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# Simultaneous Input Demands and Land Allocation in Agricultural Production under Uncertainty

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Multicrop farmers must choose variable input levels and land quantity for each crop. Economic researchers to date have analyzed these two decisions separately, either finding the best land use, given crop technologies, or solving for optimal input levels, ignoring the allocation of land. We show that both these approaches lead to suboptimal decision rules under risk aversion. An empirical example demonstrates that a risk-averse farmer who makes these decisions sequentially—first choosing input levels then allocating land—rather than simultaneously, may significantly understate the value of farming.

*Key words:* input demand, land allocation, risk.

Economists interested in agricultural production decisions under uncertainty have implicitly divided these decisions into two separate categories. The first category relates to the choice of crop-specific levels of factors of production. The second category relates to the allocation of land among crops to determine the output mix.

In the context of the first category, a number of studies indicate that a producer's choice of variable inputs affects not only the mean level of output but the shape of the statistical distribution of output, as well (e.g., Day; Roumasset). In response, economists have developed production functions with increasing generality regarding the role that inputs play in determining the random nature of output. For example, Just and Pope suggested a production function that allows input levels to

affect risk, as defined by the variance of output, independently of their effect on the expected level of output. Their specification has been used to study the relationship between variable inputs and the mean and variance of crop yields by Griffiths and Anderson and by Farnsworth and Moffitt who also studied the implied per-acre demands for variance-affecting factors of production. Later studies by Antle and by Taylor have suggested other methods of linking output distributions to variable inputs.

The second set of problems, the allocation of land, has also received much attention in the agricultural economics literature (e.g., Freund; Yassour, Zilberman, and Rausser; Collender and Zilberman; and Hazell). The best known approaches to this set of problems are stochastic dominance and mean-variance analysis. As these approaches have been applied, however, the distribution functions of crop yields, as determined by the choice of non-land inputs, are treated as determined prior to the allocation of acreage.

The first literature has focused attention on the role of input levels in determining and controlling the risks faced by agricultural producers. Comparative statics analyses (e.g., Pope and Kramer; Feder) show that a risk-averse

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firm will use more (less) of a production factor than will a risk-neutral firm if the input decreases (increases) output variance. Empirical applications which would show the magnitude of the changes in factor demands and the welfare consequences of ignoring the variance controlling effects of input decisions do not, to the authors' best knowledge, yet exist.

Applications of the second set of studies, examining the payoffs from controlling variance through asset diversification, are found throughout the economic and finance literature. But the incorporation into these problems of input choices to control the moments of the random variables under consideration—the actual decisions farmers make—has not yet been fully considered.

In this paper, we incorporate both types of decisions into the expected utility maximization framework for decision making under uncertainty. In doing so, we demonstrate that one must combine the information embedded in the stochastic production function and the land allocation problem in the same objective function in order to obtain optimal solutions in any but the most trivial of circumstances. This is true even if the per-acre production functions are independent of total acreage planted. Thus, combining the results from these two separate aspects of the literature on agricultural production under uncertainty yields additional insight into the nature of the producer's decision.

The paper makes use of the expected utility, moment-generating function approach (compare Collender and Zilberman; Collender and Chalfant). This method is applicable to both discrete and continuous choices, unlike stochastic dominance techniques, and can be used to improve upon mean-variance analysis when non-normal yield or revenue distributions are appropriate. We proceed as follows. Section 2 of the paper illustrates the basic setup of the expected utility, moment-generating function approach for the case of  $N$  crops whose distribution is multivariate normal. Crop yields are assumed to be determined according to the Just and Pope production function specification. In the third section an empirical analysis is presented. The results give guidance as to when it is important to account for the simultaneous nature of the input choice–output mix decisions, and when it may be appropriate to ignore not only the simultaneity but also the effects of input use on yield variance. The last

section summarizes the main points of the paper.

