Optimal Forest Management for Interdependent Products:
A Nested Dynamic Bioeconomic Model and Application to Bamboo

Tong Wu, Cornell University, tw494@cornell.edu
C.-Y. Cynthia Lin Lawell, Cornell University, clinlawell@cornell.edu
David R. Just, Cornell University, drj3@cornell.edu
Jiancheng Zhao, Zhejiang Academy of Forestry, jiancheng68@163.com
Zhangjun Fei, Boyce Thompson Institute and Cornell University, zf25@cornell.edu
Qiang Wei, Nanjing Forestry University, weiqiang@njfu.edu.cn

Selected Paper prepared for presentation at the 2022 Agricultural & Applied Economics Association Annual Meeting, Anaheim, CA; July 31-August 2

Optimal Forest Management for Interdependent Products:
A Nested Dynamic Bioeconomic Model and Application to Bamboo[1]

Tong Wu, C.-Y. Cynthia Lin Lawell, David R. Just, Jiancheng Zhao, Zhangjun Fei, Qiang Wei

Abstract

Sustainable forest management is a complex dynamic problem. We develop a nested dynamic bioeconomic model of forests that generate interdependent products that differ in their growth cycles, rates of growth, and potential harvest frequency. We apply our model to bamboo forest management, which involves making decisions about the timing and quantity of bamboo stem harvests and bamboo shoot harvests. Both bamboo shoots and bamboo stems are valuable products. The harvesting of bamboo stems entails cutting down the bamboo plant, while the harvesting of bamboo shoots -- which grow annually from the bamboo plant -- does not. To solve for the optimal bamboo forest management strategy, our novel nested dynamic bioeconomic model nests an inner finite-horizon within-year daily dynamic programming problem that captures daily bamboo shoot growth within a season, inside an outer finite-horizon between-year annual dynamic programming problem that captures annual bamboo stem growth from year to year. We compare the optimal bamboo stem and shoots harvesting strategy with the actual harvesting decisions made by bamboo farmers in Zhejiang province in China. Our research has important implications for the sustainable management of forests worldwide, particularly when the forests produce products that can be harvested at more frequent intervals than the trees themselves.

Keywords: forest management, bamboo, dynamic model, tree crops, forest products
***JEL* codes:** Q23
This draft: May 2022

---

[1]Wu: Cornell University; tw494@cornell.edu. Lin Lawell: Cornell University; clinlawell@cornell.edu. Just: Cornell University; drj3@cornell.edu. Zhao: Zhejiang Academy of Forestry; jiancheng68@163.com. Fei: Boyce Thompson Institute and Cornell University; zf25@cornell.edu. Wei: Nanjing Forestry University; weiqiang@njfu.edu.cn. We thank Jeff Arnold, Susanna Berkouwer, Marshall Burke, Jonah Busch, Luming Chen, Ida Djenontin, Paul Ferraro, Scott Francisco, Todd Gerarden, Dalia Ghanem, Gautam Gowrisankaran, Logan Hamilton, Jim Kiniry, Cathy Kling, Yusuke Kuwayama, Ashley Langer, Jim Lassoie, David R. Lee, Dingyi Li, Shanjun Li, Mengwei Lin, Erin Mansur, Carlos Muñoz Brenes, Ishan Nath, David Popp, Brigitte Roth Tran, Ivan Rudik, Michelle Segovia, Shuyang Si, Peter Smallidge, Nick Tsivanidis, Calum Turvey, Wayne Walker, David Weinstein, Peter Woodbury, Weiguang Wu, Tianming Yen, and Shuo Yu for detailed and helpful comments. We also benefited from comments from seminar participants at Cornell University; and conference participants at the University of Michigan conference on Forests & Livelihoods: Assessment, Research, and Engagement (FLARE). We thank Qin Li, Lina Liu, Jianping Pan, and Boqing Yuan for helping us with the data collection; for providing us information about bamboo management; and for helping to host our visits to the Zhejiang Provincial Key Laboratory of Bamboo of Zhejiang Provincial Academy of Forestry, the Anji Forestry Technology Promotion Center, the Fumin Bamboo Shoot Specialized Cooperative, and the Tianlin Bamboo Shoot Specialized Cooperative, respectively. This research was conducted with support from the Cornell University Center for Advanced Computing. We received funding for our research from the USDA National Institute of Food and Agriculture (NIFA); a Cornell TREESPEAR Research Grant; and a Cornell University Graduate School Conference Grant. Just and Lin Lawell are Faculty Fellows at the Cornell Atkinson Center for Sustainability. All errors are our own.

1. Introduction

The sustainable management of forests is a critical, timely, and important issue worldwide. Forests supply the world's population with wood and non-wood forest products, including timber, food, fuel, rubber, paper, and medicinal plants. Unfortunately, the extent of the world's forests continues to decline as human populations continue to grow and the demand for food and land increases (FAO, 2005; Matthews, 2012; FAO, 2015). The need for sustainable forest management is particularly acute in developing countries such as China, which ranks among the top countries in terms of total forest resources, accounting for 25% of the world demand for forest products, but only 5% of the world's forest area (China Forestry and Grassland Administration, 2018), and is a country where deforestation is rampant (Démurger, Hou, and Yang, 2009).

In this paper, we develop a nested dynamic bioeconomic model of the management of forests that generate interdependent products that differ in their growth cycles, rates of growth, lengths of growing periods, and potential harvest frequency. Our nested dynamic bioeconomic model has important implications for the sustainable management of forests worldwide, particularly when the forests produce products that can be harvested at more frequent intervals than the trees themselves.

We apply our novel nested dynamic bioeconomic model to bamboo forest management in China. Moso bamboo (*Phyllostachys edulis*) is the single most important bamboo species in China, accounting for 74% of China's bamboo forest area ("China Forestry and Grassland Administration", 2018), as well as the third most important source of timber in China. Both bamboo shoots and bamboo stems are harvested as valuable products: bamboo shoots are a traditional food source, and bamboo stems are used as timber for paper making, flooring, and construction (Fu, 2001).

Optimal Moso bamboo management is a complex dynamic problem. Moso bamboo forest management involves making decisions about the timing and quantity of bamboo stem harvests and bamboo shoot harvests. Both bamboo stems and bamboo shoots are products that are sold on the market. Bamboo shoots grow annually from a bamboo plant's underground rhizomes. Owing to their tender taste and to difficulties in harvesting underground shoots, winter shoots – which are young bamboo shoots that are just beginning to grow underground during the winter months – have a higher market price than the older spring shoots that emerge above ground during the later spring months. Bamboo shoots grow into bamboo plants after the end of spring shooting (Shi et al., 2013). Bamboo stems continue to grow each year until age 4-5 years (Zhang et al., 2014; Zhuang et al., 2015), while bamboo shoots only grow within a year. The harvesting of bamboo stems entails cutting down the bamboo plant, while the harvesting of bamboo shoots does not.

Various management styles have been found in bamboo forests in Asia, and the decisions of bamboo farmers can be complex and hard to understand (Yen, 2015). Chinese bamboo farmers generally follow a myopic pattern of intensively harvesting shoots when they first emerge, and then preserving the remaining shoots for later bamboo growth. In Zhejiang Province in China, bamboo forest harvest decisions and shoots harvest decisions are made according to on and off years, with guidance from biologist and forestry specialists. “On” years and “off” years are defined based on the biological growth of bamboo plants. An “on” year is a year when there is a massive emergence of bamboo shoots, and less leaf loss for a bamboo plant. An “off” year is a year with less shoots emergence and more leaf renewing, and normally comes in turn with an on year.

Shoots harvest, especially winter shoots harvest, takes place during on years, when there is a massive emergence of bamboo shoots. In order to create enough growth space for shoots to emerge, and to save space for the shoots harvest before possible decaying, bamboo stem harvests take place during on years as well. Every time a stem harvest decision is made, all mature bamboo stems are clear cut for the whole field. Due to this clear-cut pattern, massive shoots emergence and the clear cutting of mature bamboo take place simultaneously in the on year. When bamboo stems are harvested during an on year, the number of shoots the following off year will be lower. During an off year, relatively fewer shoots are harvested, and little stem cutting takes place.

The actual bamboo stem harvests and bamboo shoot harvests decisions made by Chinese bamboo farmers may be unsustainable, however, leading to profit loss and a deterioration of the bamboo forest resource. The bamboo stem price has decreased significantly in recent years, and some bamboo plants have been left unharvested when matured due to high harvest cost. In contrast, winter shoots have sometimes been over-harvested for high profit, leaving too few shoots for future bamboo forest development.

