# A Representative Market Model of Farmland Bid Prices

**Glenn D. Pederson**

A land bid-price model is formulated which integrates asset pricing models from prior studies to illustrate the singular and joint effects of ordinary and capital gains taxes, growth of returns, diseconomies of size, and risk behavior on farmland prices. An application of the model to primary data from cash grain farms illustrates that the *ceteris paribus* effect of increased marginal tax rates on a perpetual, growing income stream is to increase its present value. Larger farms in higher marginal tax brackets are shown to have a competitive advantage over smaller, lower tax bracket farms.

Analytical efforts to explain land prices have employed capital theory through the use of asset pricing models which capitalize the anticipated flow of returns from ownership and use of farmland. While the concept of discounted present value is generally accepted by agricultural economists, specification of asset pricing models has been an area of some recent controversy [Baker, Scott, Adams, Harris and Nehring]. These and other studies have incorporated taxes (ordinary and capital gains), growth, and risk into income capitalization models in an attempt to more adequately relate investor decisions to market price changes. Independently, these models have generated conflicting results concerning the effect of taxes on the ability of farmers to effectively bid for land. This paper illustrates the theoretical models developed by Harris and Nehring and later by Baker can be integrated into a single model. The integrated land bid-price model incorporates taxes, growth and risk, and extends the analytical capacity of the prior models. The model is applied to North Dakota cash grain farms of various sizes to illustrate the singular and joint effects of these variables.

Glenn D. Pederson is an assistant professor of Agricultural Economics at North Dakota State University, Fargo.

## Variations in Model Specification

Variations between suggested capitalization models can be viewed as two related issues — selection of variables and level of analysis. Individual-investor present value models relate the price of land to a particular investor's returns, tax bracket, cost of capital, and holding period and may or may not incorporate risk. Bid-price models provide a conceptually broader framework than present value models for investment decision making. In addition to the factors identified above, the bid-price varies with the perceived level of risk in the anticipated stream of returns, the decision maker's risk preferences, the degree of risk currently in the investor's portfolio, and the covariance between returns to currently-owned assets and returns to the asset being acquired (Adams). A necessary condition for the bid-price to be equivalent to present value is that the investor be either risk neutral or that the asset to be acquired have no discernible effect on the riskiness of his portfolio. Adams suggested that investors commonly lack at least some of the required information; they, therefore, concentrate upon present values.

Market models, in contrast with individual-investor models, are present value models due to obvious problems of aggregating individual investor risk preferences and portfolio characteristics. A representative market model, in which representative land market participant characteristics are specified, can conceptually generate useful comparative bid prices. A representative market model provides a framework for analyzing the competitive positions of a number of investors, each exhibiting representative financial and/or behavioral characteristics, under various assumptions about policy variables and market conditions.

Harris and Nehring attempted to show with the aid of a bid-price model that farmers operating larger farming units did not possess the greatest bidding advantage. It was suggested that the combination of higher tax rates and diseconomies of farm size reduce the competitive position of the largest farms under assumptions of either risk-averse or risk neutral behavior.

Adams later showed that the Harris-Nehring model incorrectly present valued the after-tax income stream using a before-tax discount rate. Adams also concluded that in the absence of capital gains taxes, the impact of ordinary taxes on a perpetual income stream, such as from indefinitely held farmland, can be safely ignored. Income streams which are finite, however, are effectively reduced at higher tax rates. Progressivity of the income tax adversely affects the bidding potential of a few high tax bracket investors, but need not similarly influence investors at other levels of taxable income.

Baker's market model incorporated ordinary income taxes, capital gains taxes and exclusion of a percentage of long term capital gains to analyze their joint impact on land prices. Tax effects on ordinary returns and capital gains were evaluated by allowing the terminal value of land to change as tax variables were adjusted. The model assumed a growing stream of returns and an infinite sequence of landholders, each with a finite holding period. A present value could, therefore, be computed without necessarily specifying the value of land at the end of each successive holding period. An application of the model to the Indiana farmland market indicated that farmers in higher tax brackets could pay more for land than the implied average market price. Moreover, as the marginal Federal income tax rate increased it was shown that, the price an individual could pay increased, regardless of initial tax bracket.

