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A DISTRIBUTED LAG ANALYSIS OF MILK PRODUCTION RESPONSE

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Estimates of the responsiveness of milk production to price changes provide useful information for administrators who try to provide an adequate supply of milk to consumers yet, at the same time, try to maintain a "reasonable" balance between milk production and consumption. In this paper we empirically estimate milk production response functions using a distributed lag formulation which allows a greater degree of flexibility in the lag structure than does the partial adjustment model formulation by Nerlove [13,14]. For comparison purposes we derive results using Nerlove's partial adjustment hypothesis.

Milk Production Distributed Lag Models

The quantity of milk produced in a given time period is hypothesized to be a function of the price of milk, prices of inputs used in milk production, returns obtainable from competing commodities, and the existing state of milk production technology. However, in milk production, as in many other products, there is a lagged response to a price change due to the nature of the underlying production process. Some adjustments can be made in the very short time period while others require considerably more time. The emphasis of this paper is on determining the nature of the lagged output response resulting from a change in product price.

Excluding for the moment variables other than product price, a general distributed lag model can be written as:

$$Q_t = \sum_{\tau=0}^{k-1} \beta_\tau P_{t-\tau} \tag{1}$$

where Q_t = output at time t; P_t = price at time t, k = number of periods covered by the lag function, and β_τ = the coefficients of the lag structure.

Geometrically Declining Lag

In his early work on investment, Koyck [11] assumed that β_τ in equation (1) has the special form:

$$\beta_\tau = a\lambda^\tau \tag{2}$$

where $0 \leq \lambda < 1$. Among those who have used this geometrically declining distributed lag structure are Cagan [3], Friedman [6], and Nerlove [13, 14]. Of relevance to our study is the partial adjustment model formulated by Nerlove which is:

$$Q_t - Q_{t-1} = \gamma (Q_t^* - Q_{t-1}) \qquad 0 \leq \gamma < 1 \tag{3}$$

where Q_t^* is the "desired" level of output in time t and is expressed as:

$$Q_t^* = c + bP_t. \tag{4}$$

The solution to difference equation (3) is obtained as:

$$Q_t = \sum_{\tau=0}^{k-1} \gamma (1 - \gamma)^\tau Q_{t-\tau}^* \tag{5}$$

provided the lag period, k, is sufficiently long. Substituting equation (4) into equation (5) gives:

$$Q_t = c + b \sum_{\tau=0}^{k-1} \gamma (1 - \gamma)^\tau P_{t-\tau} \tag{6}$$

That is, the weight assigned to any given price $P_{t-\tau}$ is $b\gamma(1 - \gamma)^\tau$. With reference to equation (2),

$$\beta_\tau = b\gamma (1 - \gamma)^\tau \text{ for } 0 \leq \gamma < 1 \tag{7}$$

which indicates that β_τ, as a function of τ, declines geometrically.

The major problems encountered in using a model of the above form have been discussed alsewhere by Griliches [7, 8] and Hall and Sutch [9]. One such problem is that the lag formulation itself is restricted by the geometrical specification.

More Flexible Distributed Lags

In order to obtain more flexible specifications of distributed lags, de Leeuw [4], Solow [15], Jorgenson [10], Bischoff [2], and Modigliani and Sutch [12] have proposed models which incorporate more flexible specifications of the lag structure. Recently, Almon [1] suggested a lag structure specification in which the coefficients are restricted to lie in a polynomial of low order. More recently, Hall and Sutch [9] have proposed a more direct technique of producing Almon's results which avoids using Lagrange interpolation polynomials. Hall and Sutch's model can be expressed as:

$$Q_t = \sum_{\tau=0}^{k} (\alpha_0 + \alpha_1\tau + \alpha_2\tau^2 + \dots + \alpha_N\tau^N)\, P_{t-\tau} + \alpha_{N+1} \sum_{\tau=0}^{\infty} \lambda^\tau\, P_{t-\tau} \tag{8}$$

where

$$\beta_\tau = \begin{cases} (\alpha_0 + \alpha_1\tau + \dots + \alpha_N\tau^N) + \alpha_{N+1}\, \lambda^\tau & \text{for } 0 \le \tau \le k \\ \alpha_{N+1}\, \lambda^\tau & \text{for } \tau > k \end{cases} \tag{9}$$

However, as Hall and Sutch [9] suggest, a low-order finite polynomial is appropriate for most econometric work. If a second-order finite polynomial is specified, β_τ can be expressed as:

$$\beta_\tau = \alpha_0 + \alpha_1\tau + \alpha_2\tau^2 \tag{10}$$

and equation (8) becomes:

$$Q_t = \sum_{\tau=0}^{k} (\alpha_0 + \alpha_1\tau + \alpha_2\tau^2)\, P_{t-\tau} \tag{11}$$

A further restriction on β_τ is that $\beta_\tau = 0$ when $\tau = k$. That is,

$$\alpha_0 + \alpha_1 k + \alpha_2 k^2 = 0 \tag{12}$$

Solving equation (12) for α_0 and substituting into equation (10) gives:

$$\beta_\tau = -\alpha_1 k - \alpha_2 k^2 + \alpha_1\tau + \alpha_2\tau^2 \tag{13}$$

$$= \alpha_1\,(\tau - k) + \alpha_2\,(\tau^2 - k^2). \tag{14}$$

Equation (11) can then be rewritten as:

$$Q_t = \alpha_1 \sum_{\tau=0}^{k} (\tau - k)\, P_{t-\tau} + \alpha_2 \sum_{\tau=0}^{k} (\tau^2 - k^2)\, P_{t-\tau} \tag{15}$$

For estimation purposes, equations (6) and (15) can be expanded to incorporate nonprice variables which can be estimated by ordinary least squares regression analysis.

