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Irene Valsecchi

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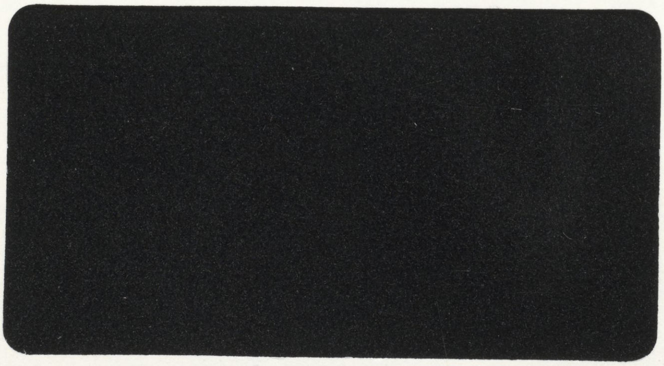

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JOB ASSIGNMENT AND PROMOTION[1]

Irene Valsecchi

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1. INTRODUCTION

In most economic models of promotion, either the worker or the firm are eventually endowed with perfect information about the agent's ability. Career is the device through which either the worker is induced to self-reveal his talent, or the firm shields its work-force from competition in labour market demand. Some recent empirical works on the internal organisation of firms (Lazear 1992, Baker-Gibbs-Holmstrom 1994, Baker-Holmstrom 1995) suggest that data reveal a very distinct and stable hierarchy with clearly identified career ladders, employees advance along a few well-defined paths, usually one at a time, and there are essentially no demotions.

Trying to set up a framework consistent with the empirical evidence, this paper tries a different approach: leaving aside issues of moral hazard, it is concerned with the case in which information on the individual agent's ability is imperfectly acquired through experiments which consist in assigning workers to jobs and recording performance in the selected tests. Past performance feeds the updating of beliefs about the individual agent's ability.

Intuitively, the idea is that each job is characterised by some probability of failure that inversely depends on the agent's unknown ability. Job i will be "easier" than job j if the expectation of good performance in job i is higher than that in job j. However, observed good performance in job i will not signal high ability as strongly as observed good performance in job j would do. In this way, optimal job assignment decisions and, consequently, career paths will have to be intertemporally consistent because current production affects what can be expected from the agent next period, thereby constraining upgrading along the job ladder.

The paper relates preliminary work on two basic issues. The first one is the choice of sequences of jobs. That is, under the assumption that there exists a variety of jobs the firm can assign the agent to, the paper sets out conditions that have to be satisfied in order to make promotion, i.e. upgrading from easier jobs to more difficult jobs, feasible. The paper shows that effort in the current period can reduce expected output in subsequent periods. It provides a reinterpretation of Peter's principle, proving that promotion may not be feasible if the agent's previous performance solves all the uncertainty on the outcomes of the job the agent is promoted to. Finally, the paper considers the question of demotion and shows that the more keen a firm is on promoting his work-force, the lower the probability of demotion is.

The second issue focuses on the choice of alternative jobs. That is, under the assumption that performance in many jobs at the current period can be a pre-requisite for promotion in later periods, the paper works out criteria of optimal assignment in the current period. The paper shows that, under some conditions, easier jobs can be preferred to more difficult jobs as screening mechanism.

2. BASIC SET UP

In order to build up a simple framework, I will assume that there do not exist spillovers across jobs performed by different agents. The basic case is so that of a single agent whose work can generate output. The case is not unfamiliar in the economic literature on promotion.

Definition 1
one job is an activity which can produce output if performed by one agent.

As stated in the Introduction, the paper is concerned with situations in which information on the agent's ability is only imperfectly acquired through time. In order to make performance reveal the agent's ability only imperfectly, a simple assumption is that output can take only two values, and that the probability of success in any job is increasing in ability.

Assumption 1: Output
- there exists a set I of feasible jobs;
- the output of job i, Y_i, can be either high, H_i, with probability $p(H_i)$, or low, L_i, with probability $[1-p(H_i)]$, where $H_i > 0$ and $L_i = 0$[1];
- $p(H_i)$ depends on two arguments: ability μ and a choice variable e, where $e \in [e_l, e_h] \in R^+$, so that $p(H_i)$ can be written as $p(H_i|e,\mu)$[2]. In particular, is continuous, monotonically increasing and concave[3].

