The Value of Public Information for Microeconomic Production Decisions

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Procedures are needed to evaluate the benefits of the provision of information. This paper shows how to apply a money metric definition of the value of information for this purpose. The application is to microeconomic input choices for agricultural production, and the information to be valued concerns the effect of fertilizer on sorghum yield. In this application both output price and output level are stochastic, and the probability distribution of output is affected by the chosen level of fertilizer.

Key words: benefit-cost, information, risk.

Better public information is cited as a need in many public policy applications. Because obtaining information is costly, there is a need for procedures to evaluate the benefits of public production of information in monetary terms. Cost-benefit analysis could then be applied to the provision of information.

Generally speaking, economic principles define information as "valuable" if it leads to decisions which are preferred. Information is then valued by comparing the outcomes obtained with and without the information. However, in the context of risky decisions and varying risk preferences, implementation of this principle is not so immediate.

With risk, optimal decisions have often been modeled as deriving from maximizing expected utility given a probability distribution over outcomes (Hirshleifer and Riley). In this context, information is considered to be the state of knowledge concerning the probability distribution used to make the decision. The value of information has then been defined in decision theory literature as the difference in expected utility (with expectation taken in terms of the probability distribution corresponding to the new information) because of the decisions made with the "more informed" and "less informed" probability distributions. However, this conceptual definition has not proved to be empirically useful because it is expressed in units of "utils" rather than in monetary terms.

Recently, Roe and Antonovitz defined a "money metric" value of information; they defined alternative money metrics in terms of "willingness to pay" and "willingness to accept." Their work applied the money metric to measuring a value of improved price forecasts in a macroeconomic context. The purpose of this paper is to extend the work of Roe and Antonovitz to the case where both price and output are stochastic. This extension is demonstrated in the context of a microeconomic production problem wherein the producer can affect the distribution of output through choice of inputs.

As an example of the value of information applied to microeconomic production, we consider the case of agricultural production. The relationship between crop yield and an input such as fertilizer is random rather than deterministic because of the effects of random factors, such as weather. Thus, at the time input decisions are made, profit is a random variable both because of uncertain yield and because the price of output is not yet determined. Farmers must choose input levels based on their preference ordering over risky outcomes, the information they have about the production relation, and the distribution of future prices. The theme of this article is that production information may be considered a pub-

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lic good. Production information collected for a given crop in a given geographic area may be relevant to many farmers growing the crop in that area. The cost of providing this information to farmers is essentially independent of the number of farmers. Thus, this information has the characteristics of a public good. As with other types of public goods, individual collection of such information is not, in general, socially optimal.

Public information regarding production has been provided by agricultural experiment stations through test plot studies. These stations have investigated the relationship between such inputs as fertilizer and yield by performing controlled experiments over long periods of time. Because it is costly to collect and analyze such test plot data, a procedure that would enable a valuation of this information would be desirable. The approach suggested here provides a practical method for performing this valuation, thereby allowing a standard cost-benefit analysis.

Value of Information

Definitions of the value of information must be briefly reviewed. In order to make the alternative definitions more comparable, the notation has been changed somewhat from that given in the original articles.

Gould and Hess defined the value of information as the difference between the expected utility of the action decision when the state of nature is known and the expected utility of the action taken when only the distribution of the state of nature is known. Noting that the expectations are taken with respect to the random state of nature, this value may be expressed mathematically as

$$E[\max_{\alpha} u(\alpha, y)] - \max_{\alpha} E[u(\alpha, y)]$$

$$= \int \max_{\alpha} u(\alpha, y)f(y)dy - \max_{\alpha} \int u(\alpha, y)f(y)dy,$$

where $E[\cdot]$ denotes the expectation operator, y denotes states of nature, $f(y)$ denotes the probability distribution of y, and α denotes actions chosen. This definition may be interpreted as the expected value (in "utils") of perfect information.

