A Monte Carlo Comparison of Alternative Estimators of Autocorrelated Simultaneous Systems Using a U.S. Pork Sector Model as the True Structure

Gopal Naik and Bruce L. Dixon

Monte Carlo analysis of the performance of alternative estimators of simultaneous system's coefficients in the presence of autocorrelation is performed. The "true" underlying model is an estimated, three-equation, monthly model of the U.S. pork market. Estimators for *ex post* forecasts are also compared. Multicollinearity is found to be a salient characteristic likely adversely affecting estimator performance. Results show that correcting for autocorrelation is desirable when levels of autocorrelation are high for both parameter accuracy and *ex post* forecasting. However, the best structural coefficient estimator for high levels of autocorrelation is not necessarily the best estimator for *ex post* forecasting.

Key words: autocorrelation, forecast accuracy, simultaneous systems, structural estimates comparison.

Considerable Monte Carlo analysis has been undertaken to evaluate the performance of the various simultaneous systems estimators as summarized in Johnston or Judge et al. A common approach of many of these experiments is to set up a simple "true" structure and choose the values of the exogenous variables to have a desired degree of multicollinearity. While such designs have the virtue of being controlled and displaying desired collinearity characteristics, they possess the defect of not being much at all like any particular real-world model. Nonetheless, the conventional wisdom is to use the results from such experiments as guides in appraising estimator performance in real-world applications.

It seems a more useful approach would be to use a real-world model and its data for the values of the exogenous variables in a Monte Carlo experiment. Because the true structural coefficients of any real-world model are almost always unknown, they would have to be generated in some fashion. The real-world coefficients could be estimated using historically observed data. The estimated coefficients of the hypothesized model could then be specified to be the "true" coefficients or structure, and historical values of the exogenous variables would be used along with randomly generated error terms to compute the values of the endogenous variables in the Monte Carlo experiments. Judgments about the comparative superiority of estimators of the hypothesized model using this approach are likely more accurate than generalizing from experiments using arbitrarily generated structures and data for the exogenous variables. Given the characteristics of the real-world model and data used in experiments, generalizations about other, similar real-world situations are likely more accurate than generalizations from experiments using the arbitrary structures and data.

In recent years simultaneous systems models

Gopal Naik is a graduate student in the Department of Agricultural Economics at the University of Illinois. Bruce L. Dixon is an associate professor in the Department of Agricultural Economics and Rural Sociology, University of Arkansas.

have been applied to commodity markets for both forecasting and structural analysis. These models are typically estimated using time-series data, and the error terms are likely to be characterized by autocorrelation. When autocorrelation is present, it can be acknowledged explicitly or ignored. For example, a study by Simister and another by Cragg (1966) examine the accuracy of parameter estimates using the standard systems estimators without correcting for autocorrelation when it is present. Kmenta suggests a limited information (single-equation) method of estimation to correct for the presence of autocorrelation. This method is applied and analyzed in the present study. In particular, this study compares the performance of two-stage least squares (2SLS) on autocorrelated data with the Kmenta estimator that corrects for autocorrelation using the Monte Carlo approach discussed above. The comparison criteria are accuracy of estimated coefficients, prediction of historical observations, and forecasts beyond the sample period. The authors are not aware of any other Monte Carlo studies which measure the benefits of correcting for autocorrelation in structural simultaneous econometric models.

A three-equation model of supply and demand of the U.S. pork market is estimated. The coefficients of the estimated model are then specified to be the true parameters for the Monte Carlo approach outlined above. The pork market is chosen because of the importance of the pork sector in the agricultural economy and because it has been frequently modeled. For example, see Martin and Zwart, Dixon and Martin, and Leuthold and Hartmann. The model is a simple three-equation system with price-dependent demand, and supply and storage equations.

This study proceeds by specifying the empirical model and presenting the method of generating the data. Then the estimators used are explained and the criteria for judging estimator and predictor performance are given. Finally, the results are analyzed by comparing estimator performance.

Econometric Model

A three-equation, simultaneous system representing monthly pork market equilibrium is utilized in this study. The specification used is similar to prior studies of the pork market, see Leuthold and Hartmann or Naik and Leuthold (1985) for example, and similar to other simple (few equations) commodity market models, particularly for live meats. The model is

$$PR_t = f(QRD_t, QBD_t, INC_t) \tag{1}$$

$$QCP_t = g(PR_t, PCORN_{t-10}, COLDPRK_t, SF_{t-6}) \tag{2}$$

$$QRD_t = QCP_t - CCLDS_t \tag{3}$$

where PR_t is retail price of pork in t (¢/lb.); QRD_t, retail demand for pork in t (million lbs.); QBD_t, retail demand for beef in t (million lbs.); INC_t, per capita income in t ($); QCP_t, current production of pork in t (million lbs.); $PCORN_{t-10}$, price of corn lagged 10 months (¢/bu.); $COLDPRK_t$, pork in storage in t (million lbs.); SF_{t-6}, sow farrowings lagged 6 months (million); and $CCLDS_t$, change in cold storage in t (storage in the current month less storage in the previous month); where the subscript t refers to the tth month, and PR_t, QCP_t and QRD_t are the endogenous variables.

