P_{t+1}, P_t, OA_{t+1}, DFA_{t+1}),$$

subject to

$$(15\text{-b}) \quad DR_{t+1} = f_1(DR_t, u_t),$$

$$(15\text{-c}) \quad P_{t+1} = f_2(DR_t, P_t, P_{t-1}),$$

$$(15\text{-d}) \quad OA_{t+1} = OA_t + DOA_t,$$

$$(15\text{-e}) \quad DFA_{t+1} = \frac{[(P_t*OA_t + OFA(OA_t))DFA_t] + INV_t - FLOW_{t+1}}{P_{t+1}*OA_{t+1} + OFA(OA_{t+1})}.$$

This model is used to recursively derive optimal farmland investment decision rules—optimal decisions for all possible state variable values. Optimal decision rules converge after a sufficient number of periods have been solved for because returns are compounded and the state transition equations are stable between years (see Bellman for a proof).

Numerical Estimates of the State Transition Equations

To numerically solve the DP model, estimates were needed for the stochastic, Markovian direct return relationship (equation (15-b)), the Markovian farmland price relationship (equation (15-c)), and the various functions and parameters within the debt-to-farm-asset ratio state transition equation (equation (15-e)).

The Stochastic, Markovian Direct Return Relationship

A direct return series from 1954 through 1984 was constructed using data from Gallager and Green, the Illinois Department of Agriculture, the Agricultural Conservation Service, and the Illinois Farm Business Farm Management (FBFM) Association. Each yearly observation was deflated by the gross national product implicit price deflator using 1984 as the base. Examination of alternative time-series models suggested that a first-order autoregressive (AR(1)) structure adequately modeled the series' time dependent nature. Fitting a linear AR(1) form $[DR_t = \alpha_1 + \alpha_2 DR_{t-1}]$ yielded residuals that were not normally distributed as judged by the Bera-Jarque test statistic. Examination of the residuals suggested a log normal distribution and a natural logarithmic form $[\ln(DR_t) = \alpha_3 + \alpha_4 \ln(DR_{t-1})]$ was fit. Normality of these residuals could not be rejected.

The natural logarithmic form had good statistical properties: autocorrelation of the error term was rejected, and heteroskedasticity was rejected for both the time dimension and the direct return level. However, the 1972 and 1973 residuals were outside a two standard deviation band from zero. These years were associated with large increases in direct returns due, most likely, to changes in grain export conditions. Similar occurrences were judged highly unlikely. Therefore, a dummy variable for 1972 and 1973 was added to the equation. Resulting parameter estimates and standard errors (in parentheses) are:

$$(16)\quad \ln(DR_t) = \underset{(.4196)}{1.0330} + \underset{(.0795)}{.79962} \ln(DR_{t-1}) + \underset{(.0795)}{.42120} DUM,$$

where DUM is an intercept dummy for 1972 and 1973. This equation has 27 degrees of freedom, a .1324 standard error of estimate, and a .8134 adjusted R^2.

The Markovian Farmland Price Relationship

The theoretical foundation of the Markovian farmland price equation was provided by Burt. He developed an econometric capitalization model in which farmland price was determined by expectations of future farmland rents with previous rents serving as the basis. This model was estimated using direct returns as a proxy for rents and a farmland price series constructed by Reiss and Scott and modified by Burt. Nonlinear least squares was used to obtain estimates, such that estimates are only asymptotically efficient. Parameter estimates and standard errors for the 1960–84 period are:

$$(17)\quad \ln(P_t) = \underset{(.0168)}{.2302} + \underset{(.0167)}{.0503} \ln(DR_t) + \underset{(.0231)}{.0934} \ln(DR_{t-1}) + \underset{(.0269)}{1.6921} E(\ln(P_{t-1})) - \underset{(.0225)}{.8208} E(\ln(P_{t-2})) + \underset{(.2910)}{.8000} MA,$$

where $E(\ln(P_t)) = [\ln(P_t) - \ln(u_t)]$, u_t is a random error, and MA is a moving average error component. The coefficient on the MA term was fixed at .8000 due to upward bias associated with maximum likelihood estimates of this parameter (Sarghan and Bhargava). This equation has 18 degrees of freedom, a .0215 standard error of estimate, and a .9957 adjusted R^2.

The Markovian farmland price relationship was treated deterministically in the DP model. Possible biases from this treatment should not be large because the standard error of estimate (36 in dollar terms) was small compared to the size of the farmland price state increment ($150); that is, a single state increment contained about 95% of the probability.

