Applications of Duality Theory to Agriculture

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Although a comprehensive framework for most of the theoretical foundations of duality has been available to economists since the seminal work by Shephard in 1953, empirical applications of duality have become popular mainly during the last ten years. The first empirical study which I am aware of that exploited duality theory is the one by Nerlove in 1963 which estimated a Cobb-Douglas cost function as an indirect way of measuring the parameters of the production function of electric utilities.

The development of the concept of flexible functional forms and its applications in the derivation of plausible functional forms for dual cost and profit functions in the early seventies [Diewert 1971; Christensen, Jorgensen and Lau] was an important step which led to the proliferation of empirical applications of duality. Several of these studies have concerned the agricultural sector. Of these, the study by Binswanger [1974a and 1974b] using U.S.A. data appears to be one of the earliest.

A reason for the increasing popularity of the use of duality in applied economic analysis is that it allows greater flexibility in the specification of factor demand and output supply response equations and permits a very close relationship between economic theory and practice.

If a transformation function dependent on factor quantities, a vector of output levels and the production technology is specified then empirical factor demand equations can be derived from the first order conditions of cost minimization. If profit maximization is assumed, the output supply response equations can also be derived from the first order conditions. Unfortunately, unless very simple and hence restrictive functional forms for the transformation function are assumed (i.e. Cobb-Douglas) these conditions frequently cannot be solved explicitly, and if that can be done, the resulting equations may not be feasible to estimate. The use of duality allows us to side-step the problems of solving first order conditions by directly specifying suitable minimum cost functions or maximum profit functions rather than production or transformation functions. From duality theory we know the set of necessary properties of the cost and profit functions which are implied by a "well behaved" production technology and by the corresponding behavioural assumptions. It is the knowledge of this set of minimum properties which has allowed the development of suitable functional forms for profit and cost functions. An advantage of starting by specifying a cost or profit function rather than the underlying transformation function is that in order to derive the estimating factor demand and output supply responses there is no need to solve any complex system of first order conditions. The behavioural response equations are obtained by simple differentiation of the dual functions with respect to input and/or output prices. The major advantage of this is that it implies less algebraic manipulations and, more importantly, it allows us to specify more complex functional forms which impose much less a priori restrictions on the estimating equations (i.e., we do not need to impose restrictions on the values of the elasticities of substitution, separability, homotheticity etc.).

In what follows what has been done on the

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use of duality in measuring agricultural factor demand and output supply responses during the last decade is examined. First the most popular approach (the cost function) is considered, which is used to estimate Hicksian input demands as well as to obtain information regarding properties of the underlying production technology. Next, applications of the profit function approach, which has allowed researchers to estimate Marshallian factor demands jointly with multioutput supply responses, are discussed. Other possible applications of duality are reviewed mainly concerning the analysis of supply responses when markets for certain inputs or outputs do not exist. That is, the study of producers behaviour when some of the prices motivating producer responses are unobservable due to the fact that many of the trade-offs in the allocation of resources occur within the farm-household and not between the farm producer and a market. This model may be relevant mainly to developing economies where a partial absence of markets in the rural sector often occurs.

The Cost Function Approach

Several studies have used this approach in measuring factor demand elasticities, elasticities of substitution and technical change in agriculture. Binswanger [1974a or 1974b] and Kako specified a translog cost function which allowed them to estimate factor shares in log linear form. The cost function specified in both studies was:

$$(1) \quad \ln C = \alpha_0 + \alpha_y \ln Y + \sum_{i=i}^{n} \nu_i \ln p_i + \tfrac{1}{2} \Sigma_i \Sigma_j \gamma_{ij} \ln p_i \ln p_j + \Sigma_i \gamma_{it} \ln p_i \ln t$$

where C is the minimum cost of production level, Y is output, p_i is the price of factor i and t is a time trend variable used as a proxy for technical change. From (1) one can obtain a specification for factor shares (S_i) via logarithmic differentiation using Shephard's lemma.

$$(2) \quad S_i = \nu_i + \Sigma_j \gamma_{ij} \ln p_j + \gamma_{it} \ln t$$

$$\text{where } \gamma_{ij} = \gamma_{ji} \qquad i = 1, \ldots, N$$

Using this specification Binswanger and Kako[1] were able to separate the effect of biased technical change (represented by the γ_{it} parameters) on factor shares from the effect of ordinary factor substitution due to factor price changes (represented by the γ_{ij} parameters in (2)). They both found that factor augmenting technical change has been very important and explains a great deal of the observed changes in factor shares in the U.S.A. and Japan.

