Relevance of Duality Theory to the Practicing Agricultural Economist: Discussion

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Ramon Lopez and Rulon Pope have succeeded in providing rigorous and relatively comprehensive summaries of duality applications and theory, respectively, in admirably concise presentations. The timeliness of this topic among agricultural economists is indicated by the publication of at least five studies using duality theory in the first three issues of the *AJAE* this year [Babin, Willis, and Clyde; Chambers; Heien; Ray; Lopez, 1982b].

Although both duality theory and agricultural applications of the theory have been with us several years, the recent surge of interest among agricultural economists in duality has been so enthusiastic that some cautionary notes are in order. Consequently, I will attempt in this discussion to supplement, and perhaps further clarify, some of the pros and cons of duality approaches discussed by the two major contributors to this session. This discussion draws upon my review of Lopez and Pope's work, of other recent duality applications to agriculture, and some recent personal experience with empirical duality analysis at the aggregate level [Rostamizadeh et al.]. Most of my more specific remarks will deal with analysis of agricultural production systems within a dual cost function approach.

As a preview of my discussion, I find neither an absolute positive nor negative answer appropriate to Pope's apt interrogative title. As with many issues, the answer is when and how. Hopefully, the discussion below will help the potential dualist select problems and techniques that minimize the risk and maximize the potential gain from his or her dualing encounters.

Terminology

The first obstacle encountered by the traditionally trained economist in considering dual approaches to familiar problems is the new verbal and mathematical vocabulary. Those familiar with production, profit, and cost functions depicted in input or output space have to shift gears mentally to conceptualize these functions in price spaces.

In addition to argument vectors, new students of duality must pay close attention to optimization constraints in dual concepts. Factor demand functions emerging from dual approaches can be either output constrained, cost constrained, or ordinary (unconstrained profit max) responses to factor price variations. Of course, constant-output, constant-cost, or ordinary factor demand curves can also be obtained by appropriate constrained or unconstrained optimization of primal cost, production, or profit functions defined in input space [Ferguson, Ch. 6]. The restrictive nature of constant-output or constant-cost factor demands derived from familiar primal functions is generally readily apparent due both to their self descriptive names and most economists' familiarity with their math-

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ematical and graphic derivation. However, it may slip by a casual reader that *partial* differentiation of the cost function (which includes output as an argument) necessarily yields constant-output input demands. These input demand properties are sometimes indicated by borrowing from consumer theory the terms Marshallian and Hicksian to represent ordinary and constant-output factor demands, respectively. Others use the term "conditional" to describe constant-output demands derived from the cost function. This term, unless specifically defined, begs the question — Conditional with respect to what?

Similar, potentially confusing, variation in terminology seems to characterize some of the major theoretical propositions in duality theory. For example, most authors [e.g. Lopez, Varian, Silberberg] refer to the derivation of constant-output input demand functions via partial differentiation with respect to input prices of a well behaved cost function as an application of Shephards's Lemma. However, Pope refers to this as an application of Hotelling's Theorem I. Most authors refer to Pope's Hotelling's Theorem II, as Hotelling's Lemma.

Advantages of Dual Approaches

Both Lopez and Pope effectively summarize the major theoretical and practical attractions of dual approaches for applied researchers. As noted in both papers, the principal theoretical advantage is that use of flexible functional forms for dual functions permits imposing many fewer restrictive assumptions about the nature of technology than does popular and mathematically tractable production function forms like the Cobb-Douglas and CES. The dual approaches "let the data speak" with regard to input substitution possibilities, homotheticity, constancy of input elasticities, and other properties. Recent empirical work by Lopez [1980] is distinguished by the rigor with which properties associated with common production function forms were tested statistically within a generalized Leontief cost function framework. His results strongly indicated that the Canadian agricultural sector could *not* be characterized by Cobb-Douglas, CES, or Leontief production technologies. These three functional forms have dominated past aggregate production function analyses.

As noted by Pope, estimation of cost or profit functions with price data may permit more precise econometric estimates of technology parameters because there often will be less multicollinearity among factor prices than among factor amounts.

Both authors praise dual approaches because of the computational convenience they offer in deriving input demand and output supply functions, demand and supply elasticities, and partial elasticities of substitution. Furthermore, because input prices are more likely to be truly exogenous to firms than are input quantities, simpler regression techniques are permitted.