As estimated, the farmland price relationship required two direct return (DR_t and DR_{t-1}) and two farmland price (P_t and P_{t-1}) state variables in the DP model. To reduce dimensionality, the DR_t was not included. Its coefficient was added to the coefficient of DR_{t-1}. Possible biases that could result from this modification were evaluated using a sequential forecasting analysis. Adding the two direct return variables together resulted in a worse prediction relationship; however, predictions generally fell within a farmland price interval.

The Debt-to-Farm-Asset Ratio State Transition Equation

Relationships and parameters within the debt-to-farm-asset ratio transition equation requiring numerical estimates were: (*a*) a function giving federal and state income tax liabilities ($TAX(\cdot)$); (*b*) functions giving other farm assets and fixed costs as a function of farm size ($OFA(\cdot)$ and $FC(\cdot)$); (*c*) a function giving interest rates on financial asset and debt holdings ($i(\cdot)$); (*d*) a function giving operating debt costs ($OD(\cdot)$); and (*e*) yearly consumption withdrawals (C_t).

Income tax liabilities were based on the 1988 federal and Illinois income tax codes. The federal tax code for a person who was married and filing jointly was used and contained four inflation-indexed marginal tax rates. Three percent was added to each tax rate to reflect expected Illinois tax requirements. Four exemptions were used in calculating deductions from income. In addition, social security taxes were included at a 12.3% rate on any farm income other than capital gains with a maximum social security tax of $5,166.

Other farm assets as a function of farm size were estimated using cross sectional FBFM data from 1983 and 1984. Results suggested that holdings of other farm assets were constant at $354 per acre for farm sizes greater than 500 acres. Similarly, fixed costs were found constant at $54 per acre for farm sizes greater than 500 acres.

Interest rates used in the model included a 3% yearly real rate of return on financial assets. Interest rates on long-term debt were based on the three-tier Federal Land Bank interest rates adopted by the St. Louis Farm Credit District in 1986. These tiers were approximated by:

$$(18)\qquad i = .055 + .13888 DFA^3$$

on debt-to-farm-asset ratios between 0 and .75. For debt-to-farm-asset ratios greater than .75, a .1189 interest rate was used.

Based on data from Gallager and Green, operating capital requirements were assumed to equal \$100 per acre. Total operating capital requirement equaled \$100 times the number of acres owned. Operating capital requirements decreased positive financial asset holdings. If positive financial asset holdings did not cover operating requirements, the remainder was financed with an operating note held for one-half year. A .075 interest rate was associated with any operating requirements exceeding financial asset holdings.

Relationships where consumption amounts depended on wealth and income levels were fit and were not significant. Consumption per year was assumed to be \$20,000.

Optimal Decisions from the DP Model

From the above numerical specification, optimal farmland investment decision rules were formulated using a numeric, value-iteration dynamic programming algorithm. This algorithm required discretizing the state and decision variables. Five direct return intervals were used which were zero, one, and two standard deviations from the asymptotic mean (i.e., \$125, \$150, \$175, \$205, and \$245). Twenty farmland price intervals ranged from \$1,000 to \$3,850 in \$150 intervals. Five lagged farmland price change intervals were used which were −\$300, −\$150, \$0, \$150, and \$300 from the current farmland price. Farm sizes ranged in 80-acre increments from 500 to 1,460 acres. A zero-acre farm size also was included to represent either farm bankruptcy or a decision to liquidate the farm. This resulted in a total of 14 farm size intervals. The debt-to-farm-asset ratio had 40 intervals which ranged in equal increments from 1 to −.5. The above state variable discretion yielded a total of 280,000 state increments. There were 14 farmland purchase (sell) decision alternatives for a beginning farm size, exactly matching the possible ending farm size increments.

Deriving optimal farmland investment decision rules began in the final year and proceeded recursively. In calculating each decision alternative's expected terminal wealth, the recursive objective function values were linearly interpolated because the ending farmland price and debt-to-farm-asset ratio values did not necessarily match the state interval midpoints. Optimal decisions were found for all state intervals associated with positive farm sizes. It was assumed that a farming operation would not be restarted if it had been liquidated. Therefore, optimal decisions were not calculated for zero-acre farm sizes. In addition, farm bankruptcy occurred when the debt-to-farm-asset ratio exceeded one and the farm was liquidated.

Optimal decision rules were generated until they converged. Convergence occurred by the tenth yearly stage, implying that the converged decision rule is applicable to all years up to the tenth year before the end of the planning horizon. If, for example, the planning horizon is 20 years, the converged decision rule is applicable for years one through 10.

