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ANALYSIS OF PRODUCTIVITY EFFECTS OF FERTILIZER

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The question, "What is the most profitable rate of commercial fertilizer to apply under given circumstances?", is one of great importance to western agronomists, economists, and farmers alike. Until recent years the use of commercial fertilizers in most of the Western states has been relatively low but they are playing an ever-increasing role in crop production. Scientific experiments, as well as farmers' experience, indicate good response to the use of fertilizer in many areas especially where adequate irrigation water is available. More and more fertilizer is being used each year and there are many indications that fertilizer consumption probably will continue to increase. From 1943 to 1950, for example, nitrogen and phosphate consumption in the West has more than tripled. By 1955, through the approval of the Defense Production Administration, it is expected that the production of nitrogen in the United States will be approximately doubled and P_2O_5 increased about 70 percent above 1950 consumption. Western farmers undoubtedly will use more commercial fertilizer, but many of them have had limited experience with it. Consequently all data available on the subject should be analyzed to the fullest extent and interpreted in terms of most effective usefulness to farm operators and others interested in commercial fertilizer and agricultural production.

This paper describes briefly a recently developed procedure utilizing the law of diminishing returns for analyzing economic returns from the application of varying rates of commercial fertilizer to a crop. The method is illustrated by the analysis of data showing the response of alfalfa to applied P_2O_5.

The hay yield data were subjected to economic analysis by use of the exponential yield curves advocated by W. J. Spillman in U.S.D.A. Technical Bulletin 348. However, the procedure used was modified according to methods proposed by S. W. Mendum and D. B. Ibach. 1/

Spillman's procedure is based upon the generally accepted assumption that as additional units of a growth factor are applied, the growth of a plant is increased at a decreasing rate (constant ratio) up to a point where additional units of that factor will no longer increase growth. Spillman held that there is a curve for each set of growing conditions. Experience may indicate need for further refinements of Spillman's techniques and of the methods used here, but the general assumption is believed sound. Use of the exponential curve permits projection of calculated yields to the right or left of reported yields. This is especially important where the range of reported yields is limited.

Certain requirements of experimental design for fertilizer rate experiments must be met to provide agronomic data suitable for economic analyses. To locate alfalfa yield response data showing the effect of increasing rates of P_2O_5 application, a large number of western agronomic experiments were reviewed. Out of these, five experiments were selected as being most suitable for economic study. The five experiments were with irrigated alfalfa and involved a series of initial rates of superphosphate application. Hay yields had been collected over a period of at least three years.

A brief illustration of the analytical procedure follows, using alfalfa yield data from a phosphate rate experiment on Superstition sand near Yuma, Arizona. The

1/ The bulletin was finally prepared for publication by S. W. Mendum after Spillman's death. More recently he and D. B. Ibach have developed certain adaptations of the procedures that greatly facilitate the work of economic interpretation, particularly when dealing with two or more variables.

first step of the procedure is to plot the reported yields, given in Table 1, on cross-section paper and draw a free hand curve to fit the data. The yield at the midpoint between 0 and 600 pounds is read at 26.7 tons and this number is then used to divide the total yield increment in two parts: $i = 26.7 - 17.03 = 9.67$ and $i' = 29.12 - 26.7 = 2.42$. Definitions of terms and computations are given below: The equation on which the procedure is based is $y = M(1-R^x)$.

$R = \frac{2.42}{9.67} = .25026$ = ratio of i' to i when size of unit is 300 pounds.

$A = \frac{i}{1-R} = \frac{9.67}{.74974} = 12.898$ tons, or the theoretical maximum increase in yield from adding P_2O_5.

$M = A + Y_o = 12.898 + 17.03 = 29.928$ tons, or the theoretical maximum yield from the use of P_2O_5 under conditions of the experiment.

$u = \frac{\log .8}{\log R}(300) = \frac{\log .8}{\log .25026}(300) = \frac{-.09691}{-.60161}(300) = .16108\ (300) = 48.324$ pounds

This is the pounds (48.324) in a unit of P_2O_5 required to render R = 0.8, and in all subsequent calculations the term "unit" refers to this quantity. This conversion, made solely for the purpose of facilitating computations, permits use of a table of values of $1-R^x$ for a range in values of x at a stated value of R. Spillman prepared such a table on the basis of R = 0.8.

p = the number of units (when R = 0.8) of P_2O_5 in the soil that are equivalent in effect to similar units applied as fertilizer, as reflected by the response curve.