We develop a novel nested dynamic bioeconomic model of the management of forests that generate interdependent products that differ in their growth cycles, and apply our model to bamboo forest management. In particular, we solve for the optimal bamboo stem harvest and bamboo shoot harvest policy using a numerical dynamic model that nests an inner finite-horizon within-year daily dynamic programming problem within an outer finite-horizon between-year annual dynamic programming problem. The inner finite-horizon within-year daily dynamic programming problem captures daily bamboo shoot growth within a year. The outer finite-horizon between-year annual dynamic programming problem captures annual bamboo stem growth from year to year. We use a Chapman-Richards growth function as our model for bamboo biomass accumulation. We compare the optimal bamboo stem harvest and bamboo shoot harvest policy with actual data on bamboo shoot and bamboo stem harvests on multiple bamboo plots in multiple townships in Zhejiang province in China. We plan to use our dynamic model to simulate, analyze, and design policies and institutions to improve Moso bamboo forest management in China.

We also use our nested dynamic bioeconomic model to develop a dynamic structural econometric model to estimate the unknown parameters in the Chapman-Richards growth function and bamboo farmer's payoff function, and apply it to a detailed daily panel data set we have collected and constructed on bamboo shoot and bamboo stem harvests on multiple bamboo plots in multiple townships in Zhejiang province in China. We also hope to incorporate the environmental benefits of bamboo forest into a social welfare function, and conduct welfare analysis.

The results of our numerical dynamic model suggest that since the number of bamboo shoots at the beginning of each year depend on the number of bamboo stem remaining at the beginning of each year and on whether the previous year was a high-precipitation year, and since bamboo stem continue to grow each year until age 4-5 years, while bamboo shoots only grow within a year, it is generally optimal to harvest bamboo shoots as winter shoots at the end of winter shooting each year that there are bamboo shoots starting the second year; and to wait to harvest all the bamboo stem until the third year or later, after their growth has begun to slow down. If the net price of winter shoots is high enough relative to the net price of spring shoots, it may be optimal to harvest the shoots as winter shoots on the last days of winter shooting, when growers can still benefit from the higher winter shoots price and the winter shoots have the most biomass they will have (since it is during the last days of winter shooting), rather than as spring shoots.

When we allow for uncertainty in rain, which increases the number of bamboo shoots in the following year, we find that when the probability of having a high-precipitation year is high enough, then if there is enough bamboo stem, it may be optimal to harvest a little bamboo stem on the first day of the third year, and harvest all the bamboo stem on the first day of a later year. Since the number of bamboo shoots at the beginning of each year depends on the number of bamboo stem remaining at the beginning of each year, and increases if the preceding summer had high precipitation, high precipitation may enable one to harvest a little bamboo stem in an earlier year without forgoing too much shoot harvest.

We find that the actual bamboo stem and bamboo shoot harvests come close to approximating the optimal harvesting strategy, but have some features that differ from what our model suggests to be optimal. Preliminary parameter estimates from our dynamic structural model suggest that bamboo farmers in China are acting as if they perceive or believe the growth rate for winter shoots to be very low and negligible, and lower than may actually be the case based on data and information on winter shoots. Our results have important implications for bamboo forest management and, to the extent that some of the differences between actual harvests and optimal bamboo harvests reflect possible sub-optimal behavior on the part of Moso bamboo forest managers, for ways to improve Moso bamboo forest management and policy.

We plan to use our dynamic model to simulate, analyze, and design policies and institutions to improve Moso bamboo forest management in China. Our novel dynamic bioeconomic model has important implications for the sustainable management of forests worldwide, particularly when the forests produce products that can be harvested at more frequent intervals than the trees themselves.

The balance of our paper proceeds as follows. Section 2 provides background information on forestry and bamboo forests in China. We discuss the previous literature in Section 3. We describe our numerical dynamic model of bamboo forest management in Section 4. We describe our chosen parameter values in Section 5. Section 6 presents the results of our numerical dynamic model. In Section 7, we compare the dynamically optimal harvesting strategies derived from our model with data on actual bamboo shoot and bamboo stem harvests on multiple bamboo plots in multiple townships in Zhejiang province in China. Section 8 presents our dynamic structural econometric model. We discuss and conclude in Section 9. We describe our ongoing and next steps in Section 10.

2. Background on Forestry and Bamboo Forests in China

China has 208 million hectares of forest area, covering 21.63% of total area of the country and constituting a total stock volume of 15,173 million cubic meters. China ranks among the top countries in terms of total forest resources, accounting for 25% of the world demand for forest products, but only 5% of the world's forest area (China Forestry and Grassland Administration, 2018).

China's forest deficiency results from a combination of various factors. Historically, China was a feudal society that relied on the brutal extraction of natural resources. Land cultivation in China started thousands of years ago, and since then there has been a continued conflict between forest and crop cultivation. Firewood collection, charcoal making, land reclamation, brick making, and house construction contributed to deforestation in the preindustrial periods (Fang and Xie, 1994). It was not until industrialization, however, when massive deforestation took place. The Great Leap forward (1958-1962) campaign destroyed forests by popularizing the usage of homemade furnaces. Even after the campaign, deforestation intensified, providing cheap logs for industrialization (Wang, Kooten, and Wilson, 2004). During the transition from a central planning system to a market economy, local households had insecure ownership rights over tress, leading to forest clearing and deforestation (Démurger, Hou, and Yang, 2009).

The forest sector acted as a base for Chinese industrial development in the first several decades after 1949. Although there are experts who appealed to ecosystem protection (Wang, Kooten, and Wilson, 2004), there was little focus on forest stock preservation. Prior to 1982, forest policies in China defined the priority of forest management was timber production (Démurger, Hou, and Yang, 2009). Although the China Forestry Administration was in place, there was no formal legislation. It was not until 1979 that the first forest law got passed and became effective in 1984 (Wang, Kooten, and Wilson, 2004). In the l990s, China started its transition to preservation and restoration-oriented forest management policies. Six major forestry conservation, restoration, expanding, and commercially developing programs were launched in order to recover and better manage forest resources in China (Démurger, Hou, and Yang, 2009).

In this paper, we focus on bamboo forests. Bamboo grows faster compared to other forest types (Wei et al., 2018), which is consistent with China's goal of forest conservation. China has abundant bamboo species and total stock ("China Forestry and Grassland Administration", 2018). Of the bamboo forest resources in China, 6.6% are in state forests, 51.4% are in collective forests, and 42.0% are in private forests (Démurger, Hou, and Yang, 2009). China has established pilot futures market in Fujian province, where bamboo change can be traced through market price (Wang et al., 2007).

Bamboo (*Bambusoideae*) is categorized as woody grass rather than a tree in plant science. Bamboo is distributed mostly in tropical areas, subtropical areas, and temperate zones in Asia. They survive even at 4000 meters elevation form sea level (Scurlock, Dayton, and Hames, 1999). There are 107 genera and 1300 species of bamboo worldwide (Zhu, 2001). China has the world's most copious bamboo forest resources, with more than 500 bamboo species in 39 genera. China has 6.01 million hectares of bamboo forest, which accounts for 3% of the country's total forest area. Eighty-nine percent of China's bamboo forests are located in eight provinces: Fujian, Jiangxi, Zhejiang, Hunan, Sichuan, Guangdong, Guangxi and Anhui. Moso bamboo (*Phyllostachys pubescens*) occurs most extensively and is the single most important bamboo species in China since it accounts for 74% of China's bamboo forest area ("China Forestry and Grassland Administration", 2018).

The bamboo market in China is arguably characterized by perfect competition. The number of bamboo farmers in China is quite high. There were 7.14 million bamboo farmers in 2010 (International Bamboo and Rattan Organisation, 2012). In Anji County of Zhejiang province alone, there were approximately 110,000 farmers growing bamboo and another 11,000 people working in the bamboo-processing industry in the county in 1999 (Pérez et al., 1999). Bamboo farmers in Zhejiang province are small peasants who own a relatively small amount of land per family. The average land area managed by a family in Anji County is 21.2 mu, of which 14.9 mu (70%) is allocated to bamboo plantations (Pérez et al., 1999).

Bamboo management standards for ensuring an appropriate tree cover and to prevent degradation of the resource stock vary by province and sometimes also by county, and are suggested by the production cooperatives. Farmers' cooperatives in China are owned and operated by their members, and are strongly committed to the development of the local communities (Chen and Scott, 2014). In the agricultural sector, cooperatives help to improve the productivity and living conditions of smallholder farmers by facilitating access to markets, credit, technology and local off-farm employment opportunities (Zhang, Wolz and Ding, 2020). The actual cultivation of bamboo forests is still done by individual bamboo farmers on their own land, however (personal communication, Mr. Jianping Pan, manager of Fumin Bamboo Shoot Specialized Cooperative, August 2018).