Figure 1 shows a portion of the converged decision rule. It consists of five panels showing the optimal decisions for a 740-acre farm having a .25 debt-to-farm-asset ratio. Each panel's horizontal axis shows the current farmland price while the vertical axis gives the optimal number of acres to purchase (positive numbers) or sell (negative numbers). The vertical axis coordinate closest to the horizontal axis, labeled "Sell All," represents a decision to liquidate the farming operation. Each panel gives decisions for a fixed farmland price movement. A farmland price movement is defined as the difference between the current farmland price and the lagged farmland price (i.e., $P_t - P_{t-1}$). A line shows optimal decisions for a fixed direct return.

This figure is used to examine the relationships among optimal farmland investment decisions and direct returns, current farmland prices, and farmland price movements (i.e., comparative dynamics). The optimal amount of farmland to purchase (sell) increases (decreases) as the:

(*a*) Direct return increases. For example, optimal investment decisions for a \$0 farmland price movement and a \$1,900 current farmland price are shown in panel C. They range from a 160-acre sale for \$125 and \$150 direct returns to a 320-acre purchase for a \$245 direct return. Higher direct returns indicate higher expected direct returns in the near future (see equation 16) and higher expected farmland prices (see equation 17).

(*b*) Current farmland price decreases. See, for example, the dashed line in panel C which shows decisions for a \$175 direct return and a \$0 farmland price movement. Optimal decisions range from a 720-acre purchase for

