A Dynamic Model for Determining Optimal Range Improvement Programs

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A Markov chain dynamic programming model is presented for determining optimal range improvement strategies as well as accompanying livestock production practices. The model specification focuses on the improved representation of rangeland dynamics and livestock response under alternative range conditions. The model is applied to range management decision making in the Cross Timbers Region of central Oklahoma. Results indicate that tebuthiuron treatments are economically feasible over the range of treatment costs evaluated. Optimal utilization of forage production following a treatment requires the conjunctive employment of prescribed burning and variable stocking rates over the treatment's life.

Key words: range improvements, rangeland dynamics, Markov chains, biophysical simulation.

Economic assessment of long-term range improvements traditionally has been conducted through application of static investment criteria. Estimates of expected forage production over the treatment's life are developed and used to derive annual net cash flows based upon some fixed managerial policy (McBryde, Conner, and Scifres; Whitson and Scifres). Measures of net present value or internal rate of return are then calculated to determine the profitability of the investment. These static approaches ignore a number of the complexities of rangeland dynamics that affect the efficiency of range improvement investments. Timing and frequency of treatment significantly influence the stream of benefits that may be realized from a range improvement program. Stochastic weather conditions following the treatment also are critical. Numerous production practices, including stocking rates and maintenance measures used to extend the life of a treatment, interact with range improvement treatments to influence the resulting forage production and animal response. Failure to coordinate these practices with range improvement treatments can lead to inefficient range improvement programs.

The objectives of this study are: (*a*) to improve upon previous range improvement studies by developing a methodology to incorporate the effects of timing and risk into the analysis of range improvement activities, and (*b*) to apply the methodology to derive optimal range improvement programs in a specified production setting. The previous work of Burt (1971) and of Karp and Pope serves as the basis from which much of this research evolves.

Review of Previous Research and Problem Statement

Burt (1971) introduced the problem of determining optimal frequencies for long-term range improvements, citing several modifications to the classic replacement problem required to represent range investment decisions. Formulations were developed under assumptions of both Markov and higher-order dependence to deduce asymptotic decision rules for treatment frequency.

Although Burt analyzed the range invest-

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ment problem using a deterministic framework, he noted that if significant stochastic variation existed in the system, numerical solution could be achieved through application of Markov chain dynamic programming. Large fluctuations in the amount and distribution of annual precipitation and resulting deviations in annual pasture productivity require that range management decisions be made in an uncertain environment. Since climate is a random variable, forage production and, hence, returns derived from range improvements are also random variables. Karp and Pope employed the theory of finite Markov chains to investigate the effect of these stochastic influences on range investment decisions. These researchers transformed the Markov chain dynamic programming model into a linear programming formulation for the purpose of determining optimal range treatment frequencies and stocking rates. A deterministic equation representing rangeland dynamics was first developed, and the deterministic control problem was solved subject to this constraint. Uncertainty was then introduced into the dynamic equation to develop the stochastic control problem.

Burt (1971) and Karp and Pope cited several simplifying assumptions required to apply their analytical frameworks and proposed a number of useful extensions to their basic models. Several of these proposed refinements focused on the representation of changes in the productivity of a range site following application of a range improvement treatment. Both studies assumed vegetative response following a treatment to be identical despite the condition of the range prior to treatment. In addition, vegetative response was assumed immediate and known with certainty. These assumptions do not reflect forage production relationships expected in a majority of rangeland settings. Previous dynamic range investment models also paid little attention to the representation of livestock response under alternative range conditions. Finally, both studies considered only a single range improvement treatment and, thus, did not address how a producer might integrate range improvement alternatives into a complete range management program.

This study incorporates these refinements and others into a stochastic dynamic programming model to improve the empirical validity of range improvement prescriptions. In the past, unavailability of response data forced researchers to take more of a methodological orientation when applying dynamic models to range investment decision making. As a result, considerable debate ensued as to the appropriateness of these models in empirical range economics research (Martin; Burt 1972). This study attempts to integrate available experimental data with results from biophysical simulation models to improve the physical representation of the range-livestock production system in dynamic optimization models. In addition to representing both the dynamic and stochastic elements of the range improvement problem, the analysis focuses on improvements in the specification of rangeland dynamics as well as more complete representation of controls available to the rangeland manager. From the model, complete range improvement programs, consisting of the time and type of treatment as well as annual livestock production practices, may be derived. The methodology is outlined and then applied to range management decision making in central Oklahoma.