Hirshleifer and Riley present an alternative definition of the value of information by using the information to revise the "prior" probability distribution of y to a "posterior" (after new information is received) distribution. The prior estimate of the probability distribution of the state of nature is denoted $f_o(y)$, and the posterior, more informed, estimate of the distribution is denoted $f_m(y)$. The associated expected utility maximization problems give rise to the prior and posterior decision problems:

$$\max_{\alpha} \int u(\alpha, y)f_o(y)\, dy, \text{ and}$$

$$\max_{\alpha} \int u(\alpha, y)f_m(y)\, dy.$$

Let α_o^* and α_m^* denote the respective solutions to the above problems. The value of being "more informed" (knowing the better estimate of the true distribution) is

$$\int u(\alpha_m^*, y)f_m(y)\, dy - \int u(\alpha_o^*, y)f_m(y)\, dy.$$

Unfortunately, this measure of value is expressed in utils.

The definition of the money metric given by Roe and Antonovitz is similar to the definition in Hirshleifer and Riley in that the more informed probability distribution is used to compare actions chosen with and without the information. However, the definition is in terms of monetary units rather than "utils." "Willingness to pay (WTP)" and "willingness to accept (WTA)" are two alternative money metric measures.

Willingness to accept (WTA) is the amount of money the decision maker is willing to accept (in every state of nature) for not being informed of the posterior distribution. That is, without knowledge of the posterior distribution, the decision maker will choose α_o^*, yielding a lower level of utility. The value WTA is the amount that must be received in every state of nature to exactly compensate the decision maker for not knowing of the "more informed" distribution. Mathematically, WTA is defined by the equation

$$\int u[\pi(\alpha_o^*, y) + WTA]f_m(y)\, dy$$

$$= \int u[\pi(\alpha_m^*, y)]f_m(y)\, dy,$$

where utility is now considered to be a function of profit, π. Similarly, willingness to pay (WTP) is defined as the amount the decision maker would be willing to pay for advance knowledge of the posterior distribution. That is, WTP is defined by the equation:

$$\int u[\pi(\alpha_m^*, y) - WTP]f_m(y)\,dy = \int u[\pi(\alpha_o^*, y)]f_m(y)\,dy.$$

Microeconomic Production Decisions

When modeling agricultural production decisions under risk, two sources of variability are noteworthy: yield and product prices. Individual producers may alter the distribution of yield via their input choices. However, assuming many small producers, they cannot alter the distribution of product price. (It is straightforward to relax this assumption of independence between product price and production decisions.) To apply the money metric definition to the case of microeconomic production decisions, the Roe-Antonovitz definition must be modified to account for the fact that the probability distribution for yield is affected by input decisions.

To define the optimal decision problem for input use, utility expectations must be taken over both price and yield. Optimal decisions are described by solutions to the problem:

$$\max_{\alpha} \int\int u[\pi(\alpha, y, p)]f(y, p; \alpha)\,dy\,dp,$$

where the joint probability density function for yield and price is $f(y, p; \alpha)$, α is the vector of inputs, y is yield, and p is the output price. The level of profit is defined by

$$\pi(\alpha, y, p) = py - w'\alpha,$$

where w is a vector of input prices. The prior (less informed) and posterior (more informed) input decision problems are

$$\max_{\alpha} \int\int u[\pi(\alpha, y, p)]f_o(y, p; \alpha)\,dy\,dp, \text{ and}$$

$$\max_{\alpha} \int\int u[\pi(\alpha, y, p)]f_m(y, p; \alpha)\,dy\,dp,$$

respectively. Denoting the prior and posterior solutions α_o and α_m, respectively, the values for willingness to accept and willingness to pay may be computed by solving

$$\int\int u[\pi(\alpha_o, y, p) + WTA]f_m(y, p; \alpha_o)\,dy\,dp = \int\int u[\pi(\alpha_m, y, p)]f_m(y, p; \alpha_m)\,dy\,dp,$$

and

$$\int\int u[\pi(\alpha_m, y, p) - WTP]f_m(y, p; \alpha_m)\,dy\,dp = \int\int u[\pi(\alpha_o, y, p)]f_m(y, p; \alpha_o)\,dy\,dp.$$

Note that (similar to an insurance premium) the value of information is an amount which is paid or received regardless of the state of nature that occurs. It is easily seen that the willingness to pay and willingness to accept are non-negative values since α_m is the optimal action with respect to $f_m(y, p; \alpha)$.