The first equation is a price-dependent demand equation with quantity of retail consumption of pork and beef, and per capita income as explanatory variables. The second equation gives current production of pork as a function of retail price of pork, price of corn, quantity of cold storage of pork, and sow farrowings. The lags of price of corn and sow farrowings selected in this equation seemed the most reasonable compared with the other models estimated. The third equation is an identity equating supply and demand where supply is production minus change in storage.

The monthly data used in this study are obtained primarily from *Livestock and Meat Statistics,* published annually by the U.S. Department of Agriculture (USDA). The data on income and population come from the U.S. Department of Commerce's *Survey of Current Business.* The coefficient values of the system (1)–(3), used for generating the Monte Carlo data on the endogenous variables, are obtained using 2SLS on the data from July 1973 through December 1983 and are as follows:

$$\begin{aligned} PR_t = {} & \underset{(7.75)}{129.85} - \underset{(-9.51)}{0.0901 QRD_t} \\ & + \underset{(1.56)}{0.0091 QBD_t} + \underset{(21.55)}{0.012 INC_t} \end{aligned} \tag{4}$$

$$R^2 = 0.82, \quad \text{D.W.} = 0.78, \quad SE = 11.5;$$

$$(5)\quad QCP_t = \underset{(-3.19)}{-347.21} + \underset{(6.59)}{3.38PR_t} - \underset{(-0.99)}{20.54PCORN_{t-10}} + \underset{(7.17)}{1.209COLDPRK_t} + \underset{(11.77)}{0.328SF_{t-6}}$$

$$R^2 = 0.68, \quad \text{D.W.} = 1.66, \quad SE = 105.5;$$

where the numbers in parentheses are the ratios of estimated coefficients to their estimated standard errors. The estimated coefficients in (4) and (5) become true parameters for the purpose of generating Monte Carlo data.

All the coefficients in the retail price equation except the coefficient of QBD_t have the expected signs and are significant at the 95% level. The positive sign on QBD_t has been obtained in previous research (Hayenga and Hacklander; Naik and Leuthold 1985). All the explanatory variables in the supply equation have the expected signs except for cold storage of pork. The seasonal movements of cold storage and current production tend to go together, and this might have yielded the positive sign. Except for the price of corn, all other coefficients are significant. The insignificance of price of corn is probably because prices are undeflated. The Durbin-Watson statistic for the demand equation indicates the presence of autocorrelation. There is substantial multicollinearity in the data. In particular, in the second stage of the 2SLS estimation, the condition index for equation (4) is 52.26 and the corresponding condition index for (5) is 30.80.[1] Hence, high precision is not expected in estimation of the parameters regardless of the impact of autocorrelation.[2] The only readily apparent source of collinearity is between QRD_t and INC_t, but the condition indices indicate that there are other sources of collinearity.

The reduced form derived from the estimated coefficients in (4) and (5) (i.e., the restricted reduced form) is

$$(6)\quad PR_t = 123.50 + 0.007QBD_t + 0.009INC_t + 1.419PCORN_{t-10} - 0.083COLDPRK_t + 0.069CCLDS_t - 0.023SF_{t-6}$$

$$(7)\quad QCP_t = 70.445 + 0.024QBD_t + 0.030INC_t - 15.745PCORN_{t-10} + 0.926COLDPRK_t + 0.234CCLDS_t + 0.252SF_{t-6}$$

$$(8)\quad QRD_t = 70.445 + 0.024QBD_t + 0.030INC_t - 15.745PCORN_{t-10} + 0.926COLDPRK_t - 0.766CCLDS_t + 0.252SF_{t-6}.$$

The 126 × 7 matrix of predetermined variables used in the estimation of the first stage of 2SLS has a condition index of 73.65. If the data were deflated by appropriate indices, the condition index would have increased to 171.84.

For simplicity, it is assumed there is no contemporaneous correlation between the error terms of the structural equations. It is also assumed that there exists first-order autocorrelation in both of the stochastic equations of the structural model.

Generation of the Monte Carlo Data

The observations on the endogenous variables are generated in the Monte Carlo experiments by using (6)–(8) to generate the means for the endogenous variables where the estimated coefficients are the "true" coefficients for the purposes of the Monte Carlo experiments. The values of the error terms, where e_{1t} and e_{2t} are the error terms in the stochastic structural equations, i.e., (1) and (2), respectively, are generated as

$$e_{it} = \rho_i e_{it-1} + u_{it} \qquad (i = 1, 2),$$

where u_{it} and ρ_i are the random disturbance term and first-order autocorrelation coefficient, respectively, for the ith equation. The u_{it}'s are generated by obtaining standard normal random numbers and multiplying them by their respective standard deviations [the values of standard deviations used are the

[1] The conditioning index is the square root of the ratio of the largest eigenvalue of $X'X$, where X is the regressor matrix, to the smallest eigenvalue of $X'X$, where X has been properly scaled. See Belsley, Kuh, and Welsch for a discussion of scaling and the use of the conditioning index as a measure of multicollinearity. They consider conditioning indices in excess of 30 to pose substantive complications.