Since fresh bamboo shoots are hard to store and transport for long distances, the majority of the fresh bamboo shoots are sold to markets in Zhejiang province, Jiangsu Province, and Shanghai. In addition, approximately 15% of the winter shoots and one third of the spring shoots are sold to local shoots processing factories (Wu et al., 2016). Consumers of bamboo shoots are from highly populated areas such as Shanghai, as well as other cites in Zhejiang and Jiangsu province including but not limited to Yongkang, Cixi, Yuyao, Dongyang, Shangyu, Fuyang, Shaoxing, Ningbo, Changzhou, Suzhou, and Hangzhou. (Shen et al., 1998; Wu et al., 2016). Most of the bamboo stem are processed locally within each county to reduce transportation costs and to contribute to local economic growth (Kusters & Belcher, 2004). Consumers of bamboo stems are generally local bamboo stem processing and manufacturing factories, due to the high transportation costs and the initiatives to contribute to local economic growth (Zhang, 2003; Kusters and Belcher, 2004). Moso bamboo stem and shoots are not only produced in Zhejiang province but also in Hunan, Fujian, Jiangxi, and Sichuan provinces. Bamboo shoots, and especially winter shoots on Zhejiang market are from all these markets, and compete for the same consumers. (People.cn, 2014).

The bamboo shoot and bamboo stem harvest cost is determined by the labor market as well as land specific characteristics such as the slope of forest land. Due to decreasing profits from bamboo forests, younger workers in rural areas have left their hometown and started to find jobs in large cities such as Hangzhou and Shanghai, leaving less labor to manage bamboo forests in rural areas of Zhejiang province; this insufficient labor supply has resulted in increasing labor costs in recent years (Jiang, 2020).

Starting in late 2018, after the time period of our data set, there have been more integrated production cooperatives that manage bamboo forests for all the members. For example, the Tianlin Production Cooperatives in Anji County owns 20,000 mu of bamboo forests, with members from 156 families (Xinhua Finance, 2020). Under an integrated cooperative, bamboo farmers transfer the right to manage their land to the cooperative, letting the cooperative hire labor and jointly manage the land from multiple farmers, and farmers receive dividends. Collective management and land transformation are responses to the diminishing profit from bamboo forests. Under this collective management regime, it is possible that an individual farmer has a larger market power, but given the total number of bamboo farmers in the province, even production cooperatives seem to be price takers in the bamboo market. In addition, integrated cooperatives did not become popular until late 2018, which means that during the span of our data, only very few farmers joined an integrated production cooperative, and most of them are still individually producing on their own land.

Moso bamboo distributes mostly in subtropical provinces include Fujian, Hunan, Zhejiang, and Jiangxi. The mean annual temperature where Moso bamboo grows well varies from 15 to 21°C (59 to 69.8°F), and the mean temperature of the coldest month is 1 to 12°C (33.8 to 53.6°F). Annual precipitation higher than 800mm (31.5 inches) and soil fertile loam deeper than 60cm (23.5 inches) with pH of 4.5 to 7.0 are ideal for Moso bamboo growth. Extreme temperature, precipitation, and soil conditions influence bamboo shoot growth for different areas (Fu, 2001). Moso bamboo reproduce by bamboo shoots grown from rhizomes. Rhizomes are mature underground stems while shoots are buds of new bamboo. The reproduction of Moso bamboo is no different from other grass, while its usage suggests bamboo to be included as tree species in China. Generally, Moso bamboo reach maturity after five years.

Moso bamboo emerges from its rhizomes, which is an underground stem system that expands out. Rhizomes are usually beneath the ground, whereas when its expansion encounter rocks they might jump out of the ground and round the hard hindrance from its top. Moso bamboo start to shoot above the ground from late March randomly. Shoots are initially covered by brown sheath with hairs. Half of the shoots rust naturally in the ground before growing into bamboo, leaving a shallow pit on the ground while another half continue to grow higher into bamboo stems. Bamboo diameter depends solely on the diameter of shoots that will not change in later growth periods (personal communication, bamboo specialist at Zhejiang Provincial Key Laboratory of Bamboo of Zhejiang Provincial Academy of Forestry, August 2018). The harvesting of bamboo stems and bamboo shoots are forms of thinning; there is little need to dig out all rhizomes and rotate the whole stand.

Bamboo shoots grow annually from a bamboo plant's underground rhizomes. As long as the rhizome has not been destroyed, bamboo shoots can still emerge from rhizomes. A bamboo plant may have rhizomes that extends massively and thus can have lots of nodes for shoots growth. The number of newly grown bamboo is the number of bamboo shoots minus number of shoots harvested. This is to say that when all the shoots emerge, they either degenerate, are harvested, or are left in the ground and grow into a newly grown bamboo stem (personal communication, bamboo specialist at Zhejiang Provincial Key Laboratory of Bamboo of Zhejiang Provincial Academy of Forestry, August 2018).

Bamboo shoots grow into a bamboo plant after the end of spring shooting (Shi et al., 2013). Generally, since Moso bamboo stems reach their maximum biomass at age 4-5 years (Zhang et al., 2014; Zhuang et al., 2015), do not increase significantly in biomass after 4.62 years (Zhuang et al., 2015), and mature at age 5-6 years (Yen and Lee, 2011).

Moso bamboo growth is in a cycle of two years in China that is called “Du”, with one Du equals to two calendar years. This cycle is determined due to biological features of a bamboo forest. In an “on” year, shoots merge rapidly and numerously. On the contrary, there are little shoots emergence during an “off” year. Due to this particular growth pattern, bamboo forest becomes an uneven stand with individuals from different Dus. First Du refers to 0-1 year bamboo. Second Du refers to 2-3 years bamboo. Third Du refers to 4-5 years bamboo. And fourth Du refers to 6-7 years bamboo. Each Du bamboo has its characteristics for specialties to distinguish them. The first Du ones are in light green with white fuzz on stems and decade sheath at stem bottom. The second Du ones are in darker colors but without fuzz. Their node thorns are harder. The third Du ones are dark green, while the fourth Du ones are in whitish green. Even with these descriptions, distinguishing bamboo ages is a difficult task for non-specialists (personal communication, bamboo specialist at Zhejiang Key Laboratory of Bamboo of Zhejiang Academy of Forestry, August 2018).

The harvesting of bamboo stems and bamboo shoots will both change the age structure of the bamboo stand, but in different patterns. Harvesting bamboo stems will reduce the amount of bamboo plants in their age class when harvested. This increases the proportion of bamboo plants in young age class.

The harvesting of bamboo shoots is a natural process of thinning since without human intervention, more than half of the shoots will degenerate and die naturally before they grow into bamboo plants. Shoots harvesting is thus a thinning activity that takes these weak shoots out before their death (Jiang, 2007). The harvesting activities will be directly influencing density of the bamboo stand and decreasing proportion of young bamboo plants compared to un-thinned bamboo stands. Harvesting one shoot eliminates one future bamboo plant from the beginning. However, harvesting shoots does not necessarily reduce total bamboo biomass in the future since thinning creates more space for other bamboo plants left in the ground to grow.

In forest management in the United States, forest thinning (silviculture) generally produces low quality logs that incur a cost due to their low market value. A unique feature of bamboo shoot thinning is that by harvesting bamboo shoots, bamboo farmers are also able to sell shoots as a by-product with a high market price.

Bamboo farmers generally follow a pattern of intensive harvest of shoots at the beginning of shoots emergence and preserve remaining shoots for later bamboo growth. In China, bamboo forest harvest decisions and shoots harvest decisions are made according to on and off years. Shoots harvest, especially winter shoots harvest take place during on years. In addition, stem harvest take place mainly during on years. This is because every time a stem harvest decision is made, all fourth Du bamboo are clear cut for the whole forest in Zhejiang province. Bamboo farmers usually clear cut sixth-year-old bamboo rather than wait until they grow into seven-year-old ones. Due to this clear cut pattern, massive shoots emergence and clear cut of forth Du bamboo take place simultaneously in the on year. When bamboo stem are harvested, which means it is going to be an off year next year, number of shoots next year is going to be less next year since bamboo stand developing more for leaves and rhizomes during an off year. For the off year, relatively less shoots harvest activities are performed and little stem cutting is operated (personal communication, bamboo specialist at Zhejiang Provincial Key Laboratory of Bamboo of Zhejiang Provincial Academy of Forestry, August 2018).

As long as the shoots are underground and have not emerged above ground, they are called winter shoots. Winter shoots remain dormant during the coldest winter days in January and February, and emerge above ground in March when temperature rises. Due to their dormancy, the nutrient contents do not change by much in these two months (Su, 2012). Technically, winter shoots could still be harvested and sold on the market for a high winter shoots price until they emerge above ground and start to be called spring shoots. Nevertheless, among bamboo managers in China, there are fewer shoots being harvested in February compared to November, September, and January based on the traditional bamboo management guidance to avoid harvesting too much winter shoots right before spring shoots emergence. The purpose for avoiding overharvesting in the month of February is based on rationales of fostering new bamboo forest (Forestry Department of Hunan Province, 2008).