A set of major conclusions drawn from the analyses of the above models can be summarized. A *ceteris paribus* increase in the tax rate will; 1) decrease the value of a constant, finite income stream, 2) not affect a constant, perpetual income stream, but 3) increase the present value of an increasing perpetual income stream. These conclusions follow from the development of each model under various parameter specifications and assumptions. Conclusions 1 and 2 can be generated by an asset pricing model which includes ordinary income tax rates, sets the growth rate on returns equal to zero, assumes a discount rate equal to the after-tax rate of return for the risk class of the asset, and varies the holding period from a finite number of years to an infinite number of years. Conclusion 3 relates to a model which makes the same discount rate assumption and incorporates a positive rate of growth in returns, ordinary income and long term capital gains taxes, and allows for a perpetual holding period. One of the problems with the perpetual holding period assumption, as it has been applied to individual-investor models (see Harris and Nehring), is that it does not fit well with what is observed in the land market (Scott). An assumption of an infinite sequence of buyers, each with a finite holding period, is a better approximation to what is observed.

## Integrating the Models

In this section it is shown that a representative market model can be formulated from the models developed by Harris and Nehring and by Baker. It begins by defining the price of land at the individual investor level. The current value of farmland to an individual is defined as the present value sum of the expected after-tax return, $R_0$, which is assumed to grow at an annual rate, g, over an m-year holding period, plus the after-tax liquidation price:

$$(1) \quad V_o = \sum_{n=1}^{m} R_o \frac{(1+g)^n}{(1+r)^n} + \frac{V_1 - c\,(V_1 - V_o)}{(1+r)^m}$$

where, $V_o$ is the present value sum, $V_1$ is the value at the end of the holding period, r is the after-tax nominal discount rate, and c is the investor's capital gains tax rate which equals 40 percent of the ordinary income marginal tax rate. The value of farmland at the end of the first holding period (m-years, hence) can be similarly be expressed as:

$$(2) \quad V_1 = \sum_{n=1}^{m} R_o \frac{(1+g)^{n+m}}{(1+r)^n} + \frac{V_2 - c\,(V_2 - V_1)}{(1+r)^m}$$

where, $V_2$ is the value at the end of the second buyer's holding period, and all other terms are as defined above.

If one extends the value concepts of equations (1) and (2) over an infinite sequence of buyers, each having the same before-tax discount rate, m-year holding period, and expecations of returns and growth of returns, the problem of specifying the value of land at the end of each successive holding period is avoided [Baker]. Although the infinite sequence of landholders implies a horizon which is equivalent to that for a single investor and an infinite holding period, the resulting present value expression is expected to generate different values as the assumed finite holding period for an individual-investor varied. Equation 1 can be rewritten to express the present value sum as a function of the growth-adjusted, after-tax discount rate, r*.

$$(3) \quad V_o = \sum_{n=1}^{m} \frac{R_o}{(1+r^*)^n} + \frac{V_1 - c\,(V_1 - V_o)}{(1+r)^m}$$

where, $(1+r^*) = (1+r) \,/\, (1+g)$, and r is an after-tax interest rate.

Under the above assumptions, Baker has shown that the implied sequence of landholders can be used to define a present value for land. The derivation of a present value expression involves several intermediate steps beyond equation (3) and is shown in the Appendix. Annual returns grow geometrically with the holding period as the number of landholders is increased. Since the land price for the first owner is a function of the discounted annual returns and the change in price over the holding period, Baker derives a parallel expression for the i-th owner. The present value of the future stream of returns to the first owner is shown to be recursively related to the annual returns stream for the i-th owner. The value of land at the end of each successive holding period is deferred to the end of the subsequent holding period. In this way the present value of the price of land when sold at the end of this long sequence eventually becomes negligible (a common assumption of the capitalization of income method). The present value of land is then a function of the expected annual return and several parameters (tax rates, the annual growth rate, a pretax nominal interest rate, and the holding period).

A simplified expression for the present value of land for the first owner is,

$$(4) \quad V_o = \frac{R_0}{r^*} \left[ 1 + \frac{1-(1-r^*)^{-m}}{c(1+r^*)^{-m} - (1+r)^{-m}} \right]$$

Equation (4) incorporates the ordinary income tax and annual growth of returns implicitly, and the long term capital gains tax rate explicitly. The effect of including the capital gains tax is to reduce the present value of land. When the capital gains tax rate

(c) is excluded, the bracketed expression equals unity. However, when the capital gains tax rate is included the condition that $r \geqq r^*$ for zero or positive rates of growth would reduce the bracketed expression to less than unity.

Several alternative values could be considered for the rate of growth of annual returns (g) and the nominal after-tax discount rate (r) in equation (4). Certain ranges are relevant. The tax-adjusted discount rate is positive ($0<r^*<r$) when the growth rate is between zero and the value of r (which is the expected condition). Yet, $r^*$ could be positive even though g is slightly greater than r.[1] Growth of annual returns could even be slightly negative and yield a value for $r^*$ which is less than r, but still positive. Two special cases could also be considered. When $g=0$, $r^*=r$, and equation 4 reduces to $V_o = R_o/r$, which is the familiar capitalization formula for a constant after-tax stream of returns discounted at an after-tax nominal discount rate. The second special case assumes that $g=r$, which implies that $r^*=0$. This latter case is not likely. While the rate of growth of annual after-tax returns may exceed the nominal after-tax discount rate temporarily, the expected relationship (which is relevant here) is that the discount rate would exceed the growth rate over a prolonged period of time.

