Relevance of Duality Theory to the Practicing Agricultural Economist: Discussion

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Many of the remarks that I have today may largely be a matter of semantics. Like most jargon and buzzwords, duality means different things to different people. To say, however, that there exists a duality between the cost and production functions, for example, means that there exists an invertible, one-to-one relationship between these functions. Put another way, the mapping that yields the cost function from the production function (line 6 in Professor Pope's Table 1) and the mapping

$$f^*(x) = \text{Max}\ \{z : C(P,z) \geqslant P \cdot x\}$$

defined in line 5 of Table 1, are mutual inverses.

Thus, the existence of a duality between the direct and indirect functions means that the individual researcher is free to use either function in his analysis since, in principle (and I emphasize the in principle part), the same economically relevant information can be obtained from both functions. Hence, the decision on whether to use the direct or indirect function in the presence of a duality is largely a matter of convenience. The decision is not trivial, especially in the case of empirical analysis. Burgess has presented convincing evidence to the effect that the choice of using either the direct of indirect functions can predetermine some of the resulting analysis, or at least lead to widely diverging results.

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Judging from the remarks of Professor Pope in his section entitled "Duality's Failings", it seems to me that he is pointing out that there are instances in which it is more appropriate to use a direct function for analysis rather than an indirect function. With this, I would certainly agree. However, I would not go so far as to say that this illustrates a failing on the part of duality. Indeed, depending upon one's point of view this underlines the strength of recent duality developments since one now has a choice to make rather than being forced to use the primal function. The existence of a choice is the most important aspect of duality theory from my point of view.

In his section on "Duality's Failings", Professor Pope raises a range of interesting theoretical and empirical problems. However, I do not feel that most of these reveal any inherent weakness in duality theory as much as they reveal a weakness in our ability to conceptualize and analyze realistic economic phenomena. Let us first turn to the case of normalized profit maximization subject to technical change that Professor Pope raises. In expression (32) Professor Pope correctly points out that the elements of the Hessian matrix of the profit function are subject to some complex non-linear restrictions which cannot be easily imposed when one uses a normalized profit function approach in estimating factor demands. I would argue, however, that there are at least as many problems associated with the use of the primal function in such a case. Since Professor Pope is clearly talking about the imposition of constraints in econometric estimation, consider the case of estimating.

$$(1) \qquad z = f(x_1, x_2, s)$$

by ordinary econometric methods. Unless expression (1) is based on purely experimental data, it is clear that there are many problems to be faced before one can be sure that he has obtained consistent and asymptotically efficient estimates. For example, efficient estimation requires use of the decision rule that generates the observations on x_1, x_2, and z. If normalized profit maximization is the rule, then efficient estimation suggests that first order conditions are of some value here. However, except under some rather restrictive assumptions on the error structure involved, consistent estimation will now be quite difficult because the observations on x_1 and x_2 will likely be correlated with the errors for equation (1) and those for the first order conditions. The reader should see de Janvry on this point. Thus, the primal approach is not problem free and in many instances may not even present an estimable alternative.

The problems Professor Pope finds when considering the use of indirect functions under uncertainty are also quite interesting. One might ask, however, what is the root cause of these problems? It seems to me that Minkowski's theorem (which Diewert has somewhat ambitiously called the heart of duality) may not be entirely applicable in these instances. In standard neoclassical analysis we are used to maximizing either non-linear functions subject to a linear constraint (utility maximization) or a non-linear function subject to non-linear constraints (e.g. cost or profit maximization). Under output and input price uncertainty or production uncertainty, we are actually maximizing a non linear function, the expected utility of profit, subject to a non-linear constraint, i.e. output and inputs are related in a non-linear fashion via the production function. Graphically, we no longer seem to be in the case when indifference curves or an isoquant can be depicted as setting on a linear constraint at the optimum so that the usual geometric interpretations of a duality which rest on Minkowski's theorem are no longer exact. Again, however, this is not so much a criticism of the dual approach as it is of the current state of economic knowledge. Comparative statics subject to non-linear constraints is still a novel enough topic to merit publication in the most prestigions economics journals as the recent work of Edlefsen demonstrates.