### **Expected Utility Maximization Using Moment-Generating Functions**

Solving expected utility maximization problems requires knowledge of an agent's utility function and the probability distribution of each random variable that affects agent's wealth. Analytical solutions do not exist for most utility functions and probability distributions. If utility depends on the entire distribution of a random variable, then the expectation of utility must be obtained by integrating utility over the range of the random variable with weights given by the appropriate probability density function. In general, solution algorithms must either make the random variable discrete (Lambert and McCarl) or rely on numerical approximations of the integral.

The expected utility, moment-generating function approach, however, allows expected utility-maximizing solutions to be found for a wide range of probability distributions. This method can be used with any probability distribution for which a moment-generating function exists as long as the agent possesses a negative exponential utility function. This utility function is often criticized since it assumes constant, rather than decreasing, absolute risk aversion. This need not be a severe drawback, however, because the risk coefficient could presumably be made a decreasing function of initial wealth. When yields are normally distributed, this approach reduces to mean-variance analysis, but the technique can easily handle non-normal distributions, provided a moment-generating function exists. We show below that this method can be adapted to solve the problem of maximizing the expected utility of profits when profits depend on both crop-specific choices about production factor levels and land allocation decisions.

The method builds on earlier results of Hammond, and Yassour, Zilberman, and Rausser. Hammond suggested the framework as a convenient method of incorporating a variety of statistical distributions into maximization of expected utility using the exponential utility function. The application of the technique by Yassour, Zilberman, and Rausser shows how it can be used to choose between crops with normal distributions and with gam-

ma distributions. Collender and Zilberman generalized the method to the case of continuous choices, in the context of the allocation of acreage to a variety of crops. Here, we incorporate the selection of variable input levels into their setup, rather than assuming that crop yields are randomly distributed with predetermined moments.

The producer's problem is to select levels of  $K$  inputs for each of  $N$  crops in the production plan and to allocate  $L$  acres of land among these  $N$  crops to maximize the expected utility of profits, where the utility function is of the negative exponential form. The optimization problem can be written as

$$(1) \quad \max EU(\pi) = E \left\{ -\exp \left[ -r \sum_{i=1}^N l_i (P_i y_i - VC_i) \right] \right\},$$

subject to

$$\sum_{i=1}^N l_i \leq L \quad \text{and} \quad l_i, x_i \geq 0, \quad i = 1, \dots, N,$$

where  $r$  is the Arrow/Pratt measure of absolute risk aversion;  $y_i = f_i(x_i, e_i)$  is the stochastic crop yield with  $e_i$  representing the random element and  $x_i$  the  $K \times 1$  vector of non-land inputs;  $VC_i = w'x_i$  gives the variable cost per acre of growing crop  $i$  with  $w$  the  $K \times 1$  vector of input prices;  $P_i$  is the price of crop  $i$  (assumed to be nonstochastic);  $l_i$  is the number of acres planted in crop  $i$ ; and  $L$  is the total acreage to be planted. Maximization of expected utility is with respect to each  $l_i$  and  $x_i$ , and is subject to the constraint that the producer cannot allocate more than  $L$  acres, plant negative acres of any crop, or apply negative inputs.

Assuming that crop yields are the only source of uncertainty, we can express the above criterion in terms of the joint moment-generating function of the stochastic crop yields:

$$(2) \quad \max EU(\pi) = -\exp \left[ r \sum_{i=1}^N l_i VC_i \right] \cdot M(t_1, t_2, \dots, t_N),$$

subject to the same restrictions as above.  $M(\cdot)$  is the joint moment-generating function of the  $N$  stochastic crop yields evaluated at  $t_i = -rl_i P_i$ . The requirements for maximizing expected utility reduce to the land availability constraint and conditions giving the efficient allocation of land and levels of non-land inputs. Efficient land allocation, assuming an interior solution, must satisfy

$$(3) \quad VC_1 - \frac{P_1 \frac{\partial M}{\partial t_1}}{M(\cdot)} = VC_j - \frac{P_j \frac{\partial M}{\partial t_j}}{M(\cdot)}, \quad j = 2, \dots, N,$$

while non-land inputs must satisfy

$$(4) \quad w_k = - \frac{\frac{\partial M}{\partial x_{jk}}}{rM(\cdot)l_j}, \quad j = 1, \dots, N; \quad k = 1, \dots, K.$$