[2] An argument could be made that the specification in (1)–(3), given the estimates in (4) and (5), could be improved. While this is probably true, a better specification would likely involve more equations and coefficients, and we wish to keep the number of coefficients to a minimum. Moreover, it is not important for the central purpose of this study that (1)–(3) be a perfect replication of reality. Our approach uses the estimates in (4) and (5) as the data. Estimates based on the Monte Carlo data are compared with values for the population parameters in generating the Monte Carlo the population parameters specified. Thus the absolute verity of (1)–(3) is not of crucial importance since the estimates based on the Monte Carlo data are being compared with population parameters that are true by hypothesis.

standard errors given in (4) and (5)]. Since the impact of autocorrelation may vary with the magnitude of ρ_i, experimental data are generated for the five levels of ρ_i. Thus, five different sets of data each consisting of fifty samples (replications) of size 50 are generated using autocorrelation coefficients of 0.0, 0.2, 0.4, 0.6, and 0.8. For simplicity, $\rho_1 = \rho_2$ in each data set.[3]

Once the matrix of observations on the structural error terms is obtained for a given sample, it is multiplied by the inverse of the matrix of endogenous variable coefficients to yield the error terms to be added to the means of the endogenous variables computed from the reduced form. The historical values of the exogenous variables from September 1977 through October 1981 (50 observations) are used for the predetermined variables in computing these means. The condition index on this subset of the predetermined variables is 64.25.

Estimation and Correction for Autocorrelation

The first forty-five observations are used in each sample to estimate the parameters. The latest five observations are used for *ex post* forecasts. First, (1)–(2) are estimated using 2SLS, and the restricted reduced form is derived (RR2SLS). Then the limited information estimator given in Kmenta (LISEM) that corrects for autocorrelation is used to estimate the structural equations, and then the restricted reduced form (RRLISEM) is derived.

The procedure suggested by Kmenta essentially requires using ordinary least squares (OLS) three times. First, each endogenous variable is regressed on the lagged values of all endogenous variables and current and lagged values of all exogenous variables present in the system to obtain the "augmented reduced-form equations," i.e., for the first endogenous variable the equation is

$$(9)\; PR_t = a_0 + a_1PR_{t-1} + a_2QCP_{t-1} + a_3QRD_{t-1} + a_4QBD_t + a_5QBD_{t-1} + a_6INC_t + a_7INC_{t-1} + a_8PCORN_{t-10} + a_9PCORN_{t-11} + a_{10}COLDPRK_t + a_{11}COLDPRK_{t-1} + a_{12}CCLDS_t + a_{13}CCLDS_{t-1} + a_{14}SF_{t-6} + a_{15}SF_{t-7}.$$

[3] There is a problem in generating the observations on $e_{i,0}$ for each sample. For each level of ρ_i an observation of $e_{i,0}$ is drawn for the first sample with the same standard deviation as u_{it}. For subsequent samples $e_{i,50}$ becomes $e_{i,0}$ for the next sample.

Equation (9) and the corresponding equations for QCP_t and QRD_t are estimated using OLS. The predicted values of endogenous variables PR_t, QCP_t, and QRD_t are substituted into the structural equations wherever they appear as explanatory variables. This allows treating each behavioral equation separately when correcting for autocorrelation. Then each stochastic structural equation is put in first-difference form using a Koyck transformation. In these equations the ρ_i become coefficients. However, since the ρ_i are unknown, they have to be estimated. Direct estimation of ρ_i requires nonlinear restrictions and thus is difficult to handle. Hence, Durbin's method is used to estimate the ρ_i. First, the autocorrelation coefficient is obtained by regressing the endogenous variable on its lagged value, estimated current and lagged values of other endogenous variables, and current and lagged values of exogenous variables present in the respective equations. For this application, the equations

$$(10)\quad PR_t = d_0 + d_1PR_{t-1} + d_2\widehat{QRD}_t + d_3\widehat{QRD}_{t-1} + d_4QBD_t + d_5QBD_{t-1} + d_6INC_t + d_7INC_{t-1},$$

and

$$(11)\quad QCP_t = f_0 + f_1QCP_{t-1} + f_2\widehat{PR}_t + f_3\widehat{PR}_{t-1} + f_4PCORN_{t-10} + f_5PCORN_{t-11} + f_6COLDPRK_t + f_7COLDPRK_{t-1} + f_8SF_{t-6} + f_9SF_{t-7}$$

are estimated. The estimates of d_1 and f_1 are the estimated autocorrelation coefficients ($\hat{\rho}_{11}$, $\hat{\rho}_2$). Using these estimates, the variables in the structural equations are transformed as follows:

$$(12)\quad PR_t - \hat{\rho}_1PR_{t-1} = g_0 + g_1(\widehat{QRD}_t - \hat{\rho}_1\widehat{QRD}_{t-1}) + g_2(QBD_t - \hat{\rho}_1QBD_{t-1}) + g_3(INC_t - \hat{\rho}_1INC_{t-1}),$$

$$(13)\quad QCP_t - \hat{\rho}_2QCP_{t-1} = h_0 + h_1(\widehat{PR}_t - \hat{\rho}_2\widehat{PR}_{t-1}) + h_2(PCORN_{t-10} - \hat{\rho}_2PCORN_{t-11}) + h_3(COLDPRK_t - \hat{\rho}_2COLDPRK_{t-1}) + h_4(SF_{t-6} - \hat{\rho}_2SF_{t-7}).$$

The OLS estimates of the g_i and h_i are the structural coefficient estimates corrected for autocorrelation.