The point which Professor Pope makes, however, is well taken. Even with the use of indirect functions it is necessary to know something about preferences before one can make any concrete inferences about the technology. Another interesting example of this problem is the measurement of the elasticity of scale which defines the percentage adjustment in output due to all inputs being multiplied by a common scalar. Denoting this elasticity as ε, it is well known [Ferguson] that it can be written as the sum of the elasticities of output, or algebraically

$$(2) \qquad \varepsilon = \sum_i \frac{\partial f}{\partial x_i} \frac{x_i}{z}.$$

Under conditions of certainty and profit maximization, first order conditions require that marginal products be equated to normalized factor prices, so that (2) becomes

$$(2') \qquad \varepsilon_c = \sum_i \frac{P_i x_i}{R \cdot Z}$$

or as the ratio of cost to revenue, which is observable from market data. Here subscript c is used to differentiate the certainty case. Problems arise, however, under uncertainty and expected utility maximization. To see this, consider Professor Pope's example of output price uncertainty. Let

$$R = \bar{R} + e$$

where e is a random variate with mean zero. First order conditions for an interior solution then require

$$(3) \quad Eu'(\pi)\{R \frac{\partial f}{\partial x_i} - P_i\} = 0. \quad i = 1, 2, \ldots, n$$

By expression (3), it follows that

$$\frac{\partial z}{\partial x_i} = \frac{P_i}{\bar{R}+\Theta}$$

where $\Theta = \text{Cov}\ \{u'(\pi),\ e\}/Eu'(\pi)$. Thus, letting subscript u denote the uncertainty case yields,

$$\varepsilon_u = \sum_i \frac{P_i x_i}{(\bar{R}+\Theta)z} \tag{4}$$

which cannot be measured without some direct knowledge about the structure of preferences. I have shown elsewhere, for similar reasons, that the usual Divisia measure of total factor productivity measures the rate of technical change correctly with probability zero [Chambers]. Further, under uncertainty and expected utility maximization one cannot obtain an appropriate measure of productivity from observed market data except in the case of constant returns to scale. In all other instances, one must know either ε or Θ to obtain an accurate productivity measure.

While these comments are meant to suggest that I share some of Professor Pope's skepticism about the universal usefulness of indirect functions, I must emphasize that there are many potential areas where an indirect approach seems to most fruitful. One of these seems to be in gauging the effect of technical change and governmental regulation on industry structure by using the methodology pioneered by Panzar and Willig.[1]

As a simple example of this latter approach, let us consider the case where the price to producers is strictly determined by governmental authorities. To allow for the existence of inframarginal firms and the existence of technical change, let the firm's cost structure be given by

$$C(z, P, t, \triangle)$$

where t denotes time, $\triangle$ is a non-negative cost decreasing parameter and each firm in the industry is characterized by a particular value of $\triangle$. Marginal firms are characterized by a $\triangle$ that allows them to earn exactly zero profit. Letting R be government controlled price then yields the following equation which defines the value of $\triangle$ associated with the marginal firms, $\triangle^*(P, R, t)$:

$$R \cdot z^* - C(z, P, t, \triangle^*) = 0 \tag{5}$$

where z^* is profit maximizing output and thus is a function of $(P, R, t, \triangle)$ such that [Panzar and Willig]:

$$\frac{\partial z}{\partial R} > 0;$$

$$\frac{\partial z}{\partial \triangle} = -\frac{\partial^2 C}{\partial z \partial \triangle} \Big/ \frac{\partial^2 C}{\partial z^2};$$

$$\frac{\partial z}{\partial t} = -\frac{\partial^2 C}{\partial z \partial t} \Big/ \frac{\partial^2 C}{\partial z^2}.$$

Now let us use these results and (5) to ascertain the effect of technical change on the level of $\triangle^*$. Implicit differentiation of (5) yields (P, R constant)

$$\left[\left(R - \frac{\partial C}{\partial z}\right)\frac{\partial z}{\partial \triangle} - \frac{\partial C}{\partial \triangle}\right] d\triangle^* + \left[\left(R - \frac{\partial C}{\partial z}\right)\frac{\partial z}{\partial t} - \frac{\partial C}{\partial t}\right] dt = 0$$

which implies by first order conditions for profit maximization:

$$\frac{\partial \triangle^*}{\partial t} = -\frac{[\partial C/\partial t]}{\partial C/\partial \triangle}. \tag{6}$$

Thus, if technical change is progressive, i.e., cost decreasing then expression (6) is negative. Hence, the minimum value of $\triangle$ which is required for firms to participate in the industry falls, as expected, under technical progression.

Now consider the effect of a change in the level of R on the lowest level of $\triangle$ consistent with participation in the industry. As Panzar and Willig have demonstrated

[1] This line of research was suggested by Lopez and Tung.

$$\frac{\partial\Delta^*}{\partial R} = \frac{z^*}{\partial C/\partial\Delta}$$

which is negative. Thus, if we consider R as a support price it follows that raising the support price lowers the level of Δ consistent with survival in the industry. In essence, this is almost the same as saying that a higher support price is consistent with a greater number of firms in the industry as one would expect.

