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A NOTE ON THE OPTIMAL LEVEL OF POLLUTION: INTEGRATED APPROACH TO ABATEMENT AND OUTPUT REDUCTION

Koichiro AKASHI and Mitsuyasu YABE

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NRIAE
NATIONAL RESEARCH INSTITUTE
OF AGRICULTURAL ECONOMICS
MINISTRY OF AGRICULTURE, FORESTRY AND FISHERIES

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1. Introduction

Environmental economics has typically adopted two approaches in determining the optimal level of pollution. The first approach investigates the conditions for Pareto optimality when polluters reduce emissions by reducing output. The second approach investigates the conditions when polluters reduce emissions by abating pollution. Studies taking the first approach were initiated by the work of Pigou (1932). Studies taking the second approach have been conducted, for example, by Baumol and Oates (1988), Watson and Ridker (1984), Fishelson (1976), and Adar and Griffin (1976).

In the past, these two approaches had been treated separetely for its graphic complication. However, Hijalte (1977), Pratt and Pearce (1979), and Pearce and Turner (1990) recently attempted to integrate them.[1]

We have decided to concentrate our attention on Pratt and Pearce (1979) (herein known as " the PP model") and on Pearce and Turner (1990) (herein known as "the PT model"), since they are representative studies of the integrated approach.

The aim of this paper is to point out a problem with both the PP model and the PT model. Basically, we have found that the conditions for optimality in these two models are not consistent with Pareto optimality. Our model offers an important correction.

2. Re-Examination of the PT Model

2.1 Summary of the PT Model

We limit our discussion on Pearce and Turner (1990), chapters 4, 5, and 6. In these chapters, they analyze the activity of competitive firms for which output price is given. Marginal net private benefit (MNPB), which is defined as the price minus the marginal private cost (MC), is marginal benefit of the firm by production activity. MNPB can be also thought of as marginal foregone profit (MFP) or opportunity cost when the firm reduces its output level.

In Figure 1, the area in which the pollution level is between *A* and *G*, the marginal abatement cost (MAC) is less than MNPB. Therefore, abatement is cheaper than output reduction to reduce emission. However, the area in which the pollution level is less than *G*, MNPB is less than MAC, and output reduction is cheaper than abatement to reduce emission. In Figure 1, the marginal external cost (MEC) curve intersects the MAC curve at point *T* where MAC is less than MNPB. As a result, the optimal level of emission is *OE*, which is determined by the intersection *T*. In such a case the firm abates pollution by *AE* units, but does not reduce output. And the optimal tax rate is *TE*.

However, if the MEC curve takes a steeper form and intersects the MAC curve at a point to the left of *G*, then MNPB is less than MAC. In this case, the optimal level of pollution is determined by the intersection of the MNPB curve and MEC curve. Hence, the firm's path of emission reduction is indicated by the arrowed line in Figure 1.

To summarize, the conditions for Pareto optimality are:

(1a) MEC = MAC where MAC is less than MNPB

and

(1b) MEC = MNPB where MNPB is less than MAC.

2.2 Problems with the PT Model

The equiliblium conditions of the PT model are not consistent with Pareto optimality. We point out two problems.

First,in the PT model the starting point of the MAC curve does not coincide with the point at which MNPB equals zero. However, if an authority does not impose taxes or regulations, the firm realizes an output to a level at which MNPB equals zero. This point where MNPB equals zero is the profit-maximizing pollution level, and when no regulations are imposed, the entire amount of pollution is emited. Thus, the point *A* should be the starting point of the MAC curve. (See Figure 2.)

Second, when the tax rate imposed on the emmision is *TE* in

Figure 2, the firm of economically rational behaviour never abates pollution from point *A* to point *E* according to the MAC curve by *AE* units because of the following reason. If the firm abates *AD* units (from point *A* to point *D* according to the MAC curve), the cost of reducing emission by last 1 unit (indicated as *DE*) is *TE*, and the cost of reducing emission by first 1 unit (indicated as *AB*) by output reduction is *PB*. As the figure shows, *PB* is less than *TE*. Hence the firm will adopt not only abatement, but also output reduction to reduce emission by *AE* units.

Next, we show the correct model in Figure 3. Output reduction is *AC* units, and the MAC curve is drawn so that it starts at the point *C*, which corresponds to *AC* units of output reduction. This MAC curve intersects the MEC curve at point *U*. In this case, MEC (*UF*) equals the MAC (*UF*) and MNPB (*QC*), and the optimal tax rate is *UF* (and *QC*), which is less than the optimal tax rate of the PT model, which is shown as *TE* in Figure 3. Thus, the equiliblium conditions for Pareto optimality should be as follows.

(2) MEC = MAC = MNPB

So far we have provided an intuitive explanation by using figures. Next, we will present a mathematically accurate model.

3. Correction of the PT Model

Herein, we present a model which avoids the problems described in the previous section.

Our first objective is to formalize the relation between the pollution generated, the pollution abated, and the pollution emitted. Our second objective is to formalize the opportunity cost by output reduction. Our third objective is to determine the minimum cost of reducing emission as a function of the amount of emission reduced. Our fourth objective is to determine the optimal amount of emission reduction in order to minimize the social loss (maximize the social benefit).

3.1 Relation between the Pollution Generated, the Pollution Abated, and the Pollution Emitted

Below we consider a case in which the abatement process is independent of the production process. Let the cost (TC) of the firm be a function of the output q, i.e. TC(q), the abatement cost (TAC) of pollution be a function of pollution abated a, i.e. TAC(a). Assume that $TC'(q) = MC(q) > 0$, $MC'(q) > 0$, $TAC'(a) = MAC(a) > 0$, $MAC'(a) > 0$.

We assume that the amount of pollution generated g is a function of the output q, i.e. $g = f(q)$, and that f is twice differentiable and monotonic increasing.[2] Therefore, the output q is also a monotonic increasing function of the pollution generated g, i.e. $q = h(g)$ and $h'(g) > 0$, where $h(g) = f^{-1}(g)$, and h is also twice differentiable.

The pollution generated g is either abated or emitted. Therefore,

(3) $g = a + e$

Where e is the amount of pollution emitted. We denote the external cost of the emission as TEC(e). We assume TEC′(e)= MEC(e) >0, MEC′(e)>0.

