The Use and Potential of Optimal Control Models in Agricultural Economics

David Zilberman

Scientific disciplines are constantly examining and refining the set of analytical tools they apply in their investigation. This process of assessment and modification has important effects on the educations of new professionals, the requirements from practitioners, the selection of research topics, the "real world" applicability and relevance of research topics, and the agenda of the profession.

At the present, I believe that the minimal arsenal of a practicing agricultural economist includes diagrammatic analysis of outcomes in competitive and monopolistic markets, basic macroeconomic models, linear programming, and simple regression. Nonlinear programming, simultaneous equation econometric models, and maybe dynamic programming are frequently applied tools and are part of the "mainstream" methods of the profession. Optimal control analysis, however, is a relatively new analytical tool; its usefulness is in the midst of a process of evaluation. The verdict is not out yet whether it becomes a mainstay of agricultural economics analysis or whether it becomes a marginal tool without widespread use (like game theory).

This presentation will argue that control theory models are here to stay; that optimal control is a powerful tool which expands the range of issues dealt with by agricultural economists and increases their effectiveness. Therefore, it should become part of the mainstream tools of agricultural economics.

After a sketchy description of optimal control models, some of their main applications and contributions to agricultural economics will be summarized, and then extremely promising avenues for future application will be presented using examples and tentative results.

Optimal Control Models and Their Use in Economics

Optimal control models describe the evolvement of a system over a time horizon and determine optimal levels of decision variables over time. The state of the system at any point in time is characterized by *state variables*. Time-dependent variables determined by the decision-maker are *control variables*. The system may also be affected by some unconrollable *random variables*. Changes over time in the state variables are according to *equations of motion* which are assumed to be functions of the state variables, control variables, and random variables at the moment of change. The decision-maker's *objective* in the general case is to maximize the expected value of the sum over time of temporal utility functions which are functions of time and levels of the control, state, and random variables at each moment of time. Thus, the optimal control problem determines the controls that maximize the value of the objective function subject to the equations of motion given the initial values of the state variables.

There are significant differences in methodology and focus of analysis between deter-

---

David Zilberman is an assistant professor in the Department of Agricultural and Resource Economics, University of California, Berkeley.

Giannini Foundation Paper No. 654 (for identification only.) Special thanks to Gordon C. Rausser, Margriet Caswell, and Jeffery La France for their comments and suggestions. The research leading to this report was supported by the University of California, Water Resources Center, as part of Water Resources Center Project UCAL-WRC-W-622.

ministic optimal control models (which assume that the system is not affected by random variables) and optimal control models with stochastic elements. Deterministic control models are used frequently as analytical tools applying the maximum principle of Pontryagin *et al.* that, under reasonable conditions, translates the intertemporal optimization problem to many temporal optimization problems of the same form and, in essence, extends the comparative static analysis to a dynamic framework. It yields qualitative results, suggests testable hypotheses, and derives relationships that can be solved numerically (many times with dynamic programming). Optimal conrol models with elements of uncertainty are much more complex than the deterministic ones and are rarely used to derive analytical results. They are mostly applied to derive numerical solutions to empirical problems and are amenable to various types of sensitivy analyses. Significant methodological research has been conducted, however, to improve the solution concepts and techniques for these models. Particular attention has been given to situations where some of the uncertainty is the result of lack of information about the true values of the parameters of the system. Recent solution concepts for these cases involve constant estimation of the system's parameters over time as more information is revealed, and the results of the process of learning are incorporated to derive better controls. The most advanced technique, namely, adaptive control, takes into account the expected gains from future learning in determining optimal control levels.[1]

While methodological developments and applications of optimization models to dynamic economic systems occurred during the 1950s, the introduction of the "maximum principle" in the 1960s elevated optimal control to prominence as a research tool in economics. It seemed especially promising for studies of economic growth and macroeconomics.

While the performance of control theory in macroeconomics has been disappointing [Wallish] and the theory of economic growth has not fulfilled all the expectations it raised [Hahn and Matthews], these applications have produced knowledge and human capital that have generated surprisingly high yields when used for other topics. By replacing time with space as the domain of the states and control variable, optimal control models have been used successfully in urban economics [Mills]. Moreover, optimal control models with human capital index as domain have been essential in the evolvement of the promising new research on optimal income tax schemes [Cooter]. The evolvement of application of optimal control models in economics should teach us a lesson regarding its potential contribution to agricultural economics. It may be that, here too, some of the most important contributions may be in analyzing systems that do not necessarily vary along a time dimension but along other dimensions.

