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Rates of Concentrate-Roughage Substitution and Economic Optimums in Feeding Yearling Steers and Heifers

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INTRODUCTION

An infinite number of combinations of concentrates and roughage can be fed in alternative beef systems to produce a given amount of beef. The concentrate-roughage ratio varies considerably between feed lots. Since the optimum combination of concentrates and roughage may be of great economic importance to the Oklahoma farmer and feeder, it appears essential that the farmer should be better informed relative to the choice of the optimum ration. If the farmer were better informed of the rate of substitution of concentrates and roughage, he would be better qualified to choose the optimum combination of feeds to market through cattle each year.

PROBLEM STATEMENT

In the production of beef some of the inputs may be fixed at a constant level. Thus, with management, labor, initial weight and other factors fixed, we may be interested in the effect on output resulting from varying concentrates and roughage.

A production function with some of the inputs fixed may be expressed as follows:

(1) $Y = f(X_1, X_2/X_3, \ldots X_n)$

Y = Beef production

X_1 = Concentrates

X_2 = Roughage

$X_3 \ldots X_n$ = Management, labor, initial weights and other relevant factors.

By holding a portion of the inputs constant and varying X_1 and X_2, it was possible to determine the rate of gain resulting from changes in the variable inputs.

COMPARISON OF THE AVAILABLE DATA AND IDEAL DATA

The ideal data needed to solve the problem are such that a relatively complete segment of the production surface could be derived. The wide range would allow a better statistical fit to the data and a wider segment of the surface is more likely to include the economically optimum combination of feeds under relevant prices.

Data were available on three different ratios of roughage and concentrate. The available ratios were 1:1, 1:2, and 1:4.[1/] It is apparent from the previous discussion that the available data may fall short of what is necessary for the "best" analysis. It is possible that the economically optimum combination may fall outside the range of the three available ratios (1:1, 1:2, 1:4). The available data will not allow as good a statistical fit to data as would be possible with a wider ratio variation.

The input-output relationship of the final weigh period was determined by averaging the time periods the animals were fed during the final interval. The other major limitation of the data was the slow gains of the cattle. It is possible a slow rate of gain could influence the marginal rate of technical substitution of roughage and concentrates; however, it is assumed such was not the case.

METHOD OF ANALYSIS

Criteria for Determining the Optimum Combination of Two Variable Inputs.

The optimum combination of the two inputs is at the point where the marginal rate of technical substitution of the X_1 and X_2 is equal to the inverse of the price

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1/ The ratios are expressed in terms of roughage to concentrate by weight pounds. For instance, 1:4 is one part roughage and four parts concentrates.

ratio. Thus, with changes in the relative prices of X_1 and X_2, the optimum combination of the variable inputs change.

The feeder needs basic information relative to the marginal rate of substitution of concentrates and roughage, and he needs a choice guide to aid in selecting the optimum ration assuming various price relationships.

Selection of the Statistical Model

Eight different algebraic forms of equations (statistical models) were selected. These equations were:

(1) $Y = a X_1^{b_1} X_2^{b_2}$

(2) $Y = a + b_1\sqrt{X_1} + b_2\sqrt{X_2} + b_3X_1X_2$

(3) $Y = a + b_1\sqrt{X_1} + b_2\sqrt{X_2}$

(4) $Y = a + b_1X_1 + b_2\sqrt{X_1} + b_3X_2 + b_4\sqrt{X_2} + b_5X_1X_2$

(5) $Y = a + b_1X_1 + b_2\sqrt{X_1} + b_3X_2 + b_4\sqrt{X_2}$

(6) $Y = a + b_1X_1 + b_2X_1^2 + b_3X_2 + b_4X_2^2 + b_5X_1X_2$

(7) $Y = a + b_1X_1 + b_2X_1^2 + b_3X_2 + b_4X_2^2$

(8) $Y = a + b_1X_1 + b_2X_2$

Certain restrictions were specified by the economic model. Hence, it was necessary to make the following assumptions about these equations:[2/] (1) diminishing returns to the variable factors, (2) diminishing MPP of each variable factor, and (3) diminishing marginal rate of substitution between the variable factors. Also, it was possible to show complementarity of inputs with four of the equations.

2/ These assumptions do not apply to equation (8).

THE RESULTS

The eight selected equations were fitted to the available data for both steers and heifers.

In selecting the final equation from those fitted, the following tests were employed.

