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and Privatization in an Oligopoly

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SUPPLY RESPONSE TO PRICE LIBERALIZATION AND PRIVATIZATION IN AN OLIGOPOLY

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A B S T R A C T

The goal of the paper is to elucidate a development of an oligopolistic industry (with symmetric technology) after price liberalization and some possible effects of privatization in it. The analysis is based on a theoretic model in the form of a system of difference equations where firms' parameters of sensitivity to profit opportunities play the role of adjustment speeds. Sufficient condition for local stability is derived.

Even in the symmetric technology case output paths of individual firms can be very different because of differences in sensitivity to profit opportunities (which is a behavioral parameter) and different pre-price-liberalization outputs (which were determined by central authorities under previous regime). This confirms that negative developments (declining output and rising price) cannot be attributed to reform measures (price liberalization itself). It also implies that current performance cannot be an indicator of firm's prospects. Therefore firm-specific economic policy measures are illusory.

Under certain restrictions on parameters of sensitivity to profit opportunities the modelled adjustment process can be rational - in the sense that, given outputs of competitors, firm's output in the current period gives it higher profit than sticking to the preceding period output - for finite number of (finite) time periods. Such restrictions can be formulated also for parameters of sensitivity to profit opportunities corresponding to Cournot tatonement.

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I. Introduction

Price liberalization and privatization are two of the most important and most essential measures in the transition from a centrally planned to a market economy. Journalists usually (having a diagram with market supply and demand curve in the case of perfect competition in mind) explain to the general public that the former will lead to elimination of (partial) disequilibrium through an increase in price bringing about an increase in industry output. The latter is then expected to increase efficiency (in production), and thereby to further increase output.

In reality (which in many industries does not approximate perfect competition) in the months following price liberalization (no matter whether privatization has already started or not) postsocialist economies witness rise of price and decline of output in many industries. These empirical facts trigger off (or strengthen) criticism of price liberalization and government's reform program as a whole, proposals to "remedy" the situation - which is mainly the result of economic policy under the previous regime - by (extensive and selective, even firm-specific) governmental intervention. Implementation of these proposals would contradict reform measures and (at least) delay the process of transformation of the economic system.

In this paper we try to elucidate, using a theoretic model in the form of the system of difference equations, a supply response in an oligopolistic industry to price liberalization and some likely effects of privatization in it.

In the following section we specify assumptions of the analysis, formulate the model and examine conditions of its local stability.

In the third section we discuss rationality of the adjustment process (dynamics) described by the model. The fourth section concludes the paper.

II. The Model and Its Properties

In this section we formulate the model and analyze its local stability.

We assume that industry is composed of n (n > 1 and integer) firms, producing a homogeneous good, with the common cost function[1] C(q) satisfying

$$C'(q)>0, \quad C''(q)\geq 0 \tag{1}$$

where $q \geq 0$ is firm's output. We denote i-th firm's output in the time period t by $q_{i,t}$, industry output in the time period t by Q_t. Of course,

$$Q_t=\sum_{j=1}^{n} q_{j,t} \tag{2}$$

We denote the inverse demand function by p(Q) (i.e. when industry output is Q, price is p(Q)) and we assume that in an interval [0, Y]

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[1]This is, of course, a restrictive assumption, but it (besides highly simplifying analysis of local stability) enables us to show in a more transparent way consequences of differing sensitivity to profit opportunities (as well as different outputs before price liberalization) among firms.

$$p'(Q)<0, \quad p''(Q)\leq 0 \qquad (3)$$

where Y is determined by the condition p(Y) = 0. We also take p(Q) = 0 for Q > Y.

As Szidarovszky and Yakowitz (1977) proved, under assumptions (1) and (3) there is unique Cournot equilibrium point. Uniqueness and the same cost function for all firms imply that the Cournot equilibrium is symmetric. We denote each firm's output in this equilibrium by μ and the industry Cournot equilibrium output by ${}^{c}Q$. Naturally, we assume $\mu > 0$.