These conditions must be solved simultaneously for an optimum, given the appearance of variable costs in (3) and of  $l_j$  in (4).<sup>1</sup>

It is easily shown that these somewhat cumbersome conditions reduce to the usual necessary conditions for expected profit maximization when the producer is risk neutral ( $r = 0$ ). In this case,  $M(\cdot)$  is equal to one and its derivative with respect to  $t_i$  equals  $\mu_i$ , the expected yield of crop  $i$ . The necessary conditions for maximizing expected utility with respect to the land allocation then reduce to

$$(5) \quad P_1 \mu_1 - VC_1 = P_j \mu_j - VC_j \quad \text{for } j = 2, \dots, N.$$

This is the familiar condition that expected profits per acre are equal for each crop grown.

We can also evaluate the first-order conditions for non-land inputs at  $r = 0$ . Recalling that when crop yields are the only stochastic elements,

$$(6) \quad M(\cdot) = E \left[ \exp \left( \sum_{i=1}^N t_i f_i(x_i, e_i) \right) \right],$$

with  $t_j = -rl_j P_j$ , we find that

$$(7) \quad \frac{\partial M}{\partial x_{jk}} = E \left[ t_j \frac{\partial f_j}{\partial x_{jk}} \exp \left( \sum_{i=1}^N t_i f_i(x_i, e_i) \right) \right]$$

as long as the expectation of the derivative equals the derivative of the expectation. This is true if the range of the output distribution does not depend on input levels. Substituting (7) into (4) and evaluating it at  $t_j = -rl_j P_j$ , gives

$$(8) \quad w_k = P_j E \left[ \frac{\partial f_j(x_j, e_j)}{\partial x_{jk}} \right], \quad k = 1, \dots, K; \quad j = 1, \dots, N.$$

This says that under risk neutrality, inputs are used up to the point where their price is equal to the expected value of their marginal product.

<sup>1</sup> The appendix to an earlier version of this paper that outlines the derivation of the first-order conditions is available on request.

To go beyond this point requires that the relationship of the production inputs,  $x_i$ , and the stochastic element,  $e_i$ , be made more explicit. This is done by picking a functional form to describe how these elements affect crop yields and by specifying a distribution function for each  $e_i$  and the stochastic relationship among the  $e_i$ 's. This defines the distribution functions of crop yields and their joint moment-generating function.

One possibility for this type of model is to treat the  $y_i$ 's as multivariate normal. To obtain a variety of relationships between higher-order moments and inputs along the lines of Antle or Taylor, as well as to account for possible nonnormality of yield distributions, one might prefer some other distribution. The method illustrated below will apply easily to any distribution for which a moment-generating function exists. However, the normality assumption serves to illustrate when it is important to account for the simultaneous nature of the land allocation and non-land production factor level decisions.

Estimation of each of the  $N$  marginal density functions independently is equivalent to assuming that each individual crop is characterized by the general form of the Just-Pope technology:

$$(9) \quad y_i = g_i(x_i) + h_i^s(x_i)e_i, \quad i = 1, \dots, N,$$

where  $E(y_i) = g_i(x_i)$ ,  $V(y_i) = h_i(x_i)$ , and the normality of  $y$  all follow from the assumption that  $e \sim N(0, 1)$ . The joint moment-generating function can be written

$$(10) \quad M(t_1, \dots, t_N) = \exp \left[ \sum_i t_i g_i(x_i) + .5 \sum_i \sum_j \rho_{ij} t_i t_j h_i^s(x_i) h_j^s(x_j) \right],$$

with  $\rho_{ii} = 1$ .

