Is The Gravity Model a Power Law?: Evidence from Colombia

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Abstract

I test a new gravity model of international trade in a developing country setting. The model predicts that larger firms export over longer distances than smaller ones; and so the distance elasticity of trade is a result of the economy's firm size distribution. This framework is appealing because it does not make any assumptions about the functional representation of trade costs at the firm-level. Although it works in French data, I find that the model fails in data on Colombian exporters in 2018. Initial regression estimates suggest that the theory underpredicts the distance elasticity of trade in Colombia, which is three times the commonly observed value of 1. An optimization problem using maximum entropy shows that to fit the model's conditions, Colombian firms could grow at a slower implied rate and export more to more distant foreign markets. The latter is unlikely to happen in reality because most of Colombian exports go to the USA, which is a relatively close and attractive export destination. A second optimization problem finds that to reduce Colombia's distance elasticity to a value of 1, its extensive margin needs to go from 50% to approximately 100%. This is improbable to happen as the largest Colombian firms prefer to concentrate their exports in North and South America. My results shed light on where the new theory fails and how it can be generalized to fit other samples. I also show how the model may be used to explain differences in firm export behavior across different countries.

Keywords: Gravity model of international trade, distance elasticity, firm size distribution, export distance, extensive margin.

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1 Introduction

Doubling the distance between two countries typically halves their bilateral trade (Head & Mayer, 2014). Conventional explanations for the unitary distance elasticity of trade[1] explain it as an outcome of two forces: *iceberg* trade costs[2] that rise rapidly with distance and a trade elasticity that parametrizes the response of traders to those iceberg trade costs (Head & Mayer, 2013). Chaney (2018) argues that the remarkable stability of the implied trade cost parameter - over time and space - is difficult to reconcile with changes in transportation technology and trade policies. He posits a new framework that sees the distance elasticity of trade as an outcome of a stochastic process of firm-level growth that produces both a Pareto distribution of firm sizes and a positive link between firm size and the average distance over which the firm exports. He shows that these hypothesized relationships hold up remarkably well in 1992 data from France.

What is still unclear is whether Chaney's prediction about firm level trade data hold true in other settings. France has extremely low trade costs with large neighboring economies, which allows small French firms to begin trading at a relatively low cost. It is also a highly developed economy that hosts multinational firms that trade across the globe.[3] Moreover, France's trade has grown in a relatively stable economic environment that saw tariffs gradually fall between it and its largest trading partners over more than half a century.[4] These conditions provided a setting that has been conducive to a relatively stable process for firm level export growth. A stable growth process for firm exports is the central component of Chaney's theory.

[1] This means that bilateral trade is inversely proportional to distance.

[2] Iceberg trade costs is the notion that part of a shipment is lost (*melts*) on the way to its final destination. Iceberg costs increase with distance, and may include more than transportation and insurance costs (Anderson, 2011).

[3] According to the Organisation for Economic Cooperation and Development (2017), multinational firms account for half of France's gross exports and foreign affiliates for a third.

[4] European Commission (2018) provides a compilation of articles that describe the history of the European Union Customs Union, of which France was a member since 1957.

In this paper, I ask whether Chaney's (2018) predictions hold up in a less conducive environment. The model is estimated on firm-level export data from Colombian in 2018. Like many developing countries, Colombia's postwar trading situation has been much more volatile than has been France's. Both Colombia's geography and the sector composition of its export are quite different than France's. The model underpredicts the observed distance elasticity in my sample, which has a value close to 3.[5] A large distance elasticity arises in the model if there are more smaller firms relative to large ones; or if export distance grows slowly with firm size. However, my sample presents a high share of large firms, and export distance grows faster with firm size than what the model suggests. Using maximum entropy as a penalty function I try to find a minimal set of changes required to make the data fit the theory's conditions and provide sound predictions. This methodology relies on concepts from information theory, and is similar to the one presented in Arndt, Robinson, and Tarp (2002). In a second problem, assuming the theory's conditions are met, I ask what is a minimum set of changes that could reduce the distance elasticity in Colombia to the commonly observed value of 1.

My paper contributes to the international trade literature by testing a novel theory of gravity put forward by Chaney (2018). Similar to Hillberry, Mahlstein, and Schropp (2020) my results unveil *where* and *when* this model fails. The model's predictions are based on an underlying firm-contact creation mechanism where firms penetrate more distant markets as they grow larger. This behavior is consistent for French firms, which enter foreign markets sequentially going to the most popular and closest markets first (Eaton, Kortum, & Kramarz, 2011). The largest Colombian exporting firms in my sample, serve as export platforms of foreign multinational corporations, and concentrate their exports in South America. My results suggest that to fit the model, these large firms should export to more distant and smaller markets in other continents. In reality, this is unlikely to occur because

[5] Head and Mayer (2014) perform a meta-analysis of over 1,800 estimates and find that the distance elasticity of trade hovers around a value of 1 across different samples and methodologies in over 150 years of data.

their parent companies have other offices that may manage exports in those regions. Moreover, some large Colombian exporting firms are cooperatives that gather the production of smaller firms. As a result, cooperatives skew the firm size distribution towards larger firms.

The distance elasticity of trade estimates from my maximum entropy results are decomposed into the *extensive* and *intensive* margins.[6] Fernandes, Klenow, Meleshchuk, Pierola, and Rodriguez-Clare (2018) study the importance of the export distance elasticity's intensive margin in the welfare gains from trade and in export variations across trading partners. They find that for developing countries the intensive margin represents nearly 50%. They suggest that in these economies it may be inappropriate to model trade using a Pareto productivity distribution where all export variation occurs in the extensive margin. In my sample the intensive margin represents nearly half of the distance elasticity of trade. Although Chaney's theory relies on the use of a Pareto firm size distribution, the share of this margin increases when I try to make the data fit the model. On the other hand, to obtain a distance elasticity of 1 in Colombia, the maximum entropy results suggests that the intensive margin must become negligible with all variation occurring in the extensive margin.

This paper also contributes to the literature on power laws. Many economic variables exhibit a power law behavior and a whole body of literature attempts to explain this phenomena (Gabaix, 2009). The Pareto distribution is a power law[7] that possesses a single shape parameter. In Chaney (2018), similar to Steindl (1965), this parameter is independent of firm size and results from a constant birth rate, and a constant growth rate of exporting firms. The instance when firm growth rate is constant and independent of firm size is referred to as Gibrat's Law after Gibrat (1931). If Gibrat's Law is satisfied, the distance elasticity of trade converges to 1.[8] Consistent with Eaton, Eslava, Kugler, and

[6] Extensive margin is the number of exporters. Intensive margin is average exports per exporter.

[7] It is the counter-cumulative distribution $(1 - CDF)$ of the Pareto that is a power law.

[8] Gabaix (1999) makes a similar point when studying the Pareto distribution of city sizes.

Tybout (2007), I find that Gibrat's Law fails in Colombia. It seems that this is one of the main reasons why the model fails in my sample and why the distance elasticity is larger than 1.

The rest of the paper is organized as follows. Section 2 describes the theoretical framework. Section 3 describes the data, the variable definitions and the parameter estimation methods. Section 4 presents estimation results and statistical tests. Sections 5 briefly describes the maximum entropy problem. Section 7 discusses the maximum entropy results. Section 8 concludes.

2 Theoretical Framework: Conditions for Gravity

The gravity equation of international trade states that bilateral trade between countries A and B ($Trade_{A,B}$), normalized by the product of their sizes, is inversely proportional to geographic distance that separates them ($Distance_{A,B}$). Mathematically, this relation is given by

$$Trade_{A,B} = \frac{1}{Distance_{A,B}^{\zeta}} \tag{1}$$

where $\zeta > 0$ is the distance elasticity of international trade.