Applications of Optimal Control to Capture Dynamic Aspects of Agricultural Economics

The higher tendency of agricultural economics, relative to other fields of economics to incorporate details of physical processes in its models, may suggest that it offers relatively more opportunities to apply dynamic control models than other fields. Indeed, there are several particular areas where the use of dynamic optimal control can improve (or has improved) significantly the quality of results obtained by agricultural economists. One of these areas is farm and production management. The use of dynamic control models in this area makes it possible to present agricultural production as a process of growth, which it is. For example, Yaron *et al.* used a dynamic framework to determine optimal irrigation policy for a field crop (wheat). In this model, yield is a function of two state varibles — soil salinity and

[1]A good exposition of optimal control methodologies under certainty and uncertainty with application to agriculture appears in Rausser and Hochman.

moisture — over the life of the crop. Irrigation is a control variable that affects the state variables and, through them, it affects yield. The model is applied using dynamic programming, and optimal (profit maximizing) irrigation strategy is selected from a feasible set.

The usefulness of optimal control is increasing as one moves from considering agricultural and resource management problems, which are totally in the private sector domain, to problems which involve public policies.

A very significant and productive research has been conducted applying dynamic optimal control models for the management of renewable and exhaustible natural resources [Dasgupta and Heal]. Special attention has been paid to optimal management of fisheries, forests [Clark], and pesticides [Howitt and Rausser]. These applications of optimal control have drastically altered the perspective and improved the quality of answers and solutions suggested by resource economists that have resulted in the revamping of long-held policies and practices of resource management.

While deterministic optimal control models have been more prominent in the economic research of natural resources management policies, optimal control models with stochastic components have been prominent in more traditional agricultural policy applications.

Stochastic and adaptive control are natural tools of planning and management for marketing boards, producer cooperatives (or cartels), and government agencies administrating commodity programs. The first application of these techniques for agricultural policies was Gustafson's dynamic programming model for optimal management of public stocks. Rausser and Hochman presented stochastic dynamic models for optimal management of marketing boards, Freebairn and Rausser used adaptive control for optimal determination of beef import quotas, and several authors recently developed optimal control models for the management of commodity programs [for example, Burt, Koo, and Dudley]. While the use of optimal control for agricultural policy analysis is increasing, it is still a limited tool in its application and impacts. This lackluster performance cannot be completely explained by the lack of training of many of the practitioners of agricultural policy analysis that prevent them from using this tool. While this is one factor, there seem to be more substantial reasons.

Because of the complexity of optimal control models with elements of uncertainty, they determine policies for specific parametric values and situations that may be temporary and of limited interest. The specificity of the results shifts the emphasis of many of the policy-oriented control articles to the technique they use. This emphasis on technique is a turn off to many people who first want to learn what they will gain from a new methodology before understanding its details. It seems that current optimal control models in agricultural policy lack a facility for qualitative sensitivity analysis over time that is able to generate generalized hypotheses and theories the way comparative statics does in economic modeling of static systems. Recent methodologies for comparative dynamics developed by Aoki may alleviate this problem somewhat.

Moreover, the perceived strength of optimal control in agricultural policy is in determining the best policies given the objective function and constraints of the economic system. The controversies in agricultural economics are mostly with regard to the exact objectives of policymakers and the constraint structure they face. Therefore, there is a higher premium to models that better clarify objectives and, in particular, constraints of the system than to models that obtain "better" numerical results for systems based on shaky ground. Thus, optimal control models will become more prominent in agricultural policy analysis if and when they will be applied to improve our understanding of the working of the agricultural economy and to overcome essential shortcomings of standard models such as badly specified aggregates, unsatisfactory modeling of processes of technological change, etc.

Use of Optimal Control When the System's Domain is Not Time

As the experience of general economics suggests, the introduction of optimal control to analyze dynamic systems may have important methodological and educational externalities. These models train the user to analyze choice problems where the key variables are not scalers but functions defined on an interval. In the dynamic control models, the array of the system is time; however, the same technique can be applied when the array is distance, product quality, or human ability. Thus, optimal control enables the economist to analyze problems where the atomistic units of the system have an essential element of heterogeneity.