3.2 Definition of Opportunity Cost Involved in Reducing Output

The total benefit (TB) and the marginal benefit (MB) by firm's production activity are given as functions of the output q;

(4a) $\mathrm{TB}(q) = p \cdot q - \mathrm{TC}(q)$

(4b) $\mathrm{MB}(q) = p - \mathrm{MC}(q)$

The output level that maximizes profit is given by

(5a) $\mathrm{MB}(q_0) = p - \mathrm{MC}(q_0) = 0$

or

(5b) $q_0 = \mathrm{MC}^{-1}(p)$

Next, we express all these relations by functions of pollution generated g, by substituting $q = h(g)$

(6a) $\mathrm{TNPB}(g) = \mathrm{TB}(h(g)) = p \cdot h(g) - \mathrm{TC}(h(g))$

(6b) $\mathrm{MNPB}(g) = [p - \mathrm{MC}(h(g))] \cdot h'(g)$

Notice that here MNPB is defined as a function of pollution generated! MNPB means "the extra net benefit from changing the level of pollution generated by 1 unit".

The profit maximizing level of pollution generated without regulations is given by

(7a) $\mathrm{MNPB}(g_0) = 0$

or

(7b) $g_0 = h^{-1}(\mathrm{MC}^{-1}(p))$

If the firm reduces the amount of pollution generated by output reduction by y units, the total opportunity cost ($\mathrm{TOC}(y)$) and marginal opportunity cost ($\mathrm{MOC}(y)=\mathrm{TOC}'(y)$) are given by [3]

(8a) $\mathrm{TOC}(y) = \mathrm{TNPB}(g_0) - \mathrm{TNPB}(g_0 - y)$

(8b) $\mathrm{MOC}(y) = \mathrm{MNPB}(g_0 - y)$

Eq.(8b) indicates that the MOC curve is the MNPB curve measured to left of point A in figure 2, 3, and 4. Then, $\mathrm{MOC}(y)>0$, and we assume $\mathrm{MOC}'(y) > 0$.[4]

3.3 Derivation of the Minimum Cost of Reducing Emission

If there are no regulations to the emission of the firm, it will generate pollution to level g_0 , and will emit entire amount of it. However, if some authority imposes taxes or any regulations to the emission, the firm is obliged to reduce emission. Now the emission can be reduced in two ways: abatement and output reduction. The former involves abatement cost, while the latter opportunity cost. For any given amount of emission reduction r, the firm seeks to minimize the cost of emission reduction which is expressed as the sum of the total abatement cost (TAC(a)) and the total opportunity cost (TOC(y)), choosing the level of x and y, where $x+y=r$. Hence, the cost-minimization problem of the firm can be stated as follows. (More detailed procedure is contained in the Appendix.)

(9a) $\min \quad TAC(a) + TOC(y)$

(9b) $\text{s.t.} \quad r = a + y$

Let the solution to this problem be $a^m(r)$, $y^m(r)$.[5] Next, substituting $a^m(r)$ and $y^m(r)$ into the objective function, we get

(10) $TRC^m(r) = TAC(a^m(r)) + TOC(y^m(r))$.

where $TRC^m(r)$ is "total reduction cost", which is the minimum cost of emission reduction r and can be thought of as a kind of cost function of emission reduction.[6] Therefore, by using the envelope theorem we can obtain the following equation.

(11) $MRC^m(r) = MAC(a^m(r)) = MOC(y^m(r))$

where $MRC^m(r)$ is the marginal reduction cost difined as

$MRC^m(r) = TRC^{m\prime}(r)$. Eq.(11) implies that the marginal cost of emission reduction should be equated to both the marginal abetment cost and the marginal opportunity cost so as to minimize the total cost of emission reduction.

Eq.(11) and eq.(9b) show that graphically the MRC^m curve is obtained by adding the MAC curve and MOC curve horizontally (See Figure 4).

3.4 Determination of the Optimal Level of the Emission

Now, the conditions for Pareto optimality are given by minimizing total social loss defined as $TEC(e)+TRC^m(r)$ with respect to e and r.[7]

(12a) $\quad \min \quad TEC(e) + TRC^m(r)$

(12b) $\quad \text{s.t.} \quad g_0 = e + r$

By the first-order conditions we can get the optimal level of emission e^* and emission reduction r^*, (graphically shown by intersection of the MRC^m curve and MEC curve in Figure 4).

(13) $\quad MEC(e^*) = MRC^m(r^*)$

The condition is "marginal external cost equals marginal reduction cost".

Next, we substitute r^* into eq.(11).

(14) $MRC^m(r^*) = MAC(a^*) = MOC(y^*)$

where $a^*=a^m(r^*)$ and $y^*=y^m(r^*)$.

Eq.(14) shows how the optimal level of abatement a^* and optimal level of emission reduced by output reduction y^* are determined (See Figure 4).

Next, we substitute eq.(13) into eq.(14).

(15) $MEC(e^*) = MAC(a^*) = MOC(y^*)$

Thus, we can confirm the validity of eq.(2).[8]

4. Re-Examination of the PP Model

4.1 Summary of the PP Model

In order to attain a Pareto optimal level of emission of industry, direct regulations or taxes must be imposed on the polluters. Taxation and regulation methods for controlling emission have been analyzed by numbers of economists, and taxes of this sort are called "Pigouvian taxes". The PP model is an important achievement which analyzes the conditions for Pareto optimality when the industry can reduce emission not only by reducing output but also by abating pollution.

When the marginal cost (MC) to the industry is given, marginal social cost (MSC) is defined as follows (See Figure 5).

(16a) $MSC_1 = MC + MEC$

(16b) MSC_2 = MC + MAC

(16c) MSC = MSC_1 where MEC is less than MAC.

(16d) MSC = MSC_2 where MAC is less than MEC.

At all points to the left of Q_X in Figure 5, MEC is less than MAC. As a result, MSC can be determined by adding MC and MEC. In contrast, all points to the right of Q_X, MAC is less than MEC. Therefore, MSC can be determined by adding MC and MAC. Hence, the MSC curve takes the form of the thick line which kinks at the level of Q_X in Figure 5. The optimal level of pollution is determined by the intersection of the MSC curve and the Demand curve.

In the case of Figure 5, the MSC curve intersects the Demand curve at point G to the right of Q_X. OS_X indicates the amount of emission, S_XS_G indicates the abatement, and S_GS_0 indicates the emission reduced by output reduction. The equilibrium condition is now "Demand=MC+MAC", and by transposing MC, we get "Demand-MC=MAC".

