PROCEEDINGS

44th Annual Meeting

WESTERN AGRICULTURAL ECONOMICS ASSOCIATION

Squaw Valley, California
July 25, 26, 27, 1971

Samuel H. Logan, Editor

EMPIRICAL INVESTIGATION OF THE RELATIONSHIP BETWEEN EXPECTED INCOME AND INCOME VARIABILITY IN FIRM DECISION MAKING

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Uncertain annual fluctuations in farm income are a major problem confronting the agricultural industry in the United States. Decisions to allocate resources to maximize farm income are tempered by the need to perpetuate the firm itself. One method of combatting this problem is through the proper combination of both crop and livestock enterprises.

The aim of this research was to develop estimates of income variance for individual enterprises and also to determine the influence on enterprise selection of total income variance. Such research will provide farm operators with information needed for selection of a profit maximizing enterprise mix, constrained by their particular aversion to income variability.

The Columbia Basin Irrigation Project of Washington was selected as the setting for the analysis. A linear programming framework used in the analysis contained the following production activities: wheat, potatoes, sugar beets, corn silage, alfalfa, cow-calf, cow-yearling, cattle feeding, ewe-lamb production, and sow-pig production. Physical, agronomic, and monetary limits established bounds for crop and livestock enterprises.

Income Variance as an Influence on Enterprise Selection

The assumption that increases in expected returns are desirable while increases in income variance are undesirable is central to this paper. An enterprise organization was preferred if it had a larger net farm income with equal or less variance of income, or if it had an equal net income, but lower income variance.

Estimates of variance provided a basis for incorporating risk into the study. Risk implies that a unique variance parameter can be empirically estimated for any probability distribution [2]. In this analysis, parameters of income variance, each estimated from a historical net income series, were used to constrain the selection of enterprises.

The nature of the income-income variance relationship can be called an efficiency frontier [3]. Each point on the efficiency frontier is where income is maximized, subject to a specific constraining level of income variance; the variance is the minimum possible consistent with that income level.

The construction of a net income variance constraint within a linear programming framework requires special handling. Beale [1] recommended using separable programming for linear approximation of nonlinear forms. Total farm variance is the summation of individual enterprise net income variance plus the covariance between enterprise net incomes. Separable programming was used to linearly approximate variance and covariance.

A linear-separable programming model was created by the inclusion of a separable programming constraint representing net income variance into a linear programming model representing a farm enterprise organization. The equation form of this model is as follows:

$$(1) \qquad \text{Max } \pi = \sum_{j=1}^{m} d_j X_j$$

$$(2) \qquad \text{Subject to } \sum_{j=1}^{m} a_{ij} X_j \leq b_i \qquad i = 1, 2, \ldots, n$$

$$(3) \qquad X_j \leq 0 \qquad j = 1, 2, \ldots, m$$

and

$$(4) \qquad \sum_j \sigma_j^2 X_j^2 + 2 \sum_{j<k} \sigma_{jk} X_j X_k \leq V_T .$$

In the above, Π is expected returns above variable costs, d_j is the annual per unit net returns from the j^{th} enterprise, X_j is the j^{th} activity level, a_{ij} and b_i are constants in the i^{th} physical constraints, σ_j^2 is the variance of annual returns from the j^{th} activity, σ_{jk} is the covariance between the annual returns of the j^{th} and k^{th} enterprises, and V_T is the total variance of annual returns.

The connection between the separable constraint (Equation 4) and the linear programming model (Equations 1, 2, and 3) was the crop and livestock sell activities. There were 11 sell activities in the model. This implied that 11 variance and 55 covariance terms were developed for the separable constraint.

A t-test was used to test the significance of each covariance term. The null hypothesis was that the covariance between net incomes of two enterprises was not significantly different from zero. The significance level used for this research was the .1 level. Out of a possible 55 covariance terms, only six were deemed significantly different from zero.

Empirical Analysis and Results

The linear-separable programming model was first run with all covariances and then rerun with only the six significant covariances, as presented in Table 1. Allowable net farm income variance was numerically developed as the right-hand side value of the separable constraint. When this RHS value was parametrically reduced, efficiency frontiers for the all covariance and the significant covariance models were numerically determined. Net income was restricted more stringently in the significant covariance model as noted by the efficiency frontiers shown in Figure 1.