An important assumption made in both studies is that the production technology is homothetic. Therefore expansion paths were assumed linear and thus changes in the scale of production would not affect factor shares. This is why the factor shares in (2) are assumed to be independent of output levels. The implication of this is that all changes in factor shares are attributed to substitution and/or factor augmenting technical change. If the production technology is not homothetic, however, a risk of overestimating the effect of factor substitution or, more likely, technical change exists. This is so because the time trend variable used as a proxy for technical change is generally positively correlated with output levels.

Lopez [1980] used a more general specification for the cost function using Canadian agricultural data. This specification allows for a non-homothetic production function but preserves the same degree of flexibility of the translog. The cost function specification used

[1]Both Binswanger and Kako considered the following inputs: land, labour, machinery, fertilizers and other intermediate inputs.

is a generalized leontief which is also a flexible functional form:

$$(3) \quad C = Y\,\Sigma_i\,\Sigma_j\,b_{ij}\,p_i^{1/2}\,p_j^{1/2} + Y^2\,\Sigma_i\,\alpha_i p_i + Yt\,\Sigma_i\,\gamma_i p_i$$

From (3) using Shepard's lemma one can obtain the factor demand equations in input/output ratio forms:

$$(4) \quad \frac{X_i}{Y} = \sum_j b_{ij} \left(\frac{p_j}{p_i}\right)^{1/2} + \alpha_i\,Y + \gamma_{it}\,t$$

where $b_{ij} = b_{ji}$ $\quad i = 1, \ldots, N$

Note that specification (4) allows one to separate the effect of relative factor price substitution, factor augmenting technical change and the scale of production on the input-output ratios (and, hence, on factor shares). In particular, equation system (4) allows, as a special case, for homotheticity. This occurs if $\alpha_i = o$ for all i, that is, when the input-output ratios are independent of output. By estimating a system of four input-output ratios (labour, capital, land and structures and other intermediate inputs) Lopez [1980] showed that the hypotehsis of homotheticity is rejected by a wide margin and that changes in the scale of production explain a very important proportion of changes in the input-output or share equations. The effect of non-neutral technical change was found to be insignificant, which was a rather surprising result. However, a recent more disaggregated study by Lopez and Tung using combined cross section and time series data for Canadian agriculture[2] found that the factor augmenting technical change parameters (γ_{it}) were jointly significant. However, the technical change effect was substantially less dramatic than those obtained by Binswanger or Kako, while the output scale effect is very strong and significant.

[2]The inputs considered were energy, energy-based, labour, capital, land and other intermediate inputs.

The own price elasticities of factor demand are quite similar for the four studies, despite using different data and models (Table 1). An overall analysis of Table 1 allows one to conclude that, in general, factor demands are inelastic; that land demand elasticity is somewhere between -0.35 and -0.50, that labour demand elasticity is roughly between -0.40 and -0.50 (Binswanger's result is an outlier). Demand for fertilizers and chemicals tends to be more elastic at least in the studies using North American data (roughly -0.9) and farm capital demand also exhibits somewhat lower values than the former. In general, one can say that the estimated demand elasticities may provide policymakers with some notion of the various degrees of price responsiveness of the inputs used in agricultural production.[3]

Unfortunately, the studies do not show the same consistency in the estimation of input substitution measures. Binswanger found that land is a substitute for labour, machinery and fertilizers. Fertilizers and land were found to be the best substitutes. These results are consistent with the findings of Lopez and Tung who found that land and energy-based inputs (largely fertilizers and other chemicals) were the best substitutes among all input pairs. Kako found that land was a substitute with all other inputs except machinery. In contrast with Binswanger's, Kako's and Lopez's results, the study by Lopez and Tung found that capital and land are complements. Labour and farm capital have been consistently found to be substitute inputs in all studies reviewed. However, labour and energy-based inputs are strong substitutes in the study by Lopez and Tung while they are complements in the studies by Binswanger and Kako.

In general, one can say that the various cost function studies have shown that (1) input demands are moderately responsive to

[3]It is important to note, however, that these are Hicksian elasticities. That is, they measure factor demand responses for *given* output levels neglecting the indirect factor demand effects associated with changes in output scale due to factor price changes.