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[1] Job i will not be equivalent to job j if either $p(H_i) \neq p(H_j)$ or $H_i \neq H_j$. The assumption that L_i is equal to zero for any i in I could be relaxed and it would result in making the outcome of any sub-optimal job assignment more disruptive.

Assumption 2: Information

- at period t1, the probability density function (p.d.f.) of ability μ, $\xi(\mu)$, is public information, where $\mu \in [\mu_l, \mu_h]$ and $0 < \mu_l < \mu_h < \infty$; $X(\mu)$ denotes the distribution function (d.f.) of μ, i.e. $X(\mu) = \int_{\mu_l}^{\mu} \xi(\tilde{\mu}) d\tilde{\mu}$;
- $\xi(\mu)$ is continuous on $[\mu_l, \mu_h]$;
- at any other period tn (with n>1), the posterior p.d.f. of μ is public information;
- no private information ever exists.

Throughout the paper it will always be assumed that the firm, and not the agent, is entitled to command over job assignment. This assumption can be justified on twofold grounds: first, promotion is an issue when labour is hired and the employed agents have little authority in selecting the tasks they are required to perform beyond the power of exit; second, the paper will not deal with incentive problems that make the initial distribution of rights between the parties of a relationship drive most of the results of the analysis.

Assumption 3: Organisation

the firm is in charge of every assignment decision through which an agent is paired to one job i out of I at the beginning of every period and for the length of the entire period.

3. CHOICE OF SEQUENCES OF JOBS

The purpose of this section is to arrive at consistent definitions of promotion and demotion, and inquire which conditions must be satisfied in order to make these events occur.

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[2] By definition $p(H_i|e,\mu)$ is always non-negative and $p(H_i|e_h,\mu_h) \le 1$.

[3] It follows that $(\partial p(H_i|e,\mu)/\partial\mu)$ is continuous and positive on $[\mu_l, \mu_h]$, $(\partial^2 p(H_i|e,\mu)/\partial\mu^2)<0$, $(\partial p(H_i|e,\mu)/\partial e)>0$, and $(\partial^2 p(H_i|e,\mu)/\partial e^2)<0$.

3.1. UPGRADING

At t1 the expected output of job i will be:

$$E(Y_i) = H_i E(p(H_i)) = H_i \int_{\mu_l}^{\mu_h} p(H_i|e,\tilde{\mu})\xi(\tilde{\mu})d\tilde{\mu}$$

At the end of t1, the posterior d.f. of μ, $X(\mu|H_i)$, will be:

$$X(\mu|H_i) = \frac{\int_{\mu_l}^{\mu} p(H_i|e,\tilde{\mu})\xi(\tilde{\mu})d\tilde{\mu}}{\int_{\mu_l}^{\mu_h} p(H_i|e,\tilde{\mu})\xi(\tilde{\mu})d\tilde{\mu}}$$

Since $p(H_i|e,\mu)$ is monotonic by Assumption 1, $\xi(\mu|H_i)$ cannot be equal to $\xi(\mu)$. Indeed, the observation of output in job i will not alter the ex-ante uncertainty over ability only if $p(H_i|e,\mu)$ is constant over μ[4].

It can be shown that $X(\mu)$ is always greater than $X(\mu|H_i)$ in (μ_l,μ_h).

Proposition 1

If H_i is the outcome of job i at t1, the expected output of any job j in I, Y_j, at t2 will be greater than the expected output of the same job j at t1. That is:

$$\int_{\mu_l}^{\mu_h} p(H_j|e,\tilde{\mu})p(H_i|e,\tilde{\mu})dX(\tilde{\mu}|H_i)d\tilde{\mu} > \int_{\mu_l}^{\mu_h} p(H_j|e,\tilde{\mu})dX(\tilde{\mu})d\tilde{\mu}$$

Proof:
applying the mean value theorem for integrals gets:

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[4] In that case, $\xi(\mu|H_i) = \dfrac{k\xi(\mu)}{\int_{\mu_l}^{\mu_h} k\xi(\tilde{\mu})d\tilde{\mu}} = \xi(\mu)$, where $k = p(H_i|e,\mu)$.