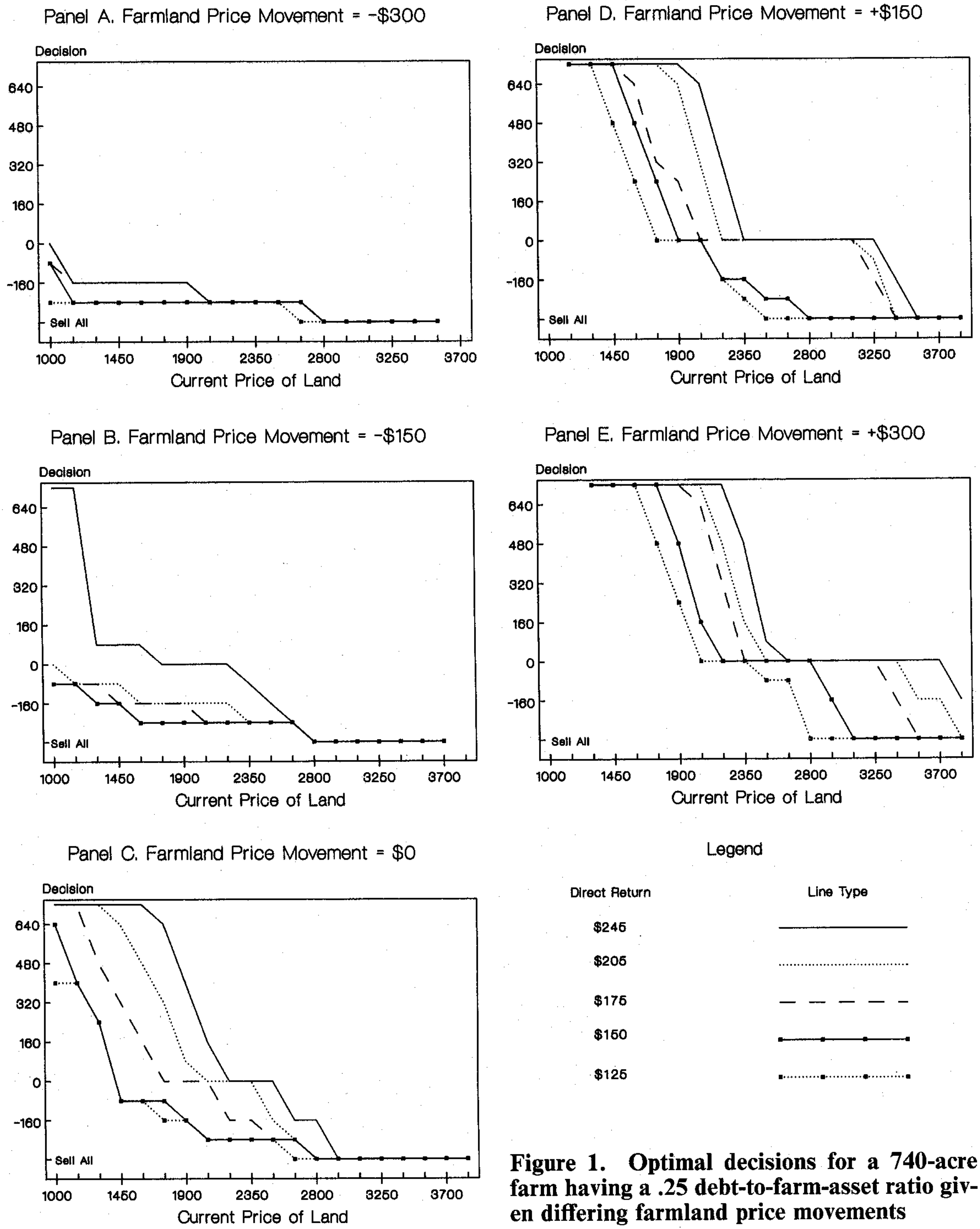

Figure 1. Optimal decisions for a 740-acre farm having a .25 debt-to-farm-asset ratio given differing farmland price movements

prices below $1,150 to a liquidation decision for current prices greater then $2,800. Higher farmland prices reduce the rate of return on farm assets. For example, a $175 direct return generates a lower return rate at a $1,900 farmland price as opposed to a $1,450 farmland price. (Note that this does not imply farmland prices are independent of returns. Returns enter into determination of farmland prices.)