Chaney (2018) links the functional representation of gravity to the literature on power laws. The intuition behind his model is simple. If larger firms usually export over longer distances than smaller ones, then the negative impact of distance on aggregate trade depends on the number of large firms relative to small ones. This effect is stronger if there are fewer large firms; and it is milder if export distance rises faster with firm size. Based on this, Chaney (2018) provides three sufficient conditions that result in a constant distance elasticity of trade over long distances. These are described in Proposition 1 of his paper, which I detail below.

Proposition 1. *If the following conditions hold:*

i) firm sizes (given by the value of firms' total exports) follows a Pareto distribution *over the support* $[K_{min}, +\infty)$ *with shape parameter* $\lambda \geq 1$*, such that*

$$P(k \leq K) = F(K) = 1 - \left(\frac{K_{min}}{K}\right)^{\lambda}$$

and

$$f(K) = \lambda \left(\frac{K_{min}^{\lambda}}{K^{\lambda+1}}\right)$$

where $F(K)$ *and* $f(K)$ *are, respectively, the C.D.F and the P.D.F of firm size* K*;*

ii) the average squared distance of exports is an increasing power function of firm size with parameter $\mu > 0$ *such that*

$$\int_0^{\infty} x^2 g_K(x)dx = K^{\mu}\left[\int_0^{\infty} x^2 g_{K_{min}}(x)dx\right]$$

where $g_K(x)$ *is the fraction of exports shipped at a distance* x *by firms of size* K*. It is further imposed that* $f_K(x)$ *and* $f'_K(x)$ *are bounded from above and that* $f_K(x)$ *is weakly decreasing above some threshold* $\bar{x}$*; and*

iii) $\lambda < 1 + \mu$

then ζ*, the elasticity of aggregate trade between two countries* A *and* B *normalized by country size* ($Trade_{A,B}$) *with respect to distance, is asymptotically constant:*

$$Trade_{A,B}(Distance_{A,B} = x) \underset{x\to\infty}{\propto} \frac{1}{x^{\zeta}} \quad \text{with } \zeta = 1 + 2(\lambda - 1)/\mu$$

This proposition further implies that if the distribution of firm sizes follows Zipf's Law, i.e. $\lambda \approx 1$, then the distance elasticity of trade is close to 1 ($\zeta \approx 1$).

The full mathematical proof of Proposition 1 may be found in Chaney (2018). For the purposes of this paper, I discuss the intuitive part of some of the proof's components.

The share of firms of every size K is given in condition (i) by $f(K)$. From condition (ii), the probability that a firm of size K exports to an average distance x is given by

$g(x|K) \equiv g_K(x)$. Using Bayes' rule, the joint probability distribution of K and x, $h(K,x)$ is

$$h(K,x) = g(x|K) \cdot f(K) = g_K(x) \cdot f(K)$$

Then, the value of aggregate exports φ shipped to a distance x, is

$$\varphi(x) \propto \int_0^\infty [K \cdot g_K(x)] \cdot f(K) dK \tag{2}$$

or using the joint probability distribution, I can rewrite is as

$$\varphi(x) \propto \int_0^\infty K \cdot h(x,K) dK \tag{2'}$$

Chaney (2018) demonstrates mathematically that as $x \to \infty$, aggregate exports are inversely proportional to distance according to the following

$$\varphi(x) \underset{x\to\infty}{\propto} \frac{1}{x^\zeta}; \quad \zeta = 1 + 2(\lambda - 1)/\mu \tag{3}$$

This result does not require any assumption about the effect of distance on trade at the firm-level. Instead, it treats gravity as a power law resulting from two growth processes that are also power laws. One is the growth of firms in the economy, and the other is the growth average squared distance with respect to firm-size.

Conditions (i) and (ii) are derived endogenously from a model of firm-contact creation and condition (iii) has a structural interpretation. This model is summarized in Appendix A.

3 Regression Specifications

In this section I describe the data set, the variables and the regressions specified for parameter estimations.

3.1 Data

To estimate the parameters of firm size distribution and average squared distance, I use firm-level export data from Colombia in 2018. This data set is publicly available from DANE (2020),[9] and for each exporting firm it details the destination, the product and the F.O.B. value of its exports. To allow for product heterogeneity, I only consider exports of differentiated goods identified with the Rauch (1999) classification.[10] Like Chaney (2018), I restrict my sample to firms that export over US$200,000 in a single year. This cutoff is close to the typical truncation point to estimate the Pareto distribution Eeckhout (2004).[11]

Aggregate bilateral exports from Colombia are calculated as the sum firm-level exports across all firms. I match this data with the distance data between countries and bilateral trade flows from CEPII.[12]

To make comparisons with my results, I use the data on French exporters in 1992 obtained from the Appendices of Chaney (2018) and Chaney (2022). For each French exporting firm I know the destination and the monetary export value (in French Francs) of its exports. This data set also contains the total aggregate imports and total GDP of French export destinations in 1992.

3.2 Variable Definition and Parameter Estimation

Data treatment, variable definitions and parameter estimation methods in this section rely heavily on Chaney (2018).

Following Axtell (2001), all Colombian exporters are ordered in an increasing order of size, where a firm's size is the F.O.B value of its total worldwide exports. I construct 50

[9] DANE stands for *Departamento Administrativo Nacional de Estadística*

[10] I use the liberal classification. Besides differentiated goods, the other categories are *homogeneous* and *referenced priced* goods. These categories are mutually exclusive.

[11] The typical truncation point is at ln size equal to 12 on the horizontal axis. In this case, $\ln(200,000) \approx 12$

[12] CEPII stands for *Centre d'Études Prospectives et d'Informations Internationales*. See Mayer and Zignago (2011) for details on the countries' bilateral distance database. See Gaulier and Zignago (2010) for details on the bilateral trade flow database.

bins of equal log width, $b = 1, ..., 50$, ranging from \$200,000 to the largest annual amount exported by a single firm in 2018.

The average firm size in bin b is given by the following equation

$$K_b = \frac{\sum_i \sum_c exports_{ic} \mathbf{1}[i \in b]}{\sum_i \mathbf{1}[i \in b]} \tag{4}$$

where $exports_{ic}$ are total exports of firm i to country c, and $\mathbf{1}[\cdot]$ is the indicator function.

The share of firms larger than K_b is given by

$$1 - F(K_b) = \frac{\sum_{b'=b}^{50} \sum_i \mathbf{1}[i \in b']}{\sum_{b''=1}^{50} \sum_i \mathbf{1}[i \in b'']} \tag{5}$$

The average squared distance of exports among firms in bin b is

$$\Delta(K_b) = \sum_c (Distance_{Col,c})^2 \left(\frac{\sum_{i \in b} \mathbf{1}[exports_{i,c} > 0]}{\sum_{c'} \sum_{i \in b} \mathbf{1}[exports_{i,c'} > 0]} \right) \tag{6}$$

where $Distance_{Col,c}$ is the distance between Colombia and country c. The intuition of (6) is the following. In each bin b there are dozens of firms. From this large number of firms, the frequency at which firms in bin b export to country c is given by the second term in parenthesis in the right hand side. This term serves as the empirical proxy for $g_{K_b}(Distance_{France,c})$.

I want to estimate the shape parameter of the FSD (λ), the size elasticity of average squared distance of exports (μ), and the export distance elasticity (ζ).