The relationship between the ordinary tax rate and the price of land is positive within the relevant ranges of the following parameters: length of holding period, annual growth rate, and nominal pretax discount rate [Baker]. Higher tax brackets are directly related to higher present values of farmland. Higher tax rates reduce the after-tax discount rate to more than offset the implied reductions in after-tax ordinary and capital gains income, and increase the present value of the capital gains exclusion.

The theoretical basis for defining a bid-price for a risky asset has been suggested by Pratt. The maximum bid-price, b, is part of a more general utility function, U(X),

$$(5) \qquad U(X) = E \quad U\,(X + \sigma - b)$$

where, X is the level of assets held by the investor, $\sigma$ is the value of the risky asset being considered, and E is the expected value operator.

Harris and Nehring used this utility framework to show that the bid-price for land can be approximated as a function of the expected value of future income from an acre of land, E(y), the variance of income, $\sigma^2(y)$, and the measure of local risk aversion, r(x). In this case y is the annual income stream and the measure of local risk aversion is a concave function of the level of net worth. The present value of an acre of land was theoretically defined using a dividend growth model which assumes a single infinite holding period and excludes capital gains considerations. Substituting the theoretically equivalent present value expressions of expected income and the variance of income for the present value of land, the bid-price expression Harris and Nehring (p. 162) derived was,

$$(6) \qquad b = d\,E(y) - \frac{1}{r(x)} \pm \left[\frac{1}{[r(x)]^2} - d^2\,\sigma^2(y)\right]^{1/2}$$

where, $d=(1-r)\,/\,(i-g)$, t is the marginal income tax rate, i is a discount rate for pure time preference, and g is the expected rate of growth of after-tax income.

Utilizing the framework provided by equation (6), a bid-price for land can be defined which incorporates capital gains taxes and a finite holding period for the investor. Using the present value expression for an acre of land from equation (4), the expected present value price of land can be defined as,

$$(7) \qquad b = E(V_o) = \frac{E(R_o)\,k}{r^*}$$

[1]Baker shows that g could exceed r by up to one-half percent and still have a limiting present value of zero.

where the parameter, k, is equal to the bracketed right side expression in equation (4). Similarly, the variability of the present value price of land can be written as,

$$(8) \qquad \sigma^2(V_o) = \frac{k^2 \, \sigma^2(R)}{r^{*2}}$$

Substituting equations (7) and (8) into equation (6) in place of the terms derived using the dividend growth model yields the following bid-price expression,

$$(9) \qquad b = \frac{E(R_o)\,k}{r^*} - \frac{1}{r(x)} \pm \left[\frac{1}{[r(x)]^2} - \frac{\sigma^2(R)\,k^2}{r^{*2}}\right]^{1/2}$$

Equation (9) defines the maximum bid-price for a risk nonneutral investor. The equivalent expression for the risk neutral investor is the same as shown in equation (7).

An evaluation of the qualitative relationships between the bid-price and the model variables reveals that most aspects of the model are unchanged from those shown by Harris and Nehring (pp. 162-3). The effect of the tax rate variable is, however, no longer ambiguous and must be expanded to include capital gains tax considerations. Tax rate increases influence the bid-price in three ways. The price increases due to a reduction of the after-tax nominal market rate of interest. Secondly, the exclusion of a percentage of long term capital gains has a positive effect on land bid prices, as has been discussed above. Additionally, the bid-price increases as the variability of after-tax income is reduced.

Equations (7) and (9) integrate existing models into a representative market model of land prices. The resulting model extends the applicability of prior models to the issue of ability of farmers in different size classes to competitively bid for farmland. Tax effects, ordinary and capital gains, are incorporated. The model is applicable within a risk framework. Effects of economies and diseconomies of size can be explicitly considered within a risk framework.

## Methodology

Data from cash grain farms in north central and northwest North Dakota were selected to make an application of the model. Financial records for 76 farms of various sizes were retrieved from the Production Credit Association's AGRIFAX record system. Cash grain operations were defined as those receiving more than 75 percent of annual gross receipts from sale of crops. Farms were categorized into four size groups according to average tillable acres operated. Tillable acres was used as the criterion for classifying farms in recognition of the theoretical relationship between number of cultivated acres and unit costs in production of grain. Acreage bounds were defined arbitrarily based upon logical breaks between farms in the reported acres operated. Characteristics of those cash grain operations, by size, are shown in Table 1.