Another area, in addition to those pioneered in Dr. Lopez's paper for this session, includes the assessment of adjustment rigidities within the agricultural sector. For example, one of the stylized facts of aggregate agricultural production and policy analysis is the existence of "asset fixity" within the agricultural sector of the United States. Although this phenomenon has widespread policy and theoretical implications, it had until recently never been subjected to empirical validation. It is possible, however, to use the methodology developed by Fuss to conduct an empirical test of "asset fixity" as Chambers and Vasavada have demonstrated at the aggregate level. Further tests, at differing levels of aggregation, seem appropriate and only appear to be waiting on implementation. In addition, it should be a straightforward matter to extend Fuss' methodology to developing a statistical method of investigating the classical notion of "enterprise flexibility" due to Heady.

Another application of already existent indirect techniques that seems to only be waiting on actual implementation is the investigation of optimal tax and policy perturbations along the lines recently developed by Diewert. Given the heavy degree of regulation in U.S. agriculture and the popular pressure to get the government out of the private sector, this type of analysis could have extremely important prescriptive policy implications.

At this point, I would like to emphasize and perhaps underline some extremely important points that are implicit in Professor Pope's paper. The first of these has to do with the "flexibility" of flexible functional forms. As we know from the work of Lau [1974] and others, there are several possible interpretations of the terminology "flexible functional form". In this analysis, we shall maintain the interpretation that a flexible form is one that is capable of being interpreted as a second order Taylor series approximation to a differentiable function. In empirical applications of flexible forms it is usually required that some restrictions be placed on the form for it to satisfy certain regularity properties. An important question is whether such restrictions impinge upon the flexibility of the form. Consider, for example, the case of the generalized Leontief approximation to a cost function consistent with a homothetic technology:

$$C(P,z) = h(z) \sum_i \sum_j \beta_{ij} (P_i P_j)^{1/2}$$

where h(z) is an increasing monotonic function and $\beta_{ij} = \beta_{ji}$. By Shephard's lemma, the i^{th} constant output derived demand is

$$x_i = h(z) \sum_j \beta_{ij} \left(\frac{P_j}{P_i}\right)^{1/2} \quad (7)$$

and thus the Allen elasticity of substitution between inputs i and j take the same sign as $\beta_{ij} = \beta_{ji}$. Now it is well known that a sufficient condition for this cost function to be globally concave as required by our usual regularity conditions is that $\beta_{ij} \geq 0$, $i \neq j$. This restriction can be imposed prior to estimation by using a methodology developed by Lau [1978].

Notice, however, that imposition of this concavity restriction prior to estimation insures that the resulting cost function will classify all input pairs as Allen substitutes since all cross input Allen elasticities are now positive. Hence, a cost function which meets this global concavity restriction cannot possibly provide a second order approximation to a technology that involves any complementarities in the Allen sense. Thus, the researcher may face a trade off between a globally concave function and one that is flexible enough to provide a second order approximation to an arbitrary cost structure that involves complementarity. A similar problem is reflected in the fact that, as Professor Pope correctly notes, a translog function cannot be made to be globally concave and hence cannot provide a second order approximation to an arbitrary cost of profit function over the entire parameter space. Clearly, flexible forms are not as flexible as we sometimes are lead to believe. A particularly interesting discussion of such topics and related problems can be found in Wales.

A second question which Professor Pope raises and which we have as yet not answered is to what do we want a close approximation — the input demand function, say, or the cost function? Using (7) we see that for a generalized Leontief cost function

$$\frac{\partial^2 x_i}{\partial P_j \partial P_k} = 0 \qquad i \neq j \neq k$$

so that the Hessian matrix associated with each input demand function for the generalized Leontief has many zeros implying (7) is only a second order approximation in a very, very limited sense. Thus, while we may have a quite satisfactory approximation to the cost function, our approximation to the input demand functions might be very poor and truly valid only up to the first order. It seems obvious that there will be instances where this state of affairs will not be acceptable. Unfortunately, aside from the well-known work surrounding the Rotterdam model very little work to date has been done in this area.

The next and final question that I shall raise as worthy of being answered is — just what are the integrability conditions under uncertainty and expected utility maximization? Further, just what are we interested in trying to recover — the utility or the production function? The results presented by Professor Pope as well as those found in expression (4) suggest that we may not be able to recover both. That is, to recover information on the structure of technology must we already know just how preferences are characterized? Until this type of question is answered, it is almost pointless to speculate on the usefulness of dual techniques for the case of uncertainty since no dualities have been formally demonstrated.

In closing, I would like to thank Professor Pope and Dr. Lopez for extremely enlightening papers on the area of duality and economics. I would also like to thank the Western Agricultural Economics Association for sponsoring a session on one of the paramount methodological approaches to economic analysis that currently exists. Further exchange along these and similar lines are clearly needed.

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