Agricultural and resource economists have applied optimal control models to systems with domains different than time. For example, optimal control models were applied to design policies for management of water quality and allocate water quantities along a river basin [Hochman, Pines, and Zilberman] where distance replaced time as the array of the system. Nevertheless, it seems that the potential of this line of research has hardly been tapped.

Explicit consideration of dimensions of hetrogeneity among key factors in agricultural economic systems have many advantages. First, it expands the range of issues and policies addressed by agricultural economists. Second, it overcomes some of the deficiencies and drawbacks of existing models, and, third, it expresses rigorously arguments that have been previously presented only heuristically. These points can be illustrated by the following very simple regional model of irrigation where land quality variations are explicitly considered.[2]

Currently, most models of agricultural production consider inputs to be homogenous. However, the effectiveness of inputs, such as fertilizer and water, depends on some measures of quality of the lands in which they are being applied. Some modern application technologies are introduced to improve the effectiveness of these inputs on certain types of lands, and they can be considered quality-augmenting technologies. For example, drip irrigation assists the soil in holding water and, therefore, improves irrigation effectiveness (compared to traditional methods of flooding and farrows) [Caswell]. Differences of land quality (measured for our purposes by water-holding capacity) affect the benefits of drip irrigation and, thus, the tendency to use it. Thus, regional models of irrigation should explicitly consider land quality differentials.

The Irrigation Example

Consider a region specializing in a crop which price is given by P. Let $q \equiv f(e)$ be a per acre production function of a certain crop under a traditional irrigation technology when q denotes output per acre, e is effective water per acre (water attains the crop), and $f(\cdot)$ has the standard properties $f'>0$, $f''<0$, and also $\lim_{e \to 0} f'(e) = \infty$. Effective water is the product of applied water per acre x and the quality measure of the land ε, $e = \varepsilon x$. The measurement of the quality of a certain piece of land is the share of irrigated water it allows to attain the crop, thus $0 \leq \varepsilon \leq 1$. Let the total land of the region be denoted by $\bar{L}$, and the density function of the land-quality distribution be $g(\varepsilon)$. Thus, for small $\Delta\varepsilon$, the amount of land with quality in the interval $(\varepsilon - \Delta\varepsilon/2, \varepsilon + \Delta\varepsilon/2)$ is approximated by $\bar{L}\ g(\varepsilon)\Delta\varepsilon$. Assume that $g(\varepsilon)>0$ for $0 \leq \varepsilon \leq 1$.

Suppose the region has one source of water for irrigation (no rain) and annual supply is given by $\bar{Z}$. Suppose also that the regional authority wants to allocate water to maximize aggregate profit. The regional optimization problem in this case is

$$(1) \qquad \max_{x(\varepsilon)} P \bar{L} \int_0^1 f(\varepsilon \cdot x)\, g(\varepsilon)\, d\varepsilon$$

subject to

[2]This model relies heavily on my joint work with Margriet Caswell.

$$(2) \qquad \bar{L} \int_0^1 g(\varepsilon)\, x(\varepsilon)\, d\varepsilon = \bar{Z}.$$

This problem is reformulated to the form of a "textbook" deterministic control problem by introducing a state variable, $Z(\varepsilon)$ — the amount of water available to lands that are not more inferior than ε. The equation of motion and initial condition associated with $Z(\varepsilon)$ are

$$(3) \qquad \dot{Z} = -g(\varepsilon)\, x(\varepsilon) \bar{L}$$

$$(4) \qquad Z(0) = \bar{Z}.$$

(Dotted variables denote derivative with respect to the domain variable, in this case, land quality).

