Elasticities: Supplementary Statistics From Interindustry Studies

Harry W. Ayer and James Baskett

Elasticities are developed to incorporate both a sector's multiplier impact and its initial relative size to give a better estimator of the sector's importance to a region's economy. A current, empirical example of the difference between multipliers and elasticities, and their use in policy is given. Elasticities, compared to multipliers, are also often easier to employ in analysis because they are used with more readily available and comprehended data.

Interindustry models of regional economies, either input-output or the similar from-to models are frequently used to examine regional growth policy.[1] The principle policy statistic of interindustry models is the sector multiplier. The multiplier shows the total, direct plus indirect, change in a region's output, income or employment which results from a one unit change in sales to final demand from one particular sector. A current example of the development of multipliers for policy use is provided by the 1977 study of the Bureau of Economic Analysis (BEA), Department of Commerce. In that work, some 56 sector multipliers for each of 173 BEA areas are listed, and multipliers for any county in the U.S. are available upon request. Since economic impact statements are often required in order to support funding and other policy, it is very likely that these multipliers will receive widespread use and influence regional policy.

It is tempting to use the magnitude of multipliers to judge a sector's relative importance and to promote particular policies. The objectives of this paper are to (1) illustrate that this use of multipliers is incorrect, and (2) develop a new statistic, an elasticity, which better reflects a sector's relative importance to the regional economy. In addition, elasticities may be easier to employ in policy analysis because they are used with more readily available and comprehended data.

Elasticities

Elasticities are easily computed from the data and statistics of interindustry models and are derived directly from the multipliers. In matrix notation, the interindustry model is formulated as

$$1) \quad AX + Y = X$$

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The research for this report was supported by the Natural Resources Economic Division and the U.S. Forest Service of the U.S. Department of Agriculture, and the Agricultural Experiment Station of the University of Arizona through Western Regional Research Project W-141.

The authors are indebted to Roger Fox, William E. Martin, Robert McKusick and journal reviewers for comments on an earlier draft of the report, and to Paul Hoyt for computational assistance. Of course, only the authors are responsible for shortcomings.

[1] A from-to model is the same as an I-O model except that no information is required on the amount of inputs purchased by the endogenous industries from the exogenous or "primary inputs" sector. The principle disadvantage of from-to versus input-output analysis is the inability to cross check interindustry transactions by determining that row totals are equal to corresponding column totals. However, nearly all the analytic power of an I-O model is retained. See Baskett and Ayer, Kalter and Lord, or Tiebout for details of from-to analysis.

where A is the matrix of technical coefficients, Y is the vector of final demands and X is the vector of total output. Thus, the total output used by processing sectors (AX) plus that sold to final demand sectors (Y) equals the total output produced in the system (X). From (1)

$$2) \quad X = (I-A)^{-1} Y$$

where I is an identity matrix and the (I-A) inverse is the direct and indirect requirements matrix. Any particular element of the matrix shows the total direct plus indirect amount of input from row sector i required to produce one more unit of output from column sector j to be sold to final demand. The sum of these elements of the *jth* producing sector over the i input supply sectors is the sector multiplier M_j. Thus, M_j shows the total change in *regional output* (ΔX_T) associated with a one unit change in sales to final demand from sector j, (ΔY_j):

$$3) \quad M_j = \frac{\Delta X_T}{\Delta Y_j}$$

The elasticity for sector j, (E_j) is:

$$4) \quad E_j = \frac{\frac{\Delta X_T}{X_T}}{\frac{\Delta Y_j}{Y_j}} = M_j \frac{Y_j}{X_T}$$

and shows the percentage change in a region's total output associated with a one percent change in final demand sales by a particular sector. No additional data, besides that needed for I-O or from-to analyses, are required for computing elasticities.

Elasticities vs. Multipliers: An Empirical Example

The importance of computing and using elasticities is illustrated in a recent study of the Salt-Verde Basin of Arizona [Baskett and Ayer]. Roughly 65 percent of the area's 22 million acres is forest and rangeland managed by the U.S. Forest Service. Timber, cattle grazing, recreation, retirement settlement, government and mining are the most important economic activities. The region is rural in nature and has some 26 communities ranging in size from 100 to one of 30,000 people. The Forest Service affects income and employment in the region through policies which increase or decrease timber cut, grazing, recreation and retirement settlement. One objective of Forest Service policy is to improve employment opportunities in local communities — especially where unemployment or underemployment is considered a problem. To analyze the potential impacts of Forest Service policies, a from-to model in employment units was formulated for each of five subareas of the Salt-Verde Basin. Very short run sector multipliers and elasticities derived for one subarea are given in Table 1.[2] Similar multipliers (M_{FD}) and elasticities (E_{FD}) were also developed for key final demand sectors.[3] Three of these policy-affected final demand sectors and their short run multipliers and elasticities are given in Table 2.