Methodology

The problem of rangeland management is modeled as a multistage decision problem using the theory of finite Markov chains. At the beginning of each year, the producer is assumed to examine the decision-making environment and select a range improvement treatment as well as accompanying livestock production practices. Optimal range improvement programs are based upon a "flexible decision criterion"; that is, an efficient set of practices is determined for each state of the system.

The "state" of the system is defined by two variables—range condition (R) and the length of time elapsed since the last treatment (V). The range condition state variable represents an index of the productivity of the range site and can take on s values, R_i, $i = 1, 2, \ldots, s$. The state variable V is used to differentiate R_i with regard to the amount of time elapsed since the last range improvement treatment. Productivity for a given R_i may differ dramatically in terms of expected forage production in future time periods for different Vs. Therefore, inclusion of the second state variable allows a more accurate description of the dynamic relationships representing the transition from one range condition to another.

It is assumed that the range manager has three types of controls. At the beginning of each period (year), the manager must select the livestock enterprise to be employed in the current grazing season (L), the stocking rate (S), as well as a range improvement treatment strategy. Available range improvement strategies include brush control by chemical application (C) and prescribed burning (B). The expected value of range production can only be increased if a chemical treatment is applied. Prescribed burning may be employed as a maintenance tool to retard the rate of range degradation following a chemical treatment (McCollum, Engle, and Rollins). Treatment variables (C and B) are binary variables that describe whether or not a treatment is applied, while stocking rate can take on r possible values, S_i, $i = 1, 2, \ldots, r$.

Previous applications (Karp and Pope; Pope and McBryde) were based upon the assumption that the effect of the treatment is independent of the current state and known with certainty. Range production was assumed to increase immediately following treatment and decrease monotonically over the remainder of the investment's life. While this assumption may be tenable when evaluating brush control on select brush species, it is not realistic for a majority of range investment applications. In most situations, forage response to chemical treatments increases over the first few years of the treatment's life, reaches an apex, and gradually declines as brush species reinvade the range site (McBryde, Conner, and Scifres). The magnitude and duration of the additional forage release from a treatment is closely related to the current condition of the range site. Also, empirical evidence suggests that variability in precipitation levels can have a significant effect on treatment effectiveness during the initial phase of the treatment's life (Van Tassel and Conner). These properties of forage response to range improvement treatments are explicitly represented in this application. The number of years required for forage production to reach a maximum following a treatment is both uncertain and conditional upon the state of the range site at treatment.

Solution of the problem requires finding the optimal control rule that maps each state (combination of R and V) into a set of controls. Controls are selected to maximize the expected present value of the sum of net returns over a given time horizon. The payoff function gives the current payoff to the decision maker's control selection given the state of the system. Thus, returns in period t are a function of the state of the system (R_t, V_t) as well as the set of controls selected, $k_t = (L_t, S_t, C_t, B_t)$, and may be expressed as $g(R_t, V_t, k_t)$. Therefore, the multistage decision problem may be expressed as

$$\max_{k_t} E \sum_{t=1}^{n} \mathrm{B}^t \cdot g(R_t, V_t, k_t), \tag{1}$$

where B represents the appropriate discount factor, $(1 + r)^{-1}$, and n is the length of the time horizon. This objective function is maximized subject to a set of relationships defining the transformation of the state variables between stages.

The Markov assumption implies that range condition in period $t + 1$ is a random variable and is dependent only upon state and control variables in period t. These interrelationships are specified using a stochastic Markov process consisting of a unique transition matrix (P) for each feasible combination of controls. Each P-matrix is a square matrix whose order is equal to the total number of possible states, m (all combinations of R and V). The probability with address ij in the rth P-matrix (P^r_{ij}) denotes the probability of moving from state i in period t to state j in period $t + 1$, given that the rth combination of controls is employed in t.

Given the above definitions, the problem can be restated by applying Bellman's Principle of Optimality. Let $f_n(i)$ be the expected return from an n-stage decision process under an optimal policy when the initial state is given by the ith combination of states R and V. The Principle of Optimality states that "an optimal policy has the property that whatever the initial state and decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision" (Bellman, p. 83). Application of this principle to the range management problem above yields the following recurrence relation:

$$f_n(i) = \max_{k} g(i, k) + \mathrm{B} \sum_{j=1}^{m} P^k_{ij} f_{n-1}(j), \tag{2}$$

where the second term in the equation gives the expected value of net returns over the remaining $n - 1$ years of the time horizon if an