$$\int_{\mu_l}^{\mu_h} p(H_i|e,\tilde{\mu})\xi(\tilde{\mu}|H_i)d\tilde{\mu} = \int_{\mu_l}^{\mu} p(H_i|e,\tilde{\mu})\xi(\tilde{\mu})d\tilde{\mu} + \int_{\mu}^{\mu_h} p(H_i|e,\tilde{\mu})\xi(\tilde{\mu})d\tilde{\mu} =$$

$$= p(H_i|e,\mu_-)\int_{\mu_l}^{\mu} \xi(\tilde{\mu})d\tilde{\mu} + p(H_i|e,\mu_+)\int_{\mu}^{\mu_h} \xi(\tilde{\mu})d\tilde{\mu}$$

where $\mu \in (\mu_l, \mu_h)$, $\mu_- \in (\mu_l, \mu)$ and $\mu_+ \in (\mu, \mu_h)$. The posterior d.f. of μ, $X(\mu|H_i)$, will first-order stochastically dominate the ex-ante d.f. of μ, $X(\mu)$ if:

$$X(\mu) > X(\mu|H_i) \qquad \text{for any } \mu \in (\mu_l, \mu_h)$$

But the last inequality always holds since $[p(H_i|e,\mu_+) > p(H_i|e,\mu_-)]$ being $p(H_i)$ monotonically increasing in μ by Assumption 1[5].
Q.E.D.

Lemma 1

$$E(Y_j|L_i) < E(Y_j)$$

Lemma 2

If H_i is the outcome of job i at tn, the expected output of any job j in I, Y_j, at the beginning of t(n+1) will be greater than the expected output of the same job j at the beginning of tn.

In moral hazard contexts, it is usually expected that effort will improve performance. Moreover, it is often assumed that there can be some substitutability between effort and ability in achieving some set standard of performance. If the choice variable e is interpreted as effort, then it can be proved that increasing effort in the current job will lower expected output in all other subsequent job assignments.

Proposition 2

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[5] If $p(H_i)$ were monotonically decreasing in μ, then: $X(\mu|H_i) < X(\mu)$.

higher e in job i at t1 will decrease the expected output of any job j in I, Y_j, at the beginning of t2 if the marginal productivity of the choice variable e in job i decreases with ability. That is:

$$\frac{\partial X(\mu|H_i)}{\partial e} > 0 \quad \text{for any } \mu \text{ if } \quad \frac{\partial^2 p(H_i|e,\mu)}{\partial\mu\partial e} < 0$$

Proof:

$$-\frac{\partial\left[\int_{\mu_l}^{\mu_h} p(H_i|e,\tilde{\mu})\xi(\tilde{\mu})d\tilde{\mu}\right]}{\partial e} = \frac{\partial p(H_i|e,\mu_-)}{\partial e}\left[\int_{\mu_l}^{\mu} \xi(\tilde{\mu})d\tilde{\mu}\right] + \frac{\partial p(H_i|e,\mu_+)}{\partial e}\left[\int_{\mu}^{\mu_h} \xi(\tilde{\mu})d\tilde{\mu}\right]$$

where $\mu \in (\mu_l, \mu_h)$, $\mu_- \in (\mu_l, \mu)$ and $\mu_+ \in (\mu, \mu_h)$.

$$\frac{\partial X(\mu|H_i)}{\partial e} > 0 \qquad \text{implies:} \qquad \frac{p(H_i|e,\mu_+)}{p(H_i|e,\mu_-)} > \frac{\dfrac{\partial p(H_i|e,\mu_+)}{\partial e}}{\dfrac{\partial p(H_i|e,\mu_-)}{\partial e}}$$

the last inequality always holds because:

$$\frac{p(H_i|e,\mu_+)}{p(H_i|e,\mu_-)} > 1 \quad \text{by Assumption 1}$$

$$\text{and} \quad \frac{\dfrac{\partial p(H_i|e,\mu_+)}{\partial e}}{\dfrac{\partial p(H_i|e,\mu_-)}{\partial e}} < 1 \quad \text{if} \quad \frac{\partial^2 p(H_i|e,\mu)}{\partial\mu\partial e} < 0$$

Q.E.D.