3. Previous Literature

Owing to intertwined feedback links between biological and economic systems, bioeconomic modeling is challenging, and there is a considerable need for studies that couple economic models of decision-making with biophysical models to provide policy-relevant implications (Kling et al., 2017). Most economic studies of forest management to date are based on mathematical optimization with a certain objective such as profit maximization (Buongiorno, and Gilless, 2003; Kant and Alavalapati, 2014), with a predominant focus on developed countries.

Forest management is a dynamic problem because trees (and bamboo) take time to grow. We build on the seminal model of the optimal rotation time for a forest developed by Faustmann (1849) and elaborated upon by Samuelson (1976). Since then, the Faustmann model has been extended in many ways (Newman, 1988), including to even-aged forest management (Jackson, 1980; Chang, 1983), uneven-aged forest management (Chang, 1981; Hall, 1983), externalities (Berck, 1981; Bowes, 1983; Calish et al., 1978; Hartman, 1976; Nguyen, 1979; Strang, 1983), taxation (Chang, 1982; Klemperer, 1979; Pearse, 1967; Rideout, 1982; Ollikainen, 1991), evolving stumpage price (Bare and Waggener, 1980; Gregersen, 1975; McConnell et al., 1983; Hardie et al., 1984; Newman et al., 1985), a one-time change of unchanged factors (Nautiyal and Williams, 1990), uncertainty (Chang, 1998), the intertemporal allocation of consumption (Deegen et al., 2011), rotation and thinning (Arimizu, 1958), optimal density (Amidon and Akin, 1968), net present discounted value of future payoff (Kilkki and Väisänen, 1969), forest production control (Hool, 1965), production control with Markov process (Hool, 1966), and thinning decisions (Amidon and Akin, 1968). The previous literature has also examined more complicated thinning decisions or combined thinning and rotation decisions (Amidon and Akin, 1968; Brodie et al., 1978; Brodie and Kao, 1979; Chen et al, 1980; Ritters et al., 1982).

In most of the forest economics literature, growth simulation models or yield models characterize the objective as the timber yield for tree species of interest in dynamic programming. Both growth simulation model and yield model describe productivity of a tree standing as a function of multiple variables such as age, temperature, soil, rainfall, slope, and rooting depth (Tyler, Macmillan, and Dutch, 1996). If the objective of the forest owners is instead profit maximization, then the market price of the timber also becomes significant factor in the payoff function.

Previous studies have some typical foci that are worth mentioning. Sophisticated studies on forestry management utilizing dynamic optimization have been done (Ritters, 1982; Haight, 1985; Yousefpour and Hanewinkel, 2009) with specific focus on developed countries. It is not surprising that dynamic optimization for forest management first suffices in more affluent areas since these areas are high in demand for sustainable development and cost effectiveness. Fewer studies are carried out in developing countries and poor areas. Pine and fir are two major types of tree species researchers interested in, due to their popularity in the western world and well developed productivity simulation models. Management of these tree species can be cost ineffective for the fact that they require intensive thinning machinery and labor.

Fast economic development in developing countries and a higher demand of sustainability generate incentive for managing forest with dynamic optimization in these regions. Different political structures, forestry contexts, objectives, and previous sivilcultural practices demonstrate various research opportunities on forest management in developing countries. The literature review done so far demonstrates the possibility of applying dynamic optimization in Chinese forestry.

Bamboo tends to be ignored in the US due to stronger interest in planting timber rich trees, low demand for bamboo shoots, and very limited history cultivating bamboo forests. Only very few studies about bamboo have been done focusing on bamboo management in the US, compared to great amount of timber forest species such as fir and oak discussed in the former sections. Introduced as a potential bio energy crop but proved to be less productive, bamboo is still been considered as a woody grass with very constrained usage in the US, since there are more other out productive trees to be cultivated (Scurlock, Dayton, and Hames, 1999). To date, no previous study used dynamic optimization in bamboo forest management as it has been done for Douglas fir or oak, which are more popular and traditional tree species for US.

We innovate on the previous literature by developing a novel nested dynamic bioeconomic model of the management of forests that generate interdependent products that differ in their growth cycles; and by analyzing bamboo forest management in particular. Our nested dynamic bioeconomic model has important implications for the sustainable management of forests worldwide, particularly when the forests produce products that can be harvested at more frequent intervals than the trees themselves.

Our paper also contributes to the literature on dynamic structural econometric models and its applications (Rust, 1987; Timmins, 2002; Iskhakov, 2010; Duflo, Hanna and Ryan, 2012; Gowrisankaran and Rysman, 2012; Scott, 2013; Aguirregabiria and Luengo, 2016; Gillingham et al., 2016; Langer and Lemoine, 2018; Carroll et al., 2019; Donna, 2019; Blundell, Gowrisankaran and Langer, 2020; Cook and Lin Lawell, 2020; Oliva et al., 2020; Feger, Pavanini and Radulescu, 2020; Ching and Osborne, 2020; Araujo, Costa, and Sant'Anna, 2020; Agarwal et al., 2021; Carroll et al., 2022b; Yeh, Gómez and Lin Lawell, 2022; Sears, Lin Lawell and Walter 2022; Carroll et al., 2022a; Li, Liu and Wei, 2021; Sambucci, Lin Lawell and Lybbert, 2022; Anderson et al., 2021).

Previous research by Misra and Nair (2011) provides evidence that dynamic structural econometric models can help significantly improve decision-making and outcomes. In their study, Misra and Nair (2011) develop and apply a dynamic structural econometric model to data on the US sales force of a large contact lens manufacturer to design sales-force compensation schemes to increase the firm's profits. Their recommendations were then implemented at the firm, resulting in an increase in annual revenues of about $12 million. Our research strives to similarly improve decision-making and outcomes by bamboo forest managers.

4. Dynamic Model of Moso Bamboo Management

Optimal Moso bamboo management is a complex dynamic problem. Moso bamboo forest management involves making decisions about the timing and quantity of bamboo stem harvests and bamboo shoot harvests. Both bamboo stems and bamboo shoots are products that are sold on the market. Bamboo shoots grow annually from a bamboo plant's underground rhizomes. Owing to their tender taste and to difficulties in harvesting underground shoots, winter shoots – which are young bamboo shoots that are just beginning to grow underground during the winter months – have a higher market price than the older spring shoots that emerge above ground during the later spring months. Bamboo shoots grow into bamboo plants after the end of spring shooting (Su, 2012). Bamboo stems continue to grow each year until age 4-5 years (Zhang et al., 2014; Zhuang et al., 2015), while bamboo shoots only grow within a year. The harvesting of bamboo stems entails cutting down the bamboo plant, while the harvesting of bamboo shoots does not.

We solve for the optimal bamboo stem harvest and bamboo shoot harvest policy using a numerical dynamic model that nests an inner finite-horizon within-year daily dynamic programming problem within an outer finite-horizon between-year annual dynamic programming problem. The inner finite-horizon within-year daily dynamic programming problem captures daily bamboo shoot growth within a year. The outer finite-horizon between-year annual dynamic programming problem captures annual bamboo stem growth from year to year.

Our dynamic programming model is a finite sequence of finite-horizon problems. In particular, we nest an inner dynamic optimization problem within an outer dynamic optimization problem. The inner dynamic optimization problem is a finite-horizon within-year daily dynamic programming problem representing the days in one bamboo growth year.

The outer finite-horizon between-year annual dynamic programming problem captures annual bamboo stem growth from year to year. Generally, Moso bamboo stems reach their maximum biomass at age 4-5 years (Zhang et al., 2014; Zhuang et al., 2015), do not increase significantly in biomass after 4.62 years (Zhuang et al., 2015), and mature at age 5-6 years (Yen and Lee, 2011). In our numerical dynamic model, we allow bamboo managers the possibility of letting bamboo stem grow to age 10 years, well past their age of maximum biomass, if it is optimal for them to do so. Since it would be very economically inefficient to harvest bamboo stem after 10 years, however, we model bamboo stem growth with a finite horizon of 10 years. We therefore have a finite sequence of 10 one-year finite horizon problems. Thus, the outer dynamic optimization problem is a finite-horizon between-year annual dynamic programming problem that runs from 0 to 10 years.

Because we have two time variables, year y and day-in-year d, in a finite horizon, nested backward iteration is applicable. We can think of the management of different age classes of bamboo that co-exist at the same time as a (possibly infinite) set of these finite-horizon dynamic programming problems (each representing a finite sequence of finite-horizon problems), one for each set of bamboo emerged as shoots at the same time, but possibly with spillovers across these problems.

We start by modeling the harvesting of bamboo that emerged from rhizomes at the same time (and therefore of the same age class). The control (action) variables are the bamboo shoots harvest decision a_s and the bamboo stem harvest decision a_b. The state variables are the number of bamboo stem per hectare n_b and the number of bamboo shoots per hectare n_s. The bamboo biomass for an individual bamboo plant is given by the bamboo biomass Y_b. The bamboo shoot biomass for an individual bamboo shoot is given by the bamboo shoot biomass Y_s. The time variables are year *y* and day-in-year *d*.