If the MSC curve intersects the Demand curve at a point to the left of Q_X, the equilibrium condition becomes either "Demand=MC+MEC" or "Demand-MC=MEC". In this case, the entire amount of pollution generated is emitted.

To summarize, the conditions for Pareto optimality are:

(17a) (Demand-MC) = MEC where MEC is less than MAC

and

(17b) (Demand-MC) = MAC where MAC is less than MEC.

4.2 Problems with the PP Model

The equilibrium conditions of the PP model are also not consistent with Pareto optimality, and several problems exist.

The first problem is that the starting point of the MAC curve is not fixed in the PP model. However, the starting point should be Q_0 which corresponds to the pollution level determined by the intersection of the MC curve and the Demand curve, since it is determined by profit maximizing activities of the polluters when no regulations are imposed to the emission.

The second problem is that the MAC curve in the PP model slopes downward, even though MAC and MEC are both components of marginal social cost. MSC_1 is marginal social cost when the entire amount of pollution generated is emitted. Then MSC_2, which is comparable to MSC_1, can be thought of as marginal social cost when the entire amount of pollution is abated. Thus, both the MAC curve and the MEC curve should actually be drawn sloping upward.

We demonstrate that the equliblium conditions for Pareto optimality should be as follows.

(18) MEC = MAC = (Demand-MC)

5. Correction of the PP Model

Once pollution is generated as by-product of output, the society has to dispose of pollution by adopting two ways, one is emitting pollution to the environment and the other is abating it. The former involves external cost, while the latter abatement cost. For a given amount of pollution generated g, society seeks to minimize the cost of disposing of pollution which is expressed as the sum of total eternal cost (TEC(e)) and the total abatement cost (TAC(a)), where g=e+a. We define total disposal cost (TDC^m) of pollution as minimum cost involved in disposing of pollution generated g. The cost-minimization problem of the society can be stated as follows.

$$\text{(19a)} \qquad \min \quad \mathrm{TEC}(e) + \mathrm{TAC}(a)$$

$$\text{(19b)} \qquad \text{s.t.} \quad g = e + a$$

Let a solution to this problem be $e^m(g)$, $a^m(g)$.[9] Next, substituting $e^m(g)$ and $a^m(g)$ into the objective function, we get the total disposal cost as function of g.

$$\text{(20)} \qquad \mathrm{TDC}^m(g) = \mathrm{TEC}(e^m(g)) + \mathrm{TAC}(a^m(g))$$

Next by using the envelope theorem, we can obtain the following equation.

$$\text{(21)} \qquad \mathrm{MDC}^m(g) = \mathrm{MEC}(e^m(g)) = \mathrm{MAC}(a^m(g))$$

where $MDC^m(g)$ is the marginal disposal cost difined as $MDC^m(g)=TDC^{m\prime}(g)$. The implied MAC curve is now upward sloping. Eq. (21) and eq.(19b) show that the MDC^m curve can be obtained by adding the MEC curve and the MAC curve horizontally. (See Figure 6).

By using this MDC^m curve, we can easily derive the conditions for Pareto optimality.

Next, we re-define marginal social cost ($MSC(g)$) by adding $MDC^m(g)$ and $MC(g)$ (See Figure 6).[10] Graphically, the MSC curve can be obtained by adding the MDC^m curve and the MC curve vertically. (This point is central to our model. Please note the differences between "MSC" as used in our model and the usual approaches, such as output reduction approach or the PP model.)

$$(22) \qquad MSC(g) = MC(g) + MDC^m(g)$$

Then, we determine the optimal level of pollution in order to maximize social welfare.

$$(23) \qquad \max \quad Demand(g)\,dg - MSC(g)\,dg$$ [11]

The first-order conditions give us the optimal level of the pollution generated. (Shown by the intersection of the MSC curve and the Demand curve in Figure 6.)

$$(24) \qquad Demand(g^*) = MSC(g^*)$$

Here, $g*$ is the optimal level of the pollution generated by the industry. By substituting $g*$ into eq.(21), we get the following equation.

(25) $$\mathrm{MDC}^{\mathrm{m}}(g^*) = \mathrm{MEC}(e^*) = \mathrm{MAC}(a^*)$$

where $e^*=e^m(g^*)$ and $a^*=a^m(g^*)$.

In this way, the optimal levels of emission e^* and pollution abated a^* are determined.

Next, by eq.(22) and eq.(24), we get

(26) $$\mathrm{Demand}(g^*)-\mathrm{MC}(g^*) = \mathrm{MDC}^{\mathrm{m}}(g^*)$$

Then by substituting eq.(25) into eq.(26), we get

(27) $$(\mathrm{Demand}(g^*)-\mathrm{MC}(g^*)) = \mathrm{MEC}(e^*) = \mathrm{MAC}(a^*)$$

Hence eq.(18) from the previous section is confirmed as valid.

The optimal tax rate to the industry is given by

(28) $$t^* = \mathrm{MDC}^{\mathrm{m}}(g^*) = \mathrm{MEC}(e^*) = \mathrm{MAC}(a^*)$$

This tax rate is lower than the rate in which the abatement process is not considerd.

Finally, we provide a short explanation of Figure 6. By dropping a perpendicular line from the intersection of the Demand curve and the MSC curve, we get point I at its intersection with the MC curve, point J at the intersection with the MDC^m curve and point G^* at the horizontal axis. Next, a horizontal line from point J intersects the MAC curve to give point K, and intersects the MEC curve to give point L. We now have the optimal level of abatement OX^*, the optimal level of emission OE^* ,and the optimal level of pollution reduced by output reduction G_0G^*. And the optimal tax rate is RI, which is equal to both the marginal abatement cost KX^* and the marginal external cost LE^* (and of course to the maarginal disposal cost JG^*). Thus our integrated model is complete.

6. Conclusion

In this paper, we analyzed the conditions for a Pareto-optimal level of pollution when emission can be reduced in two ways: output reduction and abatement. First, we surveyed former studies and pointed out that the results were not consistent with Pareto optimality. Next, we corrected them and constructed new models. We modelled both of the competitive firm and industry in general. The model of industry especially demonstrated how Pareto optimality is attained when the abatement process is introduced to the output reduction approach initiated by Pigou. Our model succeeds in demonstrating the relationship between abatement, output reduction, and emission in one diagram using a very simple method.