It has been shown that performance in the current job will affect the posterior d.f. of ability. Now, the word "promotion" has to be given an unequivocal meaning. In single-job models, promotion is an increase in the agent's wage. In multi-jobs models, promotion denotes not only a change in job assignment, but also some sort of "upgrading" from an easier job to a more difficult job. The idea of "job i being easier than job j" can have two different representations: i) the probability of success in job i is higher than the probability of success in job j; ii) the probability of failure in job i is less sensitive to changes in ability than the probability of success in job j. In order to retain both those different interpretations of the degree of facility of a job, the following definitions are needed:

Definition 2
say that job i is slacker[6] than job j if:

$$p(H_i|e,\mu) > p(H_j|e,\mu) \qquad \forall\mu$$

Definition 3
say that job j is more selective than job i if:

$$\frac{\partial\left[\dfrac{p(H_i|e,\mu)}{p(H_j|e,\mu)}\right]}{\partial\mu} < 0$$

In this paper "ability" is a general capacity to deliver output. So, in order to avoid the complication of heterogeneous attitudes to jobs, it will be assumed that, if the probability of high output in job i is higher than the probability of high output in job j when ability is at its lowest level, then good performance in job i will always be a more likely outcome than good performance in job j whatever the agent's ability.

Assumption 4
if the probability of high output in job i is higher than the probability of high output in job j for μ_1, then job i will be slacker than job j.

Given job i and j, the probability of high output in job i can be written as:

$$p(H_i|e,\mu) = p(H_j|e,\mu) + a_i(\mu)$$

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[6] That definition is adapted from Sah-Stiglitz (1986).

where $a_i(\mu)$ is monotonic in μ from Assumption 4. If $a_i(\mu_l) > 0$, job i will be slacker than job j. According to Definition 3, job j will be more selective than job i if the following holds:

$$\frac{\partial a_i(\mu)}{\partial \mu} p(H_j|e,\mu) - \frac{\partial p(H_j|e,\mu)}{\partial \mu} a_i(\mu) < 0$$

Hence, whenever $a_i(\mu_l) > 0$ and $\frac{\partial a_i(\mu)}{\partial \mu} \leq 0$, job i will be less selective than job j. When jobs i and j are equally selective, then:

$$a_i(\mu) = mp(H_j|e,\mu)$$

where m is a constant.

Now, the issue is to set up criteria that must be satisfied in order to have situations in which the agent's job assignment can change through time reasonably. That is, if the expected output in job i is higher than the expected output in job j at t, under which conditions may job j be an output-maximising job assignment at t+1?

Proposition 3

$\dfrac{\int_{\mu_l}^{\mu_h} p(H_i|e,\tilde{\mu})dX(\tilde{\mu})d\tilde{\mu}}{\int_{\mu_l}^{\mu_h} p(H_j|e,\tilde{\mu})dX(\tilde{\mu})d\tilde{\mu}}$ will decrease (stay constant/increase) as $X(\mu)$ improves in the sense of first order stochastic dominance if job j is more selective (equally selective/less selective) than job i.

Proof:

consider a first order stochastically dominated d.f. $X'(\mu)$. Then:

$$X'(\mu) = X(\mu) + K(\mu) \quad \text{where} \quad K(\mu) = \begin{cases} 0.\text{if.either}.\mu=\mu_l.\text{or}.\mu=\mu_h \\ >0.\text{otherwise} \end{cases}$$

The sign of the following expression:

$$\frac{\int_{\mu_1}^{\mu_h} p(H_i|e,\tilde{\mu})dX'(\tilde{\mu})d\tilde{\mu}}{\int_{\mu_1}^{\mu_h} p(H_j|e,\tilde{\mu})dX'(\tilde{\mu})d\tilde{\mu}} - \frac{\int_{\mu_1}^{\mu_h} p(H_i|e,\tilde{\mu})dX(\tilde{\mu})d\tilde{\mu}}{\int_{\mu_1}^{\mu_h} p(H_j|e,\tilde{\mu})dX(\tilde{\mu})d\tilde{\mu}}$$

will be the same of:

$$-\int_{\mu_1}^{\mu_h} \frac{\partial a_i(\tilde{\mu})}{\mu} K(\tilde{\mu})d\tilde{\mu} \int_{\mu_1}^{\mu_h} p(H_j|e,\tilde{\mu})dX(\tilde{\mu})d\tilde{\mu} + \int_{\mu_1}^{\mu_h} \frac{\partial p(H_j|e,\tilde{\mu})}{\mu} K(\tilde{\mu})d\tilde{\mu} \int_{\mu_1}^{\mu_h} a_i(\tilde{\mu})dX(\tilde{\mu})d\tilde{\mu}$$

which will be positive if job j is more selective than job i, zero if job j is as selective as job i, and negative if job j is less selective than job i.
Q.E.D.