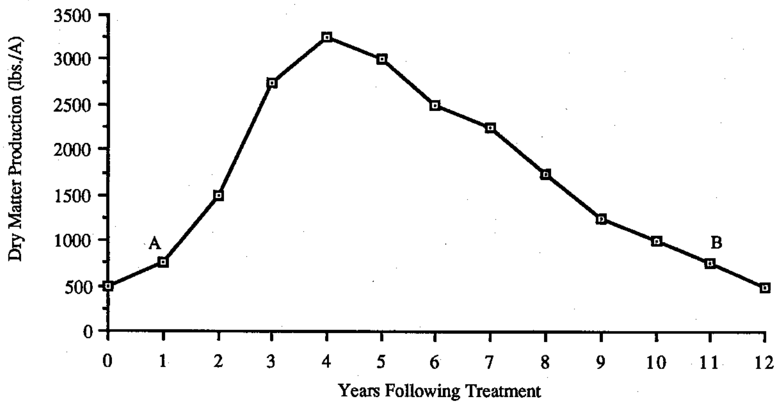

Figure 1. Response curve depicting annual forage production following application of tebuthiuron on a shallow savannah range site

optimal policy is followed, given selection of control k in period n.

Solution to the multiperiod problem is achieved in this application by employing the dynamic programming algorithm proposed by Burt and Allison. These authors illustrated that the recurrence relation converges to a constant decision rule as the time horizon is increased. Thus, the length of the time horizon is specified at a value large enough for convergence to be achieved, and the solution provides an optimal policy for all values of n. The resulting policy indicates the optimal set of controls that should be employed at each state. The dynamic programming algorithm is employed in lieu of the linear programming (LP) formulation proposed by Karp and Pope. Although ease of application makes the LP method an attractive solution technique, Karp and Pope illustrate some dimensionality problems when the procedure is applied to larger problems. Improvement in the empirical validity of the range investment model requires an increase in the dimensions of the problem. This increase in both state and control variables may be accommodated in the recursive algorithm employed.

The solution for the optimal policy defines a unique transition probability matrix, P^*, giving the transition probabilities for alternative range condition states under optimal management. The ith row of P^* corresponds to the ith row of the transition matrix $P(k_i^*)$, where k_i^* is the optimal control set at stage i. From the optimal transition probability matrix, the steady-state probability vector, π, may be derived as the row vector that solves $\pi = \pi P^*$ and $\sum_i \pi_i = 1$ (Rausser and Hochman). The steady-state probability vector indicates the long-run probability distribution of the various states and may be used to provide an estimate of the length of the treatment "renewal cycle" under the optimal management strategy. The length of the optimal renewal cycle represents the long-run expected time between treatments. In addition, a unique long-run measure of discounted expected returns from an acre of rangeland (f^*) may be estimated as the product of the steady-state vector and the vector of $f_n(i)$ values. Unlike the $f_n(i)$ values, f^* is not conditional upon the initial range condition state.

An Application

Application of the above methodology requires specification of a number of relationships defining economic and forage response to various combinations of states and controls. This analysis draws upon several different brush control and grazing studies conducted in central Oklahoma over the past decade to represent the range-livestock production system. Production relationships presented here apply to a representative shallow savannah range site located in the Cross Timbers Region of central Oklahoma.

Range condition (R) is expressed in terms of the quantity (pounds/acre) of vegetative dry matter produced annually. The state space is divided into 12 discrete states defined in 250-pound increments, ranging from 500 to 3,250 pounds of dry matter production. This range encompasses all possible levels of productivity on the shallow savannah range site, from complete brush infestation to maximum forage production resulting from brush eradication (Stritzke, Engle, and McCollum).

Livestock enterprises included in the analysis pertain to production of stocker steers during the summer grazing season (mid-April through mid-September). Two stocker enterprises, season-long stocking and intensive-early stocking, may be employed. In the season-long enterprise, calves are purchased at a weight of 450 pounds in early April and grazed for a period of 150 days. Five alternative stocking rates are available at each range condition—60%, 80%, 100%, 120%, and 140% of carrying capacity. Carrying capacity is determined by allocating 18.2 pounds of dry matter/steer/day (Brummer) when 25% of total annual dry matter production is assumed available for use by livestock (Kothmann).[1] In the intensive-early stocking enterprise, calves are grazed at twice the normal stocking density for the first 80 days of the grazing season.

Forage release in response to chemical treatments was estimated from research findings reporting annual dry matter production following an application of tebuthiuron (two pounds of active ingredient per acre) on a shallow savannah range site. A response curve describing expected annual forage production over the life of the treatment is shown in figure 1. This response curve represents a composite of results from two experiments investigating the effect of tebuthiuron on Cross Timbers rangeland (Stritzke, McMurphy, and Hammond; Engle, Stritzke, and McCollum).

