Duality, Optimization, and Microeconomic Theory: Pitfalls for the Applied Researcher

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This article graphically illustrates the one-to-one duality mapping among the production function, the product supply equation, the derived factor demand equation, and the indirect profit function for the classical profit maximization problem. This pedagogical framework is then used to illustrate how empirical application of conventional duality theory can lead to distorted empirical results if the theory (e.g., Hotelling's lemma) does not apply because the firm is not a profit maximizer or because envelope results from the wrong optimization model are used. Although the presentation is in terms of profit maximization, the basic concepts can be extended to other maintained behavioral hypotheses such as cost minimization or utility maximization. Plausible reasons why a firm, even in a competitive market, may not behave according to the neoclassical maximization paradigm are briefly reviewed.

Key words: duality theory, microeconomic theory applications, optimization.

Applications of duality theory to empirical problems are widespread. Some of the claimed advantages of a dual approach are: (*a*) it opens up a richer class of operational functional forms, especially for multiproduct, multifactor production; (*b*) it brings theoretical coherence to the analysis, especially with respect to cross-commodity relationships, that is often lacking in nondual approaches; and (*c*) it is possible to obtain factor demand and product supply equations from an indirect profit function fitted to profit and price data without having empirical observations of quantities demanded or supplied (Pope 1982b; Lau and Yotopoulos 1971, 1972; Young et al.).

Most empirical studies have used duality theory associated with the conventional static, deterministic model of perfectly competitive firm behavior (e.g., Binswanger; Lau and Yotopoulos 1971, 1972; Trosper; Lopez 1984, 1985; Weaver 1983; Collins and Taylor; Kako; Garcia, Sonka, and Yoo; Garcia and Sonka; Arif and Scott). Although the envelope theorem always holds for optimizing behavior, conventional duality results obtained by applying the envelope theorem to a particular model (i.e., Hotelling's lemma for the classical profit maximization model and Shephard's lemma for the classical cost minimization model) do not necessarily hold in the case of constraints on profit maximization (e.g., Lee and Chambers), in the case of uncertainty (Pope 1980, 1982a), or in the case of stochastic, dynamic problems (Taylor). Furthermore, there are plausible reasons for questioning the neoclassical maximization hypothesis, meaning that we should question the validity of a dual approach or a primal approach that uses first-order conditions for optimization.

This article graphically illustrates the one-to-one duality mapping among the single-input production function, the product supply equation, the derived factor demand equation, and the indirect profit function for the classical profit maximization problem. This pedagogical framework is then used to illustrate how empirical application of conventional duality theory may lead to distorted empirical results if an inappropriate duality mapping is exploited. The graphical framework gives insight into

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distortions and how they can be minimized in various cases. While pedagogical presentation in this article is in terms of the classical profit maximization problem, the same basic concepts can be applied to the classical cost minimization problem or to more complex optimization such as expected utility maximization.

For those who are not familiar with criticisms of the maximization hypothesis, the rich but partially obscure literature in this area is reviewed to establish that there are plausible arguments for not taking the hypothesis as true a priori or as a tautology and thus to establish that there are plausible reasons for questioning empirical application of duality theory. Problems in empirically testing such a hypothesis are also briefly reviewed.

Graphical Exposition of Duality

Let us now turn to a graphical presentation of duality. For a mathematical treatment of duality for the classical profit maximization problem, readers are referred to Henderson and Quandt; Varian (1984a); Beattie and Taylor; or Young et al.

To simplify graphical presentation of duality, assume that a single product, y, is produced with a single input, x, and that technology is given by the continuous, strictly concave production function, $y = f(x)$. To further simplify presentation, only product price, p, is varied while factor price, r, is held constant. Asterisks denote profit maximizing levels of the variables.

The Envelope Theorem

Since the envelope theorem as manifested by Hotelling's lemma is the heart of duality theory, it is instructive to begin with a graphical presentation of this theorem and the relationship between the indirect profit function, $\pi^* = \pi^*(p, r)$, and the direct profit function, $\pi = py - rx$. Figure 1 shows the indirect profit function as related to product price. Since direct profit does not involve optimization, there is a family of direct profit equations that can be drawn in figure 1; π_0 and π_1, which differ only by the fixed level of x and thus y, illustrate two equations in this family. The direct profit equations are linear because product price enters the direct profit equation linearly.

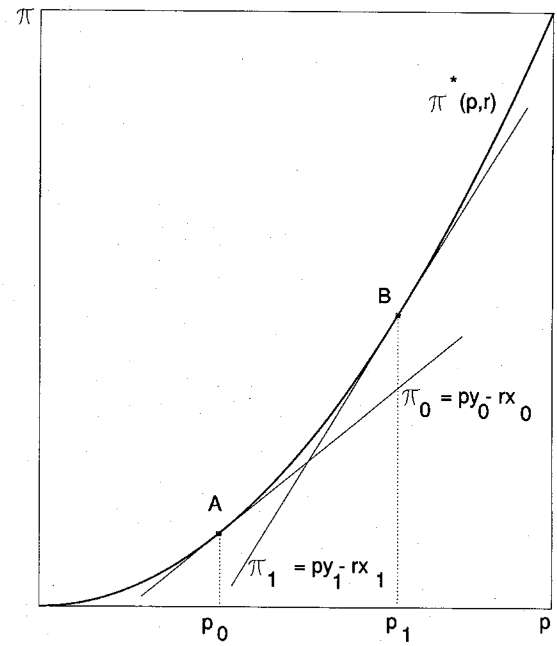

Figure 1. The envelope theorem applied to an indirect profit function

Under profit maximization, the highest profit is chosen for any (given) product price. For product price level p_0 in figure 1, this profit maximizing point is at A on the direct profit equation, π_0. If product price changes to p_1, a different direct profit equation is chosen because output and factor quantities are adjusted in response to the new product price. The new maximum profit level is at point B in figure 1.