(*c*) Farmland price movement increases. For

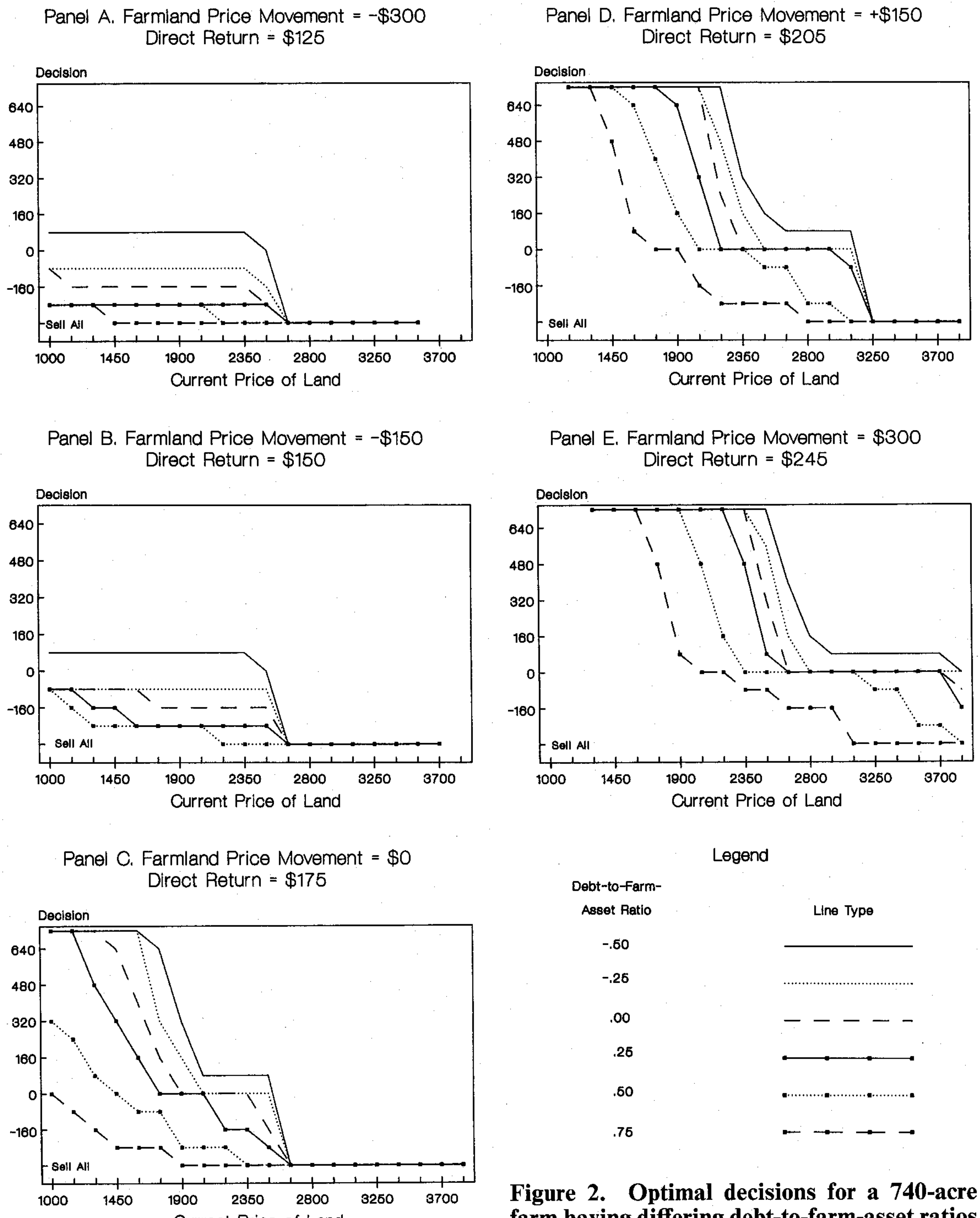

Figure 2. Optimal decisions for a 740-acre farm having differing debt-to-farm-asset ratios

example, at a $175 direct return and a $1,900 current farmland price, optimal decisions are a 240-acre sale for a −$300 farmland price movement (panel A), a zero-acre purchase at a $0 farmland price movement (panel C), and a 720-acre purchase at a $300 price movement (panel E). Higher farmland price movements increase incentives for owning farmland by increasing expected farmland prices in future years.

Optimal decisions also vary with differing debt-to-farm-asset ratios as shown in figure 2. Figure 2's axes are similar to those in figure 1. However, here each panel represents a fixed farmland price movement and a fixed direct return. The panels contain lines representing optimal decisions for differing debt-to-farm-asset ratios. Optimal decisions for lower debt-to-farm-asset ratios are greater than those for higher debt-to-farm-asset ratios. For example, optimal farmland investment decisions for $0 farmland price movement, $175 direct return, and $1,900 farmland price range from a 320-acre purchase for a −.50 debt-to-farm-asset ratio to a liquidation decision for a .75 debt-to-farm-asset ratio.

Higher beginning debt-to-farm-asset ratios and larger farmland investment decisions result in higher debt-to-farm-asset ratios. Three features within the model reduce incentives for maintaining high debt-to-farm-asset ratios. First, the increasing interest rate structure on higher positive debt-to-farm-asset ratios increases interest costs relative to farmland returns (see equation (18)). Second, the progressive income tax structure leads to a concave recursive objective function. This concavity resembles a cardinal utility function in a model containing risk aversion, thus reducing incentives for maintaining high debt-to-farm-asset ratios.

A third feature deals with the 5% transaction cost on farmland sales. When returns and farmland prices are rising, expected terminal wealth is increased by holding high debt-to-farm-asset ratios. However, if returns and farmland prices fall—which can occur with fairly high probability—high debt-to-farm-asset ratios decrease expected wealth levels. Farmland is sold when returns and prices are falling to avoid large decreases in terminal wealth. Larger sales occur for farms with higher debt-to-farm-asset ratios, increasing the farmland sales transaction costs for higher debt-to-farm-asset ratios. Thus, these costs decrease expected terminal wealth for high debt-to-farm-asset ratios.

Farm Sizes, Debt-to-Farm-Asset Ratios, and the Optimal Decision Rule

It is important to note that the optimal decisions presented above are for a point in time. Applying the optimal decision rule yearly will result in distributions of farm sizes and debt-to-farm-asset ratios. The conditional probability methods developed for DP models by Bellman and Howard were used to analyze these distributions. These methods require selecting a beginning state in an initial year. The optimal decision rule and state transition equations are then used to develop *ex ante* forecasts of distributions in future years.

Conditional probabilities were calculated using beginning direct return and farmland price rates resembling 1985 conditions: a $150 direct return, a $2,200 current farmland price, and a $2,500 lagged farmland price. The beginning farm size was 740 acres and five differing beginning debt-to-farm-asset ratios were used: .50, .25, .00, −.25, −.50. As reported on the first line of table 1, optimal decisions in the first year are to liquidate the farm when the debt-to-farm-asset ratio equals .5, sell farmland when the debt-to-asset ratio equals .25 or −.25, and purchase farmland when the debt-to-farm-asset ratio equals −.50.