The second state variable, V, describes the time elapsed since the chemical treatment (t). In this application V can take on two values: $V = 0$ describes states immediately following treatment ($t = 1, \ldots, 4$), while $V = 1$ denotes states following the apex of the response curve ($t = 5, 6, \ldots$). This state variable is needed to differentiate states such as those denoted by points A and B on the response curve. Although these states are described by the same range condition ($R = 750$ pounds/acre), they differ considerably in terms of the level of forage production expected in subsequent years. Thus, the vector of probabilities defining the transition of range condition from the same range condition state (R_i) will differ depending upon the time elapsed since the last treatment ($V = 0$ or 1).[2]

Forage response reported in figure 1 only applies if the rangeland is stocked at or below carrying capacity. Stocking in excess of carrying capacity will alter this expected rate of succession. Stocking at 120% of carrying capacity is assumed to accelerate range degradation and reduce forage carry-over such that forage production in the following year will be 250 pounds below the level reported on the response curve. Stocking rates of 140% of carrying capacity result in a 500-pound reduction in forage production in the subsequent year.[3] If the expected range condition falls to the 500-pound level, it remains constant despite the stocking rate employed.

Prescribed burning is included as a pasture maintenance tool to extend the life of a chemical treatment. Proper use of spring burning in the study area has been shown to result in increased forage production as well as improved forage quality. Burning in years following chemical treatments has been shown to increase forage production by retarding growth of brush and weedy species not adequately controlled by the herbicide (Engle, Stritzke, and McCollum). Results indicate that improvements in forage quality translate into an 8% to 15% increase in weight gain over the summer grazing season (McCollum; Wolfolk et al.). Initiation of a prescribed burn is assumed to have two effects: (*a*) maintenance of the current level of expected range productivity for an additional year, and (*b*) a 10% increase in weight gain during the current season. This response is conditional on the availability of enough vegetation during the early spring to provide sufficient fuel for an effective burning treatment. To assure that an adequate quantity of grass is carried over to provide fuel for a prescribed burn, the pasture cannot be grazed in excess of carrying capacity during the year prior to burning.

[1] This allocation reflects a moderate level of forage utilization and accounts for nonconsumptive uses (disappearance, trampling, etc.).

[2] If adequate data were available to specify a unique transition probability vector for each value of t, V could be defined by the number of years since the last treatment. This procedure would, of course, increase the dimensions of the transition probability matrices considerably.

[3] These effects were estimated from forage production data collected on overgrazed portions of the experiment described in Stritzke, Engle, and McCollum and Engle, Stritzke, and McCollum (unpublished data).

Application of the model requires estimation of the payoff function over all state/control variable combinations. A modified version of the stocker cattle growth simulation model developed by Brorsen et al. was used to estimate animal performance under alternative grazing strategies and forage production conditions. The model simulates the growth and development of stocker cattle over the grazing season using the California Net Energy System (National Research Council). Based upon daily input of forage availability, forage quality, and livestock weight, the intake of stocker cattle is estimated. Net energy provided by the forage is separated into net energy required for maintenance and gain, and daily gain is calculated as a function of metabolic weight and energy available for gain. Stocking rate effects were incorporated by modifying the model to estimate gains of stocker cattle under conditions of limited forage availability. Forage available for consumption in week $t(FA_t)$ was calculated as:

$$(3) \qquad FA_t = FA_{t-1} + NP_{t-1} - FU_{t-1},$$

where NP_{t-1} = net forage production in $t - 1$, and FU_{t-1} = forage utilization in $t - 1$ (intake and nonconsumptive use). When the quantity of available forage fell below a specified threshold value, forage was considered limiting and intake estimates were reduced below voluntary intake.

To predict the performance of stockers, the simulation model requires weekly forage data, including dry matter production, percent protein, and total digestible nutrients (*TDN*). Annual forage production corresponding to each state was divided into discrete, weekly periods to reflect an average distribution of forage availability over the grazing season (Engle, Stritzke, and McCollum; Brummer). Forage quality values (i.e., percent protein and *TDN*) were based upon the research findings of Powell, Stadler, and Claypool; and McCollum. Use of the stocker simulation model should provide more reliable estimates of livestock performance than single-equation stocking rate response functions employed in previous applications.