TABLE 1. Hicksian Input Demand Elasticities Obtained in Various Studies.

	Binswanger (Pooled time series-cross sectional)	Kako (Pooled time series-cross sectional)	Lopez (Time series)	Lopez and Tung (Pooled time series-cross sectional)
Land	−0.34	−0.49	−0.42	−0.42
Labour	−0.91	−0.46	−0.52	−0.39
Fertilizers and chemicals	−0.95	−0.32	−0.41[a]	−0.89
Farm capital	−1.09	−0.59	−0.35	−0.63

[a]Includes fertilizers, chemicals and other intermediate inputs.

prices; (2) there exist sizeable substitution possibilities among several input pairs of which energy-based inputs and land appear to exhibit the greatest potential; (3) the aggregate agricultural technology is not homothetic and (4) the more simple production function specifications such as the Cobb-Douglas or Leontief[4] are not appropriate specifications as shown by the studies by Binswanger and Lopez, respectively.

It was indicated at the outset that a nice feature of dualilty is that knowledge of the properties of the dual behavioural functions (cost or profit functions) permit a close relationshp between economic theory and empirical analysis. In particular, cost minimization behaviour implies that the functions (1) and (3) should be increasing, linear homogeneous and concave functions of prices. Moreover, its Hessian matrix must be symmetric which implies that the factor demands satisfy the well-known reciprocity conditions. The empirical results are used to either test of impose the above restrictions on the estimating demand equations. The various empirical tests of the required properties of the cost function implemented by Binswanger [1974a] and Lopez [1980] showed that in general the empirical evidence in North America does not allow one to statistically reject the parametric restrictions implied by those properties. The important implication of this is that cost minimizing behaviour is an appropriate hypothesis for North American agriculture even at the aggregate industry level.

[4]This function is widely assumed mainly in linear programming studies.

The Profit Function Approach

A common feature of the empirical studies reviewed in the previous section is that they consider a single output technology. Additionally, a serious limitation of the cost approach in general is that it assumes that output levels are not affected by factor price changes and, thus, the indirect effect of factor price changes (via output levels) on factor demands are ignored.

Moreover, the inclusion of output levels as explanatory variables may lead to simultaneous equation biases if output levels are not indeed exogeneous. This problem is certainly compounded if a multi-output cost function is estimated. In this case the input shares or input-output ratios normally used to estimate the factor demands are dependent on each of the outputs even if constant returns to scale are assumed [Hall] and, moreover, these share equations are non-linear in the various outputs. This makes it very difficult to use econometric techniques designed to tackle simultaneity problems.

The profit function approach allows one to overcome most of these problems although at the cost of requiring a stronger behavioural assumption. The profit maximization assumption may be substantially more difficult to support in agriculture than simple cost minimization because of risk related problems which are mainly related to the variability of output yields and price rather than to costs of production.

The factor demands estimated using a profit function framework allow one to measure input substitution and output scale effects of factor price changes. Additionally, one can measure the cross effects of output price changes on factor demands and vice versa as well as output supply responses and their cross price effects. A major advantage of the profit function framework is that it allows the estimation of multi-output technologies in a much simpler way than a cost function or a transformation function. The profit function, π, is defined by

$$(5)\quad \pi\ (p,w;K) \equiv \{\underset{y,w}{\text{Max}}\ py\text{-}wx{:}F(y,x;K)=0\}$$

where y is a vector of M outputs, x is a vector of N variable inputs, K is a vector of S fixed inputs, $F(\cdot)$ is a continuous, concave transformation function, p,w are vectors of M output prices and N input prices.

It has been shown that the profit function π (.) is non-decreasing in p, non-increasing in w, linear homogeneous and convex in p and w. Moreover, its Hessian matrix with respect to p and w is symmetric. As in the case of the cost function, knowledge of these properties has allowed to develop suitable functional specifications which permit to test, verify or impose the above properties. The factor demands and output supply equations are derived from the specified profit function by simple differentiation with respect to input prices and output prices, respectively (Hotelling's lemma). Furthermore, the shadow price of fixed resource K_i is the derivative of π $(\cdot)$ with respect to K_i.

Most applications of profit functions to agriculture have assumed a single output technology. The earlier works used very simple and restrictive specifications for profit functions. Among these one may mention the studies by Lau and Yotopoulos of 1972 and Yotopoulos, Lau and Lin of 1976 who used a Cobb-Douglas specification for a single output restricted profit function. They estimated output supply and input demand responses using data from India and Taiwan, respectively.