Consequently, if job i is performed at some period t, there will be:
a) no possible upgrading to any other job j in subsequent periods if job i is slacker and as selective as job j;
b) no possible upgrading to any other job j in subsequent periods if job i is slacker and more selective than job j;
c) possible upgrading to any other job j in subsequent periods if job i is slacker and less selective than job j.
Then, the following definition of promotion can be adopted:

<u>**Definition 4**</u>
promotion is an upgrading from a less selective job to a more selective job.

The argument of Peter's principle is that, since workers get promoted by being assigned to different jobs, they go up the career ladder till they reach their first level of incompetence. In the framework of analysis set up in this paper, it has already made clear that if performance in job i did not reduce the uncertainty surrounding the agent's ability at all, then promotion could not occur. On the other hand, it can be proved that, if performance in job i did solve all the uncertainty on ability, then again job i could not lead to promotion to job j.

<u>**Definition 5**</u>

say that output in job i is a perfect signal for job j when H_i is feasible only if ability belongs to some subinterval such that the probability of H_j is constant for any ability level in that subinterval and zero otherwise.

<u>Proposition 4</u>
if output in job i is a perfect signal for job j, then job i cannot be a pre-requisite for promotion to job j.
Proof:
if job i is a perfect signal for job j, then job i will be more selective than job j. At the same time, if job i is performed at some period t, it must be true that:

$$E\left[H_i \middle| X(\mu)\right] > E\left[H_j \middle| X(\mu)\right]$$

Hence:

$$\frac{E\left[H_i \middle| X(\mu|H_i)\right]}{E\left[H_j \middle| X(\mu|H_i)\right]} > \frac{E\left[H_i \middle| X(\mu)\right]}{E\left[H_j \middle| X(\mu)\right]}$$

Q.E.D.

<u>Proposition 5</u>
if there exist an ability level μ^* such that:

$$E(Y_i) > E(Y_j) \qquad \text{and} \qquad E(Y_i|e,\mu^*) = E(Y_j|e,\mu^*)$$

then, if job i is less selective than job j, the following will hold:

$$E(Y_i|e,\mu \geq \mu^*) < E(Y_j|e,\mu \geq \mu^*)$$

Proof:

$$\int_{\mu^*}^{\mu_h} p(H_i|e,\tilde{\mu})\xi(\tilde{\mu})d\tilde{\mu} = p(H_i|e,\mu_+)\int_{\mu^*}^{\mu_h} \xi(\tilde{\mu})d\tilde{\mu}$$

$$\int_{\mu^*}^{\mu_h} p(H_j|e,\tilde{\mu})\xi(\tilde{\mu})d\tilde{\mu} = p(H_j|e,\mu'_+)\int_{\mu^*}^{\mu_h} \xi(\tilde{\mu})d\tilde{\mu}$$

where: $\mu_{+}, \mu'_{+} \in (\mu^{*}, \mu_{h})$. Given: $\frac{H_j}{H_i} = \frac{p(H_i|e,\mu^{*})}{p(H_j|e,\mu^{*})}$, then:

$$\frac{\int_{\mu^{*}}^{\mu_h} p(H_i|e,\tilde{\mu})\xi(\tilde{\mu})d\tilde{\mu}}{\int_{\mu^{*}}^{\mu_h} p(H_j|e,\tilde{\mu})\xi(\tilde{\mu})d\tilde{\mu}} = \frac{p(H_i|e,\mu_{+})}{p(H_j|e,\mu'_{+})} < \frac{p(H_i|e,\mu^{*})}{p(H_j|e,\mu^{*})}$$

since job i is less selective than job j.
Q.E.D.

3.2. DOWNGRADING

There exists empirical evidence suggesting that demotion is always an unlikely case to occur. The purpose of this section is to formalise the case of demotion in a simple three period model with only two feasible jobs and two ability levels.

Assumption 5

- the agent can be active only up to three periods
- there exists only two jobs, i and j: job i is slacker and less selective than job j
- there exists only two ability levels, μ_l and μ_h with $\mu_l < \mu_h$
- $E(Y_i) > E(Y_j)$ at t1.

According to Assumption 5, the probability of high output in each job can be written as follows:

$$p(H_i|\mu_l) = p_{il} > p_{jl} = p(H_i|\mu_l) \qquad p(H_i|\mu_h) = p_{ih} > p_{jh} = p(H_j|\mu_h)$$

$$p_{il}p_{jh} - p_{ih}p_{jl} > 0$$

Accordingly to Definition 5 and Assumption 5, promotion will refer to the case in which the agent is assigned to job j in the current period after a previous assignment to job i. On the contrary, "demotion" will refer to the case in which the agent is assigned to job i in the current period after a previous assignment to job j. That is:

Definition 6

demotion is the downgrading from a more selective job to a less selective job.