The shape parameter of FSD, λ is estimated via ordinary least squares (OLS) from

$$\ln[1 - F(K_b)] = a - \lambda \ln(K_b) + \varepsilon_b \tag{7}$$

where ln is the natural log, and ε_b is an error term. Given that there is a firm export cutoff value, I need the non-zero constant term a in my regression estimation.[13]

The size elasticity of average squared distance, μ, is estimated by OLS from

$$\ln[\Delta(K_b)] = a + \mu \ln(K_b) + \varepsilon_b \tag{8}$$

where ε_b is an error term.

The aggregate export distance elasticity ζ is estimated by OLS from

$$\ln(Exports_{Col,c}) = a + b \ln(Imports_c) - \zeta \ln(Distance_{Col,c}) + \varepsilon_c \tag{9}$$

where $Exports_{Col,c}$ is the sum of all exports from Colombia to country c;[14] $Imports_c$ are the total worldwide imports of country c; $Distance_{Col,c}$ is the distance between Colombia and c; and ε_c is an error term.

Distance elasticity is decomposed into its extensive (number of exporters) and intensive (average exports per exporter) margins. Using OLS, I estimate

$$\ln(N_{Col,c}) = a + b \ln(Imports_c) - \zeta_{ext} \ln(Distance_{Col,c}) + \varepsilon_c \tag{10}$$

$$\ln\left(\frac{Exports_{Col,c}}{N_{Col,c}}\right) = a + b \ln(Imports_c) - \zeta_{int} \ln(Distance_{Col,c}) + \varepsilon_c \tag{11}$$

where $N_{Col,c}$ is the number of exporting firms that reach destination c; ζ_{ext} is the extensive margin of distance elasticity of trade; ζ_{int} is the extensive margin of distance elasticity; ε_c are error terms. Given log-additive properties $\zeta = \zeta_{ext} + \zeta_{int}$ (Hillberry & Hummels, 2008; Hillberry & McCalman, 2016; Hummels & Klenow, 2005).

Since Chaney's (2018) Proposition 1 is an asymptotic result, I estimate (9), (10) and

[13] This is the main regression I use to estimate λ and that is specified in the text of Chaney (2018). However, according to its appended code Chaney (2018) estimates λ is estimated using firm rankings: $\ln[Firm\ Rank] = a - \lambda \ln(K_b) + \varepsilon_b$. This is specification is similar to the one found in Gabaix (1999) to estimate a city size distribution. I present results using both specifications.

[14] *Exports* with capital letter denotes annual aggregate exports; and *exports* with lowercase denotes annual exports for a single firm.

(11) under two specifications: (a) considering all destinations, (b) considering destinations that are at least 2,000 km. from Colombia.

4 Testing Proposition 1 across Colombian Firms

Figure 1 shows the relationships described in conditions (i) and (ii) from Proposition 1 in Colombian and French data. French data results are replicated from Chaney (2022). The right panel plots the counter cumulative distribution (1-CDF) of firm sizes against exporting firm size in million USD. French firms are larger than Colombian firms in my sample. French FSD seems well approximated by Zipf's Law, $1 - F(K) \propto K^{-\lambda}$ with an estimated shape parameter $\lambda = 0.9707$; while Colombia has $\lambda = 0.8853$. This means that there is more larger firms relative to smaller ones in Colombia compared to France. The left panel plots the average squared distance against exporting firm size. Most Colombian firms export to longer distances than French firms of similar sizes. Average squared distance also seems to be well approximated by a power function in France as in condition (ii) with $\mu = 0.1131$; while in Colombia this relationship seems weaker with an estimated $\mu = 0.0789$. According to Proposition 1, this means that export distance frows faster with firm size in France compared to Colombia.

Statistical tests for conditions (i)-(ii) and Proposition 1 are presented in Table 1. Panel A shows results for Colombia in 2018, and Panel B shows results for France in 1992.

In Panel A, it may be seen that condition (i) is not met in the Colombian data. Although the R^2 from estimating equation (7) is 0.9478, the point estimate of the shape parameter λ is 0.8853 ($SE = 0.0313$). This means that a 95% confidence interval does not contain a value of 1 or greater as needed in condition (i). In Colombian, the relationship between firm size and average squared distance does not seem to be log-linear as stated in condition (ii). The R^2 from estimating equation (8) is 0.1573. The firm size elasticity of average squared distance, μ, is 0.0789 ($SE = 0.026$). Following Chaney (2018), I test test the parameter restriction $\lambda < 1 + \mu$ from condition (iii) by forming the probability

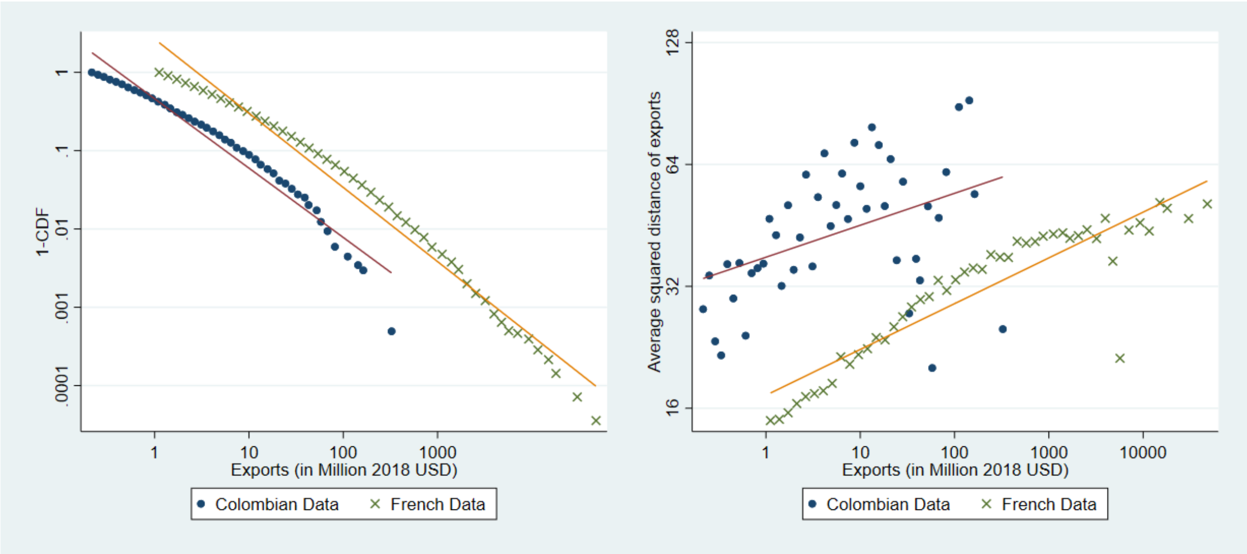

Figure 1. Conditions (i) and (ii) in Colombia and France
Data Source: DANE, CEPII, Chaney (2018)
Notes: The French export data is calculated in French Francs (FRF) in 1992. To convert it to USD I used the 12-month FRF to USD average exchange rate in 1992 published by FRED (2022). Then, to covert from 1992 USD to 2018 USD I used the CPI inflation calculator from the U.S. BLS (2022). The straight lines in the right and left panels correspond to fitted linear regression lines from estimating equations (7) and (8).

$\Pr(\lambda \leq 1 + \mu)$ by estimating λ and μ from 10,000 bootstrapped samples. Given that the shape parameter λ is less than 1 in my sample, condition (iii) is violated for 0.04% of the estimates only.