Farm physical and financial data for 1978 and 1979 were used to compute two-year individual and group averages of farm size, net worth, and average before-tax net returns per acre.[2] Federal marginal tax rates shown in Table 1 reflect estimates derived from farm income records. Annual taxable income for each farm operation was determined by subtracting total farm operating expenses (including depreciation taken) from total cash farm income and adding in reported nonfarm income. The standard deduction for a family of four was then subtracted from net cash income to provide an estimate of taxable

[2]Pooling two years of information on each farm provided more stable estimates of net income and also reduced the effect of errors in measurement or valuation of grains held in inventory for later sale on net income estimates. Resulting estimates reflect to a greater extent the typical recent income position of cash grain farms in that area of the state. Cost estimates used in computing before-tax net returns exclulde land costs.

income, and the marginal tax bracket.[3] Marginal tax rates reported in Table 1 are size class averages.

Parameters of the model were estimatd by several means. The annual rate of growth of returns per acre was estimated by computing the rate of increase in annual gross cash rent for the state as reported by the USDA. The rate of growth for the 1970-1980 period was estimated to be 9 percent. A nominal pretax discount rate equal to 18.2 percent was determined by solving Equation (4) for the pretax interest rate which would just yield an estimated average market value of $346 per acre, holding the other variables constant at the mean values for all farms.[4] Holding periods between 10 and 25 years were used to generate bid prices to reflect average long term holding periods and illustrate sensitivity of the model to this parameter.

## Effects of Risk Preferences

A bid price model such as this is highly sensitive to the risk aversion parameters which are specified. Due to the lack of empirical evidence concerning the relevant ranges for measures of local risk preferences, a range of arbitrary, yet reasonable parameters were utilized to generate land prices under conditions of risk.[5]. In their study of program participation by a representative

[3]A market value of $346 per acre is the price implied by a 6.35 percent rate of return from average cash rent figures reported for that area of the state for 1978 and 1979 (USDA). Average cash rent was equal to $21.95 per acre on land rented for wheat and barley during that two-year period (Johnson). A critical assumption incorporated into the model is that the pretax interest rate is the same for the first buyer, across all sizes of farm and for all successive investors.

[4]The method used to compute average annual taxable income uses a net income concept which is equivalent to the Census of Agriculture definition used by Harris and Nehring (p. 164). Investment credit was not deducted in the taxable income computations.

[5]The composite utility function approach used by Harris and Nehring (p. 166) could not be used to determine a single, risk aversion parameter for each farm class for two reasons. First, that function was computed using net worth as the independent variable. Net worth figures for farms reported in Table 2 generally lie outside the range of observations reported in the 1974 study by Lin, Dean, and Moore, which would make the estimated function unreliable. Second, estimated utility functions generally tend to be highly unreliable (King and Robison). Since a range of risk aversion parameters is preferred to capture the effect of risk preferences on bid prices, the measure of absolute risk aversion will be ranged over several parameter values which generally correspond with values selected by Musser and Stamoulis.

**TABLE 1. Selected Characteristics of Northwest Central North Dakota Cash Grain Farms, 1978-79.**

| Item | Size Class of Farm | | | |
|---|---|---|---|---|
| | Large | Medium | Small/Medium | Small |
| Number of Farms | 17 | 14 | 26 | 16 |
| Average Farm Size (Acres) | 2981 | 1717 | 1244 | 850 |
| Range of Farm Size (Acres) | (2028-5160) | (1506-1964) | (1087-1463) | (600-1047) |
| Average Net Worth ($000) | 560.2 | 418.7 | 262.2 | 219.6 |
| Range of Net Worth ($000) | (112.3-1739.7) | (58.6-929.7) | (37.9-600.1) | (45.1-929.7) |
| Average Net Income Per Acre ($) | 22.78 | 23.19 | 19.46 | 16.96 |
| Standard Deviation of Net Income Per Acre ($) | 11.24 | 13.12 | 9.52 | 8.05 |
| Coefficient of Variation | .493 | .566 | .489 | .475 |
| Marginal Federal Income Tax Rate | .33 | .27 | .25 | .21 |

Georgia farm, Musser and Stamoulis specified a range of risk aversion parameters to capture the effect of risk preferences on resource allocation. Risk averse behavior of investors in their land purchasing decisions was assumed in this study. Hence, bid prices were estimated by parameterizing the coefficient for risk aversion. Bid prices and risk premiums generated at each level of risk aversion are intended to be illustrative of the impact of risk within the model.