The Texas Study

A study at Texas A&M (SriRamaratnam) examined risk preferences and subjective price expectations for sorghum farmers in Texas. That study obtained risk aversion coefficients of farmers, expectations about sorghum prices, cost of nitrogen, and actual nitrogen use for the 1984 crop year. In addition, based on test plot data from the Texas A&M experiment station collected over the period 1977–84, the parameters of a "more informed" objective probability distribution for yield (pounds per acre) were estimated as a function of nitrogen applications (pounds per acre). The mean and variance of yield were functionally related to nitrogen applications as

$$\mu = 2133. + 20.2N - 0.127N^2,$$

and

$$\sigma^2 = 233105.0 - 4040.0N + 85.0N^2 - 0.4387N^3.$$

The mean relationship implies that expected yield is maximized at an application rate of 79.5 pounds of nitrogen per acre.

In the SriRamaratnam study, subjective sorghum price distributions were obtained for each farmer. Absolute risk aversion coefficients were elicited using a modified Ramsey method. That study found that a constant absolute risk aversion utility function best fit the responses. (Methods are described in more detail in SriRamaratnam.)

For the purposes of computing the value of improved nitrogen yield response information, the optimal (in the expected utility sense) level of nitrogen application was compared to the actual level reported by farmers. For consistency, the negative exponential utility function used by SriRamaratnam was employed. That is,

Table 1. Optimal Versus Actual Nitrogen Use

Farm Number	Coefficient of Absolute Risk Aversion	Acres[a]	Expected Sorghum Price	Variance Sorghum Price	Nitrogen Price	Optimal Nitrogen	Actual Use	Change in Expected Profit	Change in Cost of Nitrogen	Value of Information
	$(\times 10^{-4})$[a]		($/cwt.)[ab]	($²/cwt.²)[ab]	($/lb.)[a]	(lb./acre)	(lb./acre)[a]	($/acre)		
1	.141	1,400	5.53	.312	.22	60.2	65	−.09	−1.07	.635
2	.052	2,800	5.75	.075	.29	57.3	72	1.07	−4.27	.203
3	.097	2,700	5.75	.125	.36	50.8	68	1.14	−6.18	.390
4	.165	2,700	5.25	.125	.36	46.8	68	1.38	−7.64	.568
5	.370	3,000	5.05	.085	.11	54.9	82	−.86	−2.98	1.219
6	.118	540	5.20	.223	.30	55.7	72	1.52	−4.88	.455
7	.309	2,100	5.75	.125	.13	59.4	69	−.90	−1.25	1.254
8	.065	1,900	6.18	.132	.14	68.4	125	23.18	−7.93	.284
9	.021	19,000	5.48	.187	.20	58.3	60	−.14	−.35	.080
10	.182	820	5.33	.157	.24	59.3	66	.08	−1.60	.660
11	.284	250	5.25	.425	.14	67.8	57	.95	1.51	1.280
12	.105	3,000	5.75	.175	.19	60.9	96	6.11	−6.67	.493
13	.087	1,400	6.10	.253	.27	59.7	75	1.25	−4.12	.452
14	.221	1,400	5.18	.632	.20	57.5	75	.44	−3.51	1.264
15	.358	775	6.03	.112	.21	60.8	97	7.25	−7.60	1.724

[a] Source: SriRamaratnam.
[b] 1984 crop year.

$$u(\pi) = 1 - \exp(-\rho\pi),$$

where ρ denotes the coefficient of absolute risk aversion. For simplicity, gross revenues are assumed to be approximately normally distributed. Freund has shown that the objective of an expected utility maximizing model with the assumptions listed above is equivalent to the alternative objective:

$$\tilde{u}(\pi) = E[\pi] - \frac{\rho}{2}\operatorname{Var}[\pi].$$

where π is defined as revenue less the variable cost of nitrogen and fixed costs. Mathematically, $\pi = (P_sY - P_nN)A - B$, where P_s is the price of sorghum ($/lb.), Y is yield (lb./acre), P_n is the cost of nitrogen ($/lb.), N is the application rate (lb./acre), A is the number of acres, and B is the total fixed cost. In accordance with the assumption of many small farms, it is assumed that, given input levels, price and yield are independent random variables. Using this assumption the objective may be written

$$\begin{aligned}\tilde{u}[\pi(N)] = {} & (E[P_s]E[Y] - P_nN)A - B \\ & - \frac{\rho}{2}\{E[P_s]^2\operatorname{Var}[YA] \\ & + \operatorname{Var}[P_s]E[YA]^2 \\ & + \operatorname{Var}[P_s]\operatorname{Var}[YA]\}.\end{aligned}$$