1. Consistency of the statistical model with the economic model.

2. The models that passed the above test were examined for goodness of statistical fit.

Of the first seven equations fitted to steer data, only the coefficients of equations 1, 3, 5 and 7 were consistent with the economic model. The coefficients of equations 1, 2, 3, 5, 6 and 7 were consistent with the economic model for heifers. The statistical tests were relevant only for the equations which were consistent with the economic model.

Equation (1) ($\hat{Y} = .9454\ X_1^{.557}\ X_2^{.218}$) fitted the available data for steers better than the other consistent equations. Each of the b_i values was significant at the .01 level. Equation (3) was the only other relevant equation with all the b_i values significant at the .01 level.

As with steers, equation (1) ($\hat{Y} = .9571\ X_1^{.508}\ X_2^{.251}$) fitted the available data for heifers better than the other relevant equations. This was the only relevant equation with each of the b_i values significant at the .01 level.

ECONOMIC INTERPRETATIONS AND EVALUATIONS

Diminishing transformation of feed into beef was apparent in the regression coefficients in the Cobb-Douglas (equation) production function for both steers and heifers. The regression coefficients for steers were .557 for concentrates (X_1) and .218 for roughage (X_2). These are elasticities of production (E_p) since they indicate the percentage increases in weight resulting from a 1.0 per cent increase in feed consumed. The E_p of concentrates (.508) for heifers was slightly smaller than the E_p for steers. The E_p of roughage in the equation for heifers was .251, which was slightly larger than the .218 for steers. A significantly smaller E_p of concentrates and larger E_p of roughage would indicate steers utilized concentrates more efficiently than did heifers and that heifers utilized roughage more efficiently than steers. The sum of the E_p for heifers was equal to .759, which was slightly smaller than the sum of the E_p for steers (.775). If this difference was significant, steers utilized feed more efficiently than heifers.

Estimated isoproduct equations were derived directly from the estimated production function listed above for steers and heifers. Equation (1) was derived from the original Cobb-Douglas equation for steers ($\hat{Y} = .9454\ X_1^{.557} X_2^{.218}$). Equation (2) was derived from the original equation for heifers ($\hat{Y} = .9571\ X_1^{.508} X_2^{.251}$).

$$(1)\quad \text{Steers:}\quad X_2 = \left(\frac{\hat{Y}}{.9454\ X_1^{.557}} \right)^{\frac{1}{.218}}$$

$$(2)\quad \text{Heifers:}\quad X_2 = \left(\frac{\hat{Y}}{.9571\ X_1^{.508}} \right)^{\frac{1}{.251}}$$

Estimated "Rate of Substitution" Equations:

$$(3)\quad \text{Steers:}\quad \frac{dX_2}{dX_1} = \frac{.557X_2}{.218X_1}$$

$$(4)\quad \text{Heifers:}\quad \frac{dX_2}{dX_1} = \frac{.508X_2}{.251X_1}$$

Equations (3) and (4) represented the marginal rate of substitution of concentrates and roughage for the steers and heifers at given levels. These substitution equations yield the slope of the isoquant.

3/ A test of significance was not computed for the difference between regression coefficients for steers and heifers.

Concentrates and roughage were adequate substitutes over a relatively wide range. However, a diminishing marginal rate of substitution of the two inputs ($MRS_{X_1X_2}$) was present. The optimum combination of the two feeds was determined for a given amount of gain by equating the inverse feed price ratio for given feed prices with the $MRS_{X_1X_2}$.

APPLICATION OF THE RESULTS

Cattle feeders in general fail to adjust the proportions of concentrates and roughage to changing feed price ratios, although feed production in Oklahoma is diversified. Perhaps the lack of a simple method of determining the optimum combination of feeds is partially responsible for the failure of feeders to accept the feed price ratio as a choice rule for determining the optimum combination of feed.

The practical economic importance of feeding the optimum combination of these feeds to steers is shown in Table I. The table displays the total feed cost of producing 300 pounds of gain on steers resulting from various combinations of feeds with a wide range of feed prices.

For an illustration in using the table, concentrates priced at $2.00 per cwt. and roughage at $25.00 per ton will be assumed. These feed prices resulted in a price ratio of $1.60. This price relationship applied to the $MRS_{X_1X_2}$ in the equation for steers indicated a range of 60.3 to 63.0 per cent concentrates must be fed to obtain an economically optimum combination. The least total feed cost for this feed combination would be $53.61 to produce 300 pounds of gain on steers. Feeding any other combination of these two feeds would result in an increased total feed cost required to produce 300 pounds of grain.