Notes

[1] Hijalte (1977) is a pioneering work that adopts the integrated approach. However, that study assumed the MAC curve to be constant, even though the MAC curve is generally thought to increase as the amount of pollution abated increases. In this respect, Pratt and Pearce (1977) criticized Hijalte's work by pointing out that " Hijalte's analysis is confined to the unrealistic assumption of a constant marginal abatement cost function ". Therefore, Pratt and Pearce (1979) as well as Pearce and Turner (1990), assumed an increasing MAC curve.

[2] In both the PT model and the PP model, the pollution generated is assumed to be directly proportional to the output, i. e. $g=k \cdot q$, where k is a constant. Our assumption that the pollution generated is a monotonic increasing function of the output and twice differentiable is more general than the assumption in either of the other two models.

[3] When the amounts of pollution generated are g_0 and g_0-y, the total net private benefit of the firm are represented by TNPB(g_0), TNPB(g_0-y), respectively. If the firm reduces the level of pollution generated by y units from g_0, the total opportunity cost of the firm is TOC(y)= TNPB(g_0)-TNPB(g_0-y). Therefore, by differentiating TOC(y) with respect to y, we obtain MOC(y)=MNPB(g_0-y).

[4] Generally, MOC(y) is positive by the following reason.

$$\begin{aligned} \mathrm{MOC}(y) &= \mathrm{MNPB}(g_0-y) \\ &= [p-\mathrm{MC}(h(g_0-y))]\cdot h'(g_0-y) > 0 \end{aligned}$$

because $p-\mathrm{MC}(h(g_0-y)) > 0$ and $h'(g_0-y) > 0$ as long as $0 \le y \le g_0$.

The assumption that MOC′(y) is positive requires

$$\mathrm{MOC}'(y) = -[p-\mathrm{MC}(h(g_0-y))]\cdot h''(g_0-y) + [\mathrm{MC}'(h(g_0-y))]\cdot h'(g_0-y)^2 > 0.$$

If $q=h(g)$ is a concave function, then $h''(g)<0$ for $\forall g(0 \le g)$, hence MOC′(y) is positive. If q is directly proportional to g, i.e. $q=h(g)=k\cdot q$, where k is a constant, then $h'(g)=0$, hence MOC′(y) is positive. MOC′(y) can be negative or zero if $q=h(g)$ is a convex function and the absolute value of $[p-\mathrm{MC}(h(g_0-y))]\cdot h''(g_0-y)$ is greater than or equal to the value of $[\mathrm{MC}'(h(g_0-y))]\cdot(h'(g_0-y))^2$. Our assumption excludes this case.

[5] We assume the interior solution. Cases of corner solutions are analized in the Appendix.

In the previous subsections, we assumed MAC′(a)>0 and MOC′(y)>0. These assumptions are sufficient for the second-order conditions.

[6] In general, $\mathrm{TRC}^{\mathrm{m}}$ should be depicted as $\mathrm{TRC}^{\mathrm{m}}(r,p)$ because the

shape of TOC(y^m), which is the component of TRC^m, is thought to depend on the value of p, where p is the output price. However, our objective is to examine the firm's behaviour resulting not from changes of the output price, but from the taxes or regulations to the emission. By the above reason, we treat the output price as a constant.

[7] The maximization problem of total social benefit

$$\max \quad TSB = TNPB(g_0) - TRC^m(r) - TEC(e)$$
$$\text{s.t.} \quad g_0 = e + r$$

reduces to the minimization problem (12a) and (12b).

[8] As we have discussed above, the following equation exists for $\forall y(0 \le y \le g_0)$:

$$MOC(y) = MNPB(g_0 - y)$$

Therefore, eq.(2) and eq.(15) mean same thing.

[9] Here we assume the interior solution. As for the second-order conditions, our assuptions ($MEC'(e)>0$ and $MAC'(a)>0$) are sufficient.

[10] Here we define the marginal cost of industry as a function of pollution generated. The procedure is as follows.

First, we define the total cost and marginal cost to industry as functions of the output q, namely TC(q) and MC(q), respectively. We next substitute the relation $q=h(g)$ into TC(q), producing TC($h(g)$). Differentiating TC($h(g)$) with respect to g, we get MC($h(g)$)$\cdot h'(g)$. Now we re-write MC($h(g)$)$\cdot h'(g)$ as MC(g) for the sake of convenience. Note that the shape of MC(g) is different from that of MC(q).

[11] Here we define consumer demand as a function of the pollution generated. The procedure is as follows.

Let p=D(q) be the inverse demand function. We then substitute $q=h(g)$ into D(q), producing p=D($h(g)$)=Demand(g). As a result, Demand(g) gives consumer demand for the pollution generated.

Appendix

We solve the minimization problem including cases of corner solutions.

(A1) min $MAC(a) + MOC(y)$

(A2) s.t. $r = a + y$

(A3) $a \geq 0,\ y \geq 0$.

Let the Lagrange multiplier be λ, then the Lagrangean function is

(A4) $L = TAC(a) + TOC(y) + \lambda(r-a-y)$.

Using the Kuhn-Tucker theorem, we obtain the following first-order conditions.

(A5) $MAC(a)-\lambda \geq 0$ and $a \cdot (MAC(a)-\lambda) = 0$

(A6) $MOC(y)-\lambda \geq 0$ and $y \cdot (MOC(y)-\lambda) = 0$

(A7) $r - a - y = 0$

By our assumtion the second-order conditions are satisfied, hence a solution to eq.(A5),(A6) and (A7) is also a solution to eq.(A1),(A2) and (A3).

We examine a case of interior solutions and cases of corner solutions separetely.

A.1 A Case of Interior Solutions

Let a^m, y^m and λ^m be a solution to eq.(A5),(A6) and (A7) ,then

(A8) $\quad \mathrm{MAC}(a^m) - \lambda^m = 0$

(A9) $\quad \mathrm{MOC}(y^m) - \lambda^m = 0$

(A10) $\quad r - a^m - y^m = 0$

By the implicit function theorem, a^m, y^m and λ^m can be represented as functios of the parameter r.