The estimated distance elasticity of exports[15] for "long" distances (> 2,000 km.) in Colombia is 2.8211 ($SE = 0.4007, R^2 = 0.6249$); and when I use all distances this elasticity is 2.8785 ($SE = 0.2026, R^2 = 0.6794$). On the other hand, the predicted distance elasticity of exports $1 + 2(\lambda - 1)/\mu$ is -1.9066 ($SE = 104.91$), where its standard error is calculated using the 10,000 bootstrapped samples. The predicted value is far from the actual distance elasticities for long and all distances. Also, its sign contradicts the gravity theory. To formally test the equality of the between the actual and predicted distance elasticities of exports, I form a Wald test for the equality $\zeta = 1 + 2(\lambda - 1)/\mu$. The Wald test statistic has a high p-value (0.9641), meaning that I cannot reject the hypothesis that both values are statistically different. However, the high standard errors for the predicted distance

[15] Given that I use export data, from now I use the terms *distance elasticity of trade* and *distance elasticity of exports* interchangeably.

Table 1
Testing Proposition 1

Panel A: Test of Proposition 1 for all Colombian firms with exports above 200 thousand US dollars in 2018							
Condition (i): distribution of firm sizes	$\lambda =$	0.8853	(SE =	0.0313	,	$R^2 =$	0.9479)
Condition (ii): average squared distance of exports	$\mu =$	0.0789	(SE =	0.0260	,	$R^2 =$	0.1573)
Condition (iii) parameter restriction							
($\lambda < 1 + \mu$)	$\Pr(\lambda \geq 1 + \mu) =$	0.04%					
Distance elasticity of trade							
All distances	$\zeta_{all} =$	2.8785	(SE =	0.2026	,	$R^2 =$	0.6794)
Long distances (>2,000 km.)	$\zeta_{long} =$	2.8211	(SE =	0.4007	,	$R^2 =$	0.6249)
Predicted distance elasticity of trade	$1 + 2(\lambda - 1)/\mu =$	-1.9066	(SE =	104.9112	)		
Proposition 1:							
Wald test for $\zeta_{all} = 1 + 2(\lambda - 1)/\mu$	p-value of χ^2 test =	0.9641					
Wald test for $\zeta_{long} = 1 + 2(\lambda - 1)/\mu$	p-value of χ^2 test =	0.9641					
Panel B: Test of Proposition 1 for French firms with exports above 1 million French Francs in 1992							
Condition (i): distribution of firm sizes	$\lambda =$	0.9707	(SE =	0.0289	,	$R^2 =$	0.9770)
Condition (ii): average squared distance of exports	$\mu =$	0.1131	(SE =	0.0084	,	$R^2 =$	0.8170)
Condition (iii) parameter restriction							
($\lambda < 1 + \mu$)	$\Pr(\lambda \geq 1 + \mu) =$	0.00%					
Distance elasticity of trade							
All distances	$\zeta_{all} =$	0.7672	(SE =	0.0979	,	$R^2 =$	0.8100)
Long distances (>2,000 km.)	$\zeta_{long} =$	1.1854	(SE =	0.2498	,	$R^2 =$	0.7160)
Predicted distance elasticity of trade	$1 + 2(\lambda - 1)/\mu =$	0.4824	(SE =	0.5141	)		
Proposition 1:							
Wald test for $\zeta_{all} = 1 + 2(\lambda - 1)/\mu$	p-value of χ^2 test =	58.627%					
Wald test for $\zeta_{long} = 1 + 2(\lambda - 1)/\mu$	p-value of χ^2 test =	21.868%					

Source: DANE, CEPII, Chaney (2022)
Note: This table tests Proposition 1 from Chaney (2018) on two sets of data. Panel A shows results from using data on all 2,026 Colombian firms with total worldwide exports above 200 thousand US dollars in 2018. The parameter λ is the shape coefficient for the distribution of firm sizes, estimated from (5); μ is the size elasticity of average squared distance of exports with respect to firm size, estimated from (6); $\Pr(\lambda \geq 1 + \mu)$ is calculated using 10,000 bootstrapped estimates of λ and μ; ζ is the distance elasticity of aggregate trade (exports) from (9). The standard error of the predicted distance elasticity of trade, $1 + 2(\lambda - 1)/\mu$, is computed using 10,000 bootstrapped estimates of λ and μ. The p-value for the Wald test of $\zeta = 1 + 2(\lambda - 1)/\mu$ is computed by comparing the Wald Statistic W $= \{\zeta - 1 + 2(\lambda - 1)/\mu\}^2/\{Var(\zeta) + Var[1 + 2(\lambda - 1)/\mu]\}$ to a χ^2, where $Var[1 + 2(\lambda - 1)/\mu]$ is calculated using 10,000 bootstrapped estimates of λ and μ. Robust standard errors and adjusted R^2 are presented in parentheses. Panel B replicates results obtained from Chaney (2022) that tests Proposition 1 for all French firms with exports above 1 million French Francs in 1992.

elasticity are likely driving this result.

Panel B presented the results for French firms. I skip the analysis of these results since they are discussed in Chaney (2018) and in its subsequent paper Chaney (2022). Instead, I focus on making comparisons with the Colombian results using the model's intuition. The differences in the shape parameter of the FSD, λ means that the firm's implied net growth rate is larger than the implied exporting firm population growth[16] in Colombia compared to France ($0.8853 < 0.9707$). Similarly, a the difference shape

[16] The model assumes that the "birth" of new exporting firms occurs when a firm starts exporting.

parameter μ from condition (ii) means that the implied gross firm growth rate is smaller than the implied net growth rate of firms in Colombia compared to France ($0.0789 < 0.1131$). According to Chaney (2022), conditions (i)-(iii) from Proposition 1 are satisfied and the Wald test statistics suggest that the model's prediction is not statistically different from the actual estimated distance elasticity of trade in the French data. Based on the Wald tests, the model is not failing in Colombia either. However, if $\lambda < 1$ the first central moment of the FSD does not exist. Second, the value of λ being less than 1 is one of the reasons why the predicted distance elasticity of exports is negative. This means that exports are increasing with distance, which contradicts the gravity theory. Another problem is that condition (ii) is not precisely met. This results in μ having a large standard error. In addition, the predicted distance elasticity of exports has a very high standard error in Colombia. This is likely the result of the non-linearity of its formula and the high standard error of μ.

5 Maximum Entropy Problem

In this section I briefly explain the setup and intuition of the maximum entropy problem, and the different exercises specified for the analysis.

5.1 Problem Dimensions and Objective Function

Given that conditions (i) and (ii) do not hold in Colombian data, I try to find a set of changes I can make so these conditions hold. I prefer these changes to be as small as possible so that I can analyze a group of minimum of changes that that I could impose to my sample and make the Chaney (2018) model work. To do this, I setup an optimization problem using maximum entropy in a penalty function. Here I explain the intuition behind the problem's setup and the objective function. All problem's equations and constraints are specified in Appendix C

Aggregate exports are distributed across hundreds of firm size and export distance pairs (K, x) that are observed with probability $h(K, x)$.[17] Following Shannon (1948), these pairs (combinations) represent the amount of uncertainty (entropy) in the sample. An example helps to illustrate this point. Suppose that all Colombian exporting firms are of a single size and export only to the US, which means that $h(K, x)$ has a single point with value equal 1. In this scenario, if I randomly pick a Colombian firm there is no uncertainty about how big it is and where it exports. If I increase the number of firm size and export distance pairs, $h(K, x)$ distributes over more points increasing the sample's uncertainty. I refer to the observed value of $h(K, x)$ in my sample as the *prior*. My goal is to find a new joint distribution $\hat{h}(K, x)$ that fits condutions (i)-(iii) from Proposition 1, and a set of additional constraints, that is as close to the prior as possible.

To make the optimization problem easier, I *discretize* firm sizes and export distances. I use the same 50 firm-size bins, b, described in Section 3.2; and I use the 155 export destinations, c, reached by Colombian firms in 2018. In this way, there is a finite number of firm sizes and a finite number of distance values. Based on this, there are two dimensions for the problem: b and c. Firms in each bin may go to one or several destinations. Or, firms reaching country c may be from one or several firm size (bin) categories.