The optimal level of variable input use for this general situation of  $K$  crops may be compared with the corresponding level of variable input use that maximizes expected utility of profits from growing only one crop on  $L$  acres of land. The farmer growing  $N$  crops on  $L$  acres of land solves

$$(11) \quad w_k = P_j g_{jk} - .5rP_j^2 h_{jk} l_j - .5rP_j h_j^{-s} h_{jk} \sum_{i \neq j} \rho_{ij} h_i^s P_i l_i$$

$j = 1, \dots, N; \quad k = 1, \dots, K,$

where the subscript  $k$  denotes a partial deriv-

ative with respect to the  $k$ th input. The farmer growing only crop  $j$  solves

$$(12) \quad w_k = P_j g_{jk} - .5rP_j^2 h_{jk} L \quad k = 1, \dots, K.$$

The first terms of these two expressions are identical. They concern the effects of input decisions on the mean of output. The second terms differ if more than one crop is grown by the farmer. In this case each  $l_j$  is less than total land available. Ignoring the effect of the last term in (11) for the moment, this difference means that a farmer who plants only the  $j$ th crop will apply less (more) per acre of the  $k$ th input than a farmer who plants more than just this crop if the  $k$ th input increases (decreases) the variance of the  $j$ th output. This is an example of Zeckhauser and Keeler's size-of-risk aversion. They point out that an economic agent will act more risk averse the larger is the potential loss (and gain if the distribution is symmetric).

Now consider the last term in (11). This concerns the effect that correlations among yields have on input use. The effect on the level of  $x_{jk}$  is ambiguous except for special cases. For example, if all yields are positively correlated and the  $k$ th input increases (decreases) the variance of all  $N$  crop yields, then a farmer planting more than one crop will apply less (more) of the  $k$ th input than a farmer planting only crop  $j$ .

Another interesting aspect of these necessary conditions is revealed by assuming that the problem is solved sequentially instead of simultaneously. That is, suppose that variable inputs are chosen to maximize the per-acre expected utility of profits for each crop first and then, given these levels, the land allocation decision is made. The first step determines the  $N$  per-acre yield probability distributions. The second step determines the probability distribution of total farm profits which is the portfolio selection problem solved by Collender and Zilberman.

The  $K \cdot N$  necessary conditions for solving per-acre input levels are given by (12) (with  $L = 1$  for all crops) for a nonsimultaneous solution and by (11) for a simultaneous one.<sup>2</sup>

<sup>2</sup> Clearly, the nonsimultaneous (or sequential) solution to these problems does not require that  $L$  be set to one for all crops in the production plan. However, since crop-specific technologies (variable input levels) are chosen before land is allocated among crops, some arbitrary level of  $L$  must be chosen for each crop. To conform with how agricultural input demand problems have been solved in the past,  $L$  is set equal to one for all crops.

**Table 1. Estimated Just-Pope Production Functions for Corn and Oats**

	$y_{it} = \alpha_0 + \alpha_1 X_{it} + \alpha_2 X_{it}^2 + (\epsilon_{it} + \lambda_t) X_{it}^{2\beta}$					
	Mean			Variance		
	$\alpha_0$	$\alpha_1$	$\alpha_2$	$\sigma_\epsilon^2$	$\sigma_\lambda^2$	$\beta$
Corn	22.178 (1.63) <sup>a</sup>	.725 (.052)	-.0025 (.0004)	99.62	53.25	.100 (.145)
Oats	16.074 (1.91)	1.607 (.144)	-.0156 (.0026)	4.48	29.83	.543 (.251)

Note: Yield ( $y_{it}$ ) is a function of fertilizer  $X_{it}$  and two random components. The first,  $\epsilon_{it}$ , differs for each observation; the second,  $\lambda_t$ , affects all observations in a given year.

<sup>a</sup> The numbers in parentheses are the standard errors of the parameter estimates.

These two expressions differ by the same factors discussed above. The first consideration is on own variance. The contribution to total farm variance from the  $j$ th crop is weighted by acreage devoted to the  $j$ th crop in (11). It is weighted by a single acre in (12). In addition, the correlation between crops is given zero weight in (12), but is weighted by the acreage devoted to each of the other  $N - 1$  crops in (11).