(A11) $\quad x^m = x^m(r)$

(A12) $\quad y^m = y^m(r)$

(A13) $\quad \lambda^m = \lambda^m(r)$

therefore

(A14) $\quad \mathrm{MAC}(a^m(r)) = \mathrm{MOC}(y^m(r)) = \lambda^m(r)$

and the minimized objective function TRC^m is represented as a function of r.

(A15) $\quad \mathrm{TRC}^m(r) = \mathrm{TAC}(a^m(r)) + \mathrm{TOC}(y^m(r))$.

Then by the envelope theorem we can get

(A16) $\quad \mathrm{MRC}^m(r) = \mathrm{d}\ \mathrm{TRC}^m(r)/\ \mathrm{d}r = \lambda^m(r)$.

Therefore, the marginal cost of emission reduction ($MRC^m(r)$) equals lagrange multiplier λ^m, and by eq.(A14),(A16) following equation exist for $\forall r (0 \leq r \leq g_0)$.

$$\text{(A17)} \qquad MRC^m(r) = MAC(a^m(r)) = MOC(y^m(r))$$

A.2 Cases of Corner Solutions

First, we consider the case of $a^m=0$. Then the conditions which corresponds to eq.(A15) and (A17) are as follows.

$$\text{(A18)} \qquad TRC^m(r) = TOC(y^m)$$

$$\text{(A19)} \qquad MRC^m(r) = MOC(y^m) \leq MAC(a^m)$$

where $y^m=r$, $a^m=0$.

In the case of $y^m=0$, the conditions are given by

$$\text{(A20)} \qquad TRC^m(r) = TAC(a^m)$$

$$\text{(A21)} \qquad MRC^m(r) = MAC(a^m) \leq MOC(r^m)$$

where $a^m=r$, $y^m=0$.

Anyhow, the assumptions of corner solutions are not realistic and seem to be less important. However one should note that the corner solutions of our model are also consistent with Pareto optimality.

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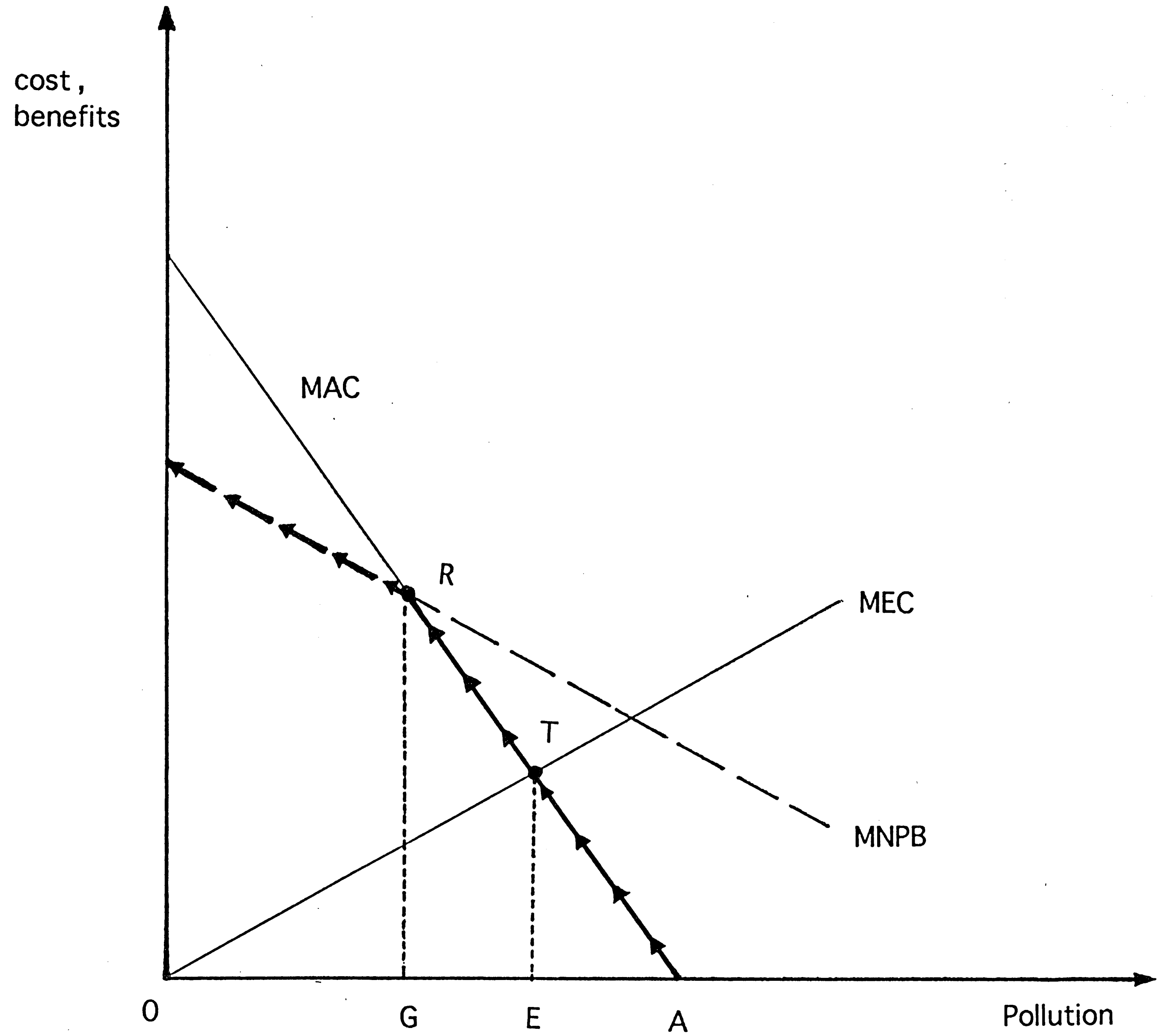

FIGURE 1. "The PT Model"