Since our aim is to analyze oligopoly industry in the process of transition from a centrally planned to a market economy, we do not assume that firms maximize profit (nevertheless, since we pay attention to effects of privatization, we do not rule out this possibility), but we assume (we are convinced that quite realistically) that they are sensitive to profit opportunities. The i-th firm's parameter of sensitivity to profit opportunities is denoted by k_i. We assume that $k_i > 0$ for all i.

Formally we assume that firms view current marginal profit as an indicator of profit opportunities. Clearly, this does not rule out profit maximization based on expectations of future (next period) output of other firms. (We touch on this issue in the next section.)

Under the assumptions made above, the model of dynamics of oligopolistic firms' output (using difference equations) has the form:

$$q_{i,t+1}=q_{i,t}+\max(k_i\pi'_{i,t},-q_{i,t}) \quad i=1,2,\ldots,n \qquad (4)$$

where $q_{i,0}$, i = 1, 2, . . ., n (i.e. initial conditions, interpreted here to be pre-price-liberalization output levels) are given and assumed to be positive. We also assume that

$$Q_0 < Y \qquad (5)$$

The symbol $\pi'_{i,t}$ stands for the marginal profit of i-th firm in the time period t.

We further analyze linear approximation of the model (4) in the neighbourhood of the Cournot equilibrium:[2]

$$q_{t+1} = q_t + KA(q_t - \mu e) \qquad (6)$$

where

q_t is a column vector of firms' outputs in the time period t,

e is column n-vector with all elements equal to unity,

K is nxn diagonal matrix with firms' parameters of profit sensitivity along the main diagonal,

A is nxn matrix whose i-th row equals to the gradient of the i-th firm's marginal profit evaluated at the Cournot equilibrium.

Due to the same cost function of all firms and symmetry of the Cournot equilibrium matrix A is symmetric and its elements are

[2]Since it is an approximation in a neighbourhood of strictly positive equilibrium point, we omit the maximum from the right hand side of (4).

$$a_{ij}=p'({}^cQ)+\mu p''({}^cQ)=\beta<0, \quad i\neq j \qquad (7)$$

$$a_{ii}=2p'({}^cQ)+\mu p''({}^cQ)-C''(\mu)=\beta+p'({}^cQ)-C''(\mu)=\beta+\alpha<0 \qquad (8)$$

It is easy to verify directly that A is negative definite.

The system (6) can be re-written as

$$q_{t+1}=(I+KA)q_t-\mu KAe \qquad (9)$$

where I is nxn identity matrix.

Thus, stability of the system (6) (i.e. local stability of the system (4)) depends on values of characteristic roots of matrix I + KA. Their analysis is summarized in the following three propositions.

Proposition 1: All characteristic roots of matrix KA are real.

Proof: Let ξ be a characteristic root of KA and z $\neq$ 0 corresponding column characteristic vector. They must satisfy

$$KAz=\xi z \qquad (10)$$

Let

$$z=z_1+iz_2 \qquad (11)$$

and

$$\xi=\xi_1+i\xi_2 \tag{12}$$

where z_1 and z_2 are real vectors. Denote by $\tilde{z}$ the complex conjugate of z. Premultiplying both sides of (10) by z^TK^{-1} (where T denotes transpose of the vector or matrix), we have

$$\tilde{z}^TAz=\xi\tilde{z}^TK^{-1}z \tag{13}$$

and using (11) and (12) and the fact that A is symmetric and K^{-1} is diagonal, we get

$$z_1^TAz_1+z_2^TAz_2=(\xi_1+i\xi_2)(z_1^TK^{-1}z_1+z_2^TK^{-1}z_2) \tag{14}$$

This implies

$$\xi_2(z_1^TK^{-1}z_1+z_2^TK^{-1}z_2)=0 \tag{15}$$

All diagonal elements of K^{-1} are positive, so it is positive definite. Therefore the expression in the parenthesis is positive, and we must have

$$\xi_2=0 \tag{16}$$ ‖

Proposition 2: All characteristic roots of KA are negative.[3]