Empirical Results

Both the geometric and polynomial distributed lag models were fitted to California quarterly milk production data. The following variables were included: prices of market milk, dairy feed, and cutter and canner cattle; an index of prices received by farmers for all farm products; average weekly gross earnings for workers in manufacturing; and, as alternative measures of technological change, time and an index number of farm production per man-hours for milk cows. Preliminary results showed that some of these variables were statistically insignificant and, hence, were excluded. Results are presented for the following geometric and polynomial lag models:

$$Q_t = \alpha_0 + \alpha_1 P_t + \alpha_2 S + \alpha_3 Q_{t-1} \tag{16}$$

$$Q_t = a_0 + a_1 \sum_{\tau=0}^{k} (\tau - k) P_{t-\tau} + a_2 \sum_{\tau=0}^{k} (\tau^2 - k^2) P_{t-\tau} + a_3 S + a_4 Z \tag{17}$$

where

Q_t = quarterly commercial production of market milk in California, 1953-1968 (in 10-million-pound units)1/

$P_{t-\tau}$ = ratio of average quarterly price paid to producers for market milk, f.o.b. ranch, California (dollars per hundredweight x 20 hundredweight ÷ tons), to average quarterly price paid by farmers for 16 percent protein dairy feed, California (dollars per ton)2/

S = 0 for the first and fourth quarters; 1 for the second and third quarters

and

Z = technology variable: T_t = time which is set equal to 1 for the first quarter in 1953; as an alternative, V_t = index of farm production per man-hour for milk cows.3/

The results based on the polynomial lag formulation are presented in Table 1 for length of lags 5 through 9. First, when a measure of technological change is included, regardless of the length of lag considered, the coefficients (W_1, W_2) of the milk-feed price ratio are statistically insignificant. However, coefficients for the seasonal effect and either measure of technology are significant at the 1 percent level. When technology is excluded, both W_1 and W_2 are significant for the 8-period lag model, while W_2 is significant for lagged periods 6 through 9. For all lagged periods, the coefficient for the seasonal effect is significant at the 5 percent level.

Results from the Nerlovian formulation are:

$$\begin{aligned} Q_t = &-39.211 + 46.564P_t + 7.642S + 0.85Q_{t-1} \\ &(4.81) \quad (4.87) \quad (6.59) \quad (30.70) \end{aligned} \tag{18}$$

Here, all the coefficients are statistically significant as indicated by the bracketed t values. When technology is included in the polynomial lag model, the results are substantially different in that the milk-feed prices have no effect on the level of milk production. This is not true, however, when Technology is excluded. There is little basis for deciding which model is "best." The equation from Table 1 where k = 8 and no technology variable is included is selected for comparison and illustrative purposes since both W_1 and W_2 were statistically significant. It can be rewritten as:

$$\begin{aligned} Q_t = &-223.960 + 7.998S + 18.774P_t + 34.049P_{t-1} + 44.289P_{t-2} \\ &\qquad (2.428) \quad (.830) \quad (3.144) \quad (15.325) \\ &+ 49.494P_{t-3} + 49.665P_{t-4} + 44.801P_{t-5} + 34.902P_{t-6} + 19.968P_{t-7} \\ &\quad (8.060) \quad (4.796) \quad (3.940) \quad (3.515) \quad (3.119) \end{aligned} \tag{19}$$

Except for the coefficient on P_t, all of the other coefficients are statistically significant at the 5 percent probability level as indicated by the bracketed t values. It appears that a price change within any given time period has little impact on production within that period. A price change at time t has a maximum effect on production four periods later. Beyond four time periods, the impact on production becomes less and less. These results appear "reasonable" in view of a priori knowledge of the dairy industry.

Table 1. Regression Estimates for California Milk Production Response, Polynomial Lag Formulations, 1953-1968[a/]