To test the forecast accuracy of various estimators, the reduced-form equations must be obtained. In addition to the two reduced-form estimators mentioned above, two other reduced-form estimators not using the structural estimates are computed. One of these unrestricted methods regresses each endogenous variable on all the predetermined variables and then corrects for autocorrelation using Durbin's method (URDUR). The other unrestricted method simply uses ordinary least squares on each equation (UROLS), thus ignoring autocorrelation when used for forecasting.

Comparison of Structural Coefficients

The estimated structural coefficients obtained by 2SLS and LISEM, which corrects for autocorrelation, are compared with respect to bias and mean squared error (MSE). The mean squared error is computed as

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(a_i - \hat{a}_i)^2$$

where a_i is the estimate of the parameter a_i and $n = 50$.

Forecast Evaluation

Historical and *ex post* forecasts for the endogenous variables are obtained using the four different reduced-form equations RR2SLS, RRLISEM, URDUR, and UROLS. The *ex post* forecasts obtained from estimators correcting for autocorrelation are adjusted by forecasting the residuals in the *ex post* periods as suggested by Goldberger. Having obtained estimates of $e_{1,45}$ and $e_{2,45}$ as given in Goldberger, the five forecasts for $e_{i,45+j}, j = 1, 5$ are computed as

$$\hat{e}_{i,45+j} = \rho^j e_{i,45}.$$

These structural residuals are then converted to forecasted reduced-form residuals using the estimated matrix of endogenous variable coefficients and are added to the forecasted means of the *ex post* observations. For the forecasts obtained from the URDUR estimator, the out-of-sample residuals are obtained similarly except that the estimates $\hat{e}_{1,45}$ and $\hat{e}_{2,45}$ are obtained using the direct estimates of the reduced-form coefficients.

Forecast accuracy evaluation is based on quantitative and qualitative evaluation methods. Percentage root mean square error (*PRMSE*) is used for quantitative measurement and a turning point method suggested by Naik and Leuthold (1986) as qualitative evaluation. The turning point method is considered qualitative because it examines only the direction of movements of actual and predicted values of the variable. It does not account for the magnitude of the difference between actual and predicted values, which is accounted for by *PRMSE*. The formula used for *PRMSE* is

$$PRMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{A_i - P_i}{A_i}\right)^2} \times 100,$$

where A_i, P_i, and n are actual values, predicted values, and number of forecast periods, respectively. The *PRMSE*s of the fifty replications are averaged for each level of autocorrelation and are used for comparison.

The turning point method suggested by Naik and Leuthold makes use of a 4 × 4 contingency table as shown in table 1. In this table a peak ($\wedge$) turning point (*PTP*) is defined as $A_{t+1} < A_t > A_{t-1}$, a trough ($\vee$) turning point (*TTP*) is $A_{t+1} > A_t < A_{t-1}$, an upward ($\nearrow$) no turning point (*UNTP*) is $A_{t+1} > A_t > A_{t-1}$ and downward ($\searrow$) no turning point (*DNTP*) is $A_{t+1} < A_t < A_{t-1}$. The elements f_{ij} in table 1 are the number of *PTP, TTP, UNTP,* or *DNTP* of actual values corresponding to the type of the direction of movements indicated by the forecast values. For example, f_{11} refers to the number of times PTP of actual values correspond to the PTP of forecast values for fifty replications for a particular estimator. The ratio of accurate-to-worst forecast (*RAWF*) measuring the qualitative accuracy is

$$RAWF = \frac{f_{11} + f_{22} + f_{33} + f_{44}}{f_{12} + f_{21} + f_{34} + f_{43}}.$$

The higher the ratio, the better is the forecast of a given estimator.

Results

The accuracy of the means of the estimated structural coefficients from the fifty samples

Table 1. 4 × 4 Contingency Table for Evaluating Qualitative Performance of Forecasts

	Forecasted Values			
	Peak (∧) Turning Point (*PTP*)	Trough (∨) Turning Point (*TTP*)	Upward (↗) No Turning Point (*UNTP*)	Downward (↘) No Turning Point (*DNTP*)
ACTUAL VALUES: Peak (∧) Turning Point (*PTP*)	f_{11}	f_{12}	f_{13}	f_{14}
Trough (∨) Turning Point (*TTP*)	f_{21}	f_{22}	f_{23}	f_{24}
Upward (↗) No Turning Point (*UNTP*)	f_{31}	f_{32}	f_{33}	f_{34}
Downward (↘) No Turning Point (*DNTP*)	f_{41}	f_{42}	f_{43}	f_{44}

Note: f_{ij}'s are the number of times *PTP, TTP, UNTP,* or *DNTP* of the actual values corresponding to the type of the direction of movements indicated by the forecast values.

for the 2SLS and LISEM estimators for different degrees of autocorrelation is shown in appendix 1. Also reported are the ratio of the mean of the fifty estimates for a given experiment less the true population parameter, this difference divided by the standard error of the mean. It is assumed that this ratio has a standard normal distribution. The ten mean autocorrelation coefficients estimated by the LISEM estimator are significantly different from the true coefficients at the 5% level in both equations. A summary of the number of mean coefficients significantly different from actual values at the 5% level is presented in table 2.