Thus, to be consistent with the objective—expected utility maximization—allocation of non-land inputs and crop acreages must be undertaken simultaneously unless the economic agent is risk neutral or all crops are stochastically independent and one acre of each crop is grown. Failure to do so means that the first-order conditions derived above for maximization of expected utility will not be met, leading to a suboptimal allocation of productive inputs. Furthermore, if producers actually behave as if they are solving these problems simultaneously, estimation of factor demand schedules and welfare analysis using a sequential approach will lead to erroneous results.

### Joint Selection of Fertilizer Levels and Acreage Planted

In this section we show empirically which factors are most important in determining the magnitudes of welfare losses and allocation errors when variable input levels and crop acreages are chosen sequentially rather than simultaneously. We examine the case of one variable input, nitrogen fertilizer, and two crops, corn and oats, each with production functions given by (9). The functional forms for the mean and variance of each crop, the

estimated parameters and their standard errors are reported in table 1.

The data used for estimation come from experiments conducted in the Yazoo–Mississippi delta from 1921 to 1960 for corn and 1928 to 1957 for oats. The yield data were generated from the application of several nitrogen fertilizer levels each growing season. The data are reported in Grissom and Spurgeon and in Day. An appropriate estimation procedure for time-series, cross-sectional data is the error components model (Judge et al.). Griffiths and Anderson give a procedure for estimating an error components model with the Just-Pope heteroscedastic error term. The estimates given in table 1 were obtained using a slight modification of their procedure to account for the absence of information concerning plot effects and the presence of unequal numbers of plots per year for corn.

The necessary conditions for a simultaneous solution for this problem are

$$(13) \quad l_c + l_o = L,$$

$$(14) \quad V_c - VC_o = P_c[g_c - h_c r P_o l_c] \\ - P_o[g_o - h_o r P_c l_o] \\ - P_c P_o h_o^5 p r (l_o - l_c),$$

$$(15) \quad w = P_c g'_c - .5 P_c^2 r l_c h'_c \\ - .5 p h_c^{-.5} h'_c h_o^5 P_c P_o l_o r,$$

$$(16) \quad w = P_o g'_o - .5 P_o^2 r l_o h'_o \\ - .5 p h_o^{-.5} h'_o h_c^5 P_o P_c l_c r,$$

where  $l_c$  and  $l_o$  represent acreage planted in corn and oats;  $x_c$  and  $x_o$  are pounds of fertilizer per acre applied to corn and oats;  $w$  is the price of fertilizer; and  $P_c$  and  $P_o$  are farmgate prices of corn and oats. These conditions are derived from (11) and (3) with a bivariate normal mo-

Table 2. Optimal Crop Acreages and Fertilizer Levels on 200 Acres

		Risk Aversion Level <sup>a</sup>			
		Risk Neutral	Slight	Moderate	Intermediate
$\rho = -.5$					
Land	Corn	200	98	72	70
	Oats	0	102	128	130
Fertilizer	Corn	135.6	135.4	134.9	134.5
	Oats	0	48.5	47.4	46.5
	Average	135.6	91.1	78.9	77.3
$\rho = .0$					
Land	Corn	200	96	59	56
	Oats	0	104	141	144
Fertilizer	Corn	135.6	135.4	134.4	133.6
	Oats	0	48.4	46.2	44.4
	Average	135.6	90.1	72.2	69.3
$\rho = .5$					
Land	Corn	200	93	25	21
	Oats	0	107	175	179
Fertilizer	Corn	135.6	135.3	133.9	132.9
	Oats	0	48.3	45.3	42.8
	Average	135.6	88.8	56.4	52.3

Note: Land is measured in acres, fertilizer in pounds per acre.  
<sup>a</sup> The values of the absolute risk aversion coefficient used for these calculations are  $3.0e-5$ ,  $2.9e-4$ , and  $5.0e-4$  for slight, moderate, and intermediate risk aversion.

ment-generating function.<sup>3</sup> To solve this system requires a correlation coefficient of crop yields ( $\rho$ ). Unlike the means and variances of the two marginal distributions,  $\rho$  is not a function of input levels but instead is a statement about the joint distribution of the random error terms in each crop's per-acre production function.<sup>4</sup> Hence, it is valid to vary  $\rho$  without varying the estimated production function parameters.