More recent studies have used flexible functional form specifications for the profit function. Binswanger and Evenson tried various single output flexible form specifications using Indian data including the generalized leontief, translog and the quadratic normalized function. They found that, in general, the results obtained using the translog specification were less compatible with the restrictions implied by economic theory than the other two forms. An undesirable feature of the specifications used by Binswanger and Evenson for both the generalized leontief and normalized quadratic forms is that the shadow prices of fixed resources are implicitly assumed constant independent of the level of fixed resources.

Another recent application of the profit function approach is the study by Sidhu and Baanante to analyze input demand and wheat supply in the Punjab region of India. They used a normalized restricted translog profit function considering wheat output, three variable inputs (labor, fertilizers and animal power) and seven fixed factors (machinery and equipments, land, various soil nutrients, schooling and irrigation area). They obtained estimates for the elasticities of wheat supply responses as well as for the three variable factor demands. They showed that the Cobb-Douglas profit function specification is not supported by the data, and that the symmetry restrictions are not rejected. They obtained a wheat supply elasticity of 0.6 and, surprisingly, they found that the output price effect is more powerful in affecting demand for labor, fertilizer and animal power than their respective prices.

As indicated before, the vast majority of the profit function applications to agriculture assume a single output technology. Since agricultural production is carried on in farm units which typically produce several outputs, this implies that either there exist no economies nor diseconomies of joint production or that an aggregate output quantity and price index exists. As it is well known the separability restrictions implied by this latter assumption are very severe. If on the other hand, only one output of the multioutput enterprise is considered then the very serious problem of separating input levels by each of the outputs needs to be faced. For this reason the use of a multi-output profit function approach to agriculture appears to be quite useful. This approach does not require any knowledge regarding the allocation of the different inputs to each output.[5]

The only research located which estimates a complete multioutput profit function applied to agriculture is reported in Lopez [1981a]. This paper reports on the estimation of a two output (animal and crop outputs), four input (land, capital, hired labour and operator labour) generalized leontief profit function using cross-sectional Canadian census data. Major distinctive features of this study are the following: (a) a simple test for existence of economies (or diseconomies) of joint production is implemented. The hypothesis of non-joint production of crop and animal outputs was not rejected. This was not unexpected given the high level of output aggregation. It is likely that at more disaggregated levels this hypothesis may be rejected. (b) a procedure to separate substitution and expansion effects for both inputs and outputs from the profit function estimates is used. This amounts to deriving the output trade-offs due to a change in one output price for given input levels, and to measure the input substitution stemming from a factor price change for given output levels. That is, the Marshallian elasticities are obtained as directly provided by estimates of the profit function and the trade-offs along the production possibility frontier and isoquants are also measured. This information is vital if one desires to understand the structure or production of the agricultural industry. (c) a third feature is the consideration of hired and operator (and family) labour as two distinct inputs. In fact, these estimates indicate that hired and operator labour respond very differently to changes in input and output prices which suggests that indeed they should be regarded as different inputs. In general, hired labour is much more responsive to price changes than operator. Moreover, our findings indicate that, surprisingly, operator and hired labour behave as complements rather than substitute inputs.

Finally, it is worthwhile to mention that the use of the profit function concept has allowed researchers to develop relatively simple tests for the existence of allocative and technical efficiency of farm production mainly in developing countries. Since the 1971 work of Lau and Yotopoulos who used a Cobb-Douglas profit specification to test for relative efficiency of Indian producers, several studies have used similar approaches. Among these one may mention the work by Sidhu and Baanante [1979] who using Punjab data found that producers do obtaian allocative efficiency and that the profit function seems to be an appropriate concept to be used in the analysis of factor demand and output supply responses.

Further Applications of Duality: Farm-Household Supply and Demand Responses

The studies reviewed in the previous sections are mainly applications of linear duality. Linear duality applies when the underlying optimization problem is characterized by having either a linear objective function or a linear constraint function. The theory is based on convex analysis. In this section an application of generalized non-linear duality is discussed [Epstein] in the context of the farm-household model [Lopez 1981b]. Non-

[5]Incidentally, it has been shown elsewhere [Lopez 1982b] that any flexible functional form specification for a single ouput profit function necessarily implies that the underlying production function is quasi-homothetic. That is, that the production function is homothetic although not necessarily with respect to the origin. This implies that the associate cost function is of the form $c = \psi(y)\,\bar{c}(w) + g(w)$, which is a very restrictive specification in the context of production theory. An analogous result for the multi-output flexible functional profit specification has not been shown.

linear duality is a generalization of conventional duality in the sense that it allows both the objective and constraint functions to be non-linear.