Some might argue that computational convenience should receive little weight in research planning decisions in an era of ready access to relatively inexpensive high speed electronic computers. However, simplicity in calculations can increase ultimate accuracy for at least two reasons. First, the opportunity for human error, no small concern as anyone who has supervised empirical research knows, is reduced. Secondly, the risk of exacerbating estimation error by rounding errors is reduced. For example, rounding errors can be significant in inverting large matrices as is required for computing Allen partial elasticities of substitution from the production function.

It is also much easier to derive the statistical properties of elasticities or other response measures that can be calculated as simple, frequently linear, functions of the estimated parameters of dual functions.

An additional practical advantage of the duality approach of considerable importance to the applied researcher, but mentioned in neither of the papers, is that data on factor and output prices, total costs, and annual profits will often be more readily available, and possibly more accurate, than data on output and input quantities.

Some Theoretical Limitations

Researchers like Binswanger, Lopez and Tung, Kako, and others who have used an aggregate cost function approach to analyze agricultural production generally calculate and report the own and cross price elasticities of demand for labor, land, and selected categories of capital inputs utilized in their analyses. Often considerable attention is devoted to the policy implications of these demand elasticity estimates.

Although the constant-output nature of these elasticities is sometimes acknowledged in the theory description, there has generally been relatively little emphasis in the interpretations/policy implications of these studies of the distinctly short-run nature of these elasticities. Over the longer run where producers have time to adjust output level as well as input combinations in response to input price changes, elasticities will often be considerably higher in absolute value. Hammonds, Yadav, and Vathana, for example, summarized results from several past demand elasticity studies of the U.S. hired farm labor market. Estimated long run elasticities were often three or four times higher than the short run elasticities estimated in the same studies. The potential danger of basing long run minimum wage or labor relations policies, for example, on short-run constant-output labor demands is obvious when long run demands are much more elastic.

Recently, Chambers described and applied a procedure for computing both constant-cost and constant-output input demand elasticities from a cost function by transforming the cost function to the indirect (cost-constrained) production function. For the U.S. meat products industry, Chambers' results showed capital, labor, and energy demand elasticities to be slightly higher, and materials demand elasticity to be several times higher, for the constant-cost input demand compared to the constant-output input demand. Constant-cost input demands represent an intermediate length of run, or adjustment potential, between constant-output and ordinary input demands.

Of course, as noted by Lopez and by Pope, longer run ordinary (Marshallian) demands can be estimated from the dual profit functions by use of Hotelling's Lemma. The key is to select the dual function that suits the particular research objectives and behavioral/institutional realities of the problem. In certain problems, behavioral objectives such as risk aversion or institutional (e.g. farm program) restrictions may make cost minimization subject to an output constraint more realistic than unconstrained profit maximization.

As a final tangential theoretical observation, the newly fashionable duality approaches to aggregate agricultural analysis do little fundamentally to resolve the venerable debate as to whether aggregate production functions (or equivalently cost functions) are really useful constructs for aggregate economic analysis to start with [Robinson; Harcourt; Pasinetti]. This debate revolves around the question of whether a meaningful measure of capital distinct from relative prices is possible in the aggregate, issues of capital switching and reswitching, and the nature of technical progress.

Empirical Problems

As noted by both principal contributors to this session, duality theory requires that cost, profit, and production functions possess certain characteristics. For example, cost functions must be increasing, linearly homogeneous, and concave in input prices. Also the estimated parameters of the cost function must satisfy required symmetry conditions. These theoretical requirements for the validity of the theory present two problems for the applied researcher. First, the imposition of parameter restrictions across input share equations, for example, induces contemporaneous correlation among error terms so GLS or maximum liklihood estimation procedures are required. Some researchers may not have ready access to or capability with such procedures. Secondly, statistical tests of the parameter restrictions may reveal that they are clearly not consistent with the data.

TABLE 1. Selected Properties of Some Aggregate Cost Function Studies of Agricultural Production.