The indirect profit function is the locus of profit maximizing points associated with all prices, thus forming an upper envelope of the family of direct profit equations.[1] Worded another way, for a given product price, say p_0, indirect profit will equal direct profit only if direct profit is evaluated at the profit maximizing levels, $x^*(p_0, r)$ and $y^*(p_0, r)$. Thus, if $x_0 = x^*(p_0, r)$ and $y_0 = y^*(p_0, r)$, then π_0 in figure 1 will equal $\pi^*(p_0, r)$ at point A. This relationship will not hold, however, if another point on π_0 is selected or if another point on a different direct profit equation, say π_1, is selected.

The envelope theorem follows from this envelope relationship. The partial derivative

[1] Note that in the multiple-input case, there exist direct profit equations that are everywhere below the indirect profit function. In the single-input case illustrated here, each direct profit function will be tangent to the indirect function at one point.

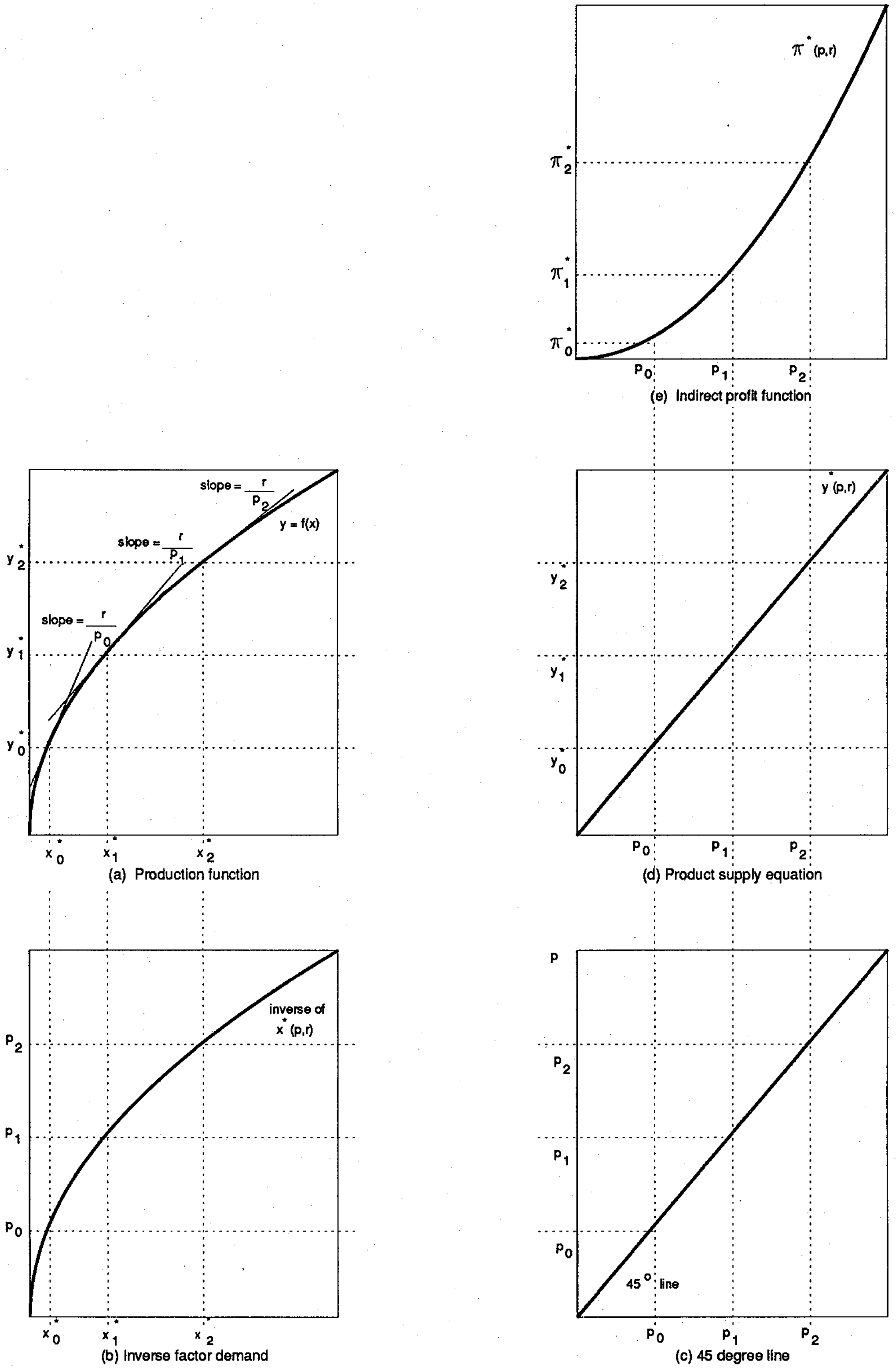

Figure 2. Relationship of the production function, product supply, factor demand, and indirect profit under profit maximization

$\partial\pi^*/\partial p = y^* = \partial\pi_0/\partial p = y_0$. In the case of the classic profit maximization problem, this relationship is referred to as the product supply property of Hotelling's lemma. Again, this envelope relationship holds only for $\partial\pi/\partial p$ evaluated at x^* and y^*. For example, if $y_1 \neq y^*(p_0, r)$, then $\partial\pi^*/\partial p \neq \partial\pi_1/\partial p = y_1$ evaluated at p_0, since π_1 is not tangent to π^* at point A (rather it is tangent at point B).