The farm-household optimizing problem can be seen as one of maximization of a non-linear utility function subject to a non-linear budget constraint. This constraint is non-linear because an important proportion of the farm-household income is given by the farm returns which is a non-linear function of household labor (which also appears in the household's utility function as leisure) and fixed factors of production.

Since the seminal work by Sen, a number of studies have analyzed the neoclassical model of the farm-household with reference to developing countries. In contrast with the conventional models of the firm and of the household, the farm-household model emphasizes the interdependences between utility maximization and profit maximization decisions which arise mainly as a consequence of the existence of endogeneous prices of labour and non-traded goods (Hymer and Resnick). It is argued that family labour and some goods which are only produced to satisfy the family's own consumption necessities are traded entirely within the farm-household complex and hence that their shadow prices are endogeneous and are dependent on the farm production technology, household preferences and prices of traded consumption goods and outputs. These endogeneous shadow prices are the main linkages between production and consumption decisions.

A number of studies [Sen, Hymer and Resnick, Barnum and Squire] have been concerned with obtaining empirically testable predictions from the farm-household model emphasizing agricultural output supply responses to price changes. Unfortunately, the farm-household model does not *a priori* provide any definite predictions with respect to output supply responses. Barnum and Squire have analyzed alternative assumptions considering situations where more than one output can be produced, the existence of non-traded goods, etc. These authors have concluded that each of the models analyzed are consistent with positive or negative output responses. This implies that if the attention is focused only on observed output supply responses it is not in general possible to empirically verify the validity of the farm-household model.

In this section we show how the use of duality may help in deriving certain expressions which allow one to empirically test the theory of the farm-household model. We also illustrate the use of duality in deriving an econometric framework appropriate to empirically test the validity of the model by estimating the farm-household behavioural equations in a non ad-hoc manner. That is, to explicitly derive the estimating equations from the theoretical model, thus fully preserving the connections between the theoretical model and the estimating equations.

We first consider a variant of the farm-household model which is a straight forward generalization of the model used by Sen. This model assumes no off-farm employment, that all outputs and inputs have exogeneous prices with the only exception being labour whose (shadow) price is endogeneously determined within the farm-household complex. It is also assumed that the household maximizes a well-defined utility function which is a function of leisure and the consumption of a vector of market-purchased goods. Thus, the utility miximization problem is

$$(6) \quad \max_{H-L,\,X} \; U(H-L, X) :$$

$$\text{(i)} \quad pX \leq \pi(q; T, L) + y$$

$$\text{(ii)} \quad H \geq H-L \geq 0, \; X \geq 0$$

where U (.) is the household utility function, H is total number of hours which household members have available for work and leisure, p is a vector of N market-purchased consumption good prices, X is a vector of N consumption goods, L is number of hours of work, π (.) is a conditional variable profit function, q is an exogeneous price vector of the S net outputs produced by the farm, using the convention of representing output prices by positive quantities and purchased input prices by negative quantities, T is a fixed factor of production, say land an y is non-labour income net of fixed obligations.

Constraint (i) in (6) indicates that the total expenditures on consumption goods cannot be greater than the income associatd with the net farm returns to labour and owned fixed resources represented by the profit (function) plus the net non-labour income which may include government transfers, asset income, etc.

The conditional variable profit function π (q;T,L) is defined as follows:

$$(7) \quad \pi(q; T, L) \equiv \underset{Q}{\text{Max}} \{qQ : (Q; T, L)\varepsilon\tau\}$$

where Q is a vector of S net outputs including M outputs and S-M inputs, and τ is the production possibilities set which is assumed to be a compact, non-empty, convex set.

It is easy to verify that π (q; T,L) is non-negative, continuous linearly homogeneous and convex in q, nondecreasing and concave in T and L for fixed q.