Daily gain estimates derived from the stocker simulation model were used in conjunction with Oklahoma State University enterprise budgets to estimate net returns for alternative state and control variable combinations. Per-acre annual net returns (*PANR*) were estimated as:

$$(4) \qquad PANR = [P_{ys}(450 + SWG) - 450\, P_{yc} - \gamma(SWG)]/S - P_m - P_C \cdot C - P_B \cdot B,$$

where P_{ys} is the price of feeder steers ($/lb.), SWG is the seasonal weight gain (lbs./head), P_{yc} is the price of calves ($/lb.), $\gamma(SWG)$ is the per-head production cost expressed as a function of seasonal weight gain (marketing and hauling costs vary as a function of weight gain), S is the stocking rate (acres/head), P_m is pasture maintenance cost ($/acre), C and B are zero-one variables indicating whether a chemical or burning treatment was initiated, and P_C and P_B are the per-acre costs of chemical and burning treatments. Prices were assumed constant at levels reflecting the average of normalized prices over the 1966–87 period, and a 2% death loss was assumed. Two costs of chemical treatment (P_C) were considered in the application, $60 and $75 per acre. This represents the range of costs a producer might pay for aerial application of tebuthiuron in the study area (Stritzke). The cost of prescribed burning (P_B) was estimated at $3.25 per acre (McCollum, Engle, and Rollins). A discount factor of .952 (r = .05) was assumed.

Specification of the transition probability matrices using historical forage production data would require several replications of brush control experiments parameterizing burning events, stocking rates, and the time of retreatment. Complete data do not exist because collection is cost prohibitive. Data unavailability has been an impediment to incorporating stochastic elements of production response into economic assessment of range improvements. Previous applications have used some assumed probability distribution of annual forage production to incorporate this source of uncertainty into their analyses. Karp and Pope noted that range improvement plans were sensitive to the assumed form of this distribution, indicating that improved specification of forage uncertainty is necessary to enhance the empirical validity of range investment models. This study employs a procedure that combines experimental data from range improvement experiments with simulated forage production data from a biophysical rangeland model to derive the necessary probability matrices.

Using experimental data reported by Stritzke, McMurphy, and Hammond; Brummer; Engle, Stritzke, and McCollum; and Stritzke, Engle, and McCollum, the expected value of the state variables in each year following a chemical treatment was estimated. The effect of alternative stocking rates and prescribed burns on forage production was incorporated using the relationships discussed above. This expected behavior was represented in the transition matrices by placing ones in the appropriate addresses. The ones were then replaced by probability distributions of annual forage production with mean annual production equivalent to the state.

A biophysical rangeland simulation model was used to calculate the probability distribution of annual forage production at each state. The simulation model, referred to as ERHYM-II, is a range site scale model developed to estimate the effect of various environmental influences on the growth and development of range plants (Wight). Daily simulation of soil water evaporation, transpiration, runoff, and soil water routing is conducted using specified climatic, edaphic, and agronomic data. Annual forage production is computed at peak standing crop as a function of actual and potential transpiration accumulated over the growing season.

Fifty years of historical weather data from Stillwater, Oklahoma, were used to estimate the probability distributions of annual forage production. To generate unique probability distributions corresponding to each state, parameters defining the production potential of the range site were specified to yield an expected value of forage production equal to the midpoint of the state. Transition probabilities were derived based upon the occurrence frequency of forage production estimates comprising each state. The resulting distribution reflects the dispersion around average annual forage production values resulting from climatic variability. By applying this randomization procedure, the noise added to the system through the stochastic specification is a function of both the prevailing state and control.

Selected Results

Results from applying the model to two different treatment cost scenarios are reported in table 1. The optimal set of controls (livestock enterprise, stocking rate, and treatment strategy) is given for each element of the state space (each combination of R and V), as well as the expected value of discounted net returns given initialization from that state (i.e., $f_n(i)$). States 1A–12A refer to states in the initial phase of the response curve ($V = 0$), while states 1B–12B apply to states following its apex ($V = 1$).

Under the high-treatment cost scenario ($75 per acre), both chemical treatment and prescribed burning are employed in the optimal management plan. Application of tebuthiuron is profitable only when the range condition falls to 500-pounds per acre (state 12B). Under this optimal control rule, the long-run expected time interval between treatments, T^*, is 18.3 years. In addition, maintenance burns are prescribed at several states in the latter years of treatment life. These annual prescribed burns serve to extend the life of the chemical treatment by retarding the reinfestation of brush species. Prescribed burns are only profitable when forage production exceeds 1,250 pounds per acre. At lower production levels, stocking rates are too low to generate sufficient income to cover the $3.25 per-acre cost of burning. By adding the long-run probabilities of states where burning is prescribed, an estimate of the frequency of prescribed burning events may be derived. In this scenario, annual prescribed burning is employed in approximately four out of every 10 years. Thus, economic returns can be enhanced through timely use of prescribed burning, rather than adopting an annual burning program as currently recommended by range researchers.