The percentage concentrates required to satisfy the equilibrium condition varies considerably with different feed price relationships. (Table II). For example, the range of percentage concentrates increases from 54.7 - 57.6 to 65.4 - 67.8 as a result of the prairie hay price increases from $20.00 to $30.00 per ton when the price of milo remains constant at $2.00 per cwt.

A significant change in the feed price ratio may be the only economic basis for varying the combination of concentrates and roughage. Therefore, a pertinent question arises. Is there significant variation in the price ratio of these feeds? With a wide variation between months and between years in the price ratio, the economic importance of adjustment is intensified.

The monthly price ratios of grain sorghum and alfalfa hay for the years 1950-1955 are shown in Table III. There was considerable difference in the yearly range for the five-year period. The year 1951 had the lowest yearly range (.59). The highest range was 1.22 found in 1954. The greatest monthly range was in June (.85), while the lowest monthly range was in December (.29). The larger yearly and monthly range in the ratios may create an incentive to vary the ratio until the $MRS_{X_1X_2}$ is equal to the inverse of the feed price ratios.

The feed price ratio between months and between years shown in Table III may indicate the economic importance in adjusting these feeds so that the $MRS_{X_1X_2}$ is equal to the inverse of the feed price ratio. When a fixed combination of concentrates and roughage is continually fed, it is impossible to maintain a least cost combination.

SUMMARY AND CONCLUSIONS

It is possible for feeders to produce beef by feeding a wide range of ratios of concentrates and roughage. Thus, choice criteria are needed to solve the problem of obtaining the optimum ration. This thesis provides a method of analysis and choice guides for solving the problem.

Experimental data were analyzed to solve the problem of optimum ration choice. The data were obtained from feeding trials conducted at the Ft. Reno Agricultural Experiment Station. Three different combinations of concentrates and roughage were fed in the feeding trial. The three rations consisted of 50:50, 65:35 and 80:20 ratios of concentrates and roughage.

TABLE I

TOTAL FEED COST OF VARIOUS COMBINATIONS OF CONCENTRATES AND ROUGHAGE REQUIRED TO PRODUCE 300 POUNDS OF GAIN FOR STEERS UNDER DIFFERENT FEED PRICES

Lbs. Concentrates	Lbs. Hay	Percentage of Concentrates	PX_1 a/ / PX_2 b/	1.00			2.00			3.00		
				15	25	35	15	25	35	15	25	35
1600	1939	45.2		30.54	40.24	49.93	46.54	56.24	65.93	62.54	72.24	81.93
1660	1765	48.5		29.84	38.66	47.49	46.44	55.26	64.09	63.04	71.86	80.69
1720	1611	51.6		29.28	37.34	45.39	46.48	54.54	62.59	63.68	71.74	79.79
1780	1475	54.7		28.86	36.24	43.61	46.66	54.04	61.41	64.46	71.84	79.21
1840	1355	57.6		28.56	35.34	42.11	46.96	53.74	60.51	65.36	72.14	78.91
1900	1249	60.3		28.37	34.61	40.86	47.37	53.61	59.86	66.37	72.61	78.86
1960	1153	63.0		28.25	34.01	39.78	47.85	53.61	59.38	67.45	73.21	78.98
2020	1067	65.4		28.20	33.54	38.87	48.40	53.74	59.07	68.60	73.94	79.27
2080	990	67.8		28.22	33.18	38.12	49.02	53.98	58.92	69.82	74.78	79.72
2140	920.5	69.9		28.30	32.91	37.51	49.70	54.31	58.91	71.10	75.71	80.31
2200	857.5	72.0		28.43	32.72	37.01	50.43	54.72	59.01	72.43	76.72	81.01
2260	800.4	73.8		28.60	32.60	36.61	51.20	55.20	59.21	73.80	77.80	81.81
2320	748.4	75.6		28.81	32.56	36.30	52.01	55.76	59.50	75.21	78.96	83.70
2380	701.0	77.2		29.06	32.56	36.07	52.86	56.36	59.87	76.66	80.16	83.67
2440	657.8	78.8		29.33	32.62	35.91	53.73	57.02	60.31	78.13	81.42	84.71
2500	618.0	80.2		29.64	32.72	35.82	54.64	57.72	60.82	79.64	82.72	85.82
2560	581.6	81.5		29.96	32.87	35.78	55.56	58.47	61.38	81.16	84.07	86.98

a/ Price of concentrates per cwt.
b/ Price of roughage per ton.