Proof:[4] A is negative definite, hence it is also negative quasi-definite. Therefore (theorem 1 and its corollary in Arrow & McManus, 1958) it is D-stable.[5] Since all diagonal elements of K are positive, the claim follows. ▌▌

Proposition 3: If for all i = 1, 2, . . ., n

$$k_i < -\frac{2}{\alpha + n\beta} \qquad (17)$$

the solution of the system (6) is stable.

Proof: Since all characteristic roots of the negative matrix KA are real and negative, the system (6) is stable if they all exceed -2, i.e. if all characteristic roots of the positive matrix -KA (which are all real and positive because all characteristic roots of KA are real and negative) are less than 2. Since -KA has all elements positive, it is indecomposable. Therefore its largest characteristic root is less or equal than the maximum of sums of its rows (theorem 5, Appendix A, Arrow & Hahn, 1971), from which the claim follows. ▌▌

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[3]Thus, the solution of system of differential equations analogical to (6) is stable. This corresponds (though our assumptions on demand and cost functions differ from his) to classical result due to Dixit (1986).

[4]This proposition can be proved also directly by equating real parts of (14).

[5]A real, square matrix M is said to be D-stable if, for diagonal matrices D, DM is stable (from the point of view of stability of the solution of system of differential equations - M.H.) if and only if $d_{ii} > 0$, where the d_{ii} denote the diagonal elements of D. (Arrow & McManus, 1958)

Condition (17) is not necessary. It follows from the fact (which can be easily verified for n=2) that for the matrix -KA where elements of A are defined by (7) and (8) the maximum of sums of rows is not a characteristic root. However - since the characteristic root of positive indecomposable matrix exceeding the moduli of all other characteristic roots is greater or equal than the minimum of sums of its rows (the above mentioned theorem in Arrow & Hahn, 1971) - it is necessary for stability of the system (6) that (17) holds for at least one i.

Thus, firms' output in any finite time period depends on Cournot equilibrium output (which is determined by the number of firms and demand and cost conditions common for all firms) as well as on their initial outputs (here assumed to be pre-price-liberalization outputs) and their sensitivity to profit opportunities. Though the Cournot equilibrium output is the same for all firms, their output paths after price liberalization can be very different. Looking at the linearized expressions for firms' marginal profits in (6), we see that (if the sum of differences between current output and Cournot equilibrium output for all remaining firms has the opposite sign than the corresponding difference for some firm or group of firms) in some periods output of some firm(s) can even further deviate from the Cournot equilibrium level. This can happen irrespective of the value(s) of he parameter(s) of sensitivity to profit opportunities, and thus even if for all i

$$k_i < -\frac{1}{\alpha + n\beta} \qquad (18)$$

e.i, even if all characteristic roots of matrix I+A are from the interval (0, 1) (and, hence, all terms in the solution of the system (6) taken separately are asymptotic), firm's output need not change monotonically. Moreover, for certain values of the parameters of sensitivity to profit opportunities (and, again, even if (18) holds), in some period(s) even industry output can further deviate from the Cournot equilibrium level ^{c}Q.

Two warnings for economic policy stem from these conclusions. First, unless demand and/or cost conditions change, not only pre-price-liberalization industry output level (of course, provided that before price liberalization industry was not in Cournot equilibrium, which was certainly the case at least in vast majority of industries) cannot be maintained, but also favourable development of industry output (rise in it) in the first period(s) after price liberalization can be reversed in the subsequent periods. However, this is a consequence of disequilibrium output levels prior to price liberalization (determined by central planners) and it is not correct to attribute it to policy measures in the process of economic reform. In particular, in many industries output would fall, as industry approaches Cournot equilibrium, even in the absence of any restrictive macroeconomic policy.