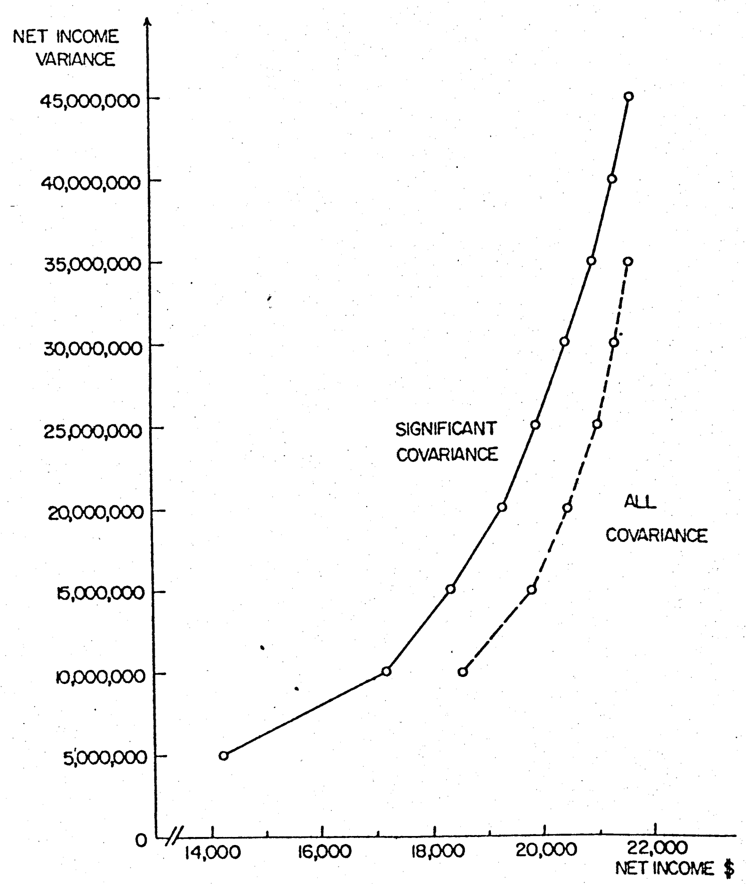

Figure 1 Efficiency Frontier Between Net Income Variance and Net Income

Table 1. Income Variance-Covariance Matrix Between Enterprises

		Enterprise										
Enterprise	Unit	Wheat	Potatoes	Sugar Beets	Alfalfa	Corn Silage	Hogs	Lambs	Spring Stocker Cattle	Fall Stocker Cattle	Summer Fin'd Cattle	Winter Fin'd Cattle
Wheat	Ac	580.7	-742.4	268.4	10.4	78.6	-13.0	-14.8	-98.6	-24.6	-77.5	-113.1
Potatoes	Ac		21,520.7	-474.2	415.8	1,316.6	-102.0	182.0	645.1	-239.8	-45.5	-539.5
Sugar Beets	Ac			984.7	5.6	222.4	3.6	41.2	-241.1*	113.6*	-37.3	-111.7
Alfalfa	Ac				222.6	197.1*	-9.7	-23.2*	15.1	15.8	-1.0	-25.3
Corn silage	Ac					587.6	-9.5	17.4	-120.8*	-7.0	-110.4	-49.5
Slaughter hogs	Hd						35.9	6.0	-14.2	3.1	-16.9	-23.2
Slaughter lambs	Hd							14.3	2.3	12.4	-6.0	-20.3
Spring stocker cattle	Hd								259.5	53.9	106.3*	-37.8
Fall stocker cattle	Hd									89.3	26.2	4.6
Summer finished cattle	Hd										166.8	47.9
Winter finished cattle	Hd											276.9

* Significant at 10% level.

Differences in enterprise mix between the all covariance and the significant covariance models were also observed (Table 2). When allowable variability was reduced to a standard deviation value of ± $3,162, important differences in levels of production between the two models were exhibited by wheat, alfalfa, corn silage, winter finished cattle, summer finished cattle, and aftermath stocker cattle enterprises.

Table 2. Enterprise Organization with all Covariance and Significant Covariance Terms at Three Levels of Income Variability

		Level of Income Variability				
		Unlimited	Intermediate(±5,000)[a]		Restrictive(+3,162)[b]	
Enterprise	Unit	All & Sig covar.	All covar.	Sig covar.	All covar.	Sig covar.
Alfalfa	acres	0	0	0	0	9
Corn silage	acres	0	0	6	10	40
Wheat	acres	80	9	94	100	61
Potatoes	acres	40	31	20	10	10
Sugar beets	acres	40	40	40	40	40
Winter-finish cattle	head	100	100	100	89	0
Summer-finish cattle	head	58	74	69	72	30
Stocker cattle	head	90	100	100	100	66
Ewes	head	147	150	169	146	169
Sows	head	16	16	16	16	16
Net Income		$ 21,839	21,004	19,886	19,511	17,200

a. Net income variance was 25,000,000.
b. Net income variance was 10,000,000.

The higher net income levels and different enterprise mix of the all covariance model are attributable to covariance buffering. The all covariance model contained 55 covariances of which 30 were negative. This allowed enterprises with high profitability and relatively larger individual variability and/or negative covariability to remain in solution at low levels of permissable total net farm income variance. The removal of all but the six significant covariances reduced covariance buffering, thereby reducing net income at lower levels of total farm income variance.

The accuracy of each linear separable model with regard to how closely linear representations of variance and covariance compare to statistically estimated variance and covariance is presented in Table 3. The acreage of each crop and number of livestock produced at a particular net income variance level were inserted into Equation 4, and actual variance was determined. This variance was compared with the separable constraint level. Errors in the all covariance model range from less than 1 percent to 3 percent. Errors in the significant covariance model ranged from less than 1 percent to 6 percent. Both models were found to be reasonably accurate.

Table 3. Error Between Separable Constraint Levels and Actual Total Farm Net Income Variance

Separable Constraint Level	Actual Net Income Variance-All Covariance Model[a]	Error (%)	Actual Net Income Variance-Significant Covariance Model[a]	Error (%)
45,000,000	...	...	44,333,534	1
40,000,000	...	...	39,142,125	2
35,000,000	34,763,418	1	34,608,418	1
30,000,000	29,437,367	2	29,077,501	3
25,000,000	25,045,797	...[b]	24,413,596	2
20,000,000	19,738,108	1	19,436,339	3
15,000,000	15,419,995	3	14,036,846	6
10,000,000	9,944,316	1	9,608,115	4
5,000,000	...[c]	...	4,980,386	...[b]

a. Calculated from Equation 4.
b. Less than .5 percent.
c. Information not available due to matrix inversion problems.

Conclusions

Developments such as substantial reductions of net income at low permissible levels of income variance suggest that knowledge of the preference function of farmers with regard to income and income variance should be researched. Policies or programs instituted without regard for the effect of this constraint on decision making may result in an undesirable product mix and/or low levels of income for farm operators.

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3. Markowitz, H. "Portfolio Selection," Journal of Finance, March, 1952, pp. 77-91.