Denoting the optimal level of nitrogen use from the expected utility model by N_o and the actual level of nitrogen use by N_a, the willingness to pay for improved yield response information may be found by solving

$$\tilde{u}[\pi(N_o) - WTP] = \tilde{u}[\pi(N_a)].$$

For this particular utility function, the solution is

$$WTP = \tilde{u}[\pi(N_o)] - \tilde{u}[\pi(N_a)].$$

The farmer-specific data and the levels of optimal nitrogen use are displayed in table 1. In all but one case, the optimal level of nitrogen was exceeded by that which was actually used. The exception to this rule was for farmer 11, whose actual level of applied nitrogen was the lowest in the sample.

The change in expected profit ranged from a high of $23.18 per acre to a low of $−0.90 per acre. The largest increase (farmer 8) resulted from a $7.93 per acre decrease in the cost of fertilizer and an increase in expected yield of 246.8 pounds per acre. (These results indicate significant overapplication of nitrogen by this farmer. However, a difference in soil type may account for some of the discrepancy between the actual and optimal levels of nitrogen use.) The largest decrease in expected profit (farmer 7) is caused by a decrease in expected yield of 37.5 pounds per acre. The decrease in fertilizer cost in this case was not sufficient to offset the decrease in expected yield. However, the corresponding decrease in the variance of returns has a large effect on utility due to the large risk aversion coefficient for farmer 7. Hence, this relatively large decrease in expected profit is consistent with the expected utility maximization objective.

The value of information (willingness to pay) for this sample of farmers ranges from a low of $0.080 per acre to a high of $1.724 per acre. The low value (farmer 9) results because of the near equality between the optimal and actual levels of nitrogen use. The high value (farmer 15) results from a large decrease in the yield variance and a large increase in expected profit.

Weighting the *WTP* values by acreage and dividing by total acres yields an average value of information of $0.439 per acre. While this seems to be a modest amount, it is noteworthy that in 1983 approximately 3.45 million acres of sorghum were grown in the state of Texas. Hence, the value of producing and disseminating information regarding the sorghum yield response to nitrogen is estimated to be on the order of $1.5 million.

While these results indicate that the dissemination of yield response research results has significant value, a few cautions are warranted regarding the estimates presented here. Because the experiment station already makes significant efforts to communicate the results of research to farmers, this valuation corresponds to an additional increase in the value of information. However, the lack of farm specific variables (e.g., soil type) in the estimated nitrogen response functions leads to an overstatement of the willingness-to-pay figures. The assumption that all farmers will internalize the research information to their planning efforts also overstates the benefits in terms of willingness to pay. Finally, the probability distribution assumptions (e.g., independence between price and yield) may be suspect. Depending on a variety of factors, this may cause the benefits from research to be either over- or understated.

The value of improved yield response information is only one component in the benefit-cost analysis. Other benefits include the value of improved production information to sorghum farmers outside Texas, the value of the skills acquired by students involved in the plot-level experiment and its analysis, and the value of other results derived from the experiment. These must be compared with the costs of producing and analyzing the test plot data, disseminating the information to farmers, and disseminating the information to the academic community. Any benefit-cost analysis would have to treat alternatives consistently with respect to overhead, the value of student training, etc. As such, the approach described here is probably best suited to the task of evaluating the benefits of dedicating extension resources to the dissemination of results of alternative applied research projects.

Conclusions

This paper has demonstrated the computation of a money metric value of information for microeconomic production choices under risk. This value of information extends the work of Roe and Antonovitz to the case where both price and output level are stochastic. In the application presented here, the producer has a direct effect on the probability distribution of output levels through the choice of input levels. This value of information can potentially be used to make benefit-cost comparisons for provision of public information.

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