TABLE II

PERCENTAGE CONCENTRATES REQUIRED TO SATISFY THE EQUILIBRIUM CONDITION FOR STEERS FOR 300 POUNDS OF GAIN

Price	Prairie Hay $10	$15	$20	$25	$30	$35	$40
Milo	Percentage Concentrates						
1.00	54.7-57.6	65.4-67.8	69.9-72.0	75.6-77.2	78.8-80.2	81.5-82.7	82.7-83.8
1.25	48.5-51.6	60.3-63.0	65.4-67.8	69.9-72.0	73.8-75.6	77.2-78.8	80.2-81.5
1.50	45.2-48.5	54.7-57.6	63.0-65.4	67.8-69.9	69.9-72.0	73.8-75.6	77.2-78.8
1.75		51.6-54.7	57.6-60.3	63.0-65.4	67.8-69.9	69.9-72.0	73.8-75.6
2.00		48.5-51.6	54.7-57.6	60.3-63.0	65.4-67.8	67.8-69.9	69.9-72.0
2.25		45.2-48.5	51.6-54.7	57.6-60.3	63.0-65.4	65.4-67.8	67.8-69.9
2.50			48.5-51.6	54.7-57.6	60.3-63.0	63.0-65.4	65.4-67.8
2.75			45.2-48.5	51.6-54.7	57.6-60.3	60.3-63.0	63.0-65.4
3.00			45.2-48.5	48.5-51.6	54.7-57.6	57.6-60.3	63.0-65.4
3.50				45.2-48.5	51.6-54.7	54.7-57.6	57.6-60.3
4.00					48.5-51.6	51.6-54.7	54.7-57.6
5.00					<45.2	45.2-48.5	48.5-51.6

a/ Only feed combinations corresponding to the rates of substitution included in the experimental data are given in this table.

TABLE III

PRICES OF GRAIN SORGHUM RELATIVE TO PRICES OF ALFALFA HAY BY MONTHS, OKLAHOMA, 1950-1955

Year	Months												Yearly Average	Range
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.		
	Grain Sorghum - Alfalfa Hay Ratios													
1950	1.75	1.93	2.02	2.05	2.11	2.34	2.34	2.05	1.80	1.61	1.55	1.52	1.93	.82
1951	1.57	1.56	1.54	1.43	1.59	1.83	1.83	1.53	1.32	1.24	1.25	1.25	1.47	.59
1952	1.25	1.25	1.39	1.59	1.98	2.05	1.69	1.43	1.36	1.31	1.30	1.24	1.42	.81
1953	1.22	1.23	1.35	1.42	1.88	1.71	1.63	1.75	1.69	1.62	1.53	1.53	1.52	.66
1954	1.57	1.61	1.77	1.82	2.17	2.56	2.20	1.69	1.47	1.34	1.38	1.44	1.75	1.22
1955	1.39	1.44	1.44	1.44	1.69	2.15	2.00	1.57	1.37	1.32	1.32	1.29	1.57	.86
Monthly Ave.	1.46	1.50	1.58	1.62	1.90	2.11	1.95	1.67	1.50	1.41	1.39	1.38	1.61	
Range	.53	.70	.67	.63	.58	.85	.72	.62	.48	.38	.30	.29	.51	

Source: Prices Received by Farmers, Agricultural Marketing Service, USDA, Oklahoma City, Oklahoma.

Several equations were fitted to the data. The Cobb-Douglas regression equation was selected as the best fitting equation for both steers and heifers. Four equations for steers were consistent with the restrictions specified by the economic model, while six equations for heifers were consistent with these restrictions.

There was a relatively large variance in the feed-price ratios which affected the optimum ration and the profitability of feeding steers and heifers. The analysis of the experimental data showed that concentrates and roughage were adequate substitutes over a relatively wide range.

The results indicated that the marginal rate of substitution of concentrates and roughage was of economic importance in feed lot operations. Wide variation was found in total feed costs for producing 300 pounds of gain with various combinations of concentrates and roughage under different price relationships.