Duality Mappings

The five panels of figure 2 show the duality mapping among the production function, the product supply equation, the derived factor demand equation related to output price, and the indirect profit function. Panel (a) of this figure shows the production function, panel (b) is the inverse factor demand equation with respect to output prices, panel (c) is a 45-degree line to transfer product price from panel (b) to panels (d) and (e), panel (d) is the product supply equation, and panel (e) is the indirect profit function. To graphically see how x^*, y^*, and π^* are derived from the production function, $y = f(x)$, consider an output price of p_0. The first-order condition for profit maximization is $dy/dx = MPP(x) = r/p_0$, where *MPP* is marginal physical productivity expressed as a function of x. Graphically, the profit maximizing level of factor usage, x_0^*, is associated with the point of tangency between the production function and a line with slope equal to r/p_0. This level of factor usage is traced from the production function in panel (a) to a point on the derived factor demand equation, $x^*(p, r)$, in panel (b). Panel (c) translates the profit maximizing output level associated with x_0^*, which is y_0^*, from the production function in panel (a) to a point on the product supply equation, $y^*(p, r)$, in panel (d). Indirect profit is given by evaluating the direct profit function at the profit maximizing input and output combination, which gives $\pi^* = py^*(p, r) - rx^*(p, r)$. Indirect profit is traced from the production function in panel (a) through panels (b), (c), and (d) to panel (e). Parameters and functions used for the relationships in figure 2 are given in the appendix.

In an empirical setting, variation in p would generate points on the production function, the derived factor demand equation, the product supply equation, and the indirect profit equation. Given observations on all relevant variables, empirical estimation of the relationships in figure 2 could be carried out, in principle, with either a traditional approach or a dual approach. With a traditional approach, observations on x and y would be used, for example, to estimate the production function. Then, given the production function, factor demand and product supply equations associated with profit maximization could be derived. The indirect profit function could then be obtained by substituting the functions $x^*(p, r)$, and $y^*(p, r)$, obtained from explicitly solving the maximization problem, into the direct profit function for x and y, respectively.

Pitfalls of the Dual Approach

Although there are many advantages of a dual approach, there are potential pitfalls associated with it (see e.g., Pope 1982b; Young; Lopez 1984; Chambers; Varian 1984a) if the maintained behavioral hypothesis is invalid, if constraint and information sets have not been correctly identified, or if the wrong duality model has been specified. The graphical framework established in figure 2 gives insight into distortions that can result from inappropriate use of duality and also gives insight into how such distortions can be eliminated or reduced in certain applications.

There are three variations of the dual approach to empirically estimating the equations shown in figure 2. The approach commonly used when there are observations on profit and prices, but no observations on the quantities x and y, is to estimate the indirect profit function, then obtain $x^*(p, r)$ and $y^*(p, r)$ by Hotelling's lemma (Binswanger; Lau and Yotopoulos 1971; Lopez 1984, 1985; Shumway, Saez, and Gottret; Moschini). Because of the one-to-one mapping between the indirect profit function and the production function, it is possible, at least in principle, to obtain the production function $y = f(x)$ from $\pi^*(p, r)$, although in practice a closed form expression of the production function can be obtained only for certain mathematical forms of the indirect profit function. A second dual approach is to estimate the set of factor demand and product supply equations (Weaver 1983; Kako) then extract technical relationships using the dual mapping. The third dual approach is to estimate the system of factor demand and product supply equations jointly with the indirect profit function (Trosper; Garcia, Sonka, and Yoo; Lau and Yotopoulos 1972; Arif and Scott). We now use the graphical framework in figure 2 to gain insight into distortions that can result from the three dual approaches mentioned above if the maintained hypothesis of profit maximization does not hold or if the specific manifestation of the envelope theorem, which in this case is Hotelling's lemma, breaks down for other reasons (see e.g., Pope 1982a, b; Taylor; Lee and Chambers; Weaver 1982).

Consider a case where the firm does not always employ the profit maximizing level of factor x. A critical assumption for this illustration is that Hotelling's lemma does not hold. Whether the assumption does not hold because (*a*) the firm is not a profit maximizer; (*b*) the firm has the wrong perception of technology; (*c*) we have not identified appropriate constraints on profit maximization; or (*d*) we have used the wrong classical profit maximization dual model rather than a constrained model (Lee and Chambers), an uncertainty model (Pope 1980, 1982a), or a stochastic dynamic model (Taylor) is not central to the graphical analysis.

Figure 3, which is based on the same framework used in figure 2, illustrates a case where the firm applies the profit maximizing level of x, x_1^*, at a price of p_1, but for a lower output price, say p_0; the firm responds by employing x_0^a units of the factor, which is more than the optimal amount, x_0^*. Similarly at a price above p_1, say p_2, the firm is assumed to employ x_2^a units of the factor, which is less than the optimal amount, x_2^*. Critical parameters associated with the relationship in figure 3 are given in the appendix.

A firm's response to variation in price would generate a time series of points along the inverse factor demand curve, $x^a(p, r)$, shown in panel (b); points on the product supply equation, $y^a(p, r)$, shown in panel (d); and points on the profit function, $\pi^a(p, r)$, shown in panel (e) of figure 3. For comparative purposes, the indirect profit function, product supply, and derived factor demand equations associated with profit maximizing behavior are also shown in panel (e) as $\pi^*(p, r)$, in panel (d) as $y^*(p, r)$, and in panel (b) as $x^*(p, r)$, respectively.

For the case illustrated in figure 3, the empirical profit function, $\pi^a(p, r)$, is equal to the indirect profit function, $\pi^*(p, r)$, at a price of p_1, because it was assumed that the firm used the profit maximizing level of x at that price, but not at other prices. If the firm does not use the profit maximizing factor level at any price, the empirical profit function will always be below the indirect profit function.