The next eight lines of table 1 show conditional probabilities for farm size and debt-to-farm-asset ratio categories after five sequential applications of the optimal decision rule and the state transition equations (i.e., in 1990). Given a .25 beginning debt-to-farm-asset ratio, the probability of being out of farm is .0000, of having a 500- to 740-acre farm is .6346, of having an 820- to 1,140-acre farm is .2198, and of having a 1,200- to 1,460-acre farm is .1456. Debt-to-farm-asset ratio probabilities for no debt, .00 to .25, .25 to .50, and greater than .50 categories are .5518, .1044, .2983, and .0440, respectively. The last two lines show the mean values for farm size and the debt-to-farm-asset ratio means. The .25 beginning debt-to-farm-asset ratio has a mean farm size of 760 acres and a mean debt-to-farm-asset ratio of .0185.

In 1990, farms with lower beginning debt-to-farm-asset ratios tend to have larger farms. This is illustrated by means of the conditional farm sizes (see table 2). The conditional mean for the .25 beginning debt-to-farm-asset ratio is 756 acres, approximately the same size as in 1985. The remaining lower debt-to-farm-asset ratios grow over the five-year period. However, farmland purchases generally require little debt financing. The majority of the probability in1990 is in the no-debt category for all beginning debt-to-farm-asset ratios.

Table 1. Conditional Probabilities in 1990 (after Five Years) for a 740-Acre Farm Having Differing Debt-to-Farm-Asset Ratios in 1985

	Debt-to-Farm-Asset-Ratio in 1985				
	.50	.25	.00	−.25	−.50
Optimal Decision in 1985	−740	−240	−160	−80	+80
Farm Size Probabilities					
Out of farming[a]	1.0000	.0000	.0000	.0000	.0000
500 to 740 acres	.0000	.6346	.4641	.0865	.0000
820 to 1,140 acres	.0000	.2198	.1834	.4888	.4305
1,200 to 1,460 acres	.0000	.1456	.3524	.4247	.5695
Debt-to-Farm-Asset Ratio Probabilities					
no debt[b]	1.0000	.5518	.5615	.5641	.6368
.00 to .25	.0000	.1044	.0858	.3124	.3632
.25 to .50	.0000	.2983	.3527	.1235	.0000
>.50	.0000	.0440	.0000	.0000	.0000
Means[c]					
Farm Size (Acres)	NA[d]	760	969	1,108	1,236
Debt-to-Farm-Asset Ratio	NA[d]	.0185	−.0276	−.0775	−.1436

[a] The farm was either liquidated or became bankrupt over the five-year period.
[b] A debt-to-farm-asset ratio less than .00.
[c] Means were computed over positive farm sizes.
[d] Not applicable.

Comparisons of the DP and Capital Budgeting Model

In essence, the DP model is a capital budgeting model that considers all possible combinations of purchasing and selling farmland in current and future periods. As traditionally applied, however, static capital budgeting models have not considered future purchase and sell decisions. Not considering future decisions may lead to nonoptimal decisions if the current decision affects the cash flows of future decisions. A current purchase or sale impacts on future decisions' cash flow by changing debt financing requirements for future purchases. Moreover, most traditional capital budgeting models rep-

Table 2. Conditional Probabilities in 1995 (after Ten Years) for a 740-Acre Farm Having Differing Debt-to-Farm-Asset Ratios in 1985

	Debt-to-Farm-Asset-Ratio in 1985				
	.50	.25	.00	−.25	−.50
Farm Size Probabilities					
Out of Farming[a]	1.0000	.0474	.0539	.0626	.0701
500 to 740 acres	.0000	.1562	.0712	.0020	.0000
820 to 1,140 acres	.0000	.1336	.1176	.1258	.0688
1,200 to 1,460 acres	.0000	.6628	.7573	.8096	.8612
Debt-to-Farm-Asset Ratio Probabilities					
No debt[b]	1.0000	.1718	.1811	.4193	.9624
.00 to .25	.0000	.1577	.5881	.5770	.0376
.25 to .50	.0000	.6518	.2308	.0030	.0000
>.50	.0000	.0187	.0000	.0000	.0000
Means[c]					
Farm Size (Acres)	NA[d]	1,235	1,327	1,373	1,413
Debt-to-Farm-Asset Ratio	NA[d]	.2541	.1348	−.0155	−.1200

[a] The farm was either liquidated or became bankrupt over the ten-year period.
[b] A debt-to-farm-asset ratio less than .00.
[c] Means were computed over positive farm sizes.
[d] Not applicable.

resent all random variables, such as farmland returns, with their expected values. This substitution results in nonoptimal decisions if certainty equivalence requirements (Simon; Theil) are not met. These requirements are violated because of progressive income taxes (Taylor).