The value function, which is the present discounted value (PDV) of the entire stream of per-period payoffs when the bamboo shoot harvest and bamboo stem harvest decisions are chosen optimally, is given by the following Bellman equation:

$$V(s,d,y) = \max_{a=(a_b,a_s)} \pi(s,a,d,y) + \beta E[V(s',d',y') \mid s,a,d,y] .$$

Since we nest an inner finite-horizon within-year daily dynamic programming problem within an outer finite-horizon between-year annual dynamic programming problem, we use two different discount factors: a daily discount factor β_d and an annual discount factor β_y. We set the annual discount factor to be $\beta_y = 0.9$. We set the daily discount factor to be $\beta_d = \exp(\ln(\beta_y)/365) = 0.9997$, which yields an annual discount factor of 0.9 over 365 days.

Since we are assuming that the same bamboo plant can be harvested for shoots and/or later for bamboo, it suffices to declare one constant to be the initial number of bamboo plants (stems) per hectare. In general, there will be around 1,750 to 3,000 bamboo stem per hectare in Zhejiang province. To make our dynamic programming problem easier to solve, we discretize action variables as proportions of the initial number of bamboo plants (stems) per hectare. Thus, a_s is expressed as a percentage of n_{b0}, the initial number of bamboo plants (stems) per hectare, and ranges from harvesting 0% to 100% of the initial number n_{b0} of bamboo stem per hectare. Similarly, a_b is expressed as a percentage of n_{b0}, the initial number of bamboo plants (stems) per hectare, and ranges from harvesting 0% to 100% of the initial number n_{b0} of bamboo stem per hectare.

For the transition density for number of bamboo shoots within a year: during each year *y*, the number of bamboo shoots will change via the bamboo shoots harvest decision a_s, which can decrease the number of bamboo plants by a percentage of the initial number of bamboo plants n_{b0}. Thus, prior to the end of spring shooting, the number of bamboo plants the next day this year is given by:

$$n_{s,y,d+1} = \max\{n_{s,y,d} - a_{s,y,d}, 0\} \quad \forall d \le d_s ,$$

where d_s is the day-in-year corresponding to the last day of spring shooting. Since bamboo shoots grow into bamboo plants after the end of spring shooting:

$$n_{s,y,d+1} = 0 \;\; \forall d > d_s .$$

For the transition density for number of bamboo plants, the number of bamboo stems n_b changes via the bamboo stem harvest decision a_b, which can decrease the number of bamboo stems by a percentage of the initial number of bamboo plants n_{b0}. Bamboo stem harvest can occur any day of year. Given a_b, we obtain the next period's number of bamboo plants by subtracting a_b from this period's number of bamboo plants. Thus, next period's number of bamboo stems is given by:

$$\begin{aligned} n_{b,y,d+1} &= \max\{n_{b,y,d} - a_{b,y,d}, 0\} \\ n_{b,y+1,0} &= \max\{n_{b,y,D} - a_{b,y,D}, 0\} \end{aligned} .$$

In addition, since bamboo shoots grow into bamboo plants after the end of spring shooting, the number of bamboo stems n_b also increases by the number of bamboo shoots that remain at the end of the last day of spring shooting. Thus, on day-in-year d_s, which is the day-in-year that corresponds to the last day of spring shooting, the transition density for the number of bamboo stems is given by:

$$n_{b,y,d_s+1} = \max\{n_{b,y,d} - a_{b,y,d} + \max\{n_{b,y,d_s} - a_{b,y,d_s}, 0\}, 0\} .$$

For now, we start by modeling the harvesting of bamboo that was all planted at the same time (and therefore of the same age class). We therefore start by modeling bamboo that start as bamboo shoots and, if they are not harvested, grow into bamboo stem at the end of spring shooting at the end of the first bamboo growth year. Thus, for the bamboo stem, we start by focusing only on bamboo stem that result from bamboo shoots that remain at the end of spring shooting in the first bamboo growth year for now.

The transition density for number of bamboo shoots between years is more complicated. Li et al. (2016) find that Moso bamboo shoots emergence is positively related to bamboo stem density: with a higher bamboo stem density, there are more rhizomes underground, and thus a larger possibility for bamboo shoots emergence. According to Zhang and Ding (1997), new bamboo shoots emergence is positively correlated with Leaf Area Index (LAI) and positively correlated with precipitation in July and August of the previous bamboo growth year, which is the period of bamboo shoots formation. LAI is defined as leaf area/ground area, and thus is an index that characterize the bamboo stem density of a Moso bamboo stand. The number of bamboo shoots at the beginning of the year depends on the number of remaining bamboo plants at the beginning of the year (remaining after bamboo stem are harvested the previous year), since remaining bamboo plants affect density of the bamboo stand. When bamboo stems are harvested, which means it is going to be an off year next year, number of shoots next year is going to be less next year since bamboo stand developing more for leaves and rhizomes during an off year.

We incorporate the positive correlation between number of shoots and bamboo plants by subtracting this year's bamboo harvest (which is equal to 0 and does not affect number of bamboo shoots if it is within the year and bamboo harvest has not taken place). We implement the positive correlation between number of bamboo shoots at the beginning of the year and on the number of remaining bamboo plants at the beginning of the year (remaining after bamboo stem are harvested the previous year) as follows:

$$n_{s,y+1,0} = \max\{n_{b,y+1,0}, 0\} \ .$$

Later, in the stochastic model, we add the positive correlation with the precipitation during the months of July and August from the previous bamboo growth year.

There are multiple available models to measure the growth and productivity of a Moso bamboo plant. Allometric equations and logistic functions have been used for characterizing bamboo growth. An allometric model predicts biomass using diameter at breast height. Biological studies suggest using the Chapman-Richards model (Richards, 1959), which is flexible growth model for plants, and has been used to predict Moso bamboo height (Yen, 2016). In addition to a model for bamboo stem growth, we also need a model for bamboo shoot growth. Bamboo shoot biomass accumulation has been described using logistic curve (Zhou, 1998). The literature constructing a growth model for bamboo shoots is sparse, however, and even less is known about undergrowth winter shoot growth. Thus, as the Chapman-Richards model is a generalized logistic curve, and since bamboo shoots are young bamboo plants, we adopt and separately parameterize separate Chapman-Richards models for winter shoot growth and spring shoot growth as well.

We therefore use a separate Chapman-Richards model for the growth of each of the three types j of bamboo products: winter shoots s_w, spring shoots s_w, and bamboo stem b. The Chapman-Richards model is given by:

$$Y_j = A_j\left(1 - Q_j e^{-\alpha_j t_j}\right)^{1/(1-v_j)},$$

where Y_j is the total biomass for bamboo product *j* in a single bamboo plant; t_j is the age of bamboo (in days for winter and spring shoots, and in years for bamboo stem); and A_j, α_j, Q_j, v_j are parameters whose interpretation and values for each of the bamboo product types *j* are discussed in more detail below.

The equation for bamboo biomass is the equation for bamboo biomass for an individual bamboo plant that has not yet been harvested. It therefore should not be a function of any of the action variables. It also does not depend on the current bamboo biomass, but only on time. We do not need to discretize or round Y_j, since it is not a state variable.

The per-period payoff function is a function of state variables, action values, day in year, and year. Given the large number of bamboo farmers – there were 110,000 bamboo farmers in one county (Anji county) in Zhejiang province alone in 1999 (Pérez et al., 1999) – and the other features of the bamboo market described in Section 2, we assume that the bamboo market is perfectly competitive and that bamboo farmers are therefore price takers. We assume that the daily fixed cost is c_0. This includes the cost of maintenance.

Since this is a finite horizon problem, the value functions and policy functions are functions of both measures of time, year *y* and day-in-year *d*. The terminal condition for the outer annual backwards iteration is that there is no continuation value after the last day of the last year. The terminal condition for the inner day-in-year backwards iteration is that, except in the last year, when there is no continuation value after the last day of the last year, the continuation in the last day of the year is the expected value of the value function on the first day of the next year. We can think of the management of different age classes of bamboo that co-exist at the same time as a (possibly infinite) set of these finite-horizon dynamic programming problems (each representing a finite sequence of finite-horizon problems), one for each set of bamboo planted at the same time, but possibly with spillovers across these problems. But for now, we start by modeling the harvesting of bamboo that was all planted at the same time (and therefore of the same age class). From Xu et al. (2017), there are on average 400 stems of bamboo in the same one-year age class per hectare.