Thus, the reformulated control problem consists of solving (1) subject to (3) and (4). Using the maximum principle, define the Hamiltonian

$$H(\varepsilon) = \bar{L}\, g(\varepsilon)\, [Pf(x \cdot \varepsilon) - \lambda(\varepsilon)\, x(\varepsilon)]$$

where $\lambda(\varepsilon)$ is the costate variable of the equation of motion (3). It can be interpreted as the shadow price of irrigated water for land of quality ε. Assume that all the water will be utilized, i.e.,

$$(5) \qquad Z(1) = 0.$$

The maximum principle suggests that an optimal solution consists of values of $x(\varepsilon)$, $\lambda(\varepsilon)$, and $Z(\varepsilon)$, $0 \leq \varepsilon \leq 1$ satisfying equations (3) to (5) and

$$(6) \qquad \dot{\lambda} = -\frac{\partial H}{\partial Z}(\varepsilon) = 0 \qquad 0 \leq \varepsilon \leq 1$$

and

$$(7) \qquad \frac{\partial H}{\partial x}(\varepsilon) = P \frac{\partial f}{\partial e}(e) \cdot \varepsilon - \lambda(\varepsilon) = 0 \qquad 0 \leq \varepsilon \leq 1.$$

Condition (6) suggests that differences in land quality does not affect the shadow price of water. (Unlike the case of allocation of resources over time, profits obtained at different land qualities are treated equally. The "discount rate" is zero.) Thus, let the shadow price of irrigated water be denoted by $\bar{\lambda}$. Using $\lambda(\varepsilon) = \bar{\lambda}$, condition (7) suggests allocation of water for irrigation such that the marginal productivity of irrigation is equalized along land qualities. Since $e = x \cdot \varepsilon$, it implies that the ratio of marginal products of effective water of two land qualities is the inverse of the land quality ratio,

$$(8) \qquad \frac{\frac{\partial f}{\partial e}(x_1 \cdot \varepsilon_1)}{\frac{\partial f}{\partial e}(x_2 \cdot \varepsilon_2)} = \frac{\varepsilon_2}{\varepsilon_1}.$$

Equation (8) and the concavity of the production function imply that lands of higher quality utilize more effective water than lower quality lands and thus produce more yield. Note, however, that actual irrigation may decline with higher quality land and the advantage of better land is reflected in lower water use as well as higher yield.

Differentiating (7) with respect to land quality yields:

$$(9) \qquad \frac{\dot{x}(\varepsilon)}{x(\varepsilon)} = -[1 + \eta_{f'_e}^{-1}]e^{-1}$$

where $\eta_{f'_e} = f'' \cdot e/f'$ is the elasticity of the marginal productivity of effective water, and it is negative. Thus, higher land qualities require more (less) irrigation when the elasticity of marginal productivity of effective irrigation ($\eta_{f'_e}$)is greater (smaller) than -1.

Assuming that farmers in the region are profit maximizers, the solution to the optimal control problem in (1), (3), and (4) suggests a Pareto efficient water-pricing rule and, alternatively, a Pareto efficient rule for direct allocation of water according to land quality. Obviously, water pricing or allocation schemes that are obtained ignoring land quality may be suboptimal or infeasible in the long run. For example, if decision-makers treat land as an homogenous input with quality that equals $\bar{\varepsilon}$, the average quality in the region, then they impose a water price of $f'(\bar{\varepsilon}\ \bar{Z}/\bar{L})$. If this price is smaller (greater) than the optimal price derived from the control problem, there will be excess demand (supply) for water. If quantities (not prices) are the allocation tool and homogeneity of land is assumed, each acre will receive $\bar{Z}/\bar{L}$ units of water, and there will be a loss of profit to the region. The cost of this error depends on the degree of variability of land quality with the region. An application of a similar model to evaluate alternative policies to control waste disposal practices of dairies found that a waste disposal rule considering heterogeneity among dairies can attain the regional environmental standard at a third of the cost of a direct regulation based on a false homogeneity assumption [Zilberman].

The above optimal control model of irrigation can be expanded to incorporate adoption of land quality augmenting technologies like drip irrigation. Assume that the annual cost of drip is k dollars per acre (independent of quality)[3]. This may be the sum of annualized investment cost and annual setup cost. The use of drip irrigation improves water effectiveness of lands from ε to $h(\varepsilon)$ where $h(0)\geq 0$, $h(\varepsilon)\geq\varepsilon$, $h'(\varepsilon)\geq 0$, $h''(\varepsilon)\leq 0$, and $h(1)=1$. Let $\gamma(\varepsilon)$ be the extent of adoption of drip irrigation for land quality ε. Its range is given by

$$(10)\qquad 0\leq\gamma(\varepsilon)\leq 1.$$

Using these definitions, the objective function for the expanded irrigation problem is

$$(11)\qquad \max_{x(\varepsilon),\gamma(\varepsilon)} \bar{L}\int_0^1 \{\gamma[Pf(x\cdot h)-k] + (1-\gamma)\, Pf(x\cdot\varepsilon)\}\, g(\varepsilon)d\varepsilon$$

(For convenience, ε was omitted from some variables that are functions of ε.) And the constraints are equations (3), (4), and (10).