$p = \frac{\log M - \log A}{\log R} = \frac{1.47608 - 1.11052}{-0.09691} = -3.772$

(The minus sign places the quantity to the left of the point of no application, but the sign may be disregarded as in subsequent work it is used to designate the first units of x starting with 0 availability)

b = the applied portion of P_2O_5. In the equation it is expressed in units, calculated as pounds applied divided by u, in pounds.

x = p + b in units, as used in the yield equation.

y = calculated yield.

Y = reported yield.

Table 1. - Calculation of yields of alfalfa at different levels of application of P_2O_5 on Superstition Sand, Yuma Mesa, Arizona

b pounds	b units	p units	p + b units	$1-R^{p+b}$ f 1/	y 2/	Y
0	0	3.772	3.772	.56902	17.03	17.03
100	2.069	3.772	5.841	.72839	21.80	21.84
200	4.138	3.772	7.910	.82882	24.80	24.31
(300) 3/	6.207	3.772	9.979	.89212	(26.70)	—
400	8.276	3.772	12.048	.93201	27.89	28.70
(500)	10.345	3.772	14.117	.95715	(28.65)	—
600	12.414	3.772	16.186	.97300	29.12	29.12
(1000)	20.690	3.772	24.462	.99574	(29.80)	—

1/ Read for p + b from Spillman's table of $1-R^x$ values when R = 0.8.
2/ $y = M(1-R^{p+b})$ e.g. y = 29.928 (.82882) = 24.80 tons. The Σd^2 (y from Y) = .8978 tons and the standard error of estimate is 0.47 tons.
3/ Data in parenthesis are associated with levels of application not used in the agronomic experiment.

Table 1 shows the calculated values for different levels of application and the yields calculated at each indicated level. In terms of this illustration the yield equation is $y = M\ (1-R^{p+b})$. The yield may be calculated for any rate of application within or beyond the range used in the experiment. This is of vital importance where the highest experimental rate is below the most profitable rate. However, in this experiment it appears that little additional yield would have resulted had more than 600 pounds of P_2O_5 been applied. Further trials located the midpoint at 26.69 tons as determined by the smallest Σd^2. This gives the approximate best fit and the resulting constants were used for the remainder of the analysis.

Some results of the yield analysis of the experiments are shown in Table 2. In 5 instances 4 rates or more were applied and in all but one instance the reported yields were on or very near the fitted curve. This close conformance is probably explained by the composition of the reported yields, which are totals for 3 years, with 4 or more replications from 3 to 10 cuttings per year. The theoretical maximum yields appear to be reasonably near known high yields in the respective areas.

Table 2. - Alfalfa yield response to P_2O_5 at four locations in the Western States.

Experiment	P_2O_5 per acre: Highest rate applied	P_2O_5 per acre: Most profitable rate	Yield of hay at most profitable rate of P_2O_5: Total	Yield of hay at most profitable rate of P_2O_5: Increase	Average increase in yield of hay per lb. of P_2O_5 by 50 pound units: First 50 lbs.	Third 50 lbs.	Fifth 50 lbs.	Tenth 50 lbs.
	Lbs.	Lbs.	Tons	Tons	Lbs.	Lbs.	Lbs.	Lbs.
1	400	843	24.765	24.165	166	117	82	35
2 1/								
Wet	600	581	37.901	18.912	189	109	63	16
Medium	600	806	34.314	13.744	80	61	46	23
Dry	600	561	28.961	11.931	106	66	41	13
3	90	98	18.152	3.022	96	7	--	--
4	90	597	21.527	14.237	125	80	51	17
5	100	378	15.484	8.246	104	52	27	4

1/ Unpublished data; Division of Soils and Irrigation, Bureau of Plant Industry.