Gradual increases in the risk aversion parameter result in decreases in the maximum bid-price and corresponding increases in the risk premium as shown in Table 2. Assuming risk neutral behavior for all farms, larger farms are able to outbid farms in the adjacent lower size class. This occurs for either of two primary reasons; 1) there exists a higher expected net income per acre, or 2) the larger farm is in a higher tax bracket so the after-tax discount rate is lower. Both factors generate greater present values of farmland. As the risk aversion parameter is increased uniformly across all farms, the risk premium increases for the large and medium size representative farms at a faster rate due to the higher variability of income. The medium-size farm is unable to outbid the small-/medium-size farm when the risk parameter equals .004. Subsequent increases in the risk parameter produce an advantageous bidding position for the small/medium and small farms. The more rapid decline of the bid-price for the medium-size farm is largely attributable to the higher coefficient of variation of net returns per acre.

A comparison of bid prices was made assuming decreasing absolute risk aversion as a function of average net worth. The composite utility function developed by Harris and Nehring (p. 166) was applied to the North Dakota cash grain farm class data. Bid prices computed by size of farm were; $424.91 (large), $355.58 (medium), $287.14 (small-/medium), and $235.14 (small). Computed risk premiums were; $32.40 (large), $44.40 (medium), $34.50 (small/medium), and $25.30 (small). Results indicate that larger

**TABLE 2. Bid Prices and Risk Premiums for North Dakota Cash Grain Farms by Size Group for Selected Levels of Risk Aversion[a].**

| Risk Aversion Parameter | Farm Size Class | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Large | | Medium | | Small/Medium | | Small | |
| | Bid Price | Risk Premium[b] | Bid Price | Risk Premium[b] | Bid Price | Risk Premium[b] | Bid Price | Risk Premium[b] |
| .0 | $457.40 | -- | $399.99 | -- | $321.70 | -- | $260.46 | -- |
| .001 | 431.60 | $ 25.80 | 374.05 | $ 25.94 | 309.23 | $ 12.47 | 252.79 | $ 7.67 |
| .002 | 403.57 | 53.83 | 345.85 | 54.14 | 296.26 | 25.44 | 244.93 | 15.53 |
| .003 | 369.37 | 88.03 | 311.40 | 88.59 | 282.17 | 39.53 | 236.68 | 23.78 |
| .004 | 314.94 | 142.46 | 256.24 | 143.75 | 265.88 | 55.82 | 227.75 | 32.71 |
| .005 | 152.83 | 304.57 | 94.11 | 305.88 | 245.02 | 76.68 | 217.67 | 42.79 |
| .006 | 138.56 | 318.84 | 80.24 | 319.75 | 209.66 | 112.04 | 205.57 | 54.89 |

[a]Bid prices per acre were computed assuming a 20-year holding period for all farm sizes.
[b]The risk premium shown is computed as the difference between the bid-price which would occur under risk neutral behavior and the bid-price generated at the indicated level of risk aversion.

farms generally place higher bid prices on land than smaller-sized farms under a situation of diminishing absolute risk aversion.

## Price Differentials by Farm Size

Bid prices are directly affected by the assumed length of holding period as shown in Table 3 under the assumption that land values do not grow independently of the growth in annual returns (e.g., speculative bidding). Longer holding periods result in higher bid prices for all size farms, *ceteris paribus*. As the holding period is reduced, however, the decline of bid prices is not uniform either in absolute or percentage terms. Also, as the length of holding period is shortened uniformly for all farms, the spread between maximum bid prices is reduced between farm sizes. A reduction of the holding period reduces the number of periods over which the greater tax advantages for large farms are compounded. Hence, the present value of the tax differential between farms is reduced. As the length of holding period is expanded, the effects of differences between the rate of growth of annual returns and the discount rate become a more significant factor in both land prices and bid price differentials between farms. For example, if the expected growth rate were to exceed the effective discount rate for a prolonged period of time, the *ceteris paribus* effect of a longer holding period would be to raise land values for all farms and increase the bid price spread between farm classes. A comparison of bid prices for a given farm size reveals that a direct, but decreasing relationship exists between the bid-price and assumed holding period.