Our per-period payoff function is:

$$\pi(s,a,d,y) = R_b(s,a,d,y) - C_b(s,a,d,y) + R_s(s,a,d,y) - C_s(s,a,d,y) - c_0 ,$$

where $R_b(s,a,d,y)$ is bamboo stem harvest revenue, $C_b(s,a,d,y)$ is bamboo stem harvest cost, $R_s(s,a,d,y)$ is bamboo shoot harvest revenue, $C_s(s,a,d,y)$ is bamboo shoot harvest revenue, and c_0 is a daily fixed cost.

Bamboo stem harvest revenue $R_b(s,a,d,y)$ is given by:

$$R_b(s,a,d,y) = p_b \tau a_b n_{b0} Y_b ,$$

where p_b is the bamboo stem price and τ is a conversion coefficient to convert bamboo stem price and bamboo stem quantity $a_b n_{b0} Y_b$ to comparable units, as explained in more detail below.

Bamboo stem harvest cost $C_b(s,a,d,y)$ is given by:

$$C_b(s,a,d,y) = c_b \tau a_b n_{b0} Y_b ,$$

where c_b is the unit cost of bamboo stem harvest and τ is a conversion coefficient to convert the unit cost of bamboo stem harvest and bamboo stem quantity $a_b n_{b0} Y_b$ to comparable units, as explained in more detail in below.

Bamboo shoot harvest revenue $R_s(s,a,d,y)$ is given by:

$$R_s(s,a,d,y) = p_s \tau a_s n_{b0} Y_s ,$$

where p_s is the bamboo shoots price and τ is a conversion coefficient to convert bamboo shoots price and bamboo shoots quantity $a_s n_{b0} Y_s$ to comparable units. We allow the bamboo shoots price p_s to differ for winter shoots and spring shoots. The biomass we predicted using Chapman-Richards model cannot been directly used to calculate total revenue in per period payoff because shoots and stem price are recorded in yuan per kilogram, where the weight in kilograms contains both biomass and water. We need coefficients that transfer biomass into kilograms for both bamboo forest products. The Chapman-Richard's model predicts biomass Y_b and Y_s in units of kilograms. The price we have are for weights rather than weights of biomass, which is dry weight. Since weight include water and biomass, we use a conversion coefficient τ to transfer biomass into its actual weight.

Bamboo shoot harvest cost $C_b(s,a,d,y)$ is given by:

$$C_s(s,a,d,y) = c_s \tau a_s n_{b0} Y_s ,$$

where c_s is the unit cost of bamboo shoot harvest and τ is a conversion coefficient to convert the unit cost of bamboo shoot harvest and bamboo shoots quantity $a_s n_{b0} Y_s$ to comparable units, as explained in more detail in below. We allow the bamboo shoots harvest cost c_s to differ for winter shoots and spring shoots.

The bamboo forest manager chooses the bamboo stem harvest strategy and the bamboo shoot harvest strategy to maximize the present discounted value of the entire stream of per-period payoffs.

In addition to a deterministic specification of our dynamic programming model, we also run our model allowing rain to be stochastic. In our stochastic rain specification, we add a third state variable, *precip*, which is a dummy for the cumulative daily precipitation over July and August of that bamboo growth year exceeding a high precipitation threshold that day. We specify a daily probability of high precipitation, which is the probability that *precip* is equal to 1 (high) that day. The daily probability of high precipitation is weakly monotonically increasing from July 1 to August 31.

We implement the positive correlation of the number of bamboo shoots per hectare with precipitation during the months of July and August of the previous bamboo growth year as a bin increment of 1 if *precip* is equal to 1 (high):

$$n_{s,y+1,0} = \min\{\max\{n_{b,y+1,0} + precip_{y-1,D}, 0\}, N_b\} \quad .$$

5. Parameter Values

5.1. Bamboo shoot price

Bamboo shoot prices vary for spring bamboo shoots and winter bamboo shoots. Due to difficulties of locating and harvesting underground winter bamboo shoots, as well as popular preference over more tender taste, winter bamboo shoots have higher market price than spring bamboo shoots. As long as the shoots are underground and have not emerged above ground, they are called winter shoots. Winter shoots remain dormant during the coldest winter days in January and February, and emerge above ground in March when the temperature rises. Due to their dormancy, the nutrient contents do not change by much in these two months (Su, 2012).

Technically, winter shoots could still be harvested and sold on the market for a high winter shoots price until they emerge above ground and start to be called spring shoots.

In addition, spring shoots taste bitter, are no longer tender, and are no longer even considered a good source of vegetable (LeBeau Bamboo Nursery, 2015) and therefore will be unpopular on the market after they exceed 30 cm (or about 1 foot), which is after around 10 days of spring shooting (Tao et al. 2020). The maximum number of days for which spring shoots are traded and spring shoots prices are recorded on the wholesale market for Zhejiang province is from March 1 to June 13, which is longer than the local number of days spring shoots are on the market because it incorporates all townships in Zhejiang province, each of which has a spring shoots market period of around 60 days that occur at different times. In Shanchuan and Sian Townships, the maximum number of days for which spring shoots are traded and spring shoots prices are recorded on the wholesale market is from March 1st to April 30, which is 61 days. These are from data and interviews.

According to data from National Agricultural Products Business Information Public Service Platform operated by China's Ministry of Commerce, bamboo shoots prices remain stable in Liaoning province, which is not a major source of bamboo, but volatile in Zhejiang province, one of the eight major provinces with bamboo forests. The bamboo shoot price in a representative market in Jiaxing in Zhejiang province varied from ¥3.06/kg to ¥24.75/kg in 2017. Bamboo shoot prices in Zhejiang province followed a similar pattern from 2014 to 2018, with the highest prices in the winter. Bamboo shoot prices are in the range of ¥2/kg to ¥40/kg, with highest price appearing in November and lowest price appearing in May in general.

In our numerical model, we vary the bamboo shoot price p_s from 25 to 40 ¥/kg for winter shoots, and from 2 to 3 ¥/kg for spring shoots until 10 days of spring shooting. After 10 days of spring shooting, we set the bamboo shoot price for spring shoots to be 0 ¥/kg.

5.2. Bamboo stem price

Unlike for bamboo shoots, daily data on bamboo stem price is not available from government operated databases. According to Wu and Cao (2016), the 2012 Moso bamboo stem price is ¥1.39/kg in Zhejiang province. Meng, Liu, and Wu (2014) find average bamboo stem price to be ¥0.79/kg. In our numerical model, the bamboo stem price p_b ranges from 0.8 to 1.4 ¥/kg.

5.3. Bamboo forest maintenance cost, harvesting costs, and planting costs

Bamboo management and harvest costs vary for different provinces. According to a recent study using randomized data from representative sampling in three provinces, the major costs of bamboo forest are labor costs, along with a minor cost of fertilizer (Wu and Cao, 2016). The average labor cost in Zhejiang province is ¥125 per worker per day. One hectare Moso bamboo is estimated to cost ¥4100 annually on labor. Increasing fertilizer usage significantly boosts bamboo shoots yields, while has little effect on stems output. This is not true for increase in labor input, which leads to higher stems and shoots quantity at the same time (Wu and Cao, 2016). Labor hours differ for timber forest and timber-shoots forest, where the former aims at bamboo stem production and the latter has two major products (Meng, Liu and Wu, 2014). Land quality such as the slope of land affects the cost of bamboo stems harvest (Wu and Cao, 2016; Dong et al., 2015).

Bamboo planting costs include land finishing costs, planting labor costs, and bamboo mother plants costs, accordingly account for 39.4, 30.9 work-day per hector, and ¥15 per mother plant. These add up to a ¥14,712.5/ha one-time cost. (Meng, Liu and Wu, 2014). All bamboo forests included in our data are naturally grown from long ago rather than planted, and thus planting cost is zero.

Maintenance costs vary in the range of 5.4 to 14.4 work-day per hectare annually, with average labor cost of ¥125/work-day in Zhejiang province. Average maintenance cost per hectare in Zhejiang province is estimated to be ¥675/ha to ¥1,800/ha annually. The difference in costs is due to age of bamboo forests. In their study, "forest formation year" is used to name the ninth year when maintenance costs decrease sharply. Intensive labor is ideal before bamboo forest formation year, and decreases sharply after forest formation year (Meng, Liu and Wu, 2014). Bamboo forest has its major cost of fertilizer, which cost 1,000 yuan per year, with an additional cost of 2,000 yuan on labor to spread fertilizer. Bamboo stem density ranges from 1,750 to 3,000 per ha for different areas. For our numerical model, we set the daily fixed cost c_0 to 2 ¥ per day.

Average shoots harvesting costs are estimated to be 1.9 work-day per hectare annually, which is ¥237.5/ha per year. Bamboo stem harvesting cost is 14 work-day per hectare annually, which is ¥1,750/ha per year (Meng, Liu and Wu, 2014).