Indirect Estimation of Demand and Supply Equations

Consider now a dual approach to estimating the profit function, $\pi^a(p, r)$ as an indirect way of obtaining product demand and factor supply equations. Note that in this case we are considering estimating only the profit equation with demand and supply equations derived by application of Hotelling's lemma to the fitted profit function, $\pi^a(p, r)$.

A distortion resulting from the dual approach to obtain demand and supply functions from an empirically estimated profit function is illustrated in figure 4, panels (a) and (b), which are expanded versions of panels (e) and (d) in figure 3. Subscripts and superscripts used in figure 4 are consistent in definition with those used in figure 3. The function $\pi^a(p, r)$ in panel (a) of figure 4 is actual profit as related to price; the true indirect profit function is $\pi^*(p, r)$. Direct profit functions for two input-output combinations are also given in figure 4; π_c is defined by the profit maximizing pair (y_0^*, x_0^*) for a price of p_0, while π_d is associated with the nonmaximizing pair (y_0^d, x_0^d). π_d is an implied direct function tangent to the fitted profit function at point D, while π_c is the direct profit function tangent to the true indirect profit function, $\pi^*(p, r)$, at point C.

Consider a price of p_0 in panel (a) of figure 4. Application of the envelope theorem to the fitted profit function, $\pi^a(p, r)$, at the price p_0 would imply a tangency with the function $\pi^a(p, r)$, at point D. The output level implied by Hotelling's lemma would thus be y_0^d, which is shown in panel (b). However, the true direct profit function for a price of p_0 is π_c with associated quantity y_0^* is, in general, different than y_0^d. As graphically illustrated (see the appendix for the mathematical functions used to construct the figures), the supply curve obtained by applying Hotelling's lemma is $y_d(p, r)$, which is neither the product supply associated with profit maximizing behavior, $y^*(p, r)$, or the actual product supply curve, $y^a(p, r)$. In general, $y_0^d \neq y_0^a \neq y_0^*$. We do not obtain the profit maximizing supply curve, $y^*(p, r)$, because π^* is not everywhere equal to π^a, and we do not obtain the actual supply curve, $y^a(p, r)$, because

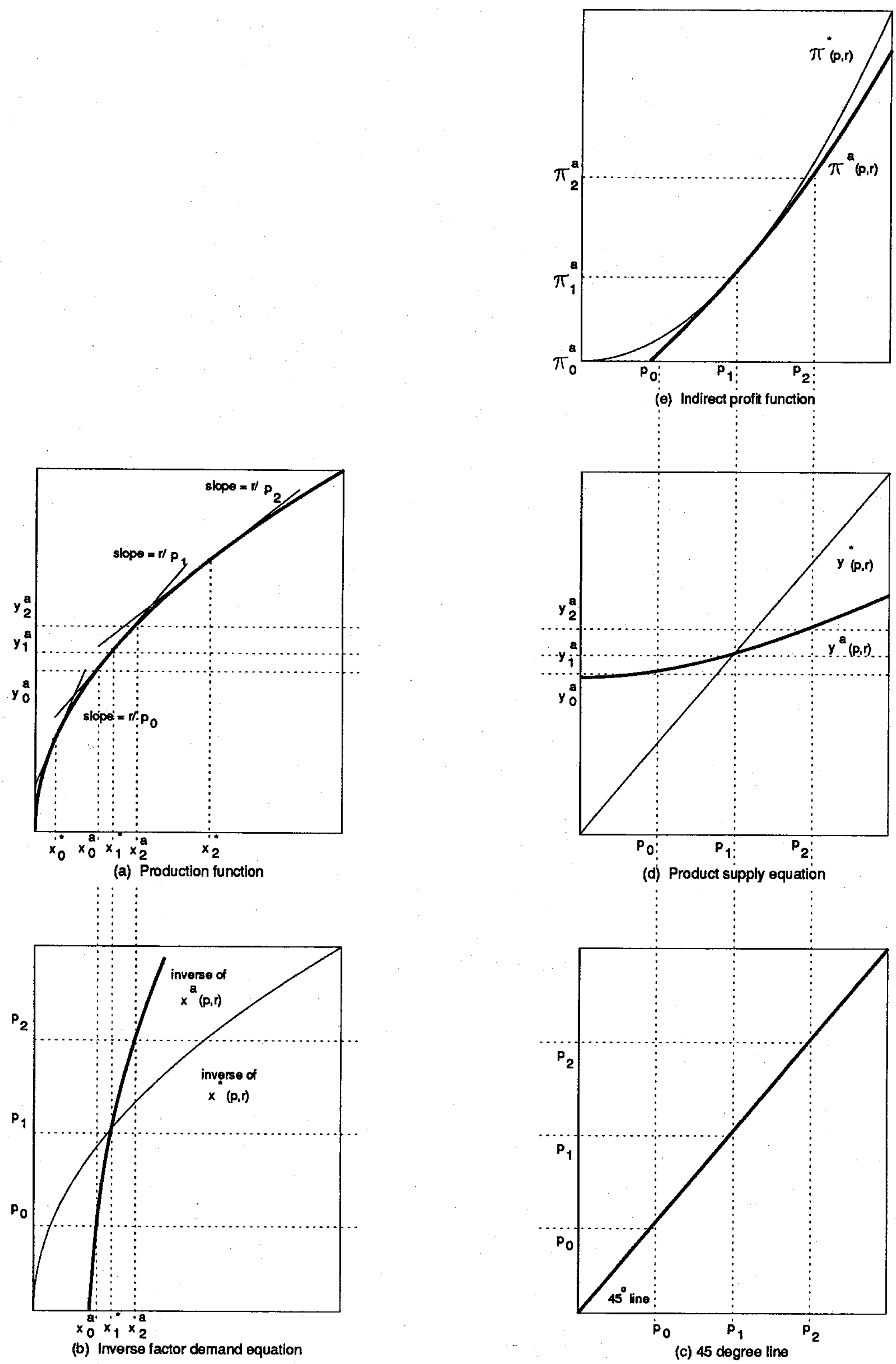

Figure 3. Relationship of the production function, product supply, factor demand, and profit when profit maximization does not hold