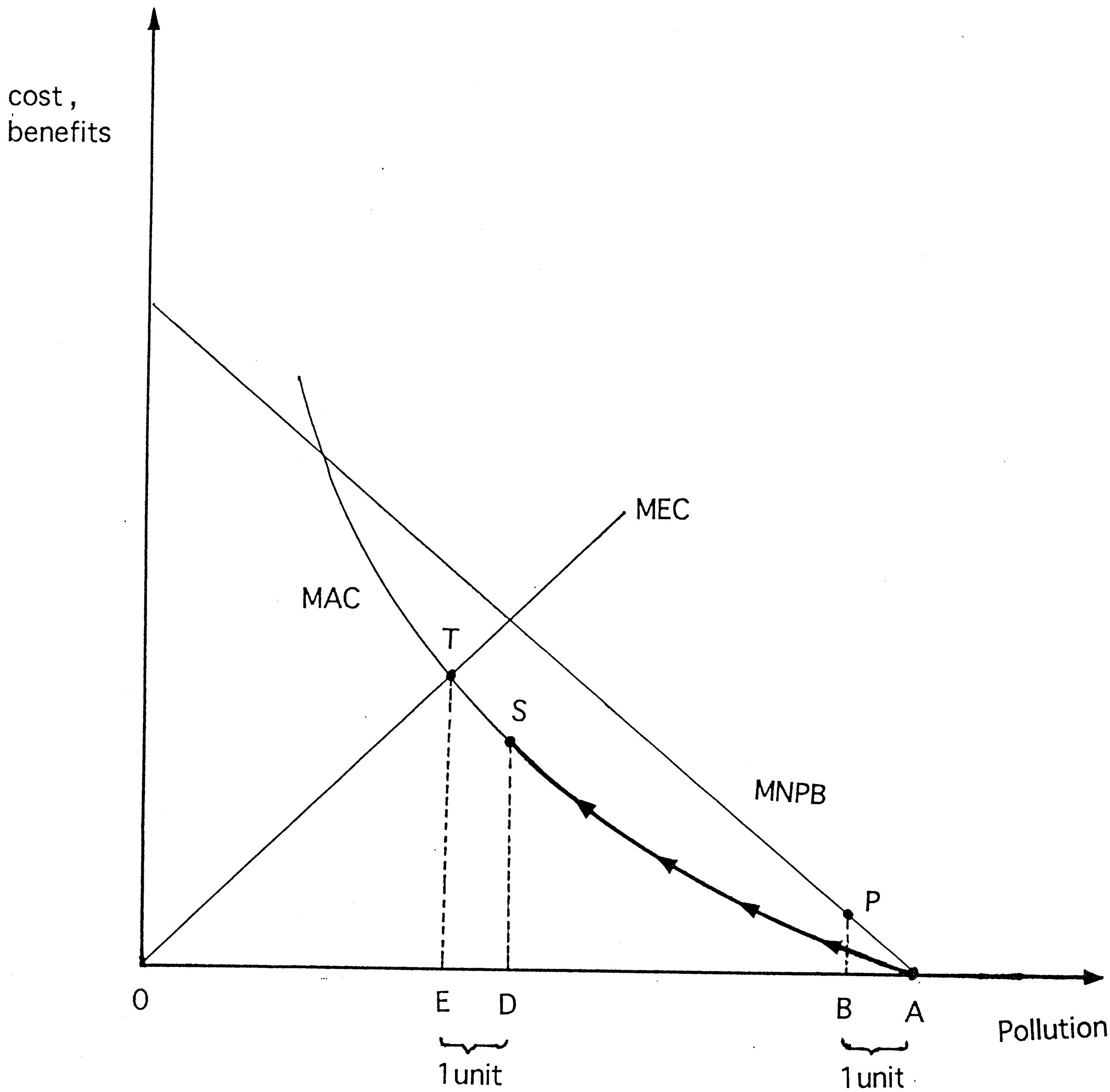

FIGURE 2. Problems of "The PT Model"

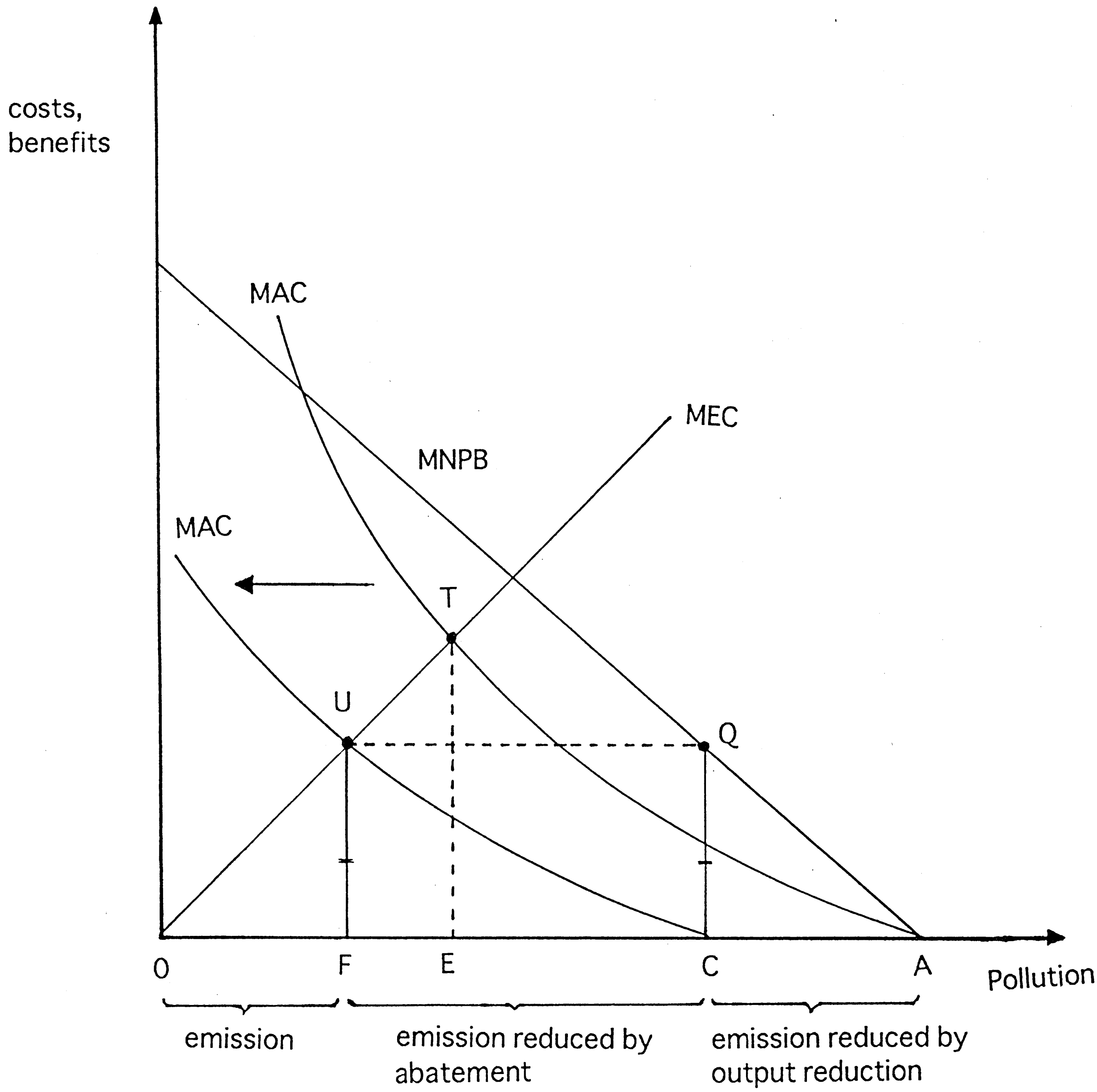

FIGURE 3. Correction of "The PT Model"