Table 3. Mean Condition Index for the Structural Equations Estimated by LISEM

Equation	Autocorrelation Coefficients .00	.20	.40	.60	.80
PR	55.36	51.18	45.78	39.87	35.55
QCP	83.93	73.98	60.30	44.59	29.19

Table 2 shows that for 2SLS estimation the percentage of mean coefficients significantly different from the actuals increases with the increase in the degree of autocorrelation, whereas the percentage increases and then decreases with the increase in the autocorrelation for LISEM estimation. Most of these changes come mainly from the demand equation (PR). The 2SLS estimator performs better for the demand equation, especially when the autocorrelation is low. In general, table 2 shows that for low autocorrelation the 2SLS estimator is better in terms of bias of the coefficients. A plausible explanation for such a high percentage of significantly different coefficients is the presence of a high degree of multicollinearity in the data. Cragg (1967) found that the presence of multicollinearity can produce substantial increases in the bias of consistent estimators. In order to assess the extent of multicollinearity present in the LISEM estimation, the condition indices are calculated for the forty-five observations in (12) and (13), and the means of these values for the fifty samples are presented in table 3. This table shows the presence of high multicollinarity for all levels of autocorrelation for both the supply and demand equations.

Table 4 gives the *MSE* of the coefficients for each level of ρ_i. In general, increases in the ρ_i are associated with increased *MSE*. However, the increase in *MSE* is greater for 2SLS than

Table 2. Number of Mean Coefficients Significantly Different from Actuals for 2SLS and LISEM for Different Degrees of Autocorrelation

	Autocorrelation Coefficients									
	.00		.20		.40		.60		.80	
Equation	2SLS	LISEM	2SLS	LISEM	2SLS	LISEM	2SLS	LISEM	2SLS	LISEM
PR	0	2	0	2	1	3	2	3	2	2
QCP	4	5	5	5	5	5	5	5	5	5
Total	4	7	5	7	6	8	7	8	7	7
Percentage	44.4	77.8	55.5	77.8	66.7	88.9	77.8	88.9	77.8	77.8

Table 4. Mean Squared Error of Coefficients for Different Degrees of Autocorrelation

Equations	Independent Variables	Autocorrelation Coefficients .00 2SLS	.00 LISEM	.20 2SLS	.20 LISEM	.40 2SLS	.40 LISEM	.60 2SLS	.60 LISEM	.80 2SLS	.80 LISEM
PR_t	QRD_t	.352E − 3	.510E − 3	.433E − 3	.635E − 3	.590E − 3	.812E − 3	.938E − 3	.106E − 2	.338E − 2	.139E − 2
	QBD_t	.146E − 3	.170E − 3	.148E − 3	.162E − 3	.164E − 3	.146E − 3	.206E − 3	.116E − 3	.320E − 3	.900E − 4
	INC_t	.210E − 4	.220E − 4	.280E − 4	.280E − 4	.410E − 4	.410E − 4	.720E − 4	.710E − 4	.190E − 3	.220E − 3
	Intercept	.142E + 4	.188E + 4	.191E + 4	.245E + 4	.280E + 4	.342E + 4	.475E + 4	.522E + 4	.124E + 5	.118E + 5
	ρ_1		.40E − 1		.46E − 1		.51E − 1		.53E − 1		.57E − 1
QCP_t	PR_t	.109E + 2	.173E + 2	.132E + 2	.204E + 2	.164E + 2	.231E + 2	.213E + 2	.241E + 2	.715E + 2	.229E + 2
	$PCORN_{t-10}$	.158E + 4	.191E + 4	.222E + 4	.247E + 4	.324E + 4	.320E + 4	.534E + 4	.439E + 4	.179E + 5	.727E + 4
	$COLDPRK_t$	.273E + 0	.359E + 0	.340E + 0	.414E + 0	.426E + 0	.463E + 0	.526E + 0	.462E + 0	.849E + 0	.373E + 0
	SF_{t-6}	.680E − 2	.778E − 2	.725E − 2	.920E − 2	.815E − 2	.106E − 1	.114E − 1	.112E − 1	.303E − 1	.113E − 1
	Intercept	.396E + 6	.591E + 6	.451E + 6	.701E + 6	.524E + 6	.792E + 6	.675E + 6	.816E + 6	.204E + 7	.753E + 6
	ρ_2		.48E − 1		.47E − 1		.47E − 1		.52E − 1		.64E − 1

for LISEM. For lower levels of the ρ_i the *MSE* is lower for 2SLS compared to LISEM, but for higher autocorrelation the *MSE* from 2SLS is higher.

The results in tables 2 and 4 indicate that for low levels of autocorrelation, ignoring the presence of autocorrelation is the best thing to do in terms of bias and *MSE*. When autocorrelation is high, LISEM appears to be biased more frequently than 2SLS, but its lower variances more than compensate in terms of *MSE*.

Forecast Accuracy

The *PRMSE*s for historical and out-of-sample forecasts are reported in table 5. These values show that for the historical (within-estimation sample) forecasts, the accuracy of the forecasts obtained using unrestricted reduced forms is greater than those from the restricted reduced forms. In particular, the forecasts from UROLS are better than those from URDUR. Highest *PRMSE*s are found for the forecasts obtained from RRLISEM. This pattern holds true for all the endogenous variables at all levels of autocorrelation.

In contrast, the pattern of estimator superiority in forecasting out of sample, which is usually of most interest, is found to be quite different from the within-sample pattern in terms of *PRMSE*. For lower levels of autocorrelation, RR2SLS performs better than the other methods; whereas for higher levels of autocorrelation URDUR outperforms other methods for the three endogenous variables, although the margin of superiority of URDUR to RRLISEM is small for $\rho_i = 0.8$.