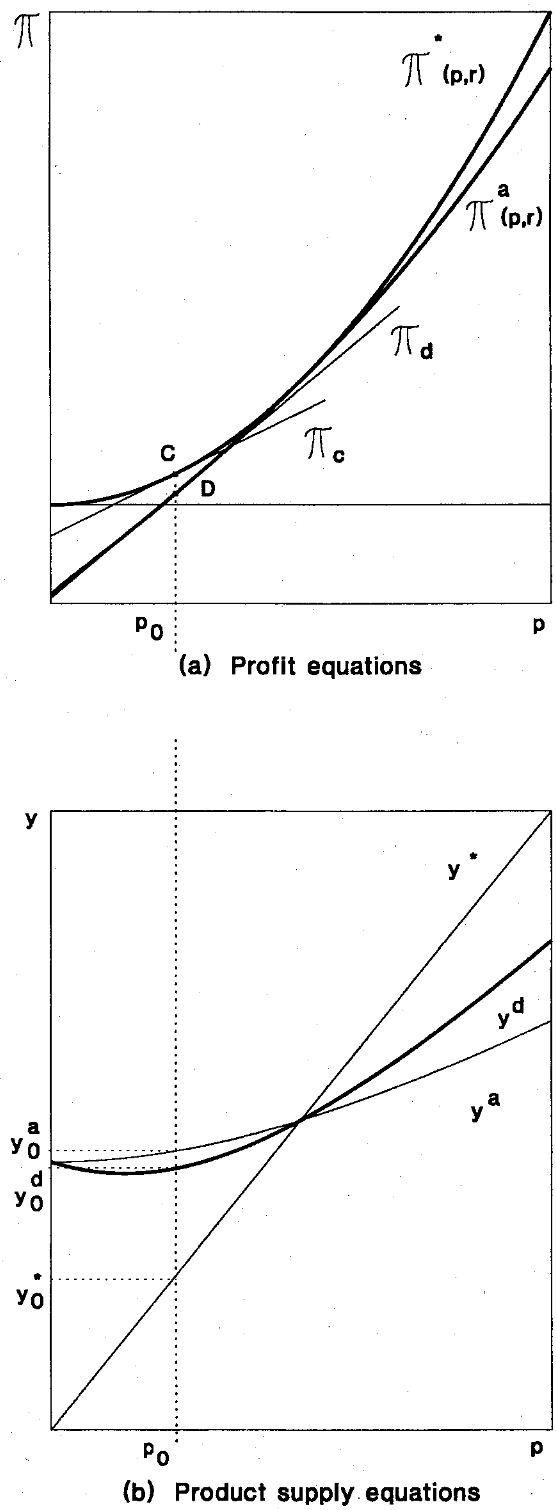

Figure 4. Application of the envelope theorem to obtain a factor demand equation in the case of nonmaximizing behavior

Hotelling's lemma is not valid in this case. Elasticities of supply based on $y^d(p, r)$ will therefore be distorted in this illustration.

Without profit maximization, we cannot make any general statements about the curvature of the profit function even with a concave production function. However, an interesting feature of this particular example is that the actual profit function, $\pi^a(p, r)$, in panel (a) of figure 4 is not globally convex, which is manifested in panel (b) by the negatively sloped supply relationship (at low prices) obtained by application of Hotelling's lemma. In empirically fitting a profit function in such a case, several errors could be made. First, we might force a convex function on the data set which would clearly lead to distorted estimates of the actual profit function. Second, data points could span the nonconvexity thereby hiding the problem. Third, we could fit a flexible functional form that would fit the data points exactly (i.e., no functional form bias), but the nonconvex range might lead us to speculation about specification and other biases rather than leading us to consider the validity of the maintained hypothesis.

Figure 5 illustrates a second departure from profit maximization. (See the appendix for numerical details.) In this case, only half the optimal input level is used, as might happen if the firm had incorrect perceptions about technology. In this case, the actual profit function is convex for all prices, but the supply function derived from the profit function by application of Hotelling's lemma results in a supply curve that lies between the actual supply curve, $y^a(p, r)$, and the supply curve associated with profit maximization, $y^*(p, r)$. Data points generated by the firm's response to varied price would generate points along $y^a(p, r)$ and not along either $y^*(p, r)$ or $y^d(p, r)$.

In the numerical example shown in figure 5, it was assumed that the firm underapplied the input resulting in a derived supply curve, $y^d(p, r)$, that was less elastic than the actual supply curve. If the firm overapplies the input (not shown), then the actual supply curve will be above the profit maximizing supply curve, while the derived supply curve will be below the actual and profit maximizing supply curves. Also, the derived curve will be more elastic than the actual supply curve in this case. This numerical example shows that without knowing $y^a(p, r)$, we cannot determine if applying Hotelling's lemma in the absence of profit maximization results in an upward or downward bias of supply elasticities.

The preceding discussion shows that fitting a profit function to observations on prices and associated profit may give distorted estimates of demand and supply functions if Hotelling's lemma does not hold. Although estimates of the actual profit function may be statistically unbiased (unless there is a functional form bias or inappropriate cross-price relationships such as symmetry implicit in the dual approach), the graphical analysis suggests that supply and demand equations derived from a fitted profit equation should be cautiously interpreted.