Given Assumption 5, at time t1 there will be no job assignment problem: the agent is required to perform job i. However at t1 the firm can choose which career path the worker may have in subsequent periods. The range of career profiles is the following:

a) no promotion at any time;

b) promotion at t3 only if high output has been achieved in job i both at t1 and t2;

c) promotion at t2 if high output has been achieved in job i at t1, demotion at t3 if low output has been scored in job j at t2;

d) promotion at t2 if high output has been achieved in job i at t1, no demotion at t3.

At t1 expected output under each policy is concave in the prior probability of high ability. Policy d), the promotion-and-no-demotion case, can immediately be ruled out because, from Assumption 5, $\xi(\mu_h)$ will have to be lower than the threshold level beyond which policy d) produces higher expected outputs than policy c) does.

Under policy c), at t2 the probability of demotion, q_2, will be:

$$q_2 = (1 - p_{jl})p_{il} + \left[(1 - p_{jh})p_{ih} - (1 - p_{jh})p_{il}\right]\xi(\mu_h)$$

while at t3, the probability of demotion, q_3, will be:

$$q_3 = \frac{(1 - p_{jl})p_{il} + \left[(1 - p_{jh})p_{ih} - (1 - p_{jh})p_{il}\right]\xi(\mu_h)}{p_{il} + \left[p_{ih} - p_{il}\right]\xi(\mu_h)}$$

Proposition 6

the higher expected output under policy c) is, the lower the probability of demotion will always be. The higher the probability of promotion at t2 is, the lower the probability of demotion at t3 will be.

Proof:

policy b) will always produce higher expected output than policy c) if the prior probability of high ability is below the following threshold level:

$$\xi^{+}(\mu_h) = \frac{p_{il}H_i - p_{jl}H_j}{(p_{jh} - p_{jl})H_j - (p_{ih} - p_{il})H_i}$$

From Assumption 5, the prior probability of high ability cannot be higher than the following threshold level:

$$\xi^{\bullet}(\mu_h) = \frac{p_{il}(1 + p_{jl} - p_{il})(p_{il}H_i - p_{jl}H_j)}{\left[(1 + p_{jh} - p_{ih})p_{ih}p_{jh} - (1 + p_{jl} - p_{il})p_{il}p_{jl}\right]H_j - \left[(1 + p_{jh} - p_{ih})p_{ih}^2 - (1 + p_{jl} - p_{il})p_{il}^2\right]H_i}$$

It follows that:

- $q_2 = 0$ if $\xi(\mu_h) \leq \xi^{\bullet}(\mu_h)$,

$$q_2[\xi^{*}(\mu_h)] \geq q_2 > q_2[\xi^{+}(\mu_h)], \quad \frac{\delta q_2}{\delta\xi(\mu_h)} < 0, \quad \frac{\delta^2 q_2}{\delta\xi^2(\mu_h)} = 0$$

- $q_3 = 0$ if $\xi(\mu_h) \leq \xi^{\bullet}(\mu_h)$,

$$q_3[\xi^{\bullet}(\mu_h)] \geq q_3 > q_3[\xi^{+}(\mu_h)], \quad \frac{\delta q_3}{\delta\xi(\mu_h)} < 0, \quad \frac{\delta^2 q_3}{\delta\xi^2(\mu_h)} > 0$$

Q.E.D.

Promotion and demotion are two faces of the same coin. From Proposition 4, upgrading can make sense only if the job at which the agent is promoted to still shows some non-zero probability of failure. At the same time, the firm can always choose to delay promotion instead of going for one of the more extreme policies like early upgrading versus no upgrading. Promotion will increase expected output only if the expectation of downgrading is sufficiently low. In a way, it would seem that the more keen firms are in promoting people, the less likely the occurrence of demotion will have to be.

4. CHOICE OF ALTERNATIVE JOBS

Up to now, the paper focused on how to switch from one job in the current period to another job in the next periods. Instead, this section is devoted to the case in which the firm can choose among many different jobs for the current period, all of the them delivering the same expected output. So optimal job assignment at the current period will depend on its implications for optimal job assignment in next periods.

