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Spatial Auto-correlation in Groundwater Levels

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Spatial Autocorrelation in Groundwater Levels

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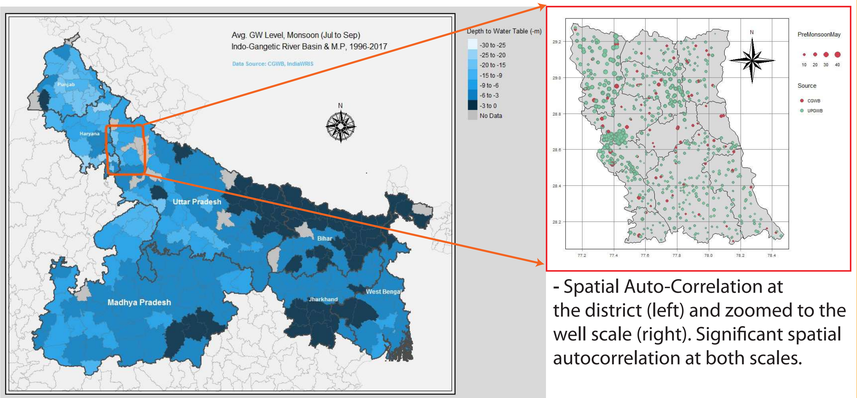
Aim and Scope

To characterize spatial auto-correlation in groundwater levels using a functional instrument called a semivariogram and use it to perform spatial prediction.
To estimate the range within which groundwater levels are spatially auto-correlated. This is useful for determining groundwater "localities" for management of groundwater resources.

Conclusions

Semivariogram was estimated using groundwater levels data for Uttar Pradesh, India in the year 2009, post-monsoon (Aug). Groundwater levels were kriged to get a high-res map for an agro-climatic sub-region of UP and were auto-correlated within a 27 km radius. There was high correlation (0.81) between predicted and observed values .

Spatial auto-correlation implies that groundwater level at a location is more similar to that at nearby locations than further away. The opposite is spatial randomness.



- Spatial Auto-Correlation at the district (left) and zoomed to the well scale (right). Significant spatial autocorrelation at both scales.

- Arises from sub-surface water flows, well interference, spatially-con tiguous climatic conditions and land use land cover (LULC)..
- Can be exploited to perform spatial prediction.
- Needs to be formally characterized in a given realization of data.
- Violates classical assumptions in econometrics if left unaccounted.

Spatial dependence model characterizes the groundwater level at each location in a domain *D* as a realization of a random variable which has a (global) spatial trend component and a (local) spatial dependence component.

A **stochastic spatial process** is a family of indefinitely many random variables indexed by location in a spatial domain *D* in R^2 expressed as *{Y(w) : w in D}* where *Y(w)* refers to the family of random variables and *w* to an arbitrary location in *D* [2].

Intrinsic spatial stationarity requires a constant mean and that the variance of the differences between two random variables separated by spatial lag *h* depend only on *h* and not on absolute positions.

$$E[Y(w)] = \mu \quad \text{for all} \quad w \text{ in } D$$

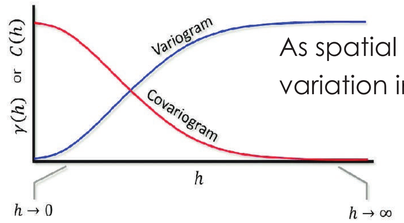
$$Var[Y(w+h) - Y(w)] = 2\gamma(h) \quad \text{for all} \quad w \text{ in } D \text{ and } h > 0$$

where *γ(h)* is called the **"semivariogram"** of the stochastic spatial process [2,3].

References
1. Bárdossy, A. (1997). Introduction to geostatistics. Institute of Hydraulic Engineering, University of Stuttgart.
2. Cressie, N., Cressie, A., & Sons, J. W. . (1991). Statistics for spatial data. J. Wiley. Retrieved from https://books.google.co.in/books?id=k_JQAAAAMAAJ
3. Cressie, N., & Moores, M. T. (2021). Spatial statistics. arXiv preprint arXiv:2105.07216.

Spatial Characterization: the Semivariogram is a measure of spatial *dissimilarity*. It is the inverse of the "covariogram" which is a measure of spatial similarity.

$$\gamma(h) = C(0) - C(h)$$

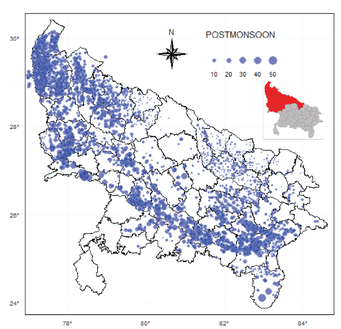


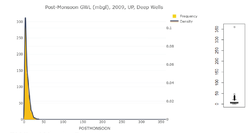
As spatial lag increases, variation in realizations increases.

Semivariogram Estimation and Model Fitting

We estimate a semivariogram from the Western half of the state of Uttar Pradesh. This sub-region is considered distinctly from the Eastern half because of greater uniformity of agro-climatic and hydro-geological conditions and groundwater levels in this region can be reasonably considered as being the realizations of a single stochastic spatial proces.