These four equations can be solved simul-

<sup>3</sup> Equations (13) to (16) assume positive fertilizer levels and full land utilization. Zero fertilizer levels are not allowed with the Cobb-Douglas specification of the error term because this would imply zero variance. There are no direct benefits from idling land in the present model so that maximum expected utility will occur with all land being planted as long as the certainty equivalent of farming marginal acreage is greater than zero. The model could be expanded by assuming that fallow land has some fixed (or random) future payoff and then treating it as another crop with associated variable costs.

<sup>4</sup> If corn and oat yields are normally distributed with means  $g_c(x_c)$  and  $g_o(x_o)$  and variances  $h_c(x_c)$  and  $h_o(x_o)$ , then their covariance is

$$E[Y_c - E(Y_c)] \cdot [Y_o - E(Y_o)] = E[h_c^s e_c h_o^s e_o],$$

or

$$h_c^s h_o^s E[e_c e_o] = \rho h_c^s h_o^s.$$

taneously using an equation-solving routine. We made use of the ZSPOW subroutine of IMSL. The prices used to obtain the solutions are \$1.89 per bushel of corn, \$1.10 per bushel of oats, and \$.10 per pound of nitrogen fertilizer. Variable cost data are derived from Paxton and Lavergne. The solutions that maximize expected utility for different levels of  $r$  and  $\rho$  using 200 planted acres are given in table 2.

The appropriate level of the coefficient of risk aversion to use in this type of study bears further discussion. We chose to set this coefficient at levels that correspond to empirically justifiable levels of partial risk aversion. The measure of partial risk aversion is defined as

$$S = \frac{-mU''(m + W)}{U'(m + W)},$$

where  $m$  is the certainty equivalent of the gamble under consideration,  $W$  is wealth, and  $U'$  and  $U''$  are the first and second derivatives of the utility function  $U$ . For negative exponential utility functions, this expression reduces to  $-m \cdot r$ .

Menezes and Hanson demonstrate that  $S$  measures the effects of changes in risk when wealth is held constant, as is implicit in our simulation. Binswanger found empirical evidence that the relevant measures of partial risk aversion for farmers in India ranged from negative or risk neutral to over 7.5, which he characterized as extreme. Young reports consistent evidence that education decreases risk aversion. Since we are primarily concerned with better educated and wealthier North American farmers, we chose levels of absolute risk aversion that range from slight to intermediate, according to Binswanger's characterization, as follows.

absolute risk aversion	partial risk aversion	Characterization
( $r$ )	( $S$ )	
$3.0e-5$	.12	slight
$2.9e-4$	.93	moderate
$5.0e-4$	1.33	intermediate

Levels of  $S$  reported are for  $\rho = -.5$  and simultaneous solutions. Because of the form of the utility function,  $S$  is not constant across gambles, but these levels are the highest attained in the reported simulations.

Under risk neutrality, only corn is grown, since the fertilizer levels that equate fertilizer

cost to the expected value of marginal product (to which the first-order conditions reduce in the risk-neutral case) result in higher per-acre expected profits for corn than for oats. As  $r$  increases, diversification into oats increases and per-acre use of the risk-increasing input (fertilizer) decrease. The decrease in per-acre fertilizer use on oats is much more dramatic than on corn. This is a result of the relatively small degree of heteroscedasticity caused by nitrogen fertilizer in corn yields. Oat yields, on the other hand, exhibit far more heteroscedasticity. Variance control with fertilizer on oats is much easier than on corn.