To account for these differences, a static deterministic capital budgeting (CB) model was constructed that differs from the DP model in only two respects. First, the CB model only considers investment decisions in the first year of the planning horizon. This differs from the DP model which considers decisions at the beginning of each year. Second, the CB model only uses the expected values of the direct return distribution in determining expected terminal wealth. This differs from the DP model which considers the entire direct return distribution.

As with the DP model, decision rules generated by the CB model converge after a sufficiently long planning horizon is specified. To find the converged decision rule, the CB model was solved for a one-year planning horizon, then a two-year planning horizon, and so on using the same state and decision variable discretion as used in the DP model. Convergence occurs by the tenth year. Converged decisions from the CB model are shown by the dotted lines in figure 3. In addition, optimal farmland investment decisions from the DP model are shown for comparison purposes. All decisions are for a 740-acre farm having a .25 debt-to-farm-asset ratio.

The CB model's decisions differ systematically from the DP model's optimal decisions. The CB model indicates larger farmland purchases than the DP model at low current farmland prices. Then the lines for the CB model's decisions cross the DP model and indicate larger farmland sales at higher current farmland prices. This phenomenon is illustrated clearly in panels A, B, and C. In panels D and E, the CB lines do not have an opportunity to cross the DP model's decisions due to the upper constraint on farm size.

The systematic differences suggest that the CB model generates decisions that are overresponsive relative to the DP model, resulting in a larger range of decisions. For example, given the farmland price movement and direct return shown in panel A, decisions from the CB model range from a 720-acre purchase to liquidation while the DP model's decisions range from a 240-acre sale to liquidation. Overresponsiveness occurs for two reasons. First, the CB model only considers decisions at the current time; thus farm size changes are presumed to occur only at that time. Second, the CB model treats a stochastic problem in a deterministic fashion. Deterministic treatment does not consider the entire Markovian direct return distribution and the effects that alternative direct return realizations have on future farmland prices and debt-to-farm-asset ratios. These factors can lead to overaggressive purchasing and selling strategies.

Following the overresponsive CB model's decisions may lead to reductions in wealth. To evaluate these reductions, 10-year conditional probabilities were calculated using the CB model's decisions. These probabilities were calculated using the same beginning state variable levels as those used in calculating conditional probability from the DP model, assuming that the CB model's converged decisions were successively applied in each year. Once the 1995 (i.e., after 10 years) conditional probabilities were found, the expected value of after-tax wealth was calculated. This value is conceptually similar to a balance sheet net worth figure.

Expected after-tax wealths in 1995 that result from following the DP and CB models' decision rules are shown respectively in the first two columns of table 3. In addition, reductions in after-tax wealth from following the CB model's decisions are given in the third column. The only debt-to-farm-asset ratio in which a reduction does not occur is the .5 debt-to-farm-asset category. In this case, both models indicate that the optimal decision in the first years is to liquidate the farming operation. Except for this debt-to-farm-asset ratio, significant reductions in expected wealth are incurred by following the CB model's decisions. These reductions can exceed $200,000.

Concluding Comments

The DP model was solved for a fairly restricted farm type and geographical area. However, this model represents a general methodology for analyzing factors inherent in most agricultural and nonagricultural land investment possibilities, namely, land returns that exhibit stochastic, dynamic structures and land prices that are linked to realized returns. As illustrated by the capital budgeting model, not accounting