The multipliers and elasticities, for both business sectors and final demand sectors, are widely disparate and present quite different inferences about the relative importance

[2]The very short run is defined as that period of time during which interindustry transactions take place; however, it does not include the short run "induced" effect of household expenditures or the long run effect of added investment expenditures. Both short and long run multipliers and elasticities were computed in the Salt-Verde research, but for brevity are not reported here. A comparison of multipliers and elasticities for the short and long runs gives the same conclusions as drawn from the comparison here of the very short run.

[3]Final demand multipliers and elasticities account for the total change in regional employment which results from a change in a particular type of final demand, such as sales to tourists, or sales to retired people. Final demand multipliers and elasticities differ from sector multipliers and elasticities in that those for final demand imply an initial change in final demand sales from several sectors. For example, if a new policy results in increased tourism, new final demand sales to tourists will be made by several sectors: department and variety stores, restaurants and bars, food stores, gasoline and service stations, and others.

TABLE 1. Sector Employment Multipliers and Elasticities, Salt-Verde Basin Arizona, 1973.

Sector	Employment Multiplier		Elasticity		Employment (Man Years)
	M_j	Rank	E_j	Rank	
1. Ag. Products, Services	1.56	2	.005	29	19
2. Beef Cattle Production	1.27	6	.055	2	319
3. Forest Operations	1.35	5	.032	8	179
4. Copper Mining	1.00	29	.043	7	2450
5. Other Mining	1.11	12	.007	26	45
6. Construction	1.60	1	.053	3	245
7. Non-Lumber Mfg.	1.08	15	.004	28	30
8. Lumber, Wood Products	1.18	9	.199	1	1263
9. Public Utilities	1.13	10	.020	14	133
10. Trans., Comm.	1.04	25	.028	11	199
11. Wholesale	1.03	28	.012	19	90
12. Dept. Variety Store	1.08	14	.022	12	153
13. Restaurants, Bars	1.05	21	.046	5	329
14. Food Stores	1.06	19	.050	4	353
15. Furniture Stores	1.07	16	.009	23	57
16. Auto., Sales, Parts	1.06	20	.018	16	124
17. Building Materials	1.22	7	.028	10	176
18. Gas Service Stations	1.06	18	.021	13	149
19. Clothing	1.03	27	.008	24	63
20. Other Retail	1.07	17	.029	9	200
21. Banking, Finance	1.04	22	.009	21	68
22. Insurance	1.42	3	.011	20	57
23. Real Estate	1.38	4	.014	17	76
24. Motels, Lodging	1.03	26	.044	6	326
25. Personal Business Svcs.	1.04	24	.020	15	139
26. Recreation	1.09	13	.009	22	58
27. Auto Repair, Retail	1.20	8	.006	27	41
28. Medical, Health Svcs.	1.04	23	.008	25	63
29. Other Services	1.11	11	.014	18	96
					Total subarea employment = 7500

Source: Computed from Baskett and Ayer.

of different sectors. An entirely different ranking of sectors, based on relative impacts on total regional employment, results from the two statistics. For example, Sector 1, Agricultural Products and Services is ranked 2nd and 29th by employing the multiplier and elasticity statistics, respectively. The Spearman statistic, showing the degree of correspondence between rankings, is .00098 and indicates that there is essentially no correspondence.[4] Rankings of final demand multipliers and elasticities shows a similar disparity, as illustrated in Table 2. The multipliers appear to indicate that home construction by seasonal tourists and retired households has the greatest regional impact, but the elasticity shows that the total regional impact from tourist consumption expenditures is currently 18 times greater than the impact from home construction, for equal percentage changes in final demand. The effect of relative sector size has simply outweighed the multiplier effect.

Another important difference between multipliers and elasticities is shown in Table 1. The relative variation among elasticities is much greater than the relative variation among multipliers. The coefficients of variation for multipliers and elasticities are 14 percent and 129 percent respectively.[5]

The variation among elasticities, in contrast to the variation among multipliers, provides a

[4]The Spearman rank difference correlation coefficient can range between +1 and −1. A "0" value indicates no relationships.