Bid-price differentials between farm sizes have a number of other potential sources in the land market. Table 4 provides for a comparison of the factors in the model which affect the bid price for farmland, assuming either risk neutral or risk averse behavior. Class comparisons are made assuming that the first-listed class of farm (e.g., large) had the tax and income characteristics of that sized-farm, and the characteristic of the second-listed class of farm (e.g., medium) shown in the source column. For example, a large farm could bid $457.40 per acre, if it had all large farm characteristics and was risk neutral. That same farm could bid only $392.92 ($64.48 less), if it had the lower tax bracket of the medium-sized farm unit. Comparison of bid price differentials between risk averse and risk neutral situations indicates that price adjustments are generally smaller when farms are assumed to exhibit a uniform level of risk aversion. Downward adjustment of the marginal federal income tax rate in conjunction with the elimination of the capital gains tax provision results in a smaller downward adjustment of the bid price. Elimination of the effect of capital gains tax provisions involves two adjustments within the model: 1) the 60 percent long term capital gains exclusion is eliminated, and 2) capital gains go untaxed. Higher bid prices which

**TABLE 3. Bid Prices for North Dakota Cash Grain Farms by Size Class Assuming Various Holding Periods.[a]**

| Farm Size Class | Holding Period (in years) | | | |
|---|---|---|---|---|
| | 25 | 20 | 15 | 10 |
| Large | $375.65 | $369.37 | $361.14 | $350.32 |
| Medium | 314.50 | 311.40 | 307.20 | 301.47 |
| Small/Medium | 285.53 | 282.17 | 277.69 | 271.71 |
| Small | 238.89 | 236.68 | 233.70 | 229.64 |

[a]All farm bid prices were computed assuming a risk aversion parameter equal to .003. Annual returns are assumed to grow at 9 percent.

**TABLE 4. Comparisons of Farm Classes and Bid Price Differentials by Source for North Dakota Cash Grain Farms[a].**

| Farm Size-Class Comparisons | Bid Price of Farm Listed First | | Source of Bid Price Differential: Marginal Federal Income Tax (with Capital Gains)[b] | | Marginal Federal Income Tax (without Capital Gains)[b] | | Before-Tax Net Income (per acre)[c] | | Standard Deviation of Net Income (per acre)[c] | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Risk Neutral (Col. 1) | Risk Averse (Col. 2) | Risk Neutral (Col. 3) | Risk Averse (Col. 4) | Risk Neutral (Col. 5) | Risk Averse (Col. 6) | Risk Neutral (Col. 7) | Risk Averse (Col. 8) | Risk Neutral (Col. 9) | Risk Averse (Col. 10) |
| Large-Medium | $457.40 | $369.37 | −64.48 | −38.63 | −34.50 | −19.86 | 8.23 | 8.23 | 0 | −41.07 |
| Large-Small/Medium | | | −80.62 | −49.25 | −56.92 | −33.78 | −66.86 | −66.86 | 0 | 27.78 |
| Large-Small | | | −107.56 | −67.71 | −92.66 | −57.39 | −116.86 | −116.86 | 0 | 46.22 |
| Medium-Small/Medium | $399.99 | $311.40 | −16.43 | −8.15 | 7.70 | 3.60 | −64.51 | −64.51 | 0 | 45.34 |
| Medium-Small | | | −43.86 | −23.06 | −28.69 | −14.62 | −107.46 | −107.46 | 0 | 58.30 |
| Small/Medium-Small | $321.70 | $282.17 | −23.00 | −17.25 | −10.28 | −7.64 | −41.18 | −41.18 | 0 | 11.78 |
| Small | 260.46 | 236.68 | | | | | | | | |

[a]Bid prices and price differentials were computed assuming a 20-year holding period. Bid price and price differentials in the risk averse column were computed using a risk aversion paraumeter equal to .003.

[b]Effect of long term capital gains taxes (using a 60 percent exclusion) was generated by first including capital gains tax rates and exclusion of gains (cols. 3 and 4), then eliminating capital gains taxes and the exclusion of gains and allowing the long term gains to go untaxed (col. 5 and 6). In this way capital gains tax effects are added to the bid price changes which occur when ordinary income marginal tax rates are switched between farm size classes.

[c]Differences in earned income per acre between farm size classes are generated by assuming that the farm size listed first had actually realized the expected before-tax net income of the second-listed farm (cols. 7 and 8), and then considering the independent effect of switching the standard deviation of before-tax net income between the same two farms (cols. 9 and 10).

result when capital gains considerations are excluded correspond to a smaller price differential between farm sizes as the tax advantage to larger farms in higher tax brackets is reduced.

Substitution of net, before-tax income of the next lower-sized farm results in bid price reductions for most farm size comparisons. Higher expected net income per acre for the medium-sized farm adjusts the bid price for the large farm upward. Level of risk aversion has no impact on bid price differentials which occur with an adjustment of expected net income. Price differentials between farms are, however, sensitive to risk behavior when the variability of net income per acre is substituted between farms as shown in the last column in Table 4. The higher standard deviation of returns experienced by the medium-sized farm had the effect of reducing the bid price for the large farm class from its former level. Other bid prices are increased as lower standard deviations of returns for adjacent, smaller farms are substituted. Table 4 generally illustrates that bid price differentials are sensitive to the tax bracket of the farm.