Without data on input and output quantities, we may have no choice but to derive factor demand and supply equations on the basis of a fitted profit function. However, it is important to recognize that the resulting demand and supply equations may not correspond to either actual behavior or to profit maximizing behavior.

Direct Estimation of Demand and Supply Equations

Consider directly estimating product supply and factor demand equations (but not the profit function) for the case depicted in figure 3. In this case a dual approach and a primal approach that uses first-order conditions are equivalent, assuming consistent functional forms. Assuming no functional form bias, the fitted demand equation would be $x^a(p, r)$ in panel (b), and the fitted product supply equation would be $y^a(p, r)$ shown in panel (d). Even though the fitted demand function and the fitted supply function differ from the functions based on profit maximizing behavior, they may nevertheless be valid supply and demand behavioral relationships.

Without functional form bias, fitting either the demand or supply curve will give an undistorted estimate of that equation. However, fitting the set of equations will give undistorted estimates only if the symmetry implicit in the dual or primal specification is appropriate. Symmetry of the behavioral relationships would imply that $\partial y^a(p, r)/\partial r = -\partial x^a(p, r)/\partial p$.

Note that there are no logical reasons to expect that, in general, symmetry holds without optimization. For example, it is well known that in the classical expected utility (EU) model of firm behavior, cross-price effects based

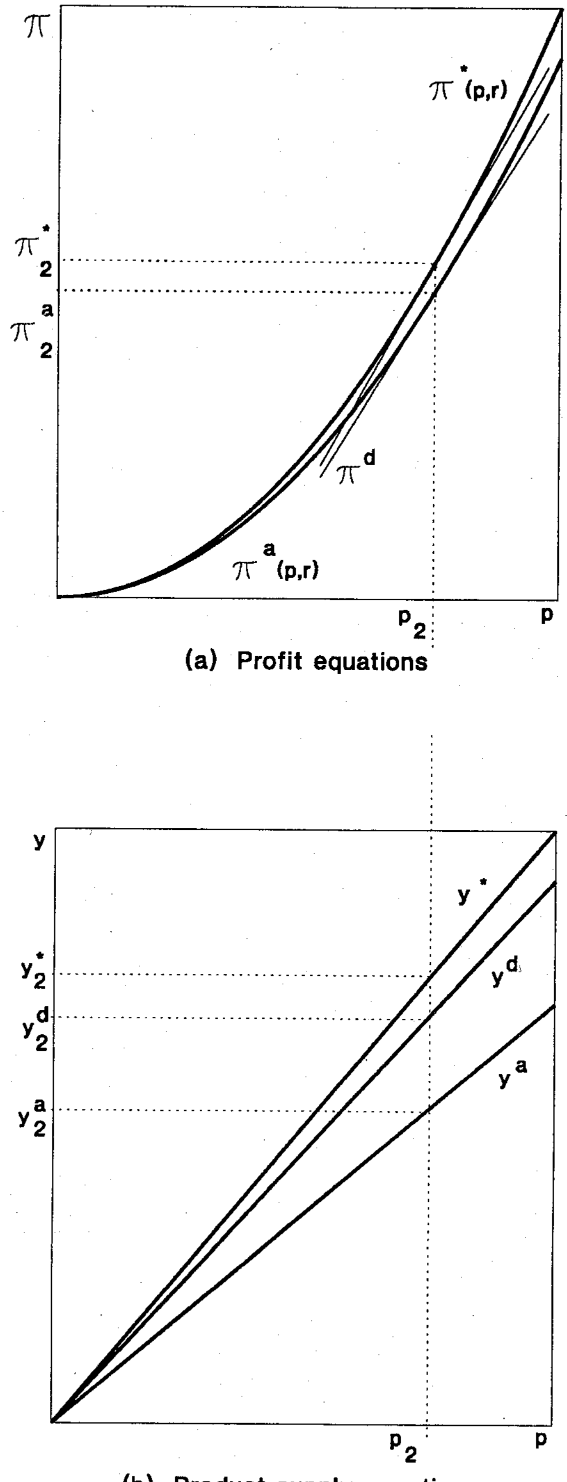

Figure 5. Application of the envelope theorem to obtain a factor demand equation in the case of nonmaximizing behavior

on ordinary demand functions are not symmetric (because of the income effect). If there is an expenditure constraint on profit maximization (Lee and Chambers), it can be shown that some cross-price relationships are not symmetric because the cost constraint has the same effect in this model as the budget constraint in the EU model. Certain stochastic, dynamic characteristics of optimization problems also introduce asymmetric cross-price effects (Taylor). That is, the reciprocity conditions that are a byproduct of the envelope theorem do not necessarily lead to symmetric cross-price effects for ordinary factor demand and product supply equations for some extensions of the classical profit maximization problem.

In any empirical application, however, we can statistically test for symmetry using standard procedures and tests. But it is important to note that if symmetry does hold, we cannot logically conclude profit maximization because certain kinds of symmetry hold for other models.

Simultaneous Estimation of All Economic Relationships

A third variation of the dual approach is to estimate simultaneously a consistent set of equations for all economic relationships. With this approach, a functional form for an indirect profit function is specified, then the functional forms for demand and supply equations are derived by application of Hotelling's lemma. The set of equations (supply, demand, and indirect profit) are estimated as a system. This approach is theoretically appealing because a consistent set of equations that satisfy symmetry and curvature properties is fitted. It could be argued that by using more equations, and thus all data, sharper estimates of parameters will be obtained.