Table 6 shows that for historical forecasts, the forecasts from the unrestricted reduced forms have generally higher *RAWF*. For high autocorrelation URDUR performs better than the other methods. The lowest ratio is for the forecasts obtained from RRLISEM except when autocorrelation is 0.8. For autocorrelation equal to 0.8 RR2SLS has the lowest ratio. This pattern is observed for all the historical endogenous variable forecasts. For out-of-sample forecasts, in general, RR2SLS performs best in terms of *RAWF* when autocorrelation is low and RRLISEM performs best when autocorrelation is 0.8.

Impact of Increased Sample Size

Since the finite sampling properties of 2SLS and LISEM are difficult at best to derive, the

Table 5. Mean *PRMSEs* for Historical and Out-of-Sample Forecasts of Endogenous Variables for Different Forecasting Methods

Variable	Method	Autocorrelation Coefficients 0.00	0.20	0.40	0.60	0.80
Historical						
PR	RR2SLS	9.31	9.53	10.22	11.41	14.35
	UROLS	8.85	8.82	9.06	9.63	10.77
	RRLISEM	9.51	9.72	10.33	11.56	14.52
	URDUR	8.90	8.90	9.21	9.98	11.79
QCP	RR2SLS	8.08	8.18	8.65	9.59	12.40
	UROLS	7.77	7.76	8.00	8.62	9.89
	RRLISEM	8.58	8.84	9.36	10.36	12.80
	URDUR	7.86	7.88	8.21	9.07	11.01
QRD	RR2SLS	8.01	8.11	8.58	9.51	12.27
	UROLS	7.71	7.70	7.95	8.58	9.85
	RRLISEM	8.48	8.73	9.25	10.25	12.67
	URDUR	7.79	7.81	8.15	9.00	10.94
Out-of-sample						
PR	RR2SLS	8.89	9.21	10.02	11.72	16.04
	UROLS	9.42	9.81	10.53	11.77	13.90
	RRLISEM	9.49	9.91	10.58	11.49	12.67
	URDUR	9.45	9.85	10.52	11.40	12.31
QCP	RR2SLS	8.83	8.97	9.61	11.08	15.31
	UROLS	9.53	9.67	10.08	10.97	13.52
	RRLISEM	9.26	9.53	10.09	10.90	12.73
	URDUR	9.62	9.71	10.05	10.82	12.45
QRD	RR2SLS	8.66	8.81	9.44	10.88	14.98
	UROLS	9.34	9.47	9.88	10.75	13.23
	RRLISEM	9.09	9.36	9.89	10.66	12.38
	URDUR	9.44	9.52	9.84	10.58	12.13

sample size of each replication was increased to 100 for one set of 50 replications and $\rho_i = 0.8$. This makes it possible to speculate about the large-sample properties of the estimators. The data used on the set of exogenous variables correspond to the period from September 1975 through December 1983. Ninety observations are used to estimate the coefficients, and the remaining ten are used for *ex post* forecasts. The mean coefficients for fifty samples and their *MSE*s are presented in table 7.

With the increased number of observations, only three coefficients have means significantly different from actual coefficients when 2SLS is used. However, the total number of coefficients that are significantly different from actual increases for the LISEM estimator. The *MSE*s, in general, are lower for LISEM compared with 2SLS. The increase in the number of observations also helps to reduce the *MSE* for both estimators. The means of the autocorrelation coefficients of both equations are closer to the actuals compared with the autocorrelation coefficients obtained for sample sizes of 50 observations. However, both coefficients are significantly different from the actuals. The *MSE*s of the autocorrelation coefficients decrease considerably with the increase in the sample size.

The *PRMSE*s of the forecasts obtained for this sample size are given in table 8. This table shows that with the increase in the number of observations, the difference in the *PRMSE* of the different methods is reduced. For historical forecasts the ranking of different methods remains almost the same. RRLISEM is the most accurate for *ex post* forecasting, but the margin of superiority over the others is small. However, the number of *ex post* forecast periods being increased to ten, we expect lesser impact of correcting for autocorrelation on forecasting accuracy of the estimators.

Summary and Conclusions

In this study the relative performances of two simultaneous system estimators are evaluated

Table 6. *RAWFs* for Historical and Out-of-Sample Forecasts of Endogenous Variables for Different Forecasting Methods

Variable	Method	Autocorrelation Coefficients 0.00	0.20	0.40	0.60	0.80
Historical						
PR	RR2SLS	2.96	3.03	3.07	3.27	2.49
	UROLS	3.09	3.01	3.33	3.26	2.92
	RRLISEM	2.88	2.90	2.96	3.07	3.29
	URDUR	2.98	3.18	3.53	3.65	4.09
QCP	RR2SLS	4.40	4.53	4.85	4.69	4.26
	UROLS	4.68	4.62	4.93	5.20	5.08
	RRLISEM	4.08	3.89	4.29	4.46	5.00
	URDUR	4.71	4.81	5.13	5.74	7.21
QRD	RR2SLS	3.41	3.66	3.89	3.62	3.00
	UROLS	3.78	3.73	3.79	3.74	3.30
	RRLISEM	2.77	2.98	3.40	3.53	3.46
	URDUR	3.67	3.37	4.06	4.37	4.46
Out-of-sample						
PR	RR2SLS	3.53	2.62	2.41	4.21	2.33
	UROLS	2.43	2.28	2.04	4.92	2.65
	RRLISEM	2.50	2.21	2.26	3.22	2.76
	URDUR	2.41	2.08	2.84	3.64	2.37
QCP	RR2SLS	5.48	5.82	6.59	4.93	4.12
	UROLS	5.33	6.00	7.89	5.75	4.92
	RRLISEM	4.75	5.33	5.25	6.09	5.41
	URDUR	5.55	6.49	7.17	5.53	5.36
QRD	RR2SLS	2.29	3.00	5.08	3.41	2.08
	UROLS	1.93	2.11	3.41	3.22	2.08
	RRLISEM	1.72	2.00	3.53	2.70	2.67
	URDUR	1.96	2.08	3.05	2.47	2.62