Study	Data	Function	Technical Change	Homotheticity of P. F.	Estimation Technique	All Demands Negative?	Locally Concave	Test of Symmetry	Test of Homogeneity
Binswanger, 1974	U.S. Ag: cross-section + time series	Translog	Nonneutral	Homothetic	RGLS	Yes	No test	Accepted	None
Kako, 1978	Japan rice farms, cross-section + time series, 1953-70	Translog	Nonneutral	Homothetic	FIQML	Yes	Accepted	None	None
Lopez, 1980	Canada Ag: time series 1946-77	General Leontief	Nonneutral (Not Sig.)	Nonhomothetic	FIML	Yes	Accepted	Accepted	NA
Chambers, 1982	U.S. Meat processing time series, 1954-76	General Leontief	Neutral	Nonhomothetic	FIML	Yes	Implicitly Rejected	None	NA
Babin, Willis and Allen, 1982	U.S. pooled industries cross section	Translog	NA	Homothetic	Zellner	No	No test	None	None
Lopez and Tung, 1982	Canada Ag: cross section + time series, 1961-79	General Leontief	Nonneutral (sig.)	Nonhomothetic	FIML	Yes	Accepted	Accepted	NA
Rostamizadeh et al., 1982, I	U.S. Ag.: time series, 1960-79	Translog	Neutral	Homothetic	JRGLS	Yes	Rejected	Rejected	
Rostamizadeh et al., 1982, II	U.S. Ag.: time series, 1960-77	Translog	Nonneutral	Homothetic	JRGLS	No	Rejected	Rejected	
Rostamizadeh et al., 1982, III	U.S. Ag.: time series 1960-77	Translog	Nonneutral	Nonhomothetic	JRGLS	No	Rejected	Rejected	
Rostamizadeh et al., 1982, IVa	U.S. Ag.: time series, 1950-79 with struc. break at 1973	Translog	Neutral	Nonhomothetic	JRGLS	No	Rejected	Rejected	
Rostamizadeh et al., 1982, IVb	U.S. Ag.: time series, 1960-79 with struc. break at 1973	Translog	Neutral	Nonhomothetic	JRGLS	No	Rejected	Rejected	

Table 1 summarizes, among other information, the results of tests of certain necessary theoretical restrictions reported in several recent studies that have utilized a cost function approach to examining agricultural production. Although necessary symmetry and homogeneity parameter restrictions were generally imposed, the compatibility of these restrictions with the data as consistently tested statistically only in the recent work by Lopez and by Rostamizadeh et al. Some researchers have also failed to test the local concavity of the cost function. This test involves checking whether the Hessian matrix of the cost function is negative semidefinite at each observation level.

Pope observes that "testing curvature (concavity) conditions is a cumbersome matter and is usually dispensed with." Our recent experience confirms that these tests are tedious, but given that these conditions are intrinsic to the theory and to the validity of the results, it is important that they be made.

As noted in Table 1, our experience with the Rostamizadeh study strongly indicates that concavity, symmetry, and homogeneity conditions certainly will not always hold, at least not with the translog specification applied to recent U.S. agriculture data. In an attempt to improve the specification, we tried alternative assumptions regarding technical change, homotheticity, and structural breaks, but with little success. Lopez's success in accepting the concavity, symmetry, and homogeneity tests using the generalized Leontief function on Canadian data provides encouraging support for the flexibility of the generalized Leontief specification.

Estimates of elasticities, technical change coefficients, and other measures of policy interest from cost function parameters seem to be quite sensitive to data composition and variable construction procedures. Lopez noted how the switch from time series to pooled cross section and time series data lead to a reversal of the conclusion that nonneutral technical progress had not been a significant factor in Canadian agriculture. The pooled cross sectional time series data for 39 states or groups of states over the 1949, 1954, 1959, and 1964 census years used by Binswanger lead to somewhat higher demand elasticities for most inputs than those observed in other studies (see Lopez's Table 1). The demand elasticities for land in our recent study were considerably lower than other estimates in the literature (Rostamizadeh et al.). This result, and possibly also our specification problems, may have been partially due to data inadequacies.

Of course, the sensitivity of results to practical data composition and variable construction problems applies equally to approaches using primal functions. The preceding discussion serves only to warn potential dualists that their "ammunition" may critically affect their success.

Conclusion: Choosing Weapons

It was noted above that dual cost and profit functions, respectively, provide an appropriate framework for derived demand analysis where short and long run adjustments, respectively, are of interest to the problem at hand. That example illustrates the importance of applying dual techniques only when they provide a good "fit" to specific research problems.

I believe Lopez's proposed framework for analyzing integrated farm-household decisions is a good example of such a good "fit." Furthermore, I find the dual profit function approach used by Lau and Yotopolous a more comprehensive device for comparing overall economic efficiency across firms than alternatives like the Farrell-Fieldhouse method (see Yotopolous and Nugent, Chs. 5 and 6, for a discussion of alternative efficiency evaluation techniques). Furthermore, the practical and theoretical arguments provided by Pope in support of duality approaches for analysing welfare impacts of changes in the economic environment are entirely convincing.

The broad spectrum of problems that can be attacked with dual approaches, plus the theoretical and practical advantages of these approaches for certain problems, indicate to me that duality will become an increasingly used component of the practicing agricultural economist's arsenal in years to come.

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