If Hotelling's lemma does not apply, however, all of the equations will be distorted because fitting the system will compromise all fitted equations. Supply and demand equations will not fit actual data because this will distort the profit equation away from data points; likewise, fitting the profit equation exactly will distort supply and demand equations away from the profit data points. Thus, fitted supply and demand equations will lie somewhere between the actual and maximizing relationships, while the fitted profit function will lie between the actual function and the true indirect function. The extent of the distortion cannot be determined a priori, or even *ex post* without knowing the actual relationships.

Derivation of the Production Function

Use of duality mappings to obtain the production function or related technical measures such as marginal physical productivity can also give distorted estimates with inappropriate use of Hotelling's lemma. This distortion can be seen in two different but equivalent ways. One way is to note that in the single-factor case, the factor demand equation is the inverse of the *MPP* function; that is, solving the first-order condition for x gives $x^* = MPP^{-1}(r/p) = x^*(p, r)$. We can thus obtain $MPP(x)$ by inverting the function $x^*(p, r)$. However, if we use the function $x^a(p, r)$ or $x^d(p, r)$ rather than $x^*(p, r)$ to obtain estimates of $MPP(x)$, it can be seen by comparing $x^*(p, r)$ to $x^a(p, r)$ or comparing $x^*(p, r)$ to $x^d(p, r)$ in panel (b) of figure 3 that these estimates are distorted.

A second way to view the distortion is that the dual approach infers that the slope of the production function is equal to the ratio of the factor price to the product price. Thus, at x_0^* the inferred slope of the production function would be r/p_0, which can be seen in panel (a) of figure 3 to be greater than the slope of the true production function at x_0^a. Under the assumed nonprofit maximizing behavioral case illustrated in figure 3, the inferred production function will be more concave than the true production function.

Validity of the Expected Utility Model

Belief in the expected utility maximization hypothesis by students of neoclassical microeconomics appears to be widespread. It could be euphemistically said that many economists seem to belong to the Austrian school of thought. As Caldwell notes, "Austrians . . . insist that the [maximization] hypothesis is the fundamental axiom of human action which is known to be true a priori but which nonetheless has empirical content." The competitive market model, which is another element of the Austrian school, is often used as an argument for profit maximization; firms that do not maximize profits are driven out of the market by competitive forces.

Due to the competitive nature of many agricultural markets and the seemingly unquestioned acceptance of the maximization hypothesis by some economists, it is appropriate to digress on why the hypothesis itself should be questioned and thus establish why empirical use of certain envelope theorem results should be questioned. For generality, the review is in terms of the EU model; expected profit maximization can, of course, be viewed as a special case of the EU model. The purposes of this review are simply to establish that there are plausible arguments for not taking the maximization hypothesis as true a priori or as a tautology and to direct the interested reader to relevant literature. Problems in empirically testing such a hypothesis, particularly in the context of applications in agricultural economics, are also briefly discussed. I begin with consideration of the competitive market argument then turn to the maximization hypothesis.

Competitive Market Argument

The classical model of perfect competition with its assumptions about perfect information and free entry and exit leads to profit maximization by all firms who remain in the market. Since there are no pure profits in a perfectly competitive market, firms who are not profit maximizers will incur losses and exit from the market.

Extension of this textbook argument to competitive agricultural markets is not direct, however, because the classical assumptions may not be appropriate for the following reasons. First, profits earned by most agricultural firms are affected by stochastic factors such as uncontrollable crop yields, price instability, and the recent instability of financial institutions. In a practical setting, the highly stochastic and largely uncontrollable nature of returns could dominate decisions that are not consistent with expected profit maximization. For example, few, if any, economists would argue that the financial crisis in agriculture in the early 1980s weeded out only those farmers who were traditionally viewed as "poor" managers. Second, wealth levels of firms can keep them viable for several years and perhaps for generations if the decision makers are especially stubborn or reluctant to move out of farming. Third, if current firms and the pool of potential new firms do not have a profit maximization objective, the market can be dominated by other kinds of behavior. Fourth, the pervasive influence of off-farm earnings may dramatically alter agricultural decisions. Thus, there are plausible conceptual reasons to find individual behavior in agricultural markets that is not consistent with profit maximization.

Even if the profit maximization hypothesis is valid, presence of imperfectly competitive market elements can obviously make duality theory break down. This can be seen in the limiting case of monopoly. Since a monopolist controls price, output price is not even an argument in the indirect profit function; rather, parameters of the demand function are arguments in the indirect profit function for a monopolist.

Maximization Hypothesis

Consider now the validity of the neoclassical maximization model, the heart of which is the EU model. After an extensive review of the variants, purposes, evidence, and limitations of the EU model, Shoemaker concluded that "EU maximization is more the exception than the rule." From a descriptive perspective, he argued that the EU model failed on three counts. First, people do not structure problems as holistically and comprehensively as EU theory suggests. Second, they do not process information, especially probabilities, according to the EU model. Burks notes that there are also philosophical problems with the classical notion of probability. Third, EU theory poorly predicts choice behavior in laboratory situations. MacCrimmon and Larson note that ". . . many careful, intelligent decision makers do seem to violate some axioms of expected utility theory, even upon reflection of their choices . . ."

From a positivistic perspective, many scholars have noted that the EU model is useful for predicting behavior, although accuracy of the prediction is often less than desirable. The same might be said of empirical applications of duality theory. Even though the EU model is acknowledged as having predictive value, underlying rationality assumptions have been questioned (Shoemaker). Even if the EU model predicts well while its assumptions are wrong, the notion that only prediction matters is epistemologically unappealing (Shoemaker; Samuelson).

The cursory review given above provides plausible reasons for questioning the EU model in general and the profit maximization model in particular; therefore, in empirical application of duality theory we must always question validity of the maintained hypothesis. Readers interested in additional reading on the neoclassical maximization hypothesis are referred to extensive references in Shoemaker and in De Alessi.