To illustrate how sensitive bid prices are to the tax rate variable, prices were computed for each farm assuming various tax brackets. Elasticities of bid price response to changes in the marginal income tax rate are shown in Table 5. Assuming risk neutral behavior, all farms exhibit identical price response elasticities since the tax rate affects net income per acre and the discount rate proportionately, and the variability of returns is not considered. Increases in the tax rate result in higher elasticities of price response, generally, and highly responsive bid-price increases at tax rates between 35 and 40 percent. High tax bracket land buyers have substantial incentives to invest in land under the assumption of risk neutrality.

Computed elasticities under an assumption of uniform risk aversion provide an indication that price response is generally reduced by the incorporation of risk averse investor behavior. Elasticities shown for individual farm sizes indicate that at some point the elasticity may turn negative as bid prices decline in response to larger reductions in after-tax returns per acre. Lower after-tax returns per acre at high tax levels affect bid prices more than the reductions in variability of returns and the after-tax discount rate.

## Conclusions and Implications

A representative land market model was developed by integrating the asset pricing model developed by Baker into a bid-price theoretical framework introduced by Pratt and developed for farmland by Harris and Nehring. The resultant model incorporates

**TABLE 5. Elasticities of Bid Price Response to Changes in the Marginal Federal Income Tax Rate by Farm Size Class for North Dakota Cash Grain Farms.**

| | Farm Size Class[a] | | | | |
|---|---|---|---|---|---|
| | All | Large | Medium | Small/Medium | Small |
| Marginal Federal Income Tax Rate | Risk Neutral | Risk Averse | Risk Averse | Risk Averse | Risk Averse |
| .05 | .05 | .04 | .04 | .05 | .05 |
| .10 | .12 | .10 | .09 | .11 | .11 |
| .15 | .21 | .18 | .16 | .18 | .19 |
| .20 | .33 | .27 | .24 | .28 | .30 |
| .25 | .51 | .40 | .32 | .43 | .45 |
| .30 | .76 | .55 | .36 | .61 | .65 |
| .35 | 1.17 | .64 | −1.65 | .85 | .95 |
| .40 | 1.86 | −1.63 | −2.91 | .58 | 1.29 |

[a]Bid prices computed under the assumption of risk aversion assumed a risk aversion parameter equal to .003.

growth of returns, ordinary and capital gains taxes, finite individual investor holding periods, and explicit consideration of economies and diseconomies of size, and risk. The combined market model allows for analysis of an increased number of factors thought to affect land investor decisions.

Application of the model to primary data from cash grain farms in northwest and north central North Dakota indicates that larger farms have a competitive bidding advantage over small-sized farms within the same land market. The model illustrates that the effect of increased marginal tax rates on a perpetual, growing income stream is to increase the present value of that stream.[6] Higher bid prices for larger farms in higher tax brackets stem primarily from three model relationships; 1) higher tax rates reduce the after-tax discount rate, 2) higher tax rates reduce the variability of after-tax net returns per acre, and 3) capital gains tax provisions which allow a portion of the increased value of land to be excluded and the remainder taxed at the preferred capital gains rate provides an incentive for investors in higher tax brackets. These factors increase the present value of land and, therefore, the bid-price.

Application of the representative market model to cash grain farms in North Dakota also indicates that diseconomies of size (to the degree and magnitude that they are captured by data on expected before-tax net returns per acre) do not greatly influence land bid prices. Rather, the tax bracket and risk aversion level of the investor are more important determinants of farmland bid prices. Implications of the application are limited by the omission of two considerations. Form of business organization has not been considered in the analysis. Partnerships and corporate farm organizations may be effective in reducing the progressiveness of the income tax and diminish the bid price differentials indicated above. Secondly, financing terms have not been incorporated into the model. Variations in financing arrangements would be conceptually difficult to model when considering a series of owners.

Generally, the impact of increasingly risk-averse investor behavior is to increase the risk premium and decrease the bid-price for farmland. At relatively low uniform levels of risk aversion large farms outbid smaller farming units. Higher uniform levels of risk aversion favor smaller farms, which exhibit lower variability of net returns per acre. When decreasing absolute risk aversion (as a function of net worth) was assumed larger farms demonstrated a competitive advantage over smaller farms in bidding for land. Bid prices generated by gradually raising the tax bracket for individual farm sizes indicated that the effect of taxes under risk-neutral investor behavior is to increase the elasticity of bid-price response. Increases in the tax rate variable under risk, however, indicate that at some point the bid-price for land begins to decline. These results differ substantially from those generated in other farmland bid-price model applications.