[5]The means and standard deviations, from which the coefficients of variation are computed ($C_X = \frac{s_X}{\bar{X}}$) are: elasticity = .028, $s_{elasticity}$ = .036; multiplier = 1.152; $s_{multiplier}$ = .1612.

TABLE 2. Final Demand Multipliers and Elasticities, Salt-Verde Basin, Arizona, 1973.

	Employment Multiplier		Elasticity	
Final Demand Sector	M_{FD}	Rank	E_{FD}	Rank
Seasonal Tourists, Consumption	1.38	3	.128	1
Retired Households, Consumption	1.39	2	.033	2
Home Construction by seasonal tourists and retired households	2.02	1	.007	3

possible basis for discriminating among sectors when selecting policies.[6] As an example, suppose that the chronic rate of unemployment in the Salt-Verde Basin is 6 percent, or .06 × 7500 = 450 people, and that public officials wish to stimulate some sector or sectors of the economy to increase total regional employment by 450 people. If added output can be exported or sold to other final demand sectors, the Forest Service may increase employment by clearing scrub vegetation to improve the range and stimulate cattle production (Sector 2), or by increasing timber exports (Lumber, Wood Products, Sector 8) through better management practices. If funds are limited and the Forest Service must choose between these strategies, the multipliers seem to suggest little difference between efforts to promote these sectors, i.e., the multipliers of 1.27 and 1.18 are nearly equal. Elasticities, on the other hand, strongly indicate that management policies favor the lumber sector: the elasticity of employment for the lumber sector, .199, is more than 3 times that for beef production, .055. To increase area job opportunities by 450, the beef sector must increase sales to final demand enough to directly hire 354 people (450 ÷ 1.27), an increase of 111 percent $(\frac{354}{319} \times 100)$ of the original beef sector work force. The lumber sector, on the other hand, must increase its direct employment by 381 people (450 ÷ 1.18), or only 30 percent $(\frac{381}{1263} \times 100)$ of the original lumber work force. There likely are a host of other factors, besides range improvement amenable to Forest Service policy, which have kept the beef sector relatively small in the past, and which will curtail the needed doubling in its size. Growth by 30 percent in the lumber sector seems more feasible, although a more careful analysis of demand and supply conditions affecting the lumber sector is needed to verify growth potential.

Tijoriwala, Martin and Bower make a similar analysis when considering the relative importance of water use in different economic sectors of Arizona. They demonstrate that even though a particular sector may have a low water multiplier, the multiplier fails to adequately reveal the relative importance of that sector in the area's water use. They develop weighted water multipliers which account for a sector's multiplier impacts, as well as the sector's relative importance in sales to final demand. The elasticities developed in this study correspond to the weighted multipliers in the Tijoriwala, Martin and Bower study.

A second reason why elasticities may be more useful to policy makers than multipliers is that elasticity analysis requires the more frequently used and comprehended percentage figures, while multiplier analysis relies on absolute numbers. Regional unemployment, for example, is usually expressed in percentage terms. The public and those setting public policy have standards, expressed in percentage terms, by which to guage the severity of the unemployment problem. For example, a 3 percent rate of unemployment is "acceptable" and little policy action is

[6]The relatively great differences among elasticities, in comparison with differences among multipliers, is not unique to the Salt-Verde area. Multipliers from dozens of interindustry studies show little size variation. For example, the recently computed multipliers for 56 industrial sectors for each of 173 Bureau of Economic Analysis (BEA of the Department of Commerce) areas throughout the U.S. show little variation, overall. Elasticities show relatively great variation because there is a great variation in sector size relative to total regional employment, and there appears to be no correlation between the size of a sector multiplier and the relative size of the sector.

deemed necessary, but corrective action is needed for unemployment rates above 6 percent. The 6 percent rate of unemployment in the example can be directly related to the elasticity measure and the relative size of sector stimulation needed to eliminate unemployment. To eliminate a 6 percent rate of regional unemployment, the lumber sector must increase sales to final demand by 30 percent

$$\left(\frac{\text{percent unemployment}}{\text{elasticity}} = \frac{.06}{.199}\right)$$

In short, it is tempting but misleading to use multipliers to suggest the relative importance of different sectors to a region's economy. Elasticities better represent a sector's relative importance by accounting for both multiplier impacts and sector size. In addition, elasticities can be used with more readily available and comprehended data.

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