Adjustments in livestock numbers over the life of the tebuthiuron treatments correspond closely to changes in expected forage production. Specific livestock enterprises produced and annual stocking rates are conditional upon the range condition as well as the treatment strategy employed at that state. Season-long stocking is employed in all states except in years prior to prescribed burning. In these years, intensive-early stocking is used to assure sufficient forage is carried over to the following year to implement a prescribed burn. In all cases, intensive-early stocking is preferred over season-long stocking below carrying capacity as a means of providing carry-over forage for burning. In states where maintenance burns are employed, benefits in the form of additional weight gain and extending the life of the chemical treatment exceed the cost of burning, as well as returns foregone from underutilization of forage produced in the previous pe-

Table 1. Optimal Range Improvement Plans under Two Treatment Costs, Flexible Management Scenario

			High Cost Treatment[b,c]			Low Cost Treatment[b,c]		
State[a]	V	R	Stocking Rate	Treatment	$f_n()$	Stocking Rate	Treatment	$f_n()$
1A	0	3,250	3.4	0	180.81	2.7	0	193.11
2A	0	3,000	3.6	0	179.51	3.6	0	192.02
3A	0	2,750	4.0	0	177.83	3.2	0	190.16
4A	0	2,500	3.5	0	176.63	3.5	0	188.96
5A	0	2,250	3.9	0	176.26	3.9	0	188.50
6A	0	2,000	4.4	0	175.06	4.4	0	187.30
7A	0	1,750	5.0	0	174.72	5.0	0	186.80
8A	0	1,500	5.8	0	173.52	5.8	0	185.60
9A	0	1,250	8.7	0	172.60	8.7	0	185.24
10A	0	1,000	10.9	0	171.50	10.9	0	184.25
11A	0	750	14.6	0	170.40	14.6	0	180.15
12A	0	500	21.8	0	165.06	21.8	0	175.58
1B	1	3,250	3.4	0	161.88	3.4	0	173.83
2B	1	3,000	3.6	2	159.50	3.6	2	170.73
3B	1	2,750	4.0	2	156.32	4.0	2	167.59
4B	1	2,500	4.4	2	152.83	4.4	2	164.62
5B	1	2,250	4.9	2	149.78	4.9	2	159.62
6B	1	2,000	5.5	2	144.57	5.5	2	153.20
7B	1	1,750	6.2	2	137.51	6.2	2	145.62
8B	1	1,500	7.3	2	129.14	5.8	0	130.68
9B	1	1,250	8.7	2	112.21	7.0	0	130.15
10B	1	1,000	10.9	0	100.60	10.9	1	123.63
11B	1	750	14.6	0	96.46	14.6	1	120.63
12B	1	500	21.8	1	95.33	21.8	1	119.63

[a] Each element of the state space represents a combination of the two state variables: R = range condition (pounds of annual dry matter production/acre), $V = 0$ or 1 and describes the time elapsed since the previous treatment.
[b] Stocking rate (acres/steer/season); treatment strategy (0 = no treatment, 1 = chemical treatment, and 2 = prescribed burning).
[c] Optimal livestock enterprise in all states is season-long stocking, except when intensive-early stocking is employed prior to prescribed burning.

riod. Stocking in excess of carrying capacity is optimal in several states of the initial phase of the treatment's life. Specifically, range conditions characterized by production levels ranging from 2,500 to 1,500 pounds per acre (states 4A–8A) are stocked at 120% of carrying capacity. Stocking in excess of 120% of carrying capacity is nonoptimal in all states.

When the cost of chemical treatment is reduced to \$60 per acre, the frequency of tebuthiuron application increases in the optimal range improvement program. Treatments are initiated whenever range condition falls to an annual productivity level of 1,000 pounds per acre or below (states 10B–12B). Thus, the length of the optimal renewal cycle decreases approximately four years ($T^* = 14.5$) in response to the lower treatment cost. Although positive economic return can be earned during the final years of the treatment's life, greater return can be earned by retreating the site before complete brush reinfestation occurs. As in the "high cost" scenario, maintenance burns are prescribed at several states to extend the life of the chemical treatment. However, prescribed burning is optimal in only six of the eight states in which burning is employed in the "high cost" scenario. When the cost of chemical treatment is reduced, there is less economic incentive to extend the life of the treatment.

Livestock management practices employed in conjunction with the range improvement strategy are similar to those derived in the high-cost treatment scenario. Intensive-early stocking is again only employed in years preceding prescribed burning treatments. Stocking rates are shown to be somewhat sensitive to the cost of chemical treatment. Optimal stocking rates exceed those derived when tebuthiuron application is assigned a higher cost for several states in the initial and later portions of the treatment's life. These stocking practices, in combination with fewer prescribed burns and higher forage production levels which trigger chemical treatments, serve to reduce the length of the renewal cycle.