[3]Fixed cost, k, could have been assumed a function of land quality. This would have complicated the analysis without adding a lot of insights.

The Hamiltonian for the new optimization problem is

$$(12)\qquad H=g(\varepsilon)\bar{L}\,\{\gamma[Pf(h\cdot x)-k] + (1-\gamma)\, Pf(\varepsilon\cdot x)-\lambda x+\eta(1-\gamma)\}$$

where $\eta(\varepsilon)$ is the shadow price of the constraint (10). Applying the maximum principle and the Kuhn-Tucker conditions for (12) yields necessary conditions for optimal solutions (assuming all the water is utilized):

$$(13)\qquad -\frac{\partial H}{\partial Z}=\dot{\lambda}(\varepsilon)=0$$

Hence, $\lambda(\varepsilon)=\bar{\lambda}\forall\varepsilon$.

$$(14)\qquad -\frac{\partial H}{\partial x}(\varepsilon)=g(\varepsilon)\ \bar{L}\cdot\{\gamma Pf'(h\cdot x)\cdot h + (1-\gamma)\ Pf'(x\cdot\varepsilon)\cdot\varepsilon-\bar{\lambda}\}=0$$

$$(15)\qquad \frac{\partial H}{\partial\gamma}(\varepsilon)=g(\varepsilon)\ \bar{L}\cdot\{Pf(h\cdot x) - Pf(\varepsilon\cdot x)-k-\eta\}\leq 0 \qquad \frac{\partial H}{\partial\gamma}\dot{\gamma}=0$$

$$(16)\qquad \frac{\partial H}{\partial h}(\varepsilon)=1-\gamma\geq 0 \qquad (1-\gamma)\ \eta=0.$$

Conditions (14) to (16) suggest that drip irrigation will be adopted fully for land quality ε if the gains from adoptions will cover the fixed cost of drip. It will not be adopted at all if the opposite is true, i.e.,

$$(17)\qquad \begin{array}{l}\gamma(\varepsilon)=1\\ 0\leq\gamma(\varepsilon)\leq 1\\ \gamma(\varepsilon)=0.\end{array} \quad \text{if } \max_x\{Pf(hx)-\bar{\lambda}x\} - \max_x\{Pf(ex)-\bar{\lambda}x-k\} \begin{array}{c}>\\=\\<\end{array} 0$$

The analysis of the region's equilibrium in Caswell has proven that, given k, P, and $\bar{Z}$, drip irrigation is adopted for a medium range of land qualities (ε, $\bar{\varepsilon}$), while the gains from the new technology will not warrant adoption on the better or worse land qualities. Moreover, increases in output price or reduction in the fixed cost of the new technology increases the range of land qualities where drip is adopted. In addition, an increase in aggregate water supply increases (decreases) the range of land qualities where drip is adopted if the elasticity of the marginal productivity or effective water is smaller (larger) than 1 in absolute value.

The effects of changes in k, P, and $\bar{Z}$ on aggregate profits, its distribution, and aggregate adoption (measured in acres) can be computed using the solution to the regional control problem. Since these parameters are affected by actual policy tools (for example, the fixed cost of drip k can be affected by a credit subsidy, extension effort reducing setup cost, research and development policy improving effectiveness of drip, etc.), this type of model extends our ability to analyze situations of technological change and to develop policies to control adoption of new technologies.

Two-Stage Optimal Control Problems

The previous sections encourage applying optimal control to temporal decision problems with heterogeneous activity units. Many real-life situations involve intertemporal decisions for systems with heterogeneous activity units. In many cases, these problems can be solved using optimal control in two stages. First, temporal optimization problems are solved for each point in time and determine the optimal behavior of activity units and the resulting temporal benefits as functions of variables that depend on time. In the second stage, the benefits functions and the dynamics of the time-dependent variables are incorporated into a dynamic optimal control problem that determines the optimal path of controllable time-dependent variables.