Assumption 6

- the agent is active for only two periods;
- the set of jobs I is so composed:

i) there exists one job z such that:

$$E(Y_z) < E(Y_i) \qquad \forall i \in I, i \neq z \quad \text{at t1}$$

$$E(Y_z|H_i) > E(Y_i|H_i) \qquad \forall i \in I, i \neq z \quad \text{at t2}$$

ii) all the other jobs are such that:

$$E(Y_i) = E(Y_j) \qquad \forall i, j \in \{I \setminus z\} \quad \text{at t1}$$

Proposition 7

for any two job, i and j in $\{I\setminus z\}$, if job i is slacker than job j, then:

$$X(\mu|L_i) > X(\mu|L_j)$$

If, in addition, job j is more selective (equally selective/less selective) than job i:

$$X(\mu|H_i) > X(\mu|H_j)$$

$$[X(\mu|H_i) = X(\mu|H_j); \quad X(\mu|H_i) < X(\mu|H_j)]$$

Proof:
when job i is slacker than job j, then:

$$-p(H_i|e,\mu_-)[1 - p(H_j|e,\mu_+)] + p(H_i|e,\mu_+)[1 - p(H_j|e,\mu_-)] > 0$$

where $\mu_- \in (\mu_l, \mu)$ and $\mu_+ \in (\mu, \mu_h)$.
The sign of the following expression:

$$p(H_i|e,\mu_-)p(H_j|e,\mu_+) - p(H_i|e,\mu_+)p(H_j|e,\mu_-)$$

will be positive if job i is less selective than job j, zero if job is as selective as job j, and negative is job i is more selective than job j
Q.E.D.

Proposition 8
whatever the job assignment at t1, if a failure has occurred at t1, there will a unique job in {I\z} that maximises expected outputs at t2 if jobs are not equally selective.
Proof:
from Lemma 1, Proposition 3 and Assumption 6, it follows that:

a) $\dfrac{H_j}{H_i} < \dfrac{\int_{\mu_l}^{\mu_h} p(H_i|e,\tilde{\mu})dX(\tilde{\mu}|L_x)d\tilde{\mu}}{\int_{\mu_l}^{\mu_h} p(H_j|e,\tilde{\mu})dX(\tilde{\mu}|L_x)d\tilde{\mu}}$, where x=i,j, if job j is more selective than job i.

b) $\dfrac{H_j}{H_i} = \dfrac{\int_{\mu_l}^{\mu_h} p(H_i|e,\tilde{\mu})dX(\tilde{\mu}|L_x)d\tilde{\mu}}{\int_{\mu_l}^{\mu_h} p(H_j|e,\tilde{\mu})dX(\tilde{\mu}|L_x)d\tilde{\mu}}$, where x=i,j, if job j is as selective as job i.

c) $\dfrac{H_j}{H_i} > \dfrac{\int_{\mu_l}^{\mu_h} p(H_i|e,\tilde{\mu})dX(\tilde{\mu}|L_x)d\tilde{\mu}}{\int_{\mu_l}^{\mu_h} p(H_j|e,\tilde{\mu})dX(\tilde{\mu}|L_x)d\tilde{\mu}}$, where x=i,j, if job j is less selective than job i.

Q.E.D.

Proposition 9
given than job i is slacker than job j, the sum of expected outputs at t1 and t2 will be higher when job i is performed at t1 if job i is not less selective than job j.
Proof:

it follows from Proposition 7 and Proposition 8.
Q.E.D.

Higher slackness will raise the probability of success at t1 and will make the posterior d.f. of ability worse in case of failure. When job i is not less selective than job j, the posterior d.f. of ability in case of success in job i will not be first-order stochastically dominated by the posterior d.f. of ability in case of success in job j. Then, although job i has a higher probability of success and can be thought of as "easier" than job j, yet it can be the best job assignment at t1

5. PRELIMINARY CONCLUSIONS

The basic framework of analysis is one in which:
a) there exist many different jobs the agent can be required to perform;
b) the firm retains the right to assign agents to jobs;
c) information on the agent's ability is only imperfectly acquired through experiments.

The preliminary results the paper achieved so far are:
i) effort at current production can reduce expected output at later periods;
ii) promotion can be feasible only if the previous job is not a perfect signal for the job the agent is promoted to;
iii) the probability of demotion has to be always lower than some threshold level below which delaying promotion is a better policy than early demotion;
iv) it can be the case that an "easy" job is a best job assignment even though there exist other, "more difficult" jobs delivering the same expected output at the initial period.

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