Second, output paths of firms can be very different even if they have the same cost function (and, hence, are, potentially, equally efficient). Thus, development of output and other indicators (e.g. profit) conveys no information about potential efficiency of firms (differences in current efficiency in production are due to different output targets assigned to them by central planners). Therefore, policy measures, based on selective approach towards individual firms (according to judgments on their "prospects") are illusory.

The model describes industry dynamics in a process of convergence to the Cournot equilibrium. Nevertheless, this equilibrium, though it plays an important role in the modelled process, is not attained in any finite time. Moreover, the model is valid only as long as its parameters, including the number of firms, do not change. Thus, the "full" (though still partial equilibrium) model of industry, taking also entry and exit into account, consists of a sequence of models of the type (6), differing in parameters (and, therefore, describing convergence to different Cournot equilibria).

The above reflections have an important specific implication. An economic policy measure with clear and unambiguous effect on values of certain variables (e.g. industry output) at Cournot equilibrium need not have the same effect on values of these variables along a path to the equilibrium. For example, "demonopolization", i.e. administrative breaking of enterprises consisting of plants capable of functioning as separate enterprises, (which according of many critics of Federal Government reform program in Czechoslovakia should have preceded price liberalization) would increase industry output (and decrease price) at Cournot equilibrium, but it could have (depending on cost and demand characteristics) quite different impact along the path to the equilibrium. (Of course, development of the industry would depend also on parameters of sensitivity to profit opportunities of new enterprises which could be different from these parameters in previous integrated enterprises.)

III. Local Stability and Rationality of Adjustment Process

As we have already noted, the model (4) does not rule out profit maximization. Of course, a firm's profit maximizing decision is based on its expectations of sum of future outputs of other firms. For some expectation formulae there is a (time invariant) parameter (set of parameters) of sensitivity to profit opportunities for which linearized model gives the same output level as profit maximization.[6] We illustrate it on the simplest case of Cournot dynamics (tatonement) based on naive expectations (sum of rivals' outputs in the next period equal to their sum in the current period).

In this case the i-th firm's optimal output in the time period t+1 is given by (assuming interior solution)

$$q^{*}_{i,t+1}=\frac{(\alpha+n\beta)\mu-\beta\sum_{j\neq i} q_j}{\alpha+\beta} \tag{19}$$

Equating this with the expression for $q_{i,t+1}$ in the linearized model (6) we have

$$k_i=-\frac{1}{\alpha+\beta} \tag{20}$$

This parameter satisfies sufficient condition of stability (17) if and only if

$$n<\frac{\alpha}{\beta}+2 \tag{21}$$

Of course, since the condition (17) is not necessary, Cournot dynamics can be (locally) stable also when condition (21) is violated for some (but not all) i.

[6]For some expectation formulae corresponding parameter of sensitivity to profit opportunities can be different for different time periods, which is, of course, inconsistent with the models (4) and (6).

Cournot tatonement is usually rejected in the literature of industrial organization (e.g., Shapiro, 1990, Vives, 1989) for two reasons. First, firms stick to expectations which are systematically falsified by the evidence. Second, it is argued that adjustment is not optimal. This criticism is , of course, justified.[7] Nevertheless, we show that for some finite number of time periods adjustment process described by the model (6) - in some cases even if it corresponds to Cournot tatonement - can be rational in the sense that, given sum of rivals' outputs, a firm's output in the time period t+1 (generated by the model) gives it higher profit than sticking to the output level from the period t. We think that - especially when firms are sensitive to profit opportunities but not profit maximizers in the strict sense (we consider this to be a stylized fact of the transition from a centrally planned to a market economy) - such a "degree of rationality" is satisfactory.