I keep the notation K_b to identify the average firm size in bin b. Distance x_c is the geographic distance between Colombia and the destination country c.[18]

The prior of the joint probability distribution of firm size and distance is calculated as

$$h(K_b, x_c) = \frac{\sum_{i \in b} \mathbf{1}[exports_{i,c} > 0]}{\sum_{c'} \sum_i \mathbf{1}[exports_{i,c'} > 0]} \tag{12}$$

I also calculate the priors of other distributions to compare them with the problem's solutions.

[17] This means $0 \leq h(K, x) \leq 1$, and $\int_0^\infty \int_0^\infty h(K, x) dx dK = 1$.

[18] This is the same as $Distance_{Col,c}$ from Section 3.2. To save space and to keep the notation from Proposition 1, I prefer to use x as geographic distance in the problem's formulation.

The prior probability distribution of distance conditional on firm size is given by

$$g(x_c|K_b) = g_{K_b}(x_c) = \frac{\sum_{i \in b} \mathbf{1}[exports_{i,c} > 0]}{\sum_{c'} \sum_{i \in b} \mathbf{1}[exports_{i,c'} > 0]} \tag{13}$$

This is the same as the second term inside the summation on the right hand side of (6).

The prior firm size distribution is calculated using

$$f(K_b) = \frac{\sum_i \mathbf{1}[i \in b]}{\sum_{b'=1}^{50} \sum_i \mathbf{1}[i \in b']} \tag{14}$$

The counter cumulative distribution of firm is given by $1 - F(K_b) = \sum_{b'=b}^{50} f(K_b)$, which is equivalent to (5).

The penalty function is a cross-entropy measure similar to the one used in Arndt et al. (2002) and is given by

$$\max_{\hat{K}_b, \hat{x}_c, \hat{g}_{K_b}(x), \hat{f}(K_b)} -\sum_c \sum_b \left[\hat{h}(K_b, x_c) \cdot \ln\left(\frac{\hat{h}(K_b, x_c)}{h(K_b, x_c)}\right)\right] \tag{15}$$

where the hat (ˆ) refers to the model's solution; and ln is the natural logarithm. This penalty function may be rewritten as

$$\min_{\hat{K}_b, \hat{x}_c, \hat{g}_{K_b}(x), \hat{f}(K_b)} \sum_c \sum_b \left[\hat{h}(K_b, x_c) \cdot \Big(\ln\big(\hat{h}(K_b, x_c)\big) - \ln\big(h(K_b, x_c)\big)\Big)\right] \tag{15$'$}$$

From here it can see that the penalty function tries to minimize the weighted difference between the logged values of $\hat{h}(K_b, x_c)$ and $h(K_b, x_c)$. This allows to interpret the value of the objective function as as the expected value of the required (logged) changes on $h(K_b, x_c)$ to make conditions (i)-(iii) from Proposition 1 hold in the original data. The larger the value of the objective function, the larger the expected change I need to make to the data to make the conditions in Proposition 1 hold.

This minimization problem is subject to a set of constraints, besides conditions (i)-(iii), some of the most important are: the gravity equation, the predicted distance elasticity formula, and the upper and lower bounds of each firm-size bin.

5.2 Exercises

The exercises will be divided in two major groups. In the first two exercises, I try to find a set of minimum changes to make conditions (i)-(iii) from Proposition 1 hold under some constraints. In the last two exercises, under the Chaney (2018) model's conditions, I try to find a group of small changes to make the distance elasticity of trade in Colombia as close as possible as the one observed in France in 1992.

the exercises differ on the value of three different measures: (1) total aggregate exports from Colombia, $Exports$; (2) the total number of exporting firms, N; (3) and the distance elasticity of trade, ζ. In my sample, total aggregate exports reach US$8,459 million in FOB terms, and there is a total of 2,026 exporting firms. Given that the model is specified for long distances, I focus on the distance elasticity of trade for long distances ($>$ 2,000 km.), ζ_{long}, in the maximum entropy exercises.

In Exercises 1 and 2, the problem's solution chooses the distance elasticity of exports under the conditions and prediction of Proposition 1. Exercise 2 is intended to be a robustness check of Exercise 2. A brief description of both exercises is

- *Exercise 1:* Total aggregate exports and the number of exporting firms in the original data is kept unchanged. The solutions picks the distance elasticity of exports to satisfy the gravity equation of international trade and its prediction formula from Proposition 1.

- *Exercise 2:* Same as Exercise 1, but the solution chooses the total value of aggregate exports and the distance elasticity to satisfy the constraints.

For the third and fourth exercises, I target the French distance elasticity of exports, which is estimated to be 1.1854 (Chaney, 2022). Targeting this distance elasticity is important because it is close to the most common value of this parameter across samples from different years and countries (see Head and Mayer (2014)). This means that in the re-arranged data, the distance elasticity of exports should reduce to about a third of the one calculated from the sample. Exercise 4 is intended to be a robustness check for Exercise 3. Both exercises are described below

- *Exercise 3:* Total aggregate exports and the number of exporters from the original sample are kept unchanged. Aggregate distance elasticity of exports is fixed at 1.1854, equal to that of France in 1992.

- *Exercise 4:* Same as Exercise 3 but here I increase the number of exporting firms by 10% from the amount in the original sample to 2,229; and total aggregate exports is chosen by the model.

The solutions to these exercises help understand the difference between the two countries, assuming that the model's conditions are met. These comparison is not trivial. Colombia is a developing country that mostly exports agricultural produce like coffee and flowers and the geography around it is complicated. France is a developed and industrialized nation, and borders other developed countries that can be easily reached.

6 Preliminary Maximum Entropy Results

Table 2 compares parameter estimates from using the original data points against those obtained from the maximum entropy (ME) results of Exercise 1. This ME problem was able to find an optimal solution and satisfy all constraints. Compared to the original data, the estimated shape parameter of the FSD, λ in the ME solutions increases 0.88 to 1.38. The size elasticity of average squared distance, increases from 0.079 to 0.381. This means that one way for the conditions in Proposition 1 to be met in the original data, is to increase the share of smaller firms relative to large ones; and that export distance increases faster with firm size. This does not mean that firm size should decrease, however.[19] These

[19] Please contact the author to obtain these results.

Table 2

Comparison of Proposition 1 parameters from Original Colombian Data vs Maximum Entropy Results (Exercise 1): Firms exporting above 200 thousand USD in 2018

		ME Results		**Colombian Data**
Proposition 1				
Condition (i): distribution of firm sizes	$\lambda =$	1.3807		0.8853
Condition (ii): average sq. distance of exports	$\mu =$	0.3808		0.0789
Predicted distance elasticity of trade	$1 + 2(\lambda - 1)/\mu =$	2.9991		-1.9066
Distance elasticity of trade				
Long Distances (>2,000 km.)	$\zeta_{long} =$	2.9991		2.8211
Extensive Margin	% of $\zeta_{long} =$	37.37%		55.83%
Intensive Margin	% of $\zeta_{long} =$	62.63%		44.17%
Aggregate Variables				
Total Aggregate Exports (US$ Million)	$Exports =$	8,459.14	(fixed)	8,459.14
Total Number of Exporters	$N =$	2,026	(fixed)	2,026

Source: DANE, CEPII

Note: This table compares the estimated parameters of Proposition 1 from the original Colombian data against those calculated from re-arranged data obtained from the Maximum Entropy Exercise 1. The Colombian data is a sample of 2,026 Colombian firms that exported more than US$200,000 in 2018. The maximum entropy problems re-arranges this data under the constraint that conditions (i)-(iii) from Proposition 1 are satisfied. The parameter λ is the shape coefficient for the distribution of firm sizes, estimated from (5). μ is the size elasticity of average squared distance of exports with respect to firm size, estimated from (6); ζ is the distance elasticity of aggregate exports from (9); the extensive and intensive margin of ζ are estimated using (10) and (11) respectively. In the ME results, the standard errors and R^2s are always zero and one respectively. The standard errors and R^2s for the Colombian data are available in Table 1.

changes increase the distance elasticity to 2.99, which is very close to the one from the original sample of 2.8211. In contrast with the Colombian data, the share of extensive margin in the distance elasticity of exports in the ME results decreases from 55.8% and to 37.4%; and the share of the intensive margin increases from 44.2% to 62.6%. In general, the ME solutions were able to make a minimum set of changes to the original data so that the condition's and prediction in Proposition 1 hold. These results are roughly similar to those obtained in Exercise 2, which are described in Appendix C.