using a real-world model and actual data on the predetermined variables. For estimating structural coefficients 2SLS and LISEM are found to be biased. For sample sizes of fifty both 2SLS and LISEM give biased estimates although 2SLS is less likely to do so at low levels of autocorrelation. However, at higher levels of autocorrelation LISEM clearly dominates in terms of *MSE,* although this is not true for low levels of autocorrelation. Thus the preferable estimator for estimating structural coefficients is a function of the degree of autocorrelation. The accuracy of the estimated ρ_i is likely sufficient to indicate the appropriate estimator to use.

For forecasting beyond the sample period, the choice of estimator is also a matter of degree of autocorrelation. For samples of size fifty RR2SLS outperforms the others for low autocorrelation, but estimators acknowledging autocorrelation (URDUR and RRLISEM) are superior for high autocorrelation. It should be observed that within-sample forecast accuracy could lead to the selection of an inferior *ex post* forecast estimator since UROLS dominates in *PRMSE* within the sample. In addition, the most preferred structural coefficients estimator for higher levels of autocorrelation is not the best *ex post* forecast estimator in small samples in terms of *PRMSE* but slightly superior to URDUR for samples of size 100.

Multicollinearity is clearly present in the samples considered. It is likely a major source of bias and variability. Such collinearity is likely present in many real-world models. The impact of multicollinearity in this study is similar to that in a more abstract study (Cragg 1967). This gives support to the notion that the implications of the results of these more abstract experiments have relevance to empirical models. It appears the search for estimators that are robust in the presence of multicollinearity seems clearly worthwhile.

A further extension of this study considering contemporaneous correlation among the error terms of different equations, varying levels of autocorrelation across equations, and the presence of lagged endogenous variables would be

Table 7. Mean Coefficients for 2SLS and LISEM When Sample Size is 100

Equation	Variable	Actual Coefficients	2SLS Coefficients	2SLS MSE	LISEM Coefficients	LISEM MSE
PR_t	QRD_t	−.090	−.079 (1.89)[a]	.00163	−.048 (18.22)	.00199
	QBD_t	.009	.013 (1.18)	.00056	.012 (3.24)	.00005
	INC_t	.012	.012 (−.22)	.00002	.010 (−4.17)	.00001
	Intercept	129.850	111.867 (1.99)	4285.92	90.432 (−10.59)	2231.99
	ρ_1	.8			.665 (−9.37)	.0280
QCP_t	PR_t	3.382	4.111 (1.29)	15.918	.144 (−18.52)	11.976
	$PCORN_{t-10}$	−20.547	−60.717 (−2.63)	13001.6	−33.370 (−1.45)	3998.27
	$COLDPRK_t$	1.209	1.241 (.34)	.44010	.991 (−4.62)	.1562
	SF_{t-6}	.328	.345 (1.29)	.00867	.279 (−6.53)	.00515
	Intercept	−347.21	−400.486 (−.58)	413103.0	289.259 (16.78)	475544.2
	ρ_2	.8			.719 (−5.74)	.016

[a] Figures in parentheses are ratios of the mean of the estimates less its true value divided by the standard error to the mean.

very useful in evaluating alternative estimators of autocorrelated simultaneous systems. The results presented above apply to correcting for autocorrelation when the underlying model is correctly specified and not for the situation where misspecification of the estimated model results in autocorrelation.

[*Received September 1985; final revision received May 1986.*]

Table 8. Mean *PRMSE*s of the Historical and Out-of-Sample Forecasts for Different Methods When Sample Size is 100

Variables	RR2SLS	UROLS	RRLISEM	URDUR
Historical				
PR	13.37	12.35	14.62	13.24
QCP	11.84	11.17	13.67	11.85
QRD	11.81	11.14	13.60	11.81
Out-of-sample				
PR	13.33	14.31	13.05	13.16
QCP	11.44	11.83	10.77	10.81
QRD	11.31	11.68	10.62	10.67

References

Belsley, D. A., E. Kuh, and R. E. Welsch. *Regression Diagnostics, Identifying Influential Data and Sources of Collinearity.* New York: John Wiley & Sons, 1980.

Cragg, J. G. "On the Relative Small Sample Properties of Several Structural Equation Estimators." *Econometrica* 35(1967):89–110.

———. "On the Sensitivity of Simultaneous Equation Estimators to the Stochastic Assumptions of the Models." *J. Amer. Statist. Assoc.* 61(1966):136–51.

Dixon, B. L., and L. J. Martin. "Forecasting U.S. Pork Production Using a Random Coefficient Model." *Amer. J. Agr. Econ.* 64(1982):530–38.