Notice that this specification allows for a rather general production technology and, in contrast with the two crop model of Barnum and Squire, for example, it allows for the existence of economies (or diseconomies) of joint production. It also allows for the existence of several purchased inputs. Also note that π $(\cdot)$ is a variable profit function conditional on a given level of work L, which is jointly determined as an equilibrium level obtained from the labour supply schedule associated with the household's preferences for leisure and the demand for labour schedule associated with the production side represented by the variable profit function.

If problem (6) is defined locally for the compact subset M, then we can define an indirect utility function associated with such a problem in the following manner:

$$(8) \quad G(p, q, T, y) \equiv \underset{H\text{-}L,X}{\text{Max}} \{U(H\text{-}L, X) :$$

$$\text{(i)} \quad pX - \pi(q; T, L) \leq y$$

$$\text{(ii)} \quad (H\text{-}L, X)\varepsilon \int \text{ and } (p, q, T, Y)\varepsilon P\}$$

where the attention is restricted the set of utility levels $\tilde{M} \equiv \{\mu: \bar{\mu} \leq \mu \leq \bar{\bar{\mu}}\}$ which implies that the corresponding commodity space $\int$ and parameter space P are compact, non-empty sets.

Epstein has shown the existence of a duality relationship in the context of a more general non-linear model of which (8) is a special case. Epstein's results imply that an indirect utility function $G(\cdot)$ exists and, moreover, that there exists a one-to-one duality relation between the function $G(\cdot)$ and $U(\cdot)$ for a given function π $(\cdot)$. The function $G(\cdot)$ is non-increasing in p, non-decreasing in q,T and y and homogeneous of degree zero in p,q and y. Moreover, minimization of $G(\cdot)$ subject to the budget constraint allows to retrieve a function U* with identical behavioural implications as U and, from the first order conditions of this minimization problem, one obtains a relationship between the indirect utility function and the farm-household behavioural equations:

$$(9) \quad \text{(i)} \; Q_i = \frac{\partial G/\partial q_i}{\partial G/\partial y} \quad i=1,\text{---},S$$

$$\text{(ii)} \; X_j = -\frac{\partial G/\partial p_j}{\partial G/\partial y} \quad j=1,\text{---},N$$

$$\text{(iii)} \; \frac{\partial \pi}{\partial T} = \frac{\partial G/\partial T}{\partial G/\partial y}$$

Notice that the net output supply equations (9i) are dependent on the structural properties of both the production technology and household's preferences. Moreover, output supply responses are affected not only by the level of net output prices, but also by the price level of those commodities consumed by the household as well as by the household non-labour income. Similarly, demand for consumer goods is jointly determined by the parameters of the production and consumption sides. It is also important to indicate that the net output supply equations specified by (9i) are unconditional, in the sense that they are evaluated at the utility maximizing level of L.

Equation (9iii) provides a specification for the shadow price of land, $\partial\pi/\partial T$. If the conditions of the implicit function theorem are satisfied by $\frac{\partial\pi(q,L,T)}{\partial T}$ then one can also derive from (9iii) a specification for the equilibrium utility maximizing level of work, L.

Lopez [1981b] has shown that the farm-household consumption demand functions, net output supply functions and the equilibrium level of hours of work are homogeneous of degree zero in consumption good prices, net output prices and non-labour income. In particular, unlike the net output supply functions of the conventional firm, the net output supply functions of the farm-household are not homogeneous of degree zero in net output prices. Also, unlike the conventional model, the farm-household consumption decisions (i.e., demand for consumer goods), production decisions and the equilibrim level of work are all dependent on parameters of both the consumption and production sides of the model.

As indicated earlier, in contrast with the conventional model of the firm, net output supply functions are not necessarily upward sloping. However, I have derived a compensated net output supply expression which can be shown to be non-negative:

$$(10) \quad \frac{\partial Q_i}{\partial q_i} - \frac{\partial Q_i}{\partial L}\frac{\partial L}{\partial y}Q_i = -e_{q_iq_i} \geq 0 \qquad i=1, \text{---} S$$

where e_{q_i} is the second partial derivative with respect to q_i of an expenditure function $e(p,q,T;\mu)$ defined by

$$(11) \quad e(p,q,T;\mu) \equiv \underset{H\text{-}L,X}{\text{Min}} \{px - \pi(q;T,L): U(H\text{-}L,x) \geq \mu\}$$

Thus, although the sign of the directly observed Marshallian output supply effect $(\frac{\partial Q_i}{\partial q_i})$ cannot be predicted, the compensated or "Hicksian" output supply effect (i.e., the left-hand-side of (10)) is non-negative. Therefore, if one can estimate $\frac{\partial Q_i}{\partial q_i}$, $\frac{\partial Q_i}{\partial L}$ and $\frac{\partial L}{\partial y}$ then it is possible to empirically verify inequality (10). This is an additional testable prediction obtained from the farm-household model.