Table 3 presents results from ME Exercise 3 and compares it with the results from using the Colombian data, and with results from using the French data. In Chaney (2022), the point estimate of the distance elasticity of trade for long distances is 1.1854 (SE = 0.2498). This estimate becomes my target and is precisely reached by the ME results. Compared to the Colombian data, in the ME results the the shape parameter λ from condition (i) increased from 0.8853 to 1.0413. This result is slightly higher than the point estimate of λ in the French data. The shape parameter μ from condition (ii) increases from

0.079 in the original Colombian data to 0.449 in the ME results. This estimate is four times higher than the point estimate of μ in the French data. Based on the model's intuition, for Colombia to be similar to France, the share of larger exporting firms relative to smaller ones could decrease; and export distance could grow faster with exporting firm sizes. Notice that the size elasticity of average squared distance μ must be higher than the one estimated in France if Colombian exporters reach the same set of destinations as it did in 2018. In the French data, the share of the extensive margin of the distance elasticity of trade is 102%, while the share of the intensive margin is -2.19%.[20] In the ME solution, the share of the extensive margin of the distance elasticity of export goes from 55.8% in the original data to 86.62%; and the share of the intensive margin goes from 44.17% to 13.38%. This suggests that for Colombian firms to be similar to French firms, they must distribute their exports over more destinations than they do in 2018. These results are roughly the same as those obtained in Exercise 4, which are described in Appendix C.

7 Preliminary Conclusions

I test a new theoretical framework proposed by Chaney (2018) that treats the gravity equation of international trade as a power law, in a developing country. This novel theory says that if the size distribution of exporting firms follows a Pareto distribution; and if average squared distance of exports is a power function of firm sizes; then the distance elasticity of trade converges to a constant over long distances. There is evidence that this framework works well in data on French exporters, where all the model's conditions hold (Chaney, 2018, 2022). However, when I apply the framework to data on Colombian exporters, the theory's conditions do not hold and the model does not provide clear predictions.

I use maximum entropy to find a group of small changes that could be applied to the Colombian data so that the model condition's hold under a set of constraints. Among the

[20] I calculated these shares using the firm-countries data set provided in the appendix of Chaney (2022). These estimates are not available in the text.

Table 3

Comparison of Proposition 1 parameters from Original Colombian Data vs Maximum Entropy Results targeting French distance elasticity of exports (Exercise 3): Firms exporting above 200 thousand USD in 2018

		ME Results		Frenh Data	Colombian Data
Proposition 1					
Condition (i): distribution of firm sizes	$\lambda =$	1.0413		0.9707	0.8853
Condition (ii): average sq. distance of exports	$\mu =$	0.4459		0.1131	0.0789
Predicted distance elasticity of trade	$1 + 2(\lambda - 1)/\mu =$	1.1854		0.4819	-1.9066
Distance elasticity of trade					
Long Distances (>2,000 km.)	$\zeta_{long} =$	1.1854	(target)	1.1854	2.8211
Extensive Margin	% of $\zeta_{long} =$	86.62%		102.19%	55.83%
Intensive Margin	% of $\zeta_{long} =$	13.38%		-2.19%	44.17%
Aggregate Variables					
Total Aggregate Exports (2018 US$ Million)	*Exports* =	8,459.14	(fixed)	57,526.29	8,459.14
Total Number of Exporters	$N =$	2,026	(fixed)	27,968	2,026

Source: DANE, CEPII, Chaney (2022)

Note: This table compares the estimated parameters of Proposition 1 from the original Colombian data against, those estimated using the French data from Chaney (2022), and those calculated from re-arranged data obtained from the Maximum Entropy Exercise 3. The Colombian data is a sample of 2,026 Colombian firms that exported more than US$200,000 in 2018. The French data is a sample of approximately 28,000 firms that exported more than 1 million French Francs in 1992 (≈ US$200,000). To convert FRF to USD I used the 12-month average exchange rate in 1992 published by FRED (2022). Then, to covert from 1992 USD to 2018 USD I used the CPI inflation calculator from the U.S. BLS. The maximum entropy problems re-arranges this data under the constraint that conditions (i)-(iii) from Proposition 1 are satisfied, and targets the estimated French distance elasticity of exports. The parameter λ is the shape coefficient for the distribution of firm sizes, estimated from (5). μ is the size elasticity of average squared distance of exports with respect to firm size, estimated from (6); ζ is the distance elasticity of aggregate exports from (9); the extensive and intensive margin of ζ are estimated using (10) and (11) respectively. In the ME results, the standard errors and R^2s are always zero and one respectively. The standard errors and R^2s for the Colombian and French data are available in Table 1.

possible changes, the share of smaller firms relative to large ones could increase in Colombia. This is unlikely to happen since many Colombian exporting firms are large cooperatives that gather the production of smaller firms. Another proposed change is that export distance increases faster with Colombian exporting firm sizes, which is also unlikely to occur in reality. I find that Colombian firms export to longer distances than French firms of similar sizes. A potential reason for this is that firms in Colombia may prefer to reach the US market before reaching other bordering countries, like Ecuador. If small exporting firms reach distant markets at an early stage, it may be difficult to reach more distant markets later. In addition, some large Colombian exporting firms are export platforms of large multinationals. It seems that these firms concentrate their exports in South America, and other foreign markets are served by other offices of the multinationals they belong to.

Then I use maximum entropy to find a set of minimal changes that could be applied to Colombian data to obtains results similar to those found in France, assuming the model conditions are met. In essence, I try to see what are possible changes to reduce the actual distance elasticity of Colombia from a value close to 3 to a value close to 1. Results suggest that to mimic French exporting behavior, Colombia could increase the share of small exporting firms relative to large ones; and export distance could increase faster with firm size. This is in line with my previous findings to make the model work. A difference is that to be like French firms, Colombian firms could export to more destinations than what they actually do. This would increase the share of the extensive margin of the aggregate distance elasticity of exports. To be like France, the share of this margin needs to be approximately 100%.

According to the micro-foundations of the model, for Colombia to fit the model, the implied net firm growth rate needs to be smaller. The new framework relies on Gibrat's Law for firm sizes. It could be that firm growth rate is not independent on firm sizes in Colombia, and so Gibrat's Law does not hold. Future research is needed to generalize the framework proposed by Chaney (2018) so that it considers these aspects that may be happening in Colombia and that do not occur in developed economics such as France. One option is to consider more flexible firm size distributions; or a more flexible relationship between firm size and export distance.

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Appendix A Model of Firm-Contact Network Creation

Chaney (2018) proposes a micro-founded model of firm-contact creation where conditions (i) and (ii) are derived endogenously, and condition (iii) has a structural interpretation. In the model, exporting firms are *born* with a mass of contacts. This means that when firms start exporting they know a group of buyers in a foreign market to which they can sell their product. With time, firms may increase their exports by meeting contacts of its contacts. Each new wave of contacts is potentially further away than the previous one. Simply put, firms become larger and export to longer distances as they grow older.