To illustrate this two-stage procedure, the irrigation problem mentioned above is extended to be a dynamic optimization problem. Assume that a region has an initial stock of water. This water stock is augmented over time by rain (in the wet season) and depleted by irrigation (in the dry season). Using two irrigation technologies (drip-flood) in manners described previously, the decision-maker's objective is to maximize the discounted aggregate profit of the region over an infinite time horizon. Thus, the regional optimization can be formulated as

$$(18)\qquad \max_{x(\varepsilon,t),\gamma(\varepsilon,t)} \int_0^{\infty} e^{-rt}\,\bar{L}\int_0^1 \{\gamma[P(t)\, f(x\cdot h) - k(t)] + (1-\gamma)\, P(t)\, f(x\cdot\varepsilon)\} g(\varepsilon)\, d\varepsilon\, dt$$

subject to

$$(19)\qquad \dot{s}(t) = G(t) - \int_0^1 x(t,\varepsilon)\, g(\varepsilon)\, d\varepsilon$$

$$0 \leq \gamma(t,\varepsilon) \leq 1 \qquad 0 \leq s(t) \qquad s(0) = \bar{s}$$

where r is the discount rate, s(t) is the stock of water at time t, and G(t) is the per period contribution of rain to the water stock. Here, the extent of adoption of drip and water use per acre depend on both time and land quality.

The problem in (18) can be solved in two stages. First, solving temporary optimal control problems defined by relations (11), (3), and (4), one derives $\pi[\bar{Z}(t), P(t), k(t)]$ which denotes aggregate net profit as a function of water consumed, prices of output, and the fixed cost at time t. This aggregate relationship replaces the temporary maximand in (18), and $\bar{Z}(t)$ is introduced to (19) to generate the second-stage optimal control problem

$$(20)\qquad \max_{Z(t)} \int_0^{\infty} e^{-rt}\, \pi[\bar{Z}(t), P(t), k(t)]\, dt$$

subject to

$$\dot{s}(t) = G(t) - \bar{Z}(t)$$

$$s(0) = \bar{s}.$$

The solution to this problem yields the optimal plan of water allocation as well as the optimal pricing scheme for water over time. Solving the optimal temporary equililbrium of (10), (3), and (4) for each time period given the optimal $\bar{Z}(t)$, one can derive the optimal water use pattern and the extent of adoption of drip irrigation over land quality and over time. These last results can project the dynamic path of the diffusion of drip irrigation under the optimal water-pricing policy.

While the dynamic model introduced in relations (18) and (19) assumes water quotas (or in a competitive economy assumes water prices) to be the only decision variables and k(t) and P(t) are given parameters, one can expand the analysis to make the fixed cost of drip irrigation a function of a government policy, say, government research and extension effort, and use the model to determine the optimal path of these policies as well as water pricing over time.

The use of the two-stage optimal control models to first determine temporal optimal behavior of an heterogeneous population and then to determine optimal dynamic behavior can overcome two shortcomings of mainstream economic models. First, it suggests a new approach to model processes of technological innovation, and, second, it derives meaningful aggregates for use in dynamic economic models. The irrigation example has illustrated both points somewhat. Discussing specific models of diffusion and growth can illuminate them further.

Up to the present, the dominant approach for modeling diffusion processes of new technologies is the one suggested by Mansfield which considers these processes to be mainly processes of imitation. Recently, however, Mansfield's model has been criticized for its weak microeconomic foundations, lack of emphasis on the role of profit maximization behavior, heterogeneity of the population of potential users and dynamic processes like learning by doing, and cheapening of capital relative to labor over time. New models of diffusion that try to overcome Mansfield's shortcomings [Feder and O'Mara; Davies] are similar in essence to the two-stage optimal control problem presented above. These models obtained S-shaped diffusion curves and other patterns of diffusion processes consistent with empirical observation. Moreover, by incorporating the assumption of negatively sloped demand to these new models of technological adoption, one can obtain a quantitative formulation of Cochrane's model of the "technological treadmill" [Kislev and Shchori-Bachrach have made some progress in this direction]. This heuristic model has been essential in depicting the distributional impacts of frequent technological changes in U.S. agriculture and in motivating and justifying government policies. Thus, two stages of optimal control have a lot of potential in modeling better processes of technological change and generating improved quantitative frameworks for policy analysis.