Another prediction from the model is that, although the effect of a change in net output price q_i on the equilibrium level of work L is in general unknown, the utility constant effect is unambiguously non-negative.

$$(12) \quad \frac{\partial Q_i}{\partial L}\left\{\frac{\partial L}{\partial q_i} - \frac{\partial L}{\partial y}Q_i\right\} = -(e_{q_iq_i} + \pi_{q_{ii}}) \geq 0 \qquad i=1,\ldots,S$$

Thus, if the equation system (9) is estimated then the left-hand-side of (12) can be calculated and the non-negative restriction implied by (12) can be tested. Notice that the sign of $\frac{\partial L}{\partial q_i}$ is in general ambiguous and that if the weak assumption that $\frac{\partial Q_i}{\partial L} > 0$ is made then a testable prediction of the model is that

$$\frac{\partial L}{\partial q_i} - \frac{\partial L}{\partial y}Q_i \geq 0, \text{ for } i=1, \ldots S.$$

Symmetry of the function e allows us to show that reciprocity conditions between production and consumption decisions also hold. Lopez [1981b] showed the following testable symmetry relationship which is obviously absent in the conventional models of the firm and the household:

$$(13)\quad \frac{\partial X_i^*}{\partial q_j} = -\frac{\partial Q_j^*(q;T,L^*)}{\partial p_i},$$

for all $i=1,\ldots,N$
$j=1,\ldots,S.$

where the compensated demand effect $\frac{\partial x_i^*}{\partial q_j} = \frac{\partial x_i}{\partial q_j} - \frac{\partial x_i}{\partial y} Q_j$ and the compensated output effect of a change in consumption goods price $\frac{\partial Q_j^*}{\partial p_i} = \frac{\partial Q_j}{\partial L}\{\frac{\partial L}{\partial p_i} + \frac{\partial L}{\partial y} X_i\}$. Thus, symmetry relations (13) can be represented in terms of expressions which can be empirically estimated and thus (13) is a testable prediction of the farm-household model.

Results similar to (10), (12) and (13) can be derived for the case in which farmers work off-farm and when the farm-household produces goods which are entirely consumed within the farm-household [Lopez 1981b]. If household members work off-farm then important questions are whether they regard on-farm and off-farm work as perfect substitutes in consumption and if there exist binding restrictions on the number of hours which they can work off-farm. If they regard on-farm and off-farm work as identical "commodities" and if they face no binding restrictions on hours of off-farm work then the shadow price of labour becomes exogeneous. If, in addition, all outputs produced by the farm-household are at least partially traded, then one can dichotomize production and consumption decisions. In this case the conventional models of the firm and household apply and the predictions discussed before no longer apply. However, if any of the above conditions are not met, then utility maximizing and profit maximization decisions are interdependent and our previous analysis holds.

A Suggested Econometric Specification for the Farm-Household Model

Using equations (9ii) and (9i) one can obtain the household demand for consumption goods, X, and the net output supply response specifications by postulating appropriate functional forms for $G(\cdot)$ and $\pi(\cdot)$. With respect to L it is necessary to indicate that we cn only obtain an implicit representation of it using equation (9iii).

A stochastic structure of the household equations (9i), (9ii) and (9iii) can be specified by assuming additive disturbances with zero means and a positive semi-definite variance-covariance matrix:

$$\text{(a)}\quad X_j = -\frac{\partial G/\partial p_j}{\partial G/\partial y} + e_{1j},$$

$$j=1,\ldots,N$$

$$(13)\quad \text{(b)}\quad Q_i = \frac{\partial G/\partial q_i}{\partial G/\partial y} + e_{2i},$$

$$i=1,\ldots,S$$

$$\text{(c)}\quad \frac{\partial \pi(q,T,L)}{\partial T} = \frac{\partial G/\partial T}{\partial G/\partial y} + e_3$$

where e_{1j}, e_{2i} and e_3 are the disturbance terms.