First we find (using linear approximation of demand and quadratic approximation of costs) the level of output which would give the i-th firm, when sum of outputs of all other firms in the time period t+1 is in accordance with the model (6), the same of profit as the output $q_{i,t}$. This is given by:

$$\tilde{q}_{i,t} = -q_{i,t} - \frac{p'({}^cQ)\,(Q_{-i,t+1} - {}^cQ - \mu) + C''(\mu)\,\mu}{p'({}^cQ) - \frac{1}{2}C''(\mu)} \qquad (22)$$

where

[7]However, the second objection, if taken literally - i.e., if we define optimal adjustment as a choice of output which maximizes profit, given sum of rivals' outputs - is too severe. Optimality in this sense would imply that firms reach Cournot equilibrium in one step, so there would be hardly any dynamics.

$$Q_{-i,t}=\sum_{j\neq i} q_{j,t} \qquad (23)$$

Equating $\tilde{q}_{i,t}$ with $q_{i,t+1}$ we get (time - varying) upper bound for rational parameter of sensitivity to profit opportunities:

$$\hat{k}_{i,t}=-\frac{2}{2p'({}^cQ)-C''(\mu)}-\frac{\chi_{i,t}}{\psi_{i,t}} \qquad (24)$$

where

$$\chi_{i,t}=2\left[\left(p'({}^cQ)\beta\sum_{j\neq i}k_j-\mu p''({}^cQ)\right)\sum_{j=1}^{n}(q_{j,t}-\mu)+p'({}^cQ)\alpha\sum_{j\neq i}k_j(q_{j,t}-\mu)\right]$$

and

$$\psi_{i,t}=\left[2p'({}^cQ)-C''(\mu)\right]\left[\alpha(q_{i,t}-\mu)+\beta\sum_{j=1}^{n}(q_{j,t}-\mu)\right] \qquad (26)$$

For any finite number of (finite) time periods[8] finite maximum of $\frac{\chi_{i,t}}{\psi_{i,t}}$ exists. If this, when subtracted from the first term in (24) gives a positive value, this difference is the supremum of i-th firm's rational parameter of sensitivity to profit opportunities (for the set of time periods in question)[9]. Any positive number less than this value satisfying sufficient stability condition (17), is an i-th firm's rational parameter of sensitivity to profit opportunities (taking other firms' parameters of sensitivity to profit opportunities as given).

The above analysis leads to two conclusions. The first one concerns theory of oligopoly. Since the value of the parameter of sensitivity to profit opportunities corresponding

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[8]This holds with one qualification: In no one from these finite number of time periods industry output equals its Cournot equilibrium output and at the same time i-th firm's output equals its Cournot equilibrium output.

[9]For some values of parameters of the model (6) and some sets of time periods rational parameter of sensitivity to profit opportunities need no exist.

to Cournot tatonement (eq. (20)) is less than the first term in (24), Cournot tatonement can be rational for some sets of time periods (and demand and cost characteristics).

The latter relates to assessment of effects of privatization. It is almost generally accepted view (and the present author also subscribes to it) that privatization leads to an increase in sensitivity to profit opportunities. However, since (or at least if) firms do not know in advance other firms' actions (i.e. if they do not collude or form a cartel), we can expect that their reaction to profit opportunities (which they have to evaluate on the basis of current and, possibly, past situation) will be restricted (especially in the case of risk aversion of entrepreneurs) in order to avoid "overshooting".

IV. Conclusion

The disappointment of many ordinary citizens with development of prices (and output and employment) after price liberalization is understandable. However, it is a consequence (besides other factors) of past prices, set by central agencies lower than equilibrium ones, and of past outputs, set by planning agencies higher than equilibrium output (in oligopolistic industries[10]). There is no other way than to go through the (painful) process of convergence to equilibrium. "Remedies" proposed by politicians and economists longing for more government intervention would not help. One of the important reasons for this is that current

[10]It is not true that under central planning there is monopoly almost in all industries; most markets are oligopolistic ones (see Klaus, 1991a, p. 34).

performance of a firm need not provide any information about its prospects, especially after a change of owner.

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