Table 2. Optimal Range Improvement Plans under Two Treatment Costs, Inflexible Management Scenario

			High Cost Treatment[b,c]			Low Cost Treatment[b,c]		
State[a]	V	R	Stocking Rate	Treatment	$f_n()$	Stocking Rate	Treatment	$f_n()$
1A	0	3,250	3.4	0	152.21	3.4	0	156.09
2A	0	3,000	3.6	0	150.13	3.6	0	154.10
3A	0	2,750	4.0	0	149.03	4.0	0	153.66
4A	0	2,500	4.4	0	148.94	4.4	0	153.21
5A	0	2,250	4.9	0	148.11	4.9	0	152.91
6A	0	2,000	5.5	0	148.21	5.5	0	152.02
7A	0	1,750	6.2	0	148.05	6.2	0	151.78
8A	0	1,500	7.3	0	147.81	7.3	0	151.43
9A	0	1,250	8.7	0	147.46	8.7	0	151.36
10A	0	1,000	10.9	0	146.38	10.9	0	150.24
11A	0	750	14.6	0	145.27	14.6	0	149.11
12A	0	500	21.8	0	144.17	21.8	0	147.98
1B	1	3,250	3.4	0	133.04	3.4	0	136.52
2B	1	3,000	3.6	0	131.95	3.6	0	135.40
3B	1	2,750	4.0	0	129.55	4.0	0	132.92
4B	1	2,500	4.4	0	126.85	4.4	0	130.16
5B	1	2,250	4.9	0	121.67	4.9	0	124.83
6B	1	2,000	5.5	0	115.08	5.5	0	118.06
7B	1	1,750	6.2	0	107.22	6.2	0	109.98
8B	1	1,500	7.3	0	98.45	7.3	0	95.91
9B	1	1,250	8.7	0	93.55	8.7	0	95.73
10B	1	1,000	10.9	0	89.03	10.9	0	91.25
11B	1	750	14.6	0	85.67	14.6	0	87.78
12B	1	500	21.8	0	84.56	21.8	1	86.65

[a] Each element of the state space represents a combination of the two state variables: R = range condition (pounds of annual dry matter production/acre), V = 0 or 1 and describes the time elapsed since the previous treatment.
[b] Stocking rate (acres/steer/season), treatment strategy (0 = no treatment and 1 = chemical treatment).
[c] Livestock enterprise is restricted to season-long stocking.

Using the steady-state probability vector, estimates of the long-run unconditional annual forage production can be derived. Under the optimal plan derived in the high cost scenario, the long-run average forage production is 1,766 pounds per acre. As a result of the increase in treatment frequency, the long-run unconditional forage production estimate is increased approximately 350 pounds in the low cost scenario.

The value of $f_n(i)$ gives the expectation of the present value of net returns obtained from one acre given an initial range condition defined by the ith combination of states R and V. Values of $f_n(i)$ range between \$95.33 and \$180.81 when a high cost treatment is assigned and \$119.63 to \$193.11 under low cost assumptions. Discounted returns are, of course, highest when production is initiated from high productivity range conditions. In these cases, the high returns earned over the initial portion of the time horizon are discounted lightly. Lower treatment costs and the ability to apply more frequent range improvement treatments translate into a 7% to 25% increase in discounted net returns, depending upon the state from which the model is initialized. Using the steady-state probabilities, an estimate of the long-run unconditional value of discounted net return (f^*) may be derived. If fixed costs such as taxes were capitalized and deducted, this value could be viewed as an approximation of the long-run expected agricultural value of an acre of rangeland, assuming optimal management. The value of f^* is \$130.98 and \$160.94 in the high- and low-cost treatment scenarios, respectively.

Comparison of the two solutions indicates that optimal range improvement plans are sensitive to the cost of chemical treatment. If treatment costs exceed \$75 per acre, tebuthiuron applications are no longer economically feasible. In this case, range condition is driven down to the low range production states, and prescribed burning is employed when forage production exceeds 1,250 pounds per acre.

Long-run unconditional forage production is reduced approximately 600 pounds per acre below average production in the high cost scenario.

Optimal Range Improvement Programs under "Inflexible Management"

A second set of results derived assuming treatment costs of $60 and $75 per acre are reported in table 2. In this scenario, identified as "inflexible management," possibilities of prescribed burning, early-season stocking, and stocking rate adjustments are not permitted as part of the optimal management plan. Stocking rates are assumed fixed at the carrying capacity determined for each range condition state. Comparison of these solutions with results derived when such adjustments are allowed indicates the consequences of evaluating range improvement investments without representing accompanying production practices.