Durbin, J. "Estimation of Parameters in Time-Series Regression Models." *J. Royal Statist. Soc.*, Series B, 22(1960):139–53.

Goldberger, A. S. "Best Linear Unbiased Prediction in the Generalized Linear Regression Model." *J. Amer. Statist. Assoc.* 57(1962):369–75.

Hayenga, M. L., and Duane Hacklander. "Monthly Supply-Demand Relationships for Fed Cattle and Hogs." *Amer. J. Agr. Econ.* 52(1970):535–44.

Judge, G. G., R. C. Hill, W. E. Griffiths, H. Lutkepohl, and T. C. Lee. *Introduction to the Theory and Practice of Econometrics.* New York: John Wiley & Sons, 1982.

Johnston, J. *Econometric Methods,* 2nd ed. New York: McGraw-Hill Book Co., 1972.

Kmenta, Jan. *Elements of Econometrics.* New York: Macmillan Co., 1971.

Leuthold, R. M., and P. A. Hartmann. "A Semi-Strong Form Evaluation of the Efficiency of the Hog Futures Market." *Amer. J. Agr. Econ.* 61(1979):482–89.

Martin, Larry, and A. C. Zwart. "A Spatial and Temporal Model of the North American Pork Sector for the Evaluation of Policy Alternatives." *Amer. J. Agr. Econ.* 57(1975):55–66.

Naik, Gopal, and R. M. Leuthold. "A Note on the Qualitative Forecast Evaluation." *Amer. J. Agr. Econ.* 68(1986):721–26.

———. "Forecasting Monthly Live-Hog Prices via Different Models." *Proceedings of Applied Commodity Price Analysis, Forecasting, and Market Risk Management,* 1985, pp. 21–39.

Simister, L. T. "Monte Carlo Studies of Simultaneous Equation Systems." Ph.D. thesis, University of Manchester, 1969.

Appendix Table 1. Mean of the Estimated Coefficients for Different Degrees of Autocorrelation Both before and after Correcting for Autocorrelation

Equations	Independent Variables	Actual Values of Coefficients	Autocorrelation Coefficients									
			.00		.20		.40		.60		.80	
			2SLS	LISEM	2SLS	LISEM	2SLS	LISEM	2SLS	LISEM	2SLS	LISEM
PR_t	QRD_t	−.090	−.087	−.080	−.086	−.078	−.084	−.073	−.079	−.066	−.074	−.059
			(.94)[a]	(3.32)	(1.33)	(3.71)	(1.91)	(5.15)	(2.64)	(7.49)	(2.07)	(10.1)
	QBD_t	.009	.011	.011	.011	.010	.010	.010	.011	.011	.013	.011
			(.89)	(.82)	(.85)	(.63)	(.84)	(.78)	(.95)	(1.11)	(1.49)	(1.34)
	INC_t	.012	.012	.012	.012	.012	.012	.013	.012	.014	.013	.015
			(.12)	(1.24)	(.29)	(1.42)	(.48)	(1.75)	(.69)	(1.81)	(.89)	(1.63)
	Intercept	129.8	123.7	112.4	121.6	108.8	117.5	100.4	109.9	88.87	95.31	74.75
			(−1.15)	(−3.06)	(−1.34)	(−3.28)	(−1.67)	(−4.07)	(−2.11)	(−4.82)	(−2.28)	(−4.11)
	ρ_1			−.060		.102		.268		.440		.609
				(−2.19)		(−3.60)		(−5.03)		(−6.69)		(−9.41)
QCP_t	PR_t	3.382	1.78	−.169	1.42	−.623	.928	−.963	.107	−1.102	−.170	−.995
			(−3.90)	(−11.5)	(−4.48)	(−13.5)	(−5.32)	(−14.7)	(−7.04)	(−15.6)	(−3.24)	(−15.7)
	$PCORN_{t-10}$	−20.54	2.77	6.38	7.10	8.96	13.6	12.5	24.1	14.6	39.2	16.4
			(5.07)	(5.48)	(5.07)	(5.16)	(5.26)	(5.02)	(5.39)	(4.38)	(3.48)	(3.37)
	$COLDPRK_t$	1.209	1.02	.751	.970	.704	.902	.675	.802	.681	.725	.729
			(−2.78)	(−8.27)	(−3.14)	(−8.84)	(−3.73)	(−8.84)	(−4.75)	(−8.62)	(−4.32)	(−8.89)
	SF_{t-6}	.328	.314	.265	.307	.257	.296	.250	.278	.246	.254	.243
			(−1.22)	(−7.02)	(−1.79)	(−7.66)	(−2.64)	(−8.10)	(−3.73)	(−8.61)	(−3.27)	(−9.18)
	Intercept	−347.2	−125.7	281.6	−65.69	362.4	19.09	418.6	160.1	438.4	215.1	410.8
			(2.63)	(9.94)	(3.23)	(11.2)	(4.10)	(11.8)	(5.49)	(12.3)	(2.99)	(12.5)
	ρ_2			−.067		.103		.262		.426		.593
				(−2.25)		(−3.48)		(−5.79)		(−8.20)		(−9.85)

[a] Figures in parentheses are the ratio of the mean of the estimates less the true value of the parameter divided by the standard error of the mean of the estimates.