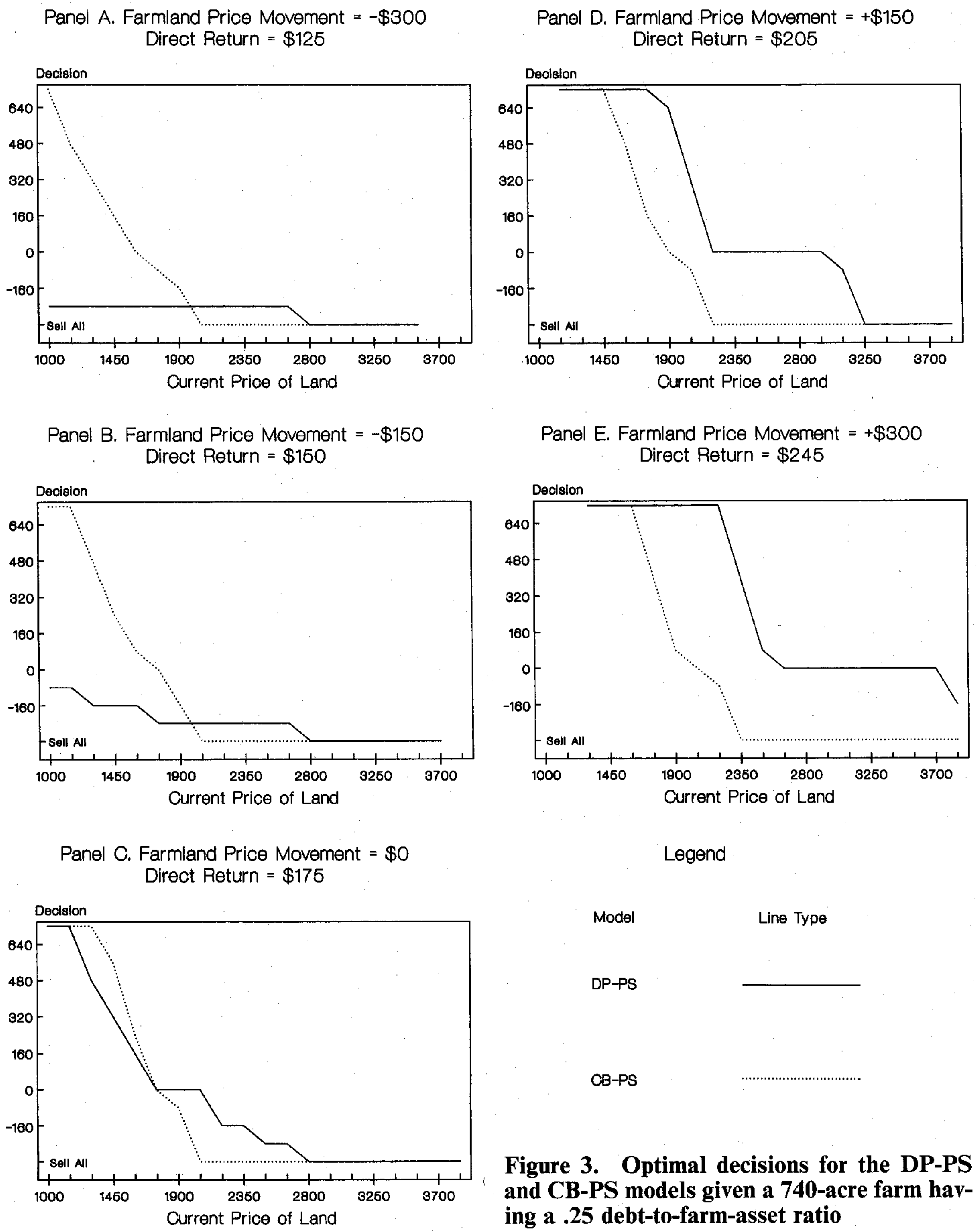

Figure 3. Optimal decisions for the DP-PS and CB-PS models given a 740-acre farm having a .25 debt-to-farm-asset ratio

for these factors may lead to nonoptimal decisions.

The comparison of the DP and CB models also suggests features desirable in an analytical and numerical optimization framework for analyzing land investments: the ability to consider future investment decisions and the ability to incorporate distribution of random variables. The former feature can be handled by many frameworks including multiperiod lin-

Table 3. Expected Wealths after Ten Years from the DP and CB Models (in 1984 Dollars)

Initial Debt-to-Farm-Asset Ratio	DP Model	CB Model	Wealth Reduction[a]
	(dollars)		
.50	1,940,080	1,940,080	0
.25	1,752,716	1,568,668	184,048
.00	2,323,062	2,107,458	215,604
−.25	2,828,857	2,642,310	186,547
−.50	3,383,204	3,264,556	118,648

[a] The DP model's wealth minus the CB model's wealth.

ear programming models, multiperiod quadratic programming models, stochastic linear programming models, and multiperiod minimization of total absolute deviations models. However, these frameworks either do not consider uncertainty or rely on certainty equivalence properties to incorporate uncertainty. This reliance may cause nonoptimal decisions. Another possibility is simulation models; however, finding optimal decisions with a simulation model requires an exhaustive search which is not efficient (Bellman). Optimal control theory is another possibility; however, at its current style of development, finding optimal solutions is difficult (Whittle).

This discussion does not suggest that dynamic programming is the only alternative. Rather, the point is that if the emphasis of the research is on dynamic factors, dynamic programming can incorporate these factors without relying on certainty equivalence requirements or any assumption concerning functional forms and types of distributions. A price may have to be paid for this flexibility. Dynamic programming models tend to require large amounts of time and may not be able to incorporate as much "detail" as other alternatives.

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