According to Mr. Jianping Pan, who is the manager of Fumin Bamboo Shoot Specialized Cooperative, bamboo harvest can be fast, one worker can harvest 1 mu (about 667 square meters) of bamboo per day. For bamboo stem, workers get paid daily with a rate of 300 yuan per day and harvest 1,250 to 2,000 kg of bamboo stem. For spring shoots, workers got paid daily, with a rate of 150 to 180 yuan per day, and can harvest 100 kg of spring shoots per day; the total harvest for each sample plot is 200-250 kg per spring shooting period. Winter shoots are more expensive and harder to find than spring shoots, and thus workers get paid for 300 yuan per day and can harvest about 15 to 20 kg per day.

For the harvesting costs in our numerical model, we calculate the unit costs of harvest by dividing estimates of harvest per worker per day by cost per worker per day. We vary the unit cost c_s of bamboo shoot harvest from 300/20 ¥/kg to 300/15 ¥/kg for winter shoots, and from 150/100 ¥/kg to 180/100 ¥/kg for spring shoots. We set the unit cost c_b of bamboo stem harvest from 300/2,000 ¥/kg to 300/1,250 ¥/kg.

5.4. Time

Since winter shooting period and the corresponding spring shooting years are in two years, we measure a "year" based on bamboo growth rather than following a strict calendar year. The first day of each bamboo growth year is the first day of winter shooting. The first day of winter shooting is September 1. As long as the shoots are underground and have not emerged above ground, they are called winter shoots. Winter shoots remain dormant during the coldest winter days in January and February, and emerge above ground in March when the temperature rises (Su, 2012). Spring shooting begins on March 1 in Zhejiang province. Thus, winter shooting is from September 1 until February 28. The number of winter shooting days is therefore 181 days. The spring shooting period starts on March 1 and ends on August 31, the last day of the bamboo growth year. In other words, shoots do not become bamboo stem until the end of the bamboo growth year. This is because, as seen in Song et al. (2016), the bamboo still seems to grow very fast following the spring shoot growth function until the end of the bamboo growth year. Thus, the number of spring shooting days we use in our numerical model is 184 days.

Since bamboo stem harvest is possible during any day throughout the year, we model the decision on each day of the year. The day in year d starts on September 1 (the first day of winter shooting) in one calendar year and ends on August 31 the following calendar year, 365 days later. Our year variable y is the bamboo growth year, not calendar year. Each bamboo growth year y starts from September 1 of one calendar year and end on August 31 of the following calendar year.

5.5. Parameters in Chapman-Richards model of bamboo shoot growth for winter shoots

To date there have not been any studies based on the biomass of winter shoots growth, and there are very few studies on Moso bamboo underground development. We use the following Chapman-Richards model for winter shoot growth:

$$Y_{s_w} = A_{s_w} \left(1 - Q_{s_w} e^{-\alpha_{s_w} t_{s_w}}\right)^{1/(1-v_{s_w})},$$

where Y_{s_w} is the total biomass of a winter shoot of age t_{w_x} days. The shoots biomass is basically the dried weight of shoots. The Chapman-Richards model for winter shoot growth yields the following equation of motion for winter shoot biomass:

$$\frac{dY_{s_w}(t_{s_w})}{dt_{s_w}} = \frac{\alpha_{s_w}}{1-v_{s_w}} A_{s_w} Q_{s_w} \left(1 - Q_{s_w} e^{-\alpha_{s_w} t_{s_w}}\right)^{\frac{1}{1-v_{s_w}}-1} e^{-\alpha_{s_w} t_{s_w}}.$$

At the inflection point,

$$\frac{d^2 Y_{s_w}(t_{s_w})}{dt_{s_w}{}^2} = 0$$

$$=> \frac{\alpha_{s_w}{}^2}{1-v_{s_w}} A_{s_w} Q_{s_w} \left(1 - Q_{s_w} e^{-\alpha_{s_w} t_{s_w}}\right)^{\frac{1}{1-v_{s_w}}-1} e^{-\alpha_{s_w} t_{s_w}} \cdot \left(\frac{v_{s_w}}{1-v_{s_w}} Q_{s_w} (1 - Q_{s_w} e^{-\alpha_{s_w} t_{s_w}})^{-1} e^{-\alpha_{s_w} t_{s_w}} - 1 \right) = 0$$

$$=> \frac{v_{s_w}}{1-v_{s_w}} Q_{s_w} (1 - Q_{s_w} e^{-\alpha_{s_w} t_{s_w}})^{-1} e^{-\alpha_{s_w} t_{s_w}} = 1$$

$$=> v_{s_w} = 1 - Q_{s_w} e^{-\alpha_{s_w} t_{s_w}}$$

For the age t_{s_w} of winter shoots, due to its relatively short period of growth, age of bamboo shoots is measured in days rather than years. Winter shooting is from September 1 until February 28. The number of winter shooting days $t_{s_w}^{\max}$ is therefore 181 days.

The parameter A_{s_w} is related to the maximum possible winter shoot biomass for a single winter shoot. According to a video from Zhejiang province of winter shoots in late November 2020 ("Zhejiang Local Winter Shoots Trading on Site"), it is very rare to have winter shoots that is 0.75 kg in Zhejiang province, which is 0.375 kg in dry biomass (using our transition of 0.5 between weight and biomass). According to Yonghua Qiu, a senior engineer from Suichang Bureau of Forestry, where Suichang is a township in Zhejiang, the maximum possible winter shoots weight could be go up to as large as more than 0.5 kg in previous years, while since 2013 is a dry year, winter shoots only grow up to 0.25 kg. It is also rare to harvest winter shoots that is more than 1.5 kg (Zeng and Peng, 2013). In our numerical model, we set $Y_{s_w}^{\max}$, the maximum possible winter shoots biomass at the end of winter shooting (day $t_{s_w}^{\max}$), to be 0.75 kg. This is to say, the maximum possible winter shoots biomass at the end of winter shooting will be 1.5 kg per shoot in weight, and thus 0.75 kg in biomass. We then calibrate A_{s_w}, which is the maximum possible winter shoot biomass as the number of days goes to infinity (which is well past the end of winter shooting) as follows:

$$A_{s_w} = Y_{s_w}^{\max} / \left(1 - Q_{s_w} e^{-\alpha_{s_w} t_{s_w}^{\max}}\right)^{1/(1-v_{s_w})}.$$

For the growth rate α_{s_w} for winter shoots, the growth rate for bamboo shoots is more rapid than that for bamboo stem (Song et al., 2016). To date there have not been any studies based on the biomass of winter shoots growth, and there are very few studies on Moso bamboo underground development. Wei et al. (2017) describes underground bamboo shoots development, but only have a time trend of growth of winter shoots in terms of individual height, not biomass. Hu et al. (2020) study gene expression for each month of shoots growth from September to the following year's April. The number of genes expressed in the shoots is a measure of shoots growth activity level, as well as biomass accumulation. Since Hu et al. (2020) find the winter shoots express fewer genes than spring shoots do, we choose a growth rate α_{s_w} for winter shoots that is slightly lower than the growth rate α_{s_s} for spring shoots that we specify below. In particular, since we set the growth rate α_{s_s} for spring shoots to 0.036 below, and winter shoots is expressing less genes compared to spring shoots, we set the growth rate α_{s_w} for winter shoots to 0.03.

For the biological constant Q_{s_w}, which is related to the initial winter shoot biomass at the beginning of winter shooting, we set Q_{s_w} to 1 because we want the biomass of winter shoots to be equal to 0 on day $t_{s_w} = 0$.

The parameter v_{s_w} is related to the inflection point of the Chapman-Richards growth function, where the time rate of change in winter shoot biomass reaches its maximum. This allometric constant lies between zero and one for the Chapman-Richards growth model

(Fekedulegn et al., 1999; Liu and Li, 2003). Wei et al. (2017) measure individual shoot diameter from August to February the following year find that the diameter change of Moso bamboo is slow until late November and then slows down again. Sun et al. (2017) find that underground shoots formed in September; developed into underground shoots in October and November. Shoot growth rate slowed down and almost stopped in December until February next year. Shoots growth resumes in March and emerged from ground (Sun et al. 2017). The fastest growth happened between early and late November. Bamboo shoots start to dormant from December because of the cold weather until next March. (Wei et al., 2017; Hu et al., 2020). This is to say, the fastest growing time is around day 76 (mid November) of the entire winter shoot growth process. We therefore set the winter day of inflection $t_{s_w}^{\text{infl}}$ to be 76. We calculate v_{s_w} using

$$v_{s_w} = 1 - Q_{s_w} e^{-\alpha_{s_w} t_{s_w}^{\text{infl}}} .$$

and iterating on v_{s_w} until convergence.

5.6. Parameters in Chapman-Richards model of bamboo shoot growth for spring shoots

We use the following Chapman-Richards model for spring shoot growth:

$$Y_{s_s} = A_{s_s} \left(1 - Q_{s_s} e^{-\alpha_{s_s} t_{s_s}}\right)^{1/(1-v_{s_s})} ,$$

where Y_{s_s} is the total biomass of a spring shoot of age t_{s_s} days.