The complete setup, proofs and derivations may be found in Chaney (2018). I only discuss the model's component that are relevant for my purposes. The main model's assumption are the following:

- Firms are uniformly distributed over an one-dimensional continuous space $\mathbb{R}$. The model is symmetric, so I can focus on firms located at the origin ($x = 0$).

- Time is assumed to be continuous. New exporting firms are born at a rate γ. So, at time t there is the same density of firms $e^{\gamma t}$.

- When a firm is born, it samples a mass K_0 of new contacts among other newborn firms only. K_0 is distributed geographically according to the symmetric density function $k_0(\cdot)$, i.e. the mass of contacts in the interval $[a, b]$ is given by $\int_a^b k_0(x)dx$. Firms are infinitely lived.

- New contacts are continuously created. At any point in time, each existing contact may reveal one of its own contacts according to a Poisson process wit arrival rate β. Existing contacts are continuously lost to an exogenous Poisson shock with rate δ.

- A firm sells its output only to its existing contacts. For simplicity, the value of individual shipments is normalized to one. This means that the number of contacts of a firm K is also a measure of its size.

- It is assumed that $\delta > \beta - \delta > 0$. The assumption $\gamma > \beta - \delta$ rules out a degenerate equilibrium in which older firms become "too" large. The assumption $\beta - \delta > 0$ rules out the counterfactual

Additionally, the model defines the function $k_a(x) : \mathbb{R} \to \mathbb{R}^+$ as the distribution of the contacts of a firm of age a. The total mass of contacts that a firm of age a has worldwide, K_a is thus given by

$$K_a \equiv \int_{\mathbb{R}} k_a(x)dx$$

where $k(a)$ is the density of contacts of firm of age a has in location x.

The law of motion of contact creation is similar to the one in Duffie and Manso (2007) and Duffie, Giroux, and Manso (2010). It says that the mass of contacts evolves recursively according to the partial differential equation

$$\frac{\partial}{\partial a} k_a(x) = \beta \int_{\mathbb{R}} \frac{k_a(x-y)}{K_a} k_a(y)dy - \delta k_a(x) \tag{A.1}$$

with the initial condition $k_0(x)$.

From this model, Chaney (2018) proposes two additional propositions.

Proposition 2. *For any initial distribution k_0 that is symmetric and admits a finite second moment, the normalized distribution of contacts of a firm of age a, $f_a = k_a/K_a$, converges when age a grows large to a Laplace distribution (two-sided exponential),*

$$f_a(x) \underset{a\to\infty}{\sim} \frac{1}{2\sqrt{\Delta_0/2}e^{\beta a/2}} \exp\left(-\frac{|x|}{\sqrt{\Delta_0/2}e^{\beta a/2}}\right)$$

This property holds exactly for all a's if

$$f_0(x) = \frac{1}{2\sqrt{\Delta_0/2}} \exp\left(-\frac{|x|}{\sqrt{\Delta_0/2}}\right)$$

Proposition 3. *If the population of firms grow at a constant rate γ and the contacts of individual firms evolve according to (A.1), then the distribution of firm sizes is Pareto,*

$$F(K) = 1 - \left(\frac{K}{K_0}\right)^{-\gamma/(\beta-\delta)} \quad ; \quad for K \geq K_0$$

, and the average squared distance from a firm's contacts is a power function of its size,

$$\Delta(K) \equiv \int_{\mathbb{R}} x^2 f_K(x) dx = \Delta_0 \left(\frac{K}{K_0}\right)^{\beta/(\beta-\delta)}$$

, where f_K is the distribution of firm with K contacts ($f_K = k_{a(K)}/K$ with $a(K)$ such that $K_{a(K)} = K$) and $\Delta_0 \equiv \int_{\mathbb{R}} x^2 f_0(x) dx$ is the average squared distance from initial contacts.

Appendix B Maximum Entropy Problem

The maximum entropy problem was coded in the General Algebraic Modeling System (GAMS) developed by GAMS Development Corp. (2022). The general setup of the program code is described below.

Dimensions: The variables in my maximum entropy problem have two dimensions listed below

- Bins of firm sizes b: As discussed in Section 3.2, firms are placed in 50 bins according to the total value of their worldwide exports.
- Destination country c: Firms in each bin may export to a total of 155 countries. These are the same export destinations of Colombian firms in 2018.

Choice Variables: For the Maximum Entropy Problem I use the same variables described in 3.2 plus some more variables to describe the solution. In the following list I order these variables according their dimension:

- the exports from bins of bin b to country c, $exports_{bc}$;
- the number of firms from bin b that reach country c, m_{bc};
- total number of firms in bin b, n_b;
- Average Firm Size in bin b, K_b;
- upper and lower bound of firm size of bin b, $ubound_b$ and $lbound_b$;
- cumulative Distribution Function of Firm Sizes, $F(K_b)$;
- average Squared Distance of Exports, $\Delta(K_b)$;
- the firm size distribution parameter, λ; and the size elasticity of average squared distance, μ;

- the distance elasticity of trade ζ; and the size elasticity of trade β

Parameters: There are some values that remain unchanged in the problem, these are described below

- Distance from Colombia to country c in kilometers, x_c.

- Total imports of country c, $Imports_c$

Functions: There is one objective function and several constraints.

Objective function: The objective function used is (15) described in Section 5.

Variable definition identities: I include functions to define the choice variables. To define average bin size, K_b I use equation (4). The firm size counter cumulative distribution (1-CDF) is defined using (5). Average squared distance is defined with equation (6).

Total exports constraints: I define three constraints of export sums. One for the total amount of exports from Colombia, $Exports$; a second one for the total amount of exports from all firms in each bin, $exports_b$; and a third one for the total exports from Colombia to each country, $exports_c$. They are given by

$$Exports = \sum_{b,c} exports_{bc} \tag{B.1}$$

$$exports_b = \sum_{c} exports_{bc} \tag{B.2}$$

$$exports_c = \sum_{b} exports_{bc} \tag{B.3}$$

Total number exporting firms constraints: I define a group of constraints for the number of exporting firms in the economy, N; the number of exporting firms that reach country c, M_c; and two additional constraints to make the solution sound. First, I need that the number of firms in each bin-destination pair to be smaller than or equal to the number of firms in each bin. Second, within in each bin, the total number of firms in the bin-destination pairs needs to be greater than or equal to the number of firms in each bin.