The irrigation example illustrates the usefulness optimal control procedures have in deriving well-defined temporary aggregates. One of the justified criticisms of the neoclassical theory of economic growth, introduced as part of the "Cambridge Controversy" [Harcourt] is the use of an ill-specified aggregate production function and capital. The "putty clay" approach and vintage models are examples of how the use of optimal control for populations with hetergenous activity units can remedy this problem.

Under reasonable assumptions about the distributions of the production parameters and properties of the micro production functions, Houthakker and Sato have obtained aggregate relationships of the C.E.S. and Cobb-Douglas forms; thus, suggesting a sound theoretical explanation to the good empirical fit of these functional forms and supplying insights regarding their use and misuse. Moreover, the putty-clay vintage models, which extend the range of issues dealth with by models of economic growth, suggest new measures of technological change and supply new policy insights in the static [Hochman and Zilberman] and dynamic (using two-stage optimal control) settings [Zilberman].

Empirical Considerations and Conclusions

While one may agree in principle that optimal control has immense potential when applied to temporary decision problems of a heterogeneous population and when solutions of such problems are introduced to dynamic optimization problems, he may consider wide-scale application of this approach impractical because of data limitations.

Empirical application of the temporary analysis requires estimation of the rules guiding the performance of activity units and the capacity distribution of activity units. Extension to dynamic analysis require estimation or prediction of these parameters over time. The dynamic extension is a lot simpler when the distribution of the activity units stay constant over time. For example, application of the temporal model of irrigation defined by relations (10), (3), and (4) involves estimating the production function of the crop as a function of effective water, estimation of water effectiveness of drip irrigation as a function of land quality, and estimation of land distribution according to correctly defined quality measures.

The example indicates the enormity of the data requirement of the approach advocated here. There is evidence, however, that identification of discriminating variables, the prediction of their effects on agents' behavior, and the estimation of the distribution of population according to these variables is feasible and profitable. Marketing research studies, public opinion polls, etc., have data requirements that crudely resemble the ones required by the proposed framework. Obviously such studies obtain the data they need, and the mushrooming of public opinion research institutes is a "revealed preference" proof to the profitability of their activities. The giant production linear programming models, such as the ones developed by Heady at Iowa State University apply (see Heady and Srivastava), in essence, an approach closely relaed to the one suggested here, and the increase in detail and size of data bases used for these models should be a source for optimism.

Nevertheless, lack of data is still the main deterrent to the widespread application of temporal control models for heterogeneous populations and the continued reliance on neoclassical models assuming homogeneity of population. It seems, however, that these data problems reflect constraints of the past that will be removed in the future, as data-handling technologies are improving. This assessment follows the casual observation that specification of models in economics and data availability are determined more or less simultaneously, given data-processing technology. Most of the empirical economic models and data basis we use reflect the data-processing technology of the 1960s and 1970s. Namely, they reflect the availability of large and fast computers with strong computing capabilities, relatively expensive data collection and storage cost, and almost no intercomputer communication. The current proliferation of microcomputers, establishment of communication networks, and the cheapening of storage costs suggest that the application of more data-intensive analytical methods will become easier and cheaper; and temporary optimal control models will benefit from this perceived reality.

This presentation has demonstrated that optimal control has made many inroads in agricultural economics. In some areas (notably, resource economics), it became a mainstream tool of analysis. In other areas, it is still peripheral in use. In the areas where optimal control has risen to prominence, its application has made a quantal difference by allowing analysis of new topics and by drastically altering basic theories and beliefs. It has been suggested that optimal control has enormous potential in other areas, if it will be applied to explicitly address the impacts of heterogeneity of activity units. This will allow better understanding (and prediction) of reality, addressing issues of equity and distribution, and generate meaningful aggreagtes over time. While in the past data restrictions have prevented such applications, they seem to become less of a problem as time goes on, making this application of optimal control promising and feasible.

References

Aoki, N. *Dynamic Analysis of Open Economies*. New York: Academic Press, 1981.

Burt, O., W. Koo, and W. Dudley. "Optimal Stochastic Control of U.S. Wheat Stock and Exports." Department of Agricultural Economics and Economics, Staff Paper No. 79-2, Montana State University, 1979.

Caswell, M. "The Diffusion of Low Volume Irrigation Technology in California Agriculture." Unpublished doctoral dissertation (in progress).