Lag period	Constant term	W_1 [b/]	W_2 [c/]	S	V_t	T_t	R^2
k = 5	-209.025	-3.450 (-0.123)	3.495 (0.777)	11.023 (2.705)*			0.803
	161.731	6.392 (0.348)	-1.433 (-0.614)	7.463 (3.769)*	0.816 (9.096)*		0.926
	136.912	-6.348 (-0.532)	0.600 (0.405)	7.875 (6.016)*		1.563 (16.105)*	0.968
k = 6	-218.138	-17.050 (-0.773)	4.015 (1.373)+	9.669 (2.815)*			0.820
	140.073	-1.222 (-0.076)	-0.177 (-9.102)	8.085 (4.193)*	0.766 (8.071)*		0.925
	125.959	-10.614 (-1.017)	1.052 (0.943)	7.982 (6.227)*		1.516 (14.574)*	0.967
k = 7	-223.977	-21.210 (-1.226)	3.525 (1.801)‡	8.316 (2.445)ό			0.837
	139.459	-2.991 (-0.258)	0.077 (0.070)	8.788 (4.760)*	0.750 (7.758)*		0.930
	121.956	-7.487 (-0.958)	0.667 (9.907)	8.141 (6.484)*		1.475 (13.639)*	0.967
k = 8	-223.960	-17.792 (-1.340)+	2.517 (1.921)‡	7.998 (2.428)ό			0.851
	147.521	-1.153 (-0.030)	-0.054 (-0.072)	8.263 (4.548)*	0.751 (7.530)*		0.929
	126.649	-3.152 (-0.509)	0.241 (0.463)	7.987 (6.240)*		1.470 (12.818)*	0.964
k = 9	141.390	-0.192 (-0.025)	-0.080 (-0.135)	7.529 (4.189)*	0.723 (7.265)*		0.930
	122.426	-1.777 (-0.323)	0.128 (0.308)	7.668 (5.895)*		1.430 (12.086)*	0.963

a/ Numbers in parentheses are t values.

b/ $W_1 = \sum_{\tau=0}^{k} (k - \tau) P_{t-\tau}$

c/ $W_2 = \sum_{\tau=0}^{k} (k^2 - \tau^2) P_{t-\tau}$

* Statistically significant at the 1% level

ό Statistically significant at the 5% level

‡ Statistically significant at the 10% level

\+ Statistically significant at the 20% level

Price Elasticities and Lagged Response

For the Nerlovian formulation the short- and long-run price elasticities are 0.381 and 2.541, respectively. However, for the polynomial lag, it is possible to derive a price elasticity for each time period as well as an elasticity which covers the entire response period considered. The elasticity of supply associated with a change in relative prices at any point in time t can be calculated as:

$$e_\tau = \frac{\partial Q_t}{\partial P_{t-\tau}} \cdot \frac{\overline{P}}{\overline{Q}} \cdot \qquad \tau = 0, 1, \ldots, N. \tag{21}$$

The cumulated price elasticity for all k quarters is:

$$e_s = \sum_{\tau=0}^{k-1} \frac{\partial Q_t}{\partial P_{t-\tau}} \frac{\overline{P}}{\overline{Q}} \tag{22}$$

P and Q are set at their respective means for the entire time period considered. The derived elasticities are given in Table 2.

Table 2. Estimated Price Elasticities Derived from the Polynomial Lag Formulation

Time period	e_τ	Time period	e_τ
t	0.16	t + 4	0.42
t + 1	0.29	t + 5	0.38
t + 2	0.38	t + 6	0.30
t + 3	0.42	t + 7	0.17
	e_s = 2.53		

With reference to Table 2, for a 1 percent change in the milk-feed price ratio at time t, the supply elasticity for t is 0.16. This value reaches a maximum of 0.42 at t + 3 and t + 4 and declines to 0.17 at t + 7. However, the total increase in supply from a 1 percent change in price at time t is given by the cumulated elasticity, which is 2.53. It is interesting to note that the short-run elasticity of 0.38 derived from the Nerlovian model is that obtained from the polynomial lag formulation for the time periods t + 2 and t + 5. However, the elasticities computed for periods t + 3 and t + 4 are greater than this. Also, the long-run elasticity of 2.541 based on the geometrically declining lag structure is approximately equal to the cumulated elasticity derived from the polynomial lag model. Hence, for the models selected, the long-run response estimates do not depend critically on whether a geometrically declining or a polynomial lag price structure is assumed. However, the response for each period of time and for various short-run intervals does. Because of its specification, the geometric lag continually declines; hence, output response decreases with time. However, the polynomial lag model suggests that the rate of output response from a price change first increases and then declines (as indicated by the coefficients of $P_{t-\tau}$ in equation 19). This type of output response does not seem to be unrealistic for milk production since it appears unlikely that the greatest marginal output from a given change in price is forthcoming in the immediate period after the price change.

Conclusions

We estimated milk production response for both a polynomial and a geometrically declining distributed lag price structure. Specifying a price lag structure in a polynomial fashion has the advantage that the shape of the price lag structure is not specified *a priori*; this allows for considerably more flexibility than does the geometrically declining lag structure used in many supply estimation studies. We feel that for much of agricultural production output response to some given price change first increases through time and then declines. This certainly appears to be true for dairying. However, only if a flexible price lag structure such as the polynomial formulation is used can this type of output response be detected.

FOOTNOTES

1/ Data obtained from California Crop and Livestock Reporting Service, Manufactured Dairy Products, Milk Production Utilization, and Prices, (Sacramento), various issues.

2/ Milk prices same as above. Feed prices obtained from U.S. Department of Agriculture, Agricultural Marketing Service, Agricultural Prices, various issues.

3/ Index of farm production obtained from U.S. Department of Agriculture, Changes in Farm Production and Efficiency, Statistical Bulletin 233, 1969.

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