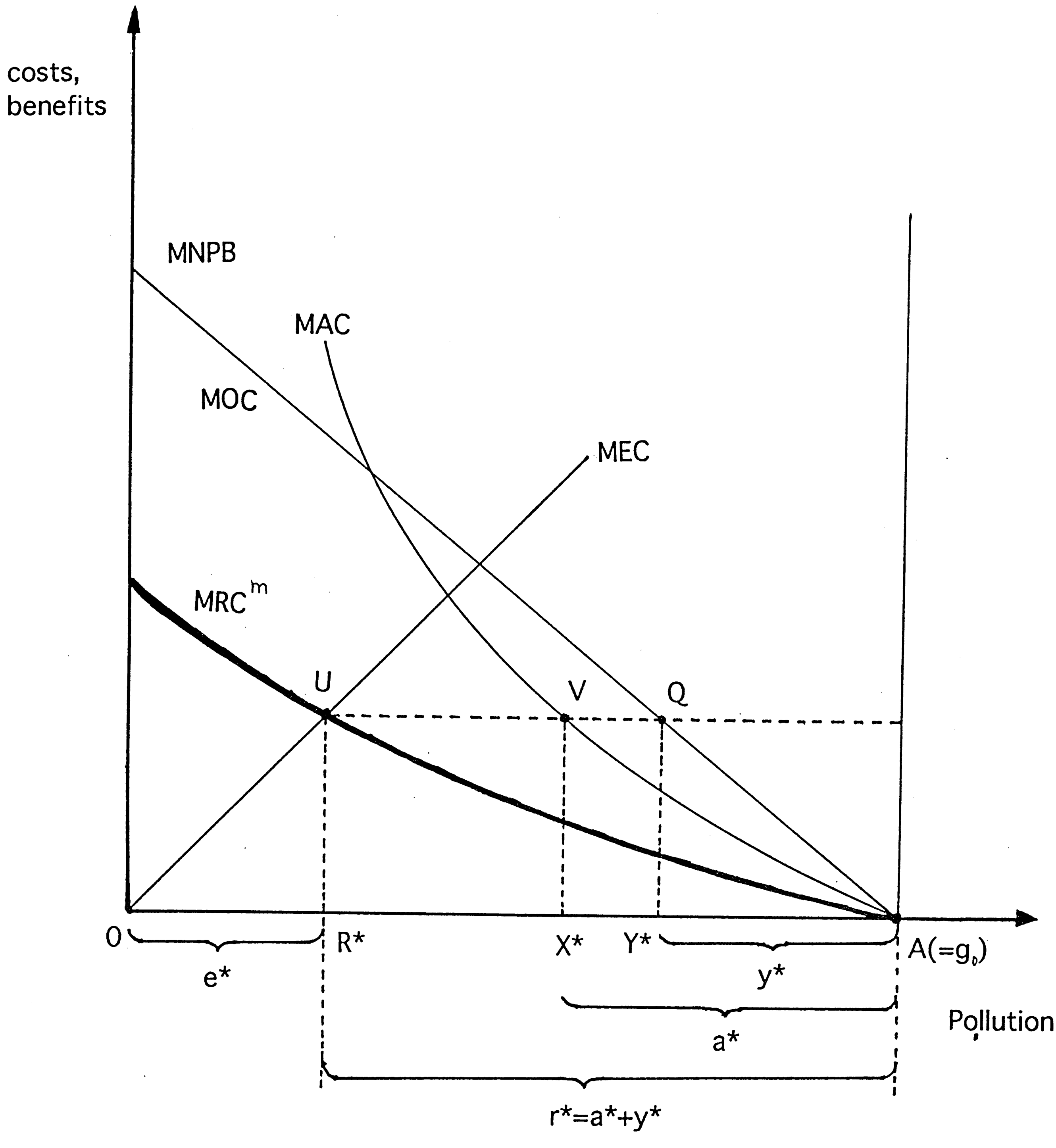

FIGURE 4. Correction of "The PT Model"