Chemical treatments remain optimal in the low cost scenario; however, the frequency of their use is reduced. Treatments are initiated only when brush encroachment has progressed to the point where forage production is reduced to the lowest range production state (500 pounds per acre). This modification of the optimal control rule translates into a 3.2 year reduction in the optimal frequency of chemical treatments and a 540-pound reduction (to 1,556 pounds per acre) in long-run unconditional forage production. As a result of eliminating stocking rate adjustments and prescribed burning, a portion of the additional value contributed by the chemical applications is no longer attainable. Therefore, treatment frequency must be reduced to provide adequate income over the treatment cycle to cover the cost of treatment. Since stocking rates are set at carrying capacity, stocking levels at several states differ from the "flexible management" solution. The expected values of discounted net returns range from $86.65 to $156.09 and are reduced 19% to 28% below those derived when burning and stocking rate adjustments are permitted. In addition, the unconditional value of discounted net returns is reduced 37% below the value derived under "flexible management" (f^* = $101.88). Thus, failure to represent the complete range improvement program when evaluating range investments significantly affects treatment prescriptions, as well as expected net returns derived from those treatments.

When tebuthiuron applications are assigned a cost of $75 per acre, chemical treatments are not adopted by decision makers employing "inflexible management strategies." Through continual grazing without range improvement expenditure, range condition is eventually driven down to the range site's minimum production level. The long-run unconditional forage production is estimated at 766 pounds per acre. This result is consistent with the range management practices of several study area producers over the past several years. As a result of a perceived lack of profitability among available range improvement alternatives, many managers have opted to accept range productivity levels well below the site potential. Livestock production on these pastures provides the producer a low but stable annual income. The expected values of discounted net returns range from 11% to 16% below those reported in table 1. Also, the long-run unconditional value of discounted net returns is reduced $41.95 below that derived in the flexible management scenario, indicating the total contribution of the range improvement program to producer income.

Summary and Conclusions

Range managers operate in a dynamic and stochastic production environment. As a result, range investment prescriptions should be based upon models that incorporate the influence of these complexities. The dynamic programming model presented here expands upon previous research in this area by incorporating the effects of timing and risk into the analysis of range management decision making. The analysis also attempts to improve upon past studies by more completely representing the decision-making environment facing the range manager. Specific attention was focused on the improved specification of rangeland dynamics as well as a more complete representation of the controls available to the range manager. Improvements in the empirical validity of the production relationships employed in this analysis, as compared to previous applications, center on three areas. First, biophysical simulation is used in lieu of single-equation response models to determine livestock response under alternative range conditions and production practices. Second, an additional state variable is employed to more accurately represent the dynamic relationships defining the transition between range condition states. Finally, historical weather data is used in combination with a biophysical range site production model to estimate the variability observed in annual forage production.

The formulation may be used to derive optimal range improvement programs consisting of the timing of chemical treatments and prescribed burns as well as accompanying stocking rates. The model was applied to range management decision making on a representative shallow savannah range site in the Cross Timbers Region of central Oklahoma. Several range improvement experiments and grazing studies conducted in the Cross Timbers Region provided data necessary for specification of the dynamic production model.

Results from the analysis indicate that tebuthiuron treatments on shallow savannah range sites are economically feasible brush control practices for the treatment costs evaluated ($60 and $75 per acre). Optimal utilization of forage released from tebuthiuron applications requires the conjunctive employment of prescribed burning and variable stocking rates over the life of the treatment. Results also indicate that the profitability of range improvement investments are influenced by accompanying production practices. Optimal treatment frequency and estimates of potential returns derived from treatments are significantly affected by livestock enterprises, stocking rates, and prescribed burning programs employed in subsequent years. Failure to include these practices in range improvement analyses may result in underestimating the profitability of a range investment.

The formulation presented provides improved guidance for range management decision making in a dynamic production environment. Although the analysis employs data from numerous range improvement and grazing studies conducted in the study area, several assumptions were required to specify the production relationships necessary to apply the dynamic programming formulation. Research is needed to provide additional information concerning rangeland dynamics and interactions among treatments, stocking rates, range condition, and climatic influences. Several of the critical production relationships specified in the model are site specific and would require reestimation prior to applying the model to alternative range management situations. The current specification, however, does provide general prescriptions concerning the use of chemical treatments and the efficient employment of accompanying production practices.

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