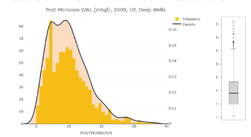
Groundwater Level (mbgl), 2009, Post-Monsoon, Uttar Pradesh (Deep Wells, NA omitted)





GWL - Whole State

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	NA's
0.03	3.0	5.4	7.5	10.0	360.0	26437



GWL - Western Half

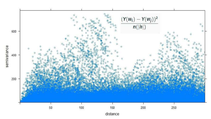
Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
0.4	5.0	9.1	10.0	13.4	36.3

Step 1: Variogram cloud

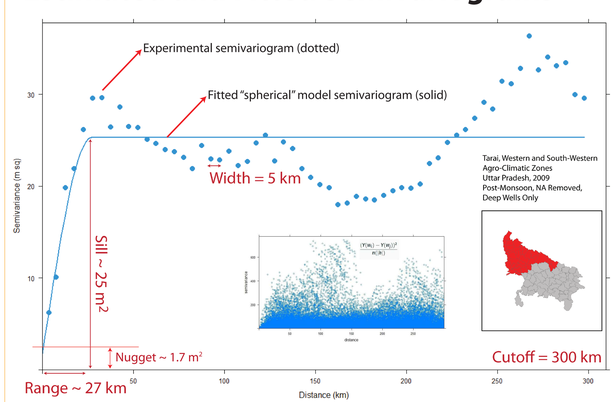
Classical estimator for an isotropic semivariogram: (N. A. Cressie, 1993).

$$2\hat{\gamma}(h) = \sum_{|h|} \frac{(Y(w_i) - Y(w_j))^2}{n(|h|)} \quad (5)$$

$|h|$ - all pairs of wells w_i, w_j distance *h* apart
$n(|h|)$ number of such wells



Estimated and Fitted Semivariograms



Step 2: Experimental Semivariogam

The experimental semivariogram is obtained by binning the points in the cloud and taking expected values within each bin.

Step 3: Fitted Model Semivariogam

A closed form model is fitted to the experimental semivariogram using either MLE, restricted MLE or OLS fitting criteria (Cressie, 1993).

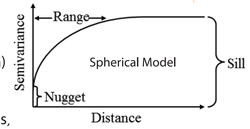
A fitted model can be used to perform spatial prediction as it provides a measure of variance at arbitrary values of spatial lag *h*.

The important parameters of a fitted semivariogram are the nugget, sill and range which are defined below-left and visualized in the figure to the left and below.

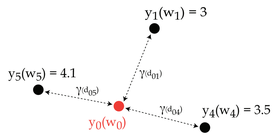
Spherical Semivariogram Model (Cressie, 1993)

$$\gamma(\mathbf{h}) = \begin{cases} 0, & \mathbf{h} = \mathbf{0}, \\ c_0 + c_s[(3/2)(\|\mathbf{h}\|/a_s) - (1/2)(\|\mathbf{h}\|/a_s)^3], & 0 < \|\mathbf{h}\| \le a_s, \\ c_0 + c_s, & \|\mathbf{h}\| \ge a_s, \end{cases}$$

c_0 - Nugget effect (micro-scale variation)
c_s - Partial Sill (max. variance)
a_s - Range (distance of max. variance)
h - Spatial lag (separation between wells,



Spatial Prediction: the Kriging Estimator



$$y_0 = \sum \lambda_i y_i$$

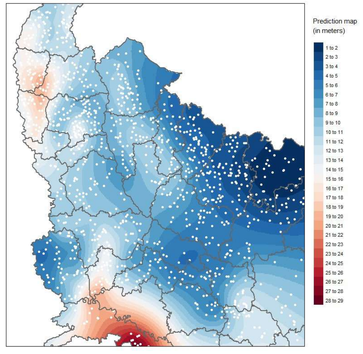
$$\lambda_i = \gamma(d_{0i})^2 / \sum \gamma(d_{0i})^2$$

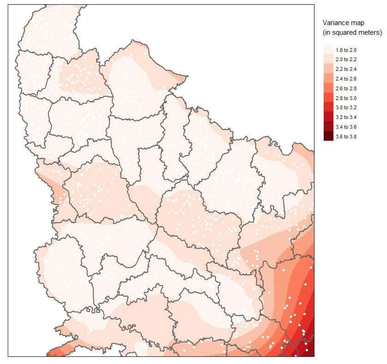
Predict unknown value as a weighted sum of neighboring values using semivariogram, subject to:

1. **Unbiasedness**
$E[\hat{Y}(w0)] = E[Y(w0)] = \mu$
Satisfied if $\sum \lambda_i = 1$ and the mean is stationary i.e $E[Y(wi)] = \mu$ for all *i*, where μ is a constant.
2. **Minimum Variance**
Find weights that minimize prediction variance, σ^2 subject to unbiasedness constraints
$\sigma^2 = Var[\hat{Y}(w0) - Y(w0)]$

Kriging Vs Deterministic Spatial Interpolation

- Accounts for data redundancy due to spatial clustering.
- Provides confidence interval for estimates.

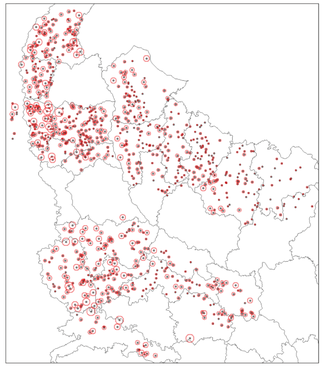




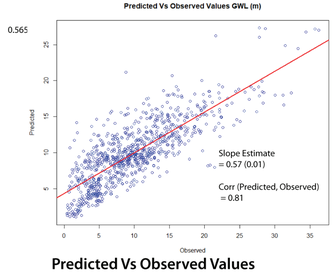
Above Left: spatial prediction of groundwater levels using administrative data for UP, 2009. Right: Prediction variance. White dots represent the wells where observations were made by the monitoring agency. Source of data: UPGWD

Variogram Model Cross Validation

Spherical Variogram Model, Range ~ 27 km, Sill ~ 25 m², Nugget ~ 1.7 m²



Spatial Distribution of Residuals,



Predicted Vs Observed Values