Testing the Maximization Hypothesis

From a philosophical standpoint, there is no apparent agreement on whether the maximization hypothesis is testable, as evidenced by an exchange between Boland (1981, 1983) and Caldwell. Some view the hypothesis as a tautology which is by definition untestable. Boland (1983) argues that the hypothesis is not a tautology but that no criticism of it will ever be successful. Caldwell agrees that the EU model is untestable. He states:

> There are a number of problems associated with testing the hypothesis; perhaps the most telling is that any direct test, including the revealed preference approach, requires that assumptions be made concerning the stability of preferences of the choosing agent, as well as the states of information confronting him. Since the content of these assumptions . . . are subject to change but are not themselves directly testable, test results . . . are not unambiguously interpretable. (pages 824–825)

Caldwell further argues that the EU model is untestable because "utility" is an undefined theoretical term, but since "profit" is measurable, profit maximization is logically testable. However, the above cited problems associated with assumptions about the states of information confronting the decision maker apply to testing absolute (as opposed to relative) profit maximization as well as testing the general EU model.

Various parametric and nonparametric approaches to testing for profit maximization have been investigated. The parametric approach tests for departures from the first-order conditions for profit maximization; that is, the test is implicitly or explicitly based on a comparison of price ratios to marginal physical productivities (e.g., Dillon and Anderson). A weakness of the parametric approach is that the test is conditional on the functional form selected for the production function. Since technical or biological theory rarely indicates the appropriate functional form to represent technology, we are left with the difficult problem of nonnested hypothesis testing. Nonnested model selection rules have a heuristic base, and small sample properties of the rules are virtually unknown (Judge et al.).

The nonparametric approach allows tests of profit maximization without any maintained hypothesis of functional form for technology (Varian 1984b; Chavas and Cox; Hanoch and Rothschild; Fawson and Shumway). However, the nonparametric tests are not ideal because they have a heuristic base and because they only determine whether observed behavior is consistent or inconsistent with the null hypothesis.

Parametric and nonparametric tests of profit maximization obviously complement each other. Information gained from the tests, while not definitive, would complement empirical application of duality theory and perhaps give a better intuitive understanding of whether the maintained hypothesis is valid. However, in an empirical setting seldom can we definitively discriminate among alternative behavioral hypotheses such as (*a*) unconstrained profit maximization, (*b*) profit maximization subject to a cost constraint (Lee and Chambers), and (*c*) expected profit maximization in a dynamic setting (Antle; Taylor). Therefore, which duality model, if any, should be applied cannot be definitively established for any given empirical problem.

Summary and Concluding Remarks

Graphical analysis presented in this article suggests that potential distortions resulting from application of duality theory can be minimized or even eliminated in certain cases. One case is where the aim of the empirical research is to estimate factor demand and supply equations directly as behavioral relationships (even if profit maximization does not hold) and where the profit function is not of direct interest. In such a case, use of functional forms obtained by applying Hotelling's lemma to a prespecified indirect profit function may not result in a distortion as long as the profit function is not fitted along with the demand and supply equations. Distortions can still result if symmetry of cross-price effects, which is implicit in the dual approach for the profit maximization model, does not hold. However, symmetry can be empirically tested using standard statistical procedures.

A second case is where the aim is to establish only the profit function, perhaps for comparison to profit functions for other firms in the market. In this case, distortions are minimized by directly fitting only the profit function. Finally, if the aim of the research is to establish supply and demand equations and the profit function, then it must be recognized that all of the equations will be distorted if the maintained hypothesis is not valid.

Major advantages of a dual approach to empirical problems are: (*a*) it opens up a richer class of operational functional forms, especially for multiproduct, multifactor production; (*b*) it brings coherence to the analysis, especially with respect to cross-commodity relationships, that is often lacking in nondual approaches; and (*c*) it is possible to obtain factor demand and product supply equations from an indirect profit function fitted to profit and price data without having empirical observations on the quantities demanded or supplied (Pope 1982b; Lau and Yotopoulos 1971, 1972; Young et al.). These advantages can indeed be empirically exploited if the maintained behavioral hypothesis is valid. Unfortunately, we cannot definitively establish validity of the maintained hypothesis in most if not all empirical studies.

It must be recognized that there are also weaknesses with some nondual approaches. For example, the nondual approach is not operational without observations on quantities. A nondual approach also has pitfalls, especially if profit maximization (or other maintained hypothesis) is valid, but a set of functional forms for factor demand and product supply equations that are inconsistent (with respect to curvature or cross-price relationships) with profit maximization is used; empirical results would therefore be distorted if in fact profit maximization held. Additional appraisal of advantages and disadvantages of dual approaches relative to other approaches is given in Young et al. Selection among various dual approaches and between dual and nondual approaches for empirical application appears to be more art than science. The graphical framework presented in this article allows analysis of distortions associated with different ways of using duality associated with classical profit maximization. The framework can be extended to other optimization models.

Duality is indeed useful, but empirical results based on this theory should be cautiously interpreted—much more cautiously than is apparent from recent literature.

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APPENDIX

Relationships shown in figures 1–5 were generated from the following model:

(A.1) $y = 2x^{.5}$ production function,

(A.2) $\pi = py - rx$ direct profit function,

where $r = .2$ and the graphs are for $0 \leq p \leq 1$. The decision function (which is also the factor demand equation) assumed for the suboptimization case shown in figures 3 and 4 is

(A.3) $$x^a = .25(x^* - 6.25) + 6.25,$$

where x^* is the profit maximizing level of x. Note that for $p = .5$, $x^a = x^*$. The suboptimization case shown in figure 5 is for

(A.4) $$x_a = .5x^*.$$