The correlation coefficient affects both the optimal crop mix and fertilizer demand. First, consider the land allocation decision. As crop yields become more positively correlated, the expected utility-maximizing decision maker must diversify more to obtain the desired variance reduction. Thus, for a given level of risk aversion, less of the riskier crop, corn, is planted, the more yields are positively correlated.

The correlation coefficient also changes fertilizer demand. A *ceteris paribus* increase in  $\rho$  increases the variance of farm profits, thereby decreasing per-acre fertilizer use for each crop. Average per-acre fertilizer use also decreases because this effect is reinforced by the acreage choice, since more of the crop with lower fertilizer rates is planted as  $\rho$  increases. However, if corn used less fertilizer than oats, it is likely that average per-acre fertilizer use would show an increase as  $\rho$  increased. Similarly, if the less risky crop (oats) required more fertilizer than the risky crop (corn), one would see total farm use of fertilizer increase with  $r$  as oats entered the solution. Thus, aggregate farm data may lead to erroneous conclusions regarding the role inputs play in controlling risk. To better understand the behavior of underlying aggregate factor demand it is important to gather data on crop-specific input levels.

The results of solving equations (13)–(16) sequentially rather than simultaneously—by first solving equations (15) and (16) for each crop's fertilizer level (with own-crop acreage set to one and the other acreage set to zero) and then using these levels to solve (13) and (14)—are given in table 3.

The most apparent difference in the sequential results is how the size-of-risk effect influences the amount of variable input use. The change in variance from a change in fertilizer use is weighted by a single acre with the se-

**Table 3. Optimal Crop Mix on 200 Acres, Given Per-Acre Fertilizer Levels Which Maximize Per-Acre Expected Utility of Profits**

		Risk Aversion Level <sup>a</sup>			
		Risk Neutral	Slight	Moderate	Intermediate
$\rho = -.5$					
Land	Corn	200	98	72	71
	Oats	0	102	128	129
Fertilizer	Corn	135.6	135.6	135.6	135.5
	Oats	0	48.6	48.6	48.6
	Average	135.6	91.2	79.9	79.4
$\rho = .0$					
Land	Corn	200	96	59	58
	Oats	0	104	141	142
Fertilizer	Corn	135.6	135.6	135.6	135.5
	Oats	0	48.6	48.6	48.6
	Average	135.6	90.4	74.3	73.8
$\rho = .5$					
Land	Corn	200	94	27	23
	Oats	0	106	173	177
Fertilizer	Corn	135.6	135.6	135.6	135.5
	Oats	0	48.6	48.6	48.6
	Average	135.6	88.5	60.3	58.6

<sup>a</sup> See table 2 for notes.

quential solution. Thus, the optimal level of input use for each crop is quite insensitive to both risk aversion levels and  $\rho$ .<sup>5</sup> This makes this type of solution analogous to information farmers receive concerning the amount of fertilizer to use. Typically, fertilizer use recommendations from public and private sources do not take into account differences in farmer attitudes towards risk. The differences in the two solutions can be thought of as the differences that would occur from two farmers using different information sources.

Allocation errors are important if they cause a significant welfare loss. By definition, the solutions given in table 2 lead to greater expected

<sup>5</sup> Rounding off the levels of fertilizer use on corn and oats makes them appear almost constant over the range of  $r$  and  $\rho$  in table 3. Actually, the comparative statics predictions of Pope and Kramer, that a risk-averse producer will use less fertilizer than a risk-neutral farmer, are supported by the unrounded figures. The differences are quite small, however—less than .05 pounds per acre for corn and .08 pounds per acre for oats. This illustrates two points. First, although comparative statics analyses are useful for signing changes, they usually give no indication of the importance of the changes. Second, these small changes are the result of not correctly specifying how fertilizer levels affect variance. The risk-averse producer who does not take into account total farm variance (by weighting the variance change by the number of acres planted) will underestimate the amount of variance control available from inputs.