This last constraint allows a single firm to go to different destinations. The constrains are given by the following

$$N = \sum_b n_b \tag{B.4}$$

$$M_c = \sum_b m_{bc} \tag{B.5}$$

$$m_{bc} \leq n_b, \quad \sum_c m_{bc} \geq n_b \tag{B.6}$$

Probability density function (PDF) constraints: The probability density functions are based on the formulas from Section 15. These are described below

$$\begin{aligned} f_b &= \frac{n_b}{N} \\ f_b &\geq 0 \\ \sum_b f_b &= 1 \end{aligned} \tag{B.7}$$

$$\begin{aligned} g_{K_b}(x) = g_{bc} &= \frac{m_{bc}}{\sum_c m_{bc}} \\ g_{bc} &\geq 0 \\ \sum_c g_{bc} &= 1 \end{aligned} \tag{B.8}$$

$$\begin{aligned} h_{bc} &= g_{bc} \times f_b \\ h_{bc} &\geq 0 \\ \sum_{b,c} h_{bc} &= 1, \end{aligned} \tag{B.9}$$

Gravity Equation and Proposition 1 constraints: I add the gravity equation and the following equations from Chaney (2018) as constraints

$$\ln Exports_c = a_g - \zeta \ln x_c + \beta \ln Imports_c, \quad x_c \geq 2000km. \tag{B.10}$$

$$\ln\left(1 - F(K_b)\right) = a_f - \lambda \ln\left(K_b\right) \tag{B.11}$$

$$\ln\left(\Delta(K_b)\right) = a_d - \lambda \ln\left(K_b\right) \tag{B.12}$$

$$\lambda \leq 1 + \mu \tag{B.13}$$

$$\zeta = 1 + 2(\lambda - 1)/\mu \tag{B.14}$$

Bin range constraints: Firms in the problem are allowed to move between bins. To manage this, I impose the following constraints on bin size ranges

$$range_b = ubound_b - lbound_b = ubound_b - ubound_{b-1} \tag{B.15}$$

$$K_b \leq ubound_b, \quad K_b \geq lbound_b \tag{B.16}$$

Additional constraints: I also impose additional constraints that are included in the GAMS program code.

Output evaluation: I compare the problem's solution with the original data. This provides some evidence of where more changes are needed to make the Colombian data fit the model.

Solver: The problem is solved using CONOPT 4, which is a large-scale Non-Linear Programming (NLP) solver.

Appendix C   Additional Maximum Entropy Results

This appendix shows the results for the Maximum Entropy Exercises 2 and 4 which are intended to be robustness checks for Exercises 1 and 3 respectively.

Table C.1 compares parameter estimates from Proposition 1 using the original Colombian data with those from the ME solutions of Exercise 2. This exercise is the same as Exercise 1 but here the solution chooses the value of total aggregate exports to maximize the objective function. In this scenario, total aggregate exports increased 22% approximately compared to the original data, going from US$8,460 million to US$10,345 million. The shape parameter of the FSD λ from condition (i) increases from 0.885 in the original data set to 1.371. The elasticity μ from condition (ii) increases from 0.079 in the original sample to 0.3781 . These results are similar to those obtained in Exercise 1 and can be interpreted in the same way. One difference with Exercise 1 is that the point estimate of the distance elasticity of exports for long distances, ζ_{long} in the maximum entropy solution is 2.962, which is closer to the its value in the original data 2.8211. In this exercise, the shares of the extensive and intensive margins of the distance elasticity change roughly in the same amounts as in Exercise 1.

Table C.2 shows results of parameter estimates obtained from Exercise 4 and compares it with estimates calculated from using the Colombian data, and with those from using the French data. This exercise is similar to Exercise 3, but here I increase the number of exporters by 10% and let the solution pick the value of aggregate exports. Total exports increase about 66% going from $8,459.14 millions to $14,106 millions. The maximum entropy solutions reach an estimated distance elasticity of 1.171, which compared to Exercise 3 is close to my target of 1.1854. In this exercise, the shape parameter λ from condition (i) increases from 0.8853 in the Colombian data to 1.0242, which is close to its value in the French estimates. The parameter μ from condition (ii) increases from 0.079 in the Colombian data to 0.2825 which is more then twice as much as its estimated value in the French data. Also, compared to Exercise 3 here the share of the extensive margin of

Table C.1

Comparison of Proposition 1 parameters from Original Colombian Data vs Maximum Entropy Results (Exercise 2): Firms exporting above 200 thousand USD in 2018

		ME Results		**Colombian Data**
Proposition 1				
Condition (i): distribution of firm sizes	$\lambda =$	1.3709		0.8853
Condition (ii): average sq. distance of exports	$\mu =$	0.3781		0.0789
Predicted distance elasticity of trade	$1 + 2(\lambda - 1)/\mu =$	2.9618		-1.9066
Distance elasticity of trade				
Long Distances (>2,000 km.)	$\zeta_{long} =$	2.9618		2.8211
Extensive Margin	% of $\zeta_{long} =$	38.013%		55.83%
Intensive Margin	% of $\zeta_{long} =$	61.987%		44.17%
Aggregate Variables				
Total Aggregate Exports (US$ Million)	$Exports =$	10,345.21	(free)	8,459.14
Total Number of Exporters	$N =$	2,026	(fixed)	2,026

Source: DANE, CEPII

Note: This table compares the estimated parameters of Proposition 1 from the original Colombian data against those calculated from re-arranged data obtained from the Maximum Entropy Exercise 1. The Colombian data is a sample of 2,026 Colombian firms that exported more than US$200,000 in 2018. The maximum entropy problems re-arranges this data under the constraint that conditions (i)-(iii) from Proposition 1 are satisfied. The parameter λ is the shape coefficient for the distribution of firm sizes, estimated from (5). μ is the size elasticity of average squared distance of exports with respect to firm size, estimated from (6); ζ is the distance elasticity of aggregate exports from (9); the extensive and intensive margin of ζ are estimated using (10) and (11) respectively. In the ME results, the standard errors and R^2s are always zero and one respectively. The standard errors and R^2s for the Colombian data are available in Table 1.

the distance elasticity of export increases from 55.8% in the Colombian data to 95.45%; and the share of the intensive margin reduces from 44.17% in the original Colombian data to 4.55%. These results are similar to those from Exercise 3 and thus confirm them.

Table C.2

Comparison of Proposition 1 parameters from Original Colombian Data vs Maximum Entropy Results targeting French distance elasticity of exports (Exercise 4): Firms exporting above 200 thousand USD in 2018

		ME Results		Frenh Data	Colombian Results
Proposition 1					
Condition (i): distribution of firm sizes	$\lambda =$	1.0242		0.9707	0.8853
Condition (ii): average sq. distance of exports	$\mu =$	0.2825		0.1131	0.0789
Predicted distance elasticity of trade	$1 + 2(\lambda - 1)/\mu =$	1.1713		0.4819	-1.9066
Distance elasticity of trade					
Long Distances (>2,000 km.)	$\zeta_{long} =$	1.1713	(target)	1.1854	2.8211
Extensive Margin	% of $\zeta_{long} =$	95.45%		102.19%	55.83%
Intensive Margin	% of $\zeta_{long} =$	4.55%		-2.19%	44.17%
Aggregate Variables					
Total Aggregate Exports (2018 US$ Million)	$Exports =$	14,106.57	(free)	57,526.29	8,459.14
Total Number of Exporters	$N =$	2,229	(fixed)	27,968	2,026

Source: DANE, CEPII, Chaney (2022)

Note: This table compares the estimated parameters of Proposition 1 from the original Colombian data against, those estimated using the French data from Chaney (2022), and those calculated from re-arranged data obtained from the Maximum Entropy Exercise 4. The Colombian data is a sample of 2,026 Colombian firms that exported more than US$200,000 in 2018. The French data is a sample of approximately 28,000 firms that exported more than 1 million French Francs in 1992 ($\approx$ US$200,000). To convert FRF to USD I used the 12-month average exchange rate in 1992 published by FRED (2022). Then, to covert from 1992 USD to 2018 USD I used the CPI inflation calculator from the U.S. BLS. The maximum entropy problems re-arranges this data under the constraint that conditions (i)-(iii) from Proposition 1 are satisfied, and targets the estimated French distance elasticity of exports. The parameter λ is the shape coefficient for the distribution of firm sizes, estimated from (5). μ is the size elasticity of average squared distance of exports with respect to firm size, estimated from (6); ζ is the distance elasticity of aggregate exports from (9); the extensive and intensive margin of ζ are estimated using (10) and (11) respectively. In the ME results, the standard errors and R^2s are always zero and one respectively. The standard errors and R^2s for the Colombian and French data are available in Table 1.