Substantial differences in the efficiency of P_2O_5 at different units of application are shown in Table 2. The first 50 pounds produced several times as much alfalfa as the tenth 50 pound unit. Efficiency at the tenth 50 pound unit was low, but application of this unit would be profitable in 5 of 7 cases. Differences in moisture treatment in Experiment 2 indicate this factor affects P_2O_5 efficiency. The data in Table 2 indicate that the fields tested can profitably use high rates of P_2O_5. The most profitable rate was greater than the highest rate tested in 5 instances, in the other two it was near the high rate.

Constants determined for the Arizona trial with 26.69 tons as the mid-point were used to calculate the most profitable rate. Definition of terms and calculation of the most profitable rate of P_2O_5 to apply under conditions of the experiment are as follows:

v = value in dollars per ton of hay standing in the field.

$(-l_nR)$ = .22314 ______. This is the natural logarithm of 0.8.

$$Q = \frac{1}{v\ M(-l_nR)} = \frac{1}{20\ (29.937)\ (.22314)} = \frac{1}{133.60284} = .00748$$

r'' = cost of 1 unit of P_2O_5 = cost per lb. times u = .09 x 48.504 = \$4.3654

$Qr'' = 0.03265$. This is equivalent to R^{p+b} at the point where $R^{p+b} - Qr'' = 0$.
Hence: $1 - R^{p+b} = 0.96735$.

The value of .96735 in Spillman's Table 19, of values for $1-R^x$ (when $x = p+b$) is 15.335 new units. Since p is calculated at 3.770; b the most profitable rate to apply is 15.335 - 3.770 = 11.565 units of 48.504 pounds each or 561 pounds.

The calculated most profitable rate for a given situation is a point of departure to be used with good judgment taking into consideration the quality of the data used to make the approximation, probable growing conditions, probable price-cost conditions and other factors that may come under the general heading of "risk". Many factors may alter returns expected but nevertheless decisions must be made on rate of application. Since phosphate fertilizer is nontoxic and remains more or less available in non acid Western soils until used, there is little risk in applying calculated most profitable rates under most Western conditions. Some soil scientists subscribe to the idea of liberal P_2O_5 treatments to remove it as a limiting factor, but several aspects of this problem remain to be explored. It is believed that "p" will be useful in predicting yield response, but further studies correlating it with yield response under various conditions are needed. It is but a relative figure based on the efficiency of "b", i.e. applied P_2O_5 and is not comparable to results of chemical analysis.

The procedure briefly explained here is exploratory and deals only with one variable; the rate of P_2O_5 applied. It could also be used for nitrogen on non-leguminous crops or for that matter for several other nutrient elements on most crops. Much of the procedure used here is essential when two or more variables are studied simultaneously. Yield is the only benefit considered, but other items such as quality of crop, nitrogen fixation, response by following crops to residual P_2O_5, and general soil improvement may justify evaluation.

Most of the completed fertilizer research in the West deals with individual crops, and in many cases additional information by crops is needed on fertilizer rates, irrigation practice, and other growth factors. In the absence of complete detailed data by crops and by rotations, available information must be integrated into the development of a synthesized farm organization in efforts to ascertain the most profitable management practices.

A field experiment with fertilizer rates should include enough rates to ascertain the shape of the curve which should extend to the point where the increases in yield are of no economic importance. A minimum of 6 rates is suggested until experience dictates otherwise. Selection of test sites fairly typical of soil fertility levels and conditions on farms is essential so that results may be extended to as many farms as possible. Approximation of the degree nutrient deficiency and the relation of this to yield response is especially important.

Experience has shown the desirability of close cooperative effort by agronomists and economists in planning and analyzing fertilizer experiments. An economist who has the advisory assistance of a soil scientist available on short notice is fortunate. Those who attempt economic analysis of fertility problems can greatly increase the accuracy and acceptability of their work by close collaboration with soil scientists and agronomists. Physical scientists can increase the effectiveness of their findings by giving the economist a clear understanding of the data with which he deals. The economist can render valuable assistance in planning experiments suitable for economic analysis and by making the analysis. Cooperative planning and analysis of fertility experiments shortens the time and greatly reduces the cost of answering the mutual question "Does it pay?".