**Table 4. The Increase in Certainty Equivalent from Simultaneous Solution**

	Risk Aversion Level <sup>a</sup>		
	Slight	Moderate	Intermediate
$\rho = -.5$			
Dollar increase	.02	2.90	9.32
Percent increase	.0	.09	.35
$\rho = .0$			
Dollar increase	.06	13.10	40.94
Percent increase	.0	.53	2.84
$\rho = .5$			
Dollar increase	.20	30.77	96.36
Percent increase	.01	1.61	19.02

<sup>a</sup> See table 2 for notes.

utility than those in table 3, since the sequential solutions can always be chosen when making decisions simultaneously. A convenient means of measuring welfare changes is the difference in the certainty equivalents implied by the two sets of solutions. The certainty equivalent of relevance here is the minimum amount of money that, if received with certainty, would be sufficient to induce a farmer to forego farming for a year. The differences in certainty equivalents reported in table 4 are the maximum values a farmer allocating land and fertilizer sequentially would be willing to pay for the ability to allocate the two simultaneously. Thus, it is a measure of the welfare loss from not correctly considering the interaction between input use and acreage allocation to control total farm variance.

As risk aversion increases, the ability to control variance matters more. The welfare losses grow with risk aversion because the sequential solution controls variance primarily through the land allocation decision. Fertilizer use for the sequential solution is relatively insensitive to increases in risk aversion. If yields are negatively correlated, this loss of variance control is less severe because total farm variance is lower. The losses increase as yields become more positively correlated. The contribution of fertilizer to total variance is given very little weight by the sequential solutions. This neglect matters most when the variance is highest, that is, when there is a positive covariance term.

The welfare losses reported in table 4 are quite small when risk aversion is slight or moderate. But when risk aversion increases to intermediate levels, the changes in certainty

equivalents are quite substantial. For moderate and slight levels of risk aversion, the sequential solution approximates the optimal solutions quite well. This is due to the almost constant per-acre demands for fertilizer when risk aversion is not severe. In their study of fertilizer and risk, Rosegrant and Roumasset also found that nitrogen fertilizer demands are almost constant for individuals that exhibit moderate to no risk aversion. These findings are due to the relatively small contribution of fertilizer to yield variance in the crops considered. If the heteroscedastic portion of the error terms were larger, the per-acre fertilizer demands would respond more to risk levels. Or, if the coefficients of variation for the two crops (defined as the ratio of the standard deviation of output to the mean of output) in this study were larger, the sequential solution would track the simultaneous solution less well, and the welfare losses would be more significant at lower levels of risk aversion.

## Conclusions

In this paper, we examined jointly two problems for risk-averse decision makers faced with an uncertain environment. The first problem involves the choice of crop-specific variable input levels that affect the distribution of output; the second involves the choice of the output mix. We demonstrated that the failure to solve these two problems simultaneously will lead to suboptimal decisions unless the producer is risk neutral or all crop yields are stochastically independent and only one acre of each crop is grown. Welfare losses from the inefficient resource allocation increase as producers become more risk averse and as yields become more positively correlated. The magnitude of these losses is directly dependent on the degree of control farmers have over the relative riskiness of their farming ventures through their input use. The reported welfare losses understate the actual losses from acting suboptimally because it is implicitly assumed that all other factors which affect the variance are held constant. The gain from allocating all inputs optimally can only be greater than that from allocating nitrogen fertilizer optimally since leaving all other inputs fixed would always be an option.

Finally, it should be emphasized that the conclusions reached in this paper are not de-

pendent on the assumptions of normality or constant absolute risk aversion or the form of the production function used in the examples. In general, the expected utility-maximizing quantities of risk affecting inputs employed will depend on the total level of risk in the production plan. The model presented here can be easily modified to accommodate other forms of production functions or statistical distributions for output so long as the joint moment-generating function of crop yields exists with moments dependent on production inputs. Further extensions to other discrete choices such as the choice of marketing strategies or participation in government programs can easily be incorporated.

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