It is evident that equation (13c) cannot be estimated unless the shadow price of the fixed factor of production T is observed. Unfortunately, the shadow price of T is rarely observed if a rental market for factor T does not exist. Although the shadow price of factor T, $\frac{\partial \pi}{\partial T}$, cannot be observed, the variable π ("profit") can at least be calculated; it is simply the net farm returns after payments for all variable inputs (except L) are deducted from the gross sales. Hence, given that q, T and L are also observed one could in principle estimate the vector of parameters, α, which characterizes the conditional variable profit function by estimating

$$(14)\quad \pi = \pi\,(q,T,L;\alpha) + \mu$$

where μ is a disturbance term.

The problem with estimating (14) is that the variable L may be correlated with the disturbance term and hence the estimates of α would not be consistent. However, if $\pi(\cdot)$ is linear in the parameters (as is usually the case when flexible functional forms are used) then one can use an instrumental variable technique and thus obtain consistent estimates of α. Therefore, if an appropriate instrumental variable for L exists, then one can estimate equation (14) and obtain consistent estimates ($\hat{\alpha}$) for the parameters of the conditional variable profit function. Using the estimated vector $\hat{\alpha}$ one can evaluate the function $\frac{\partial\pi(q;T,L;\hat{\alpha})}{\partial T}$ which is the "true" shadow price of land measured with an error. Thus the true shadow price of T is equal to the estimated shadow price plus an error term assumed to be stochastic. That is:

$$(15) \quad \frac{\partial\pi(q,T,L;\alpha)}{\partial T} = \frac{\partial\pi(q,T,L;\hat{\alpha})}{\partial T} + \mu_T.$$

Notice that equation (15) is not estimated; the $\hat{\alpha}$ parameters are obtained by estimating equation (14) and substituted in $\frac{\partial\pi}{\partial T}$. In other words, by estimating $\hat{\alpha}$ in (14) one obtains a measure of the "true" shadow price of capital subject to an error, μ_T. If (15) is used in (13.c) then one may interpret equation (13.c) as an error of measurement of the dependent variable situation which offers no estimation problems:

$$(16) \quad \frac{\partial\pi(q,T,L;\hat{\alpha})}{\partial T} = \frac{\partial G/\partial T}{\partial G/\partial y} + \tilde{e}_3$$

where $\tilde{e}_3 \equiv e_3 - \mu_T$

If e_3 and μ_T are normally distributed and independent of p,q,T and y in (13 c) and (15) then $\tilde{e}_3$ will possess the same properties. Thus, there is no problem with using the predicted rather than the actual shadow price of T in estimating (13 c).

Although an explicit labour equation cannot be estimated, if the parameters of (14) and (16) are estimated then one obtains an implicit representation of L on the left-hand-side of (16) and hence all the relevant economic information regarding the factors determining the equilibrium level of family labour can be derived.

In summary, it appears that estimation of a complete system of production and consumption equations for the farm-household is feasible and desirable. This system should be jointly estimated given the interdependences of the production and consumption sectors emphasized by the fact that all behavioural equations are derived from the same indirect utility function. Finally, the testable implications of the farm-household model derived above can be imposed or tested in the estimating model.

Looking Forward

There are at least three additional potentially fruitful areas of research where duality may prove to be a useful approach. The use of duality in the context of dynamic models and on modelling supply responses under risk which, I understand, is covered in the other paper presented in this session, are certainly important areas of further applications of duality in agriculture.

The use of duality has also helped to simplify the analysis of competitive market equilibrium analysis and allows one to use less restrictive a priori assumptions on the derivation and characterization of competitive market equilibrium. An example of this approach is the analysis of the land market and agricultural supply and demand responses to exogeneous changes in factor or output prices in the context of a small open economy [Lopez 1982a]. I think that further work in this area appears quite promising.

A third direction of research using duality may be in the context of the analysis of non-competitive behaviour mainly at the food processing, distribution and retailing (PDR) sector. This sector is, in general, highly concentrated in North America and one could expect that the use of conventional models based on price taking behaviour might not be very appropriate. In Agriculture Canada we are at an exploratory phase of the development of a model which will allow us to estimate food demand, production and food retail price equations and to simultaneously measure market power and mark-up ratios. The model yields as special cases the perfectly competitive and perfect collusion (or monopoly) situations. We have found that the use of duality theory has been useful in the derivation of the empirical model and comparative static analysis.

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