Clark, C. W. *Mathematical Bioeconomics: The Optimal Management of Renewable Resources*. New York: John Wiley and Sons, 1976.

Cochrane, W. W. *Farm Prices: Myth and Reality*. Minneapolis: University of Minnesota Press, 1958.

Cooter, R. "Optimal Tax Schedules and Rates: Mirlees and Ramsey." *American Economic Review*, 68(1978):756-68.

Dasgupta, P. and G. Heal. "The Optimal Depletion of Exhaustible Resource." *Review of Economic Studies Symposium*, 41(1974):3-28.

Davies, W. S. "Inter-Firm Diffusion of Process Innovation." *European Economic Review*, 12(1979):279-317.

Feder, G. and G. O'Mara. "Farm Size and the Adoption of 'Green Revolution Technology.'" *Economic Development and Cultural Change*, 30(1981):59-76.

Freebairn, John W. and Gordon C. Rausser. "An Adoptive Control Approach to Agricultural Policy." *Australian Journal of Agricultural Economics*, 18(1974):208-20.

Gustafson, R. L. *Carryover Levels for Grains: A Method for Determining Amounts That are Optimal Under Specified Conditions*. U.S. Department of Agriculture, Technical Bulletin No. 1178, 1958.

Hahn, F. H. and O. W. Mathews. "The Theory of Economic Growth: A Survey." *Economic Journal*, 84(1964):779-902.

Harcourt, G. C. *Some Cambridge Controversies in the Theory of Capital*. Cambridge: Cambridge University Press, 1974.

Heady, E. O. and V. K. Srivastava. *Spatial Sector Programming Models in Agriculture*. Iowa: Iowa State University Press, 1975.

Hochman, Eithan and David Zilberman. "Examination of Environmental Policies Using Production and Pollution Microparameter Distribution." *Econometrica*, 46(1978):739-60.

Hochman, Eithan, David Pines, and David Zilberman. "The Effects of Pollution Taxation on the Pattern of Resource Allocation: The Downstream Diffusion Case." *Quarterly Journal of Economics*, 91(1977):625-38.

Howitt, R. and G. C. Rausser. "Pesticide Externalities, Noisy Measurements, and Public Regulation." *Journal of Economic Dynamics and Control* (forthcoming).

Houthakker, H. S. "The Pareto Distribution and the Cobb-Douglas Production Function in Activity Analysis." *Review of Economic Studies*, 23(1955-56):27-31.

Huffman, W. E. "Allocative Efficiency: The Role of Human Capital." *Quarterly Journal of Economics*, 91(1977):59-80.

Kislev, Y. and N. Shchori-Bachrach. "The Process of an Innovation Cycle." *American Journal of Agricultural Economics*, 55(1973):28-37.

Mansfield, E. "Technical Change and the Rate of Imitation." *Econometrica*, 29(1961):741-66.

Mills, F. S. *Studies in the Structure of the Urban Economy*. Baltimore: The Johns Hopkins University Press, 1972.

Moffitt, L. Joe, David Zilberman, and Richard E. Just. "A 'Putty Clay' Approach to Aggregation to Production/Pollution Possibilities: An Application in Dairy Waste Control." *American Journal of Agricultural Economics*, 60(1978):452-59.

Pontryagin, L. S., R. V. Boltyanshii, R. V. Gambreldzi, and E. F. Mishenken. *The Mathematical Theory of Optimal Processes*. Oxford: Pergamon Press, 1964.

Rausser, Gordon C. and Eithan Hochman. *Dynamic Agricultural Systems: Economic Prediction and Control*. New York: Elsevier/North Holland Inc., 1979.

Sato, K. *Production Functions and Aggregation*. New York: North Holland Publishing Company, 1975.

Wallish, Henry C. "Discussion: Application of Optimal Control to Problems of Economic Stabilization." *American Economic Review*, 66(1976):356-58.

Yaron, D., G. Strateener, D. Shimshi, and M. Weigbord. "Wheat Response to Soil Moisture and the Optimal Irrigation Policy Under Conditions of Unstable Rainfall." *Water Resources Research*, 9(1973):1145-54.

Zilberman, David. "A Putty-Clay Approach to Environmental Quality Control." Unpublished Ph.D. dissertation, Department of Agricultural and Resource Economics, University of California, Berkeley, 1979.