[6]Conversely, the model can be used to show that the effect of increased marginal tax rates on a perpetual, declining income stream is to decrease the present value of the land resource.

## References

Adams, Roy D. "The Effect of Income Tax Progressivity on Valuations of Income Streams by Individuals." *American Journal of Agricultural Economics*, 59(1977):538-42.

Baker, Timothy G. "An Income Capitalization Model of Land Value With Income Tax Considerations." *American Journal of Agricultural Economics*, forthcoming, 1982.

Harris, Duane G. and Richard F. Nehring. "Impact of Farm Size of the Bidding Potential for Agricultural Land." *American Journal of Agricultural Economics*, 58(1976):161-69.

Johnson, Jerome E. "Farmland Leasing Arrangements and Rentals, 1979." *North Dakota Farm Research*, 28(1980):3-4.

Johnson, Jerome E. "Farmland Values, 1979." *North Dakota Farm Research*, 37(1980):24-28.

King, Robert P. and Lindon J. Robison. "An Interval Approach to Measuring Decision Maker Preferences." *American Journal of Agricultural Economics*, 63(1981):510-520.

Musser, Wesley N. and Kostas G. Stamoulis. "Evaluating the Food and Agriculture Act of 1977 With Firm Quadratic Risk Programming." *American Journal of Agricultural Economics*, 63(1981):447-457.

Pratt, John W. "Risk Aversion in the Small and in the Large." *Econometrica*, 32(1964):122-36.

Scott, John T., Jr. *Effects of Income Tax Progressivity on Land Buy and Sell Decisions and the Tax Act of 1981*. Paper presented at the American Agricultural Economics Association Meetings, Clemson, SC, 1981.

U.S. Department of Agriculture. *Farm Real Estate Market Developments*. CD86, Washington, D.C., August 1981.

## Appendix

Given the present value expression for the first buyer in equation (3), it is necessary to show that an equivalent expression can be derived for the (i+1)th owner. Following Baker, equation (3) can be rewritten as,

$$\text{(A1)} \quad V_o\,(1-c\,(1+r)^{-m}) = \sum_{n=1}^{m} \frac{R_o}{(1+r^*)^n} + \frac{(1-c)\,V_1}{(1+r)^m}$$

The price paid by the second buyer is then,

$$\text{(A2)} \quad V_1\,(1-c\,(1+r)^{-m}) = \sum_{n=1}^{m} \frac{R_1}{(1+r^*)^n} + \frac{(1-c)\,V_2}{(1+r)^m}$$

where, $R_i = R_o\,(1+g)^{im}$

The price expression for the (i+1)th buyer is, therefore,

$$\text{(A3)} \quad V_i\,(1-c\,(1+r)^{-m}) = \sum_{n=1}^{m} \frac{R_i}{(1+r^*)^n} + \frac{V_{i+1}\,(1-c)}{(1+r)^m}$$

under the assumption that all buyers in the sequence hold land for m-periods and have the same discount rate, and income tax rate.

The price to the (i+1)th buyer can be rewritten in a simple form as,

$$\text{(A4)} \quad V_i = \alpha\,R_i + \Theta\,V_{i+1}$$

where,

$$\alpha = \frac{1-(1+r^*)^{-m}}{r^*\,(1-c(1+r)^{-m})}$$

$$\Theta = \frac{(1-c)\,(1+r)^{-m}}{(1-c(1+r)^{-m})}$$

Analogously, the present value expression for $V_o$ is,

$$\text{(A5)} \quad V_o = \alpha\,R_o + \Theta\,V_1$$

Recursively substituting for $V_i$ on the right side of (A5) until the number of buyers becomes quite large,

$$\text{(A6)} \quad V_o = \alpha \sum_{i=0}^{\infty} \Theta^i R_i$$

Equation 6 states that as the sequence of landowners become quite long the present value of land is approximated by the sequence of discounted annual after-tax returns. Substituting the value of $R_i$ from (A2) into (A6), the revised present value expression is,

$$\text{(A7)} \quad V_o = \alpha\,R_o \sum_{i=0}^{\infty} (\Theta(1+g)^m)^i$$

To simplify (A7) the limit of the right side expression is taken to define the present value for the first buyer as,

$$\text{(A8)} \qquad V_o = \frac{\alpha R_o}{1-\Theta(1+g)^m}$$

Substitution for $\alpha$ and $\Theta$ from (A4), and simplification of the resulting equation yields,

$$\text{(A9)} \qquad V_o = \frac{R_0}{r^*}\left[1 + \frac{1-(1+r^*)^{-m}}{c(1+r^*)^{-m}-c(1+r)^{-m}}\right]$$