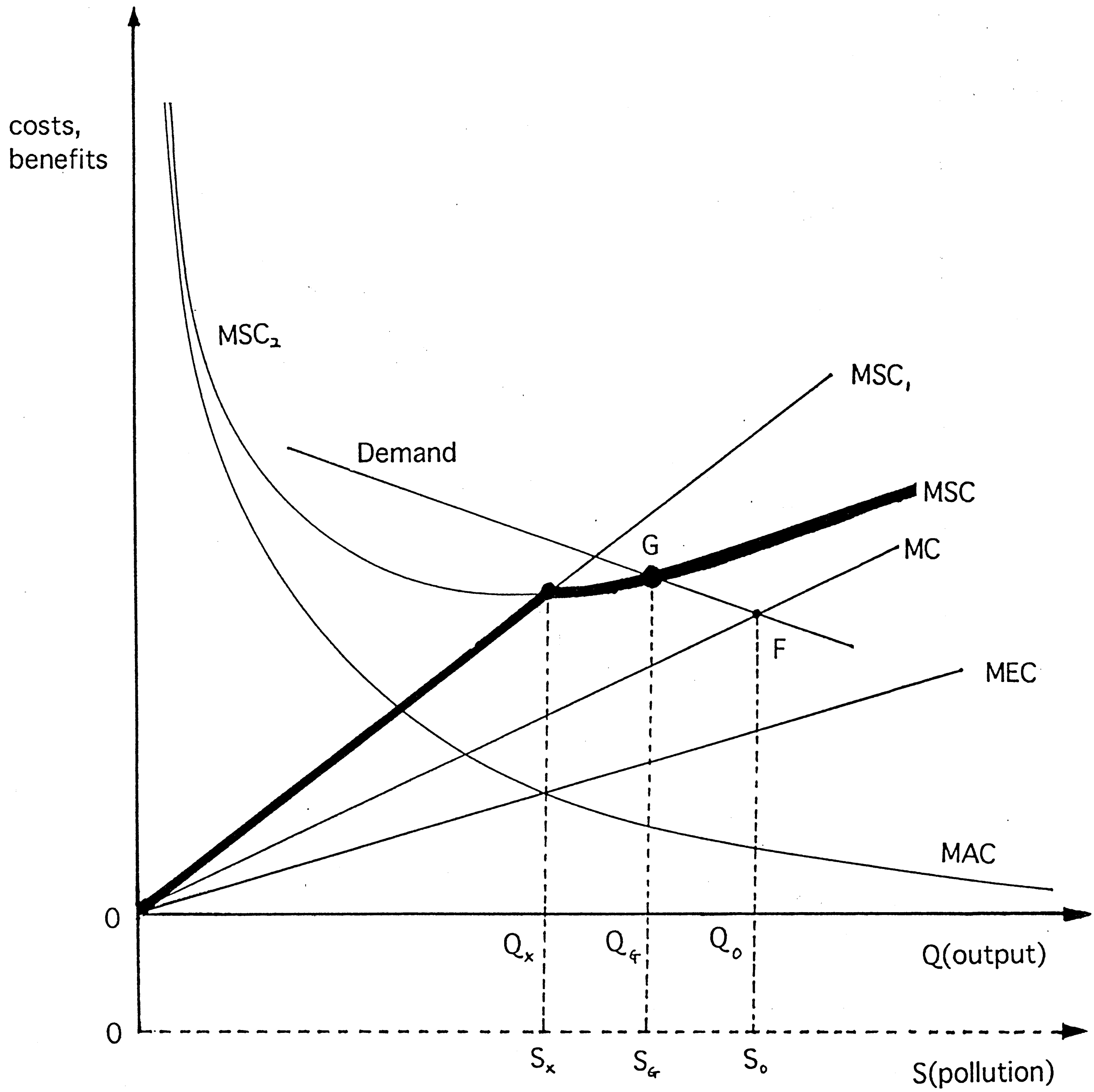

FIGURE 5. "The PP Model"

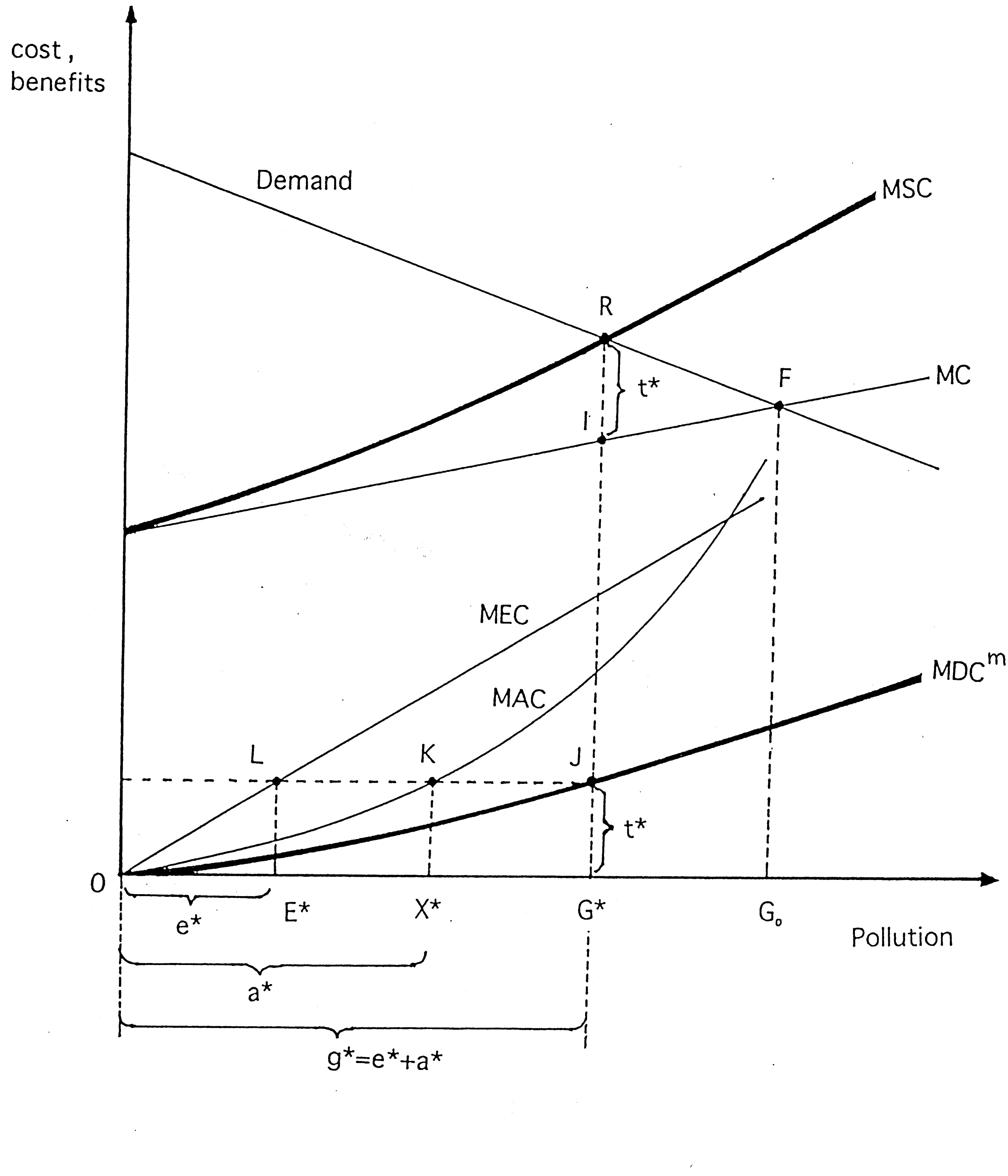

FIGURE 6. Correction of "The PP Model"