For the age t_{s_s} of spring shoots, due to its relatively short period of growth, age of bamboo shoots is measured in spring shooting days rather than years. The spring shooting period starts on March 1 and ends on August 31, the last day of the bamboo growth year. In other words, shoots do not become bamboo stem until the end of the bamboo growth year. This is because, as seen in Song et al. (2016), the bamboo still seems to grow very fast following the spring shoot growth function until the end of the bamboo growth year. Thus, the number of spring shooting days $t_{s_s}^{\max}$ we use in our numerical model is 184 days. Bamboo shoots grow into a bamboo plant after the end of spring shooting (Shi et al., 2013).

The parameter A_{s_w} is related to the maximum possible spring shoot biomass for a single spring shoot. Xu et al. (2011) study the time trend of above ground biomass in Lin'an city, Zhejiang Province, and find that on spring shooting day 88, the spring shoot biomass is approximately 8.25 kg in dry weight. Song et al. (2016) shows shoots biomass at the end of August to be ~8 kg. In our numerical model, we set $Y_{s_s}^{\max}$, the maximum possible spring shoots biomass at the end of spring shooting (day $t_{s_s}^{\max}$), to be 8 kg. We then calibrate A_{s_s}, which is the maximum possible spring shoot biomass as the number of spring shooting days goes to infinity (which is well past the end of spring shooting) as follows:

$$A_{s_s} = Y_{s_s}^{\max} / \left(1 - Q_{s_s} e^{-\alpha_{s_s} t_{s_s}^{\max}}\right)^{1/(1-v_{s_s})} .$$

For the growth rate α_{s_s} for spring shoots, the growth rate for bamboo shoots is more rapid than that for bamboo stem (Song et al., 2016). Based on Song et al. (2016), the growth rate for spring shoots at the end of April is 0.036 per day. We therefore set set our spring shoot growth rate α_{s_s} to 0.036.

The biological constant Q_{s_s} is related to the initial spring shoot biomass at the beginning of spring shooting. Since Q_{s_s} is based on the biomass of spring shoots at the beginning of spring shooting, then this should be calculated based on the biomass at the end of winter shooting. In other words, we use the biomass on the last day of winter shooting to calculate Q_{s_s}. The biomass on the last day of winter shooting, $Y_{s_w}^{\max}$, is the Chapman-Richards growth function for winter shoots evaluated on the last day of winter shooting. We then calculate Q_{s_s} as:

$$Q_{s_s} = \frac{1 - \left(Y_{s_w}^{\max} / Y_{s_s}^{\max}\right)^{1-v_{s_s}}}{1 - \left(Y_{s_w}^{\max} / Y_{s_s}^{\max}\right)^{1-v_{s_s}} e^{-\alpha_{s_s} t_{s_s}^{\max}}} .$$

The parameter v_s is related to the inflection point of the Chapman-Richards growth function, where the time rate of change in spring shoot biomass reaches its maximum. According to Song et al. (2016), the maximum growth rate occurs at the end of April, which is around 60 days of spring shooting. We take the maximum growth rate for spring shoots as occurring at ~60 days of spring shooting. We therefore set the spring day of inflection $t_{s_s}^{\text{infl}}$ to be 60. We calculate v_{s_s} using:

$$v_{s_s} = 1 - Q_{s_s} e^{-\alpha_{s_s} t_{s_s}^{\text{infl}}} .$$

and iterating on v_{s_s} until convergence.

5.7. Parameters in Chapman-Richards model of bamboo stem growth

We use the following Chapman-Richards model for bamboo stem growth:

$$Y_b = A_b\left(1-Q_b e^{-\alpha_b t_b}\right)^{1/(1-\nu_{s_s})},$$

where Y_b is the total biomass of a bamboo stem of age t_b years.

For the age t_b of bamboo forest in years, Moso bamboo stems reach their maximum biomass at age 4-5 years (Zhang et al., 2014; Zhuang et al., 2015), do not increase significantly in biomass after 4.62 years (Zhuang et al., 2015), and mature at age 5-6 years (Yen and Lee, 2011). We assume Moso bamboo stem biomass does not increase after $t_b^{\max}$ years, and set $t_b^{\max}$ to 8 years.

For A_b, which is related to the maximum possible bamboo stem biomass for a single bamboo plant in the specific area, the maximum possible bamboo biomass for a single bamboo plant depends on land quality such as slope, precipitation, soil type, and temperature of the bamboo field we are interested in. Yen (2016) calculate maximize stem biomass for Moso bamboo in central Taiwan in its 5th year growth to be 15.88 kg per plant with standard deviation of 2.51 kg. Zhang et al. (2014) find that the maximum stem biomass for an eight-year-old Moso bamboo has average biomass of 15.06 kg, with a standard deviation of 6.58 kg. Moso bamboo with longer age will have higher maximum stem biomass, while stem biomass accumulation slow down in the mature age for Moso bamboo, which is generally at age 5-6 years (Yen and Lee, 2011). In our numerical model, based on the means in the previous literature, we set $Y_b^{\max}$, the maximum possible bamboo stem biomass at the end of $t_b^{\max}$ years, to be 15.5 kg. We then calibrate A_b, which is the maximum possible bamboo stem biomass as the number of years goes to infinity (which is well past $t_b^{\max}$) as follows:

$$A_b = Y_b^{\max} / \left(1-Q_b e^{-\alpha_b t_b^{\max}}\right)^{1/(1-\nu_b)}.$$

For the growth rate α_b for bamboo stem, the growth rate for Moso bamboo differs with studies as well. According to Xu et al. (2011), the major biomass accumulation occurred along with the fast elongation of bamboo stem in the early stage of bamboo growth. In the stage where first shoot shell detached and branch emergence, bamboo biomass tripled. To estimate the biomass accumulation rate for Moso bamboo, we compare bamboo stem biomass in different age groups.

According to Zhang et al. (2014), the growth rate for bamboo stem biomass over four 2-year stages is in the range of 0.060 to 0.196 per 2-year stage, or an average of 0.03 to 0.098 per year. Based on Song et al. (2016), the growth rate after 4 months of shooting (in August before the first full bamboo growth year) is 0.75 per year. In our numerical model, we set the growth rate α_b for bamboo stem to 0.75.

The biological constant Q_b, which is related to the initial bamboo stem biomass at the beginning of the first bamboo growth year. For bamboo stem, we model the growth of bamboo stem starting from the end of spring shooting, when bamboo shoots become bamboo stem. At the beginning of its full bamboo growth year (i.e., at the beginning of bamboo growth year age 1), the initial bamboo stem biomass is the maximum bamboo shoot biomass at the end of spring shooting. The end of spring shooting in years is $t_{b0} = \left(t_{s_w}^{\max} + t_{s_s}^{\max}\right)/365$. The initial bamboo stem biomass at the end of spring shooting (year t_{b0}) is the maximum bamboo shoot biomass $Y_{s_s}^{\max}$ at the end of spring shooting. We then calculate Q_{s_s} as:

$$Q_b = \frac{1-\left(Y_{s_s}^{\max} / Y_b^{\max}\right)^{1-v_b}}{e^{-\alpha_b t_{b0}} - \left(Y_{s_s}^{\max} / Y_b^{\max}\right)^{1-v_b} e^{-\alpha_b t_b^{\max}}} .$$

The parameter v_b is related to the inflection point of the Chapman-Richards growth function, where the time rate of change in bamboo stem biomass reaches its maximum. In Song et al. (2016), the biomass accumulation is fastest after in September following spring shooting. Since the bamboo growth year starts September 1, this means that the inflection point takes place the first month of the first full bamboo growth year (bamboo growth year age 1). We therefore set the year of inflection t_b^{infl} to be 1. We calculate v_b using:

$$v_b = 1 - Q_b e^{-\alpha_b t_b^{\text{infl}}} .$$

and iterating on v_b until convergence.

5.8. Daily probability of high precipitation

In our stochastic rain specification, we add a third state variable, *precip*, which is a dummy for the cumulative daily precipitation over July and August of that bamboo growth year exceeding a high precipitation threshold that day. We use 400 mm as the cutoff to determine if *precip* is high (*precip* =1) or not (*precip* = 0).

Since cumulative daily precipitation over July and August of a bamboo growth year varies within July and August of a year (and is weakly monotically increasing), the state variable *precip* is not necessarily constant for all of July and August. For some townships and some years, it is possible that *precip* = 0 at the beginning of July but then becomes 1 closer to the end of August.

The daily probability of high precipitation is the probability that *precip* is equal to 1 (high) that day. The daily probability of high precipitation is weakly monotonically increasing from July 1 to August 31. For each township, for each day in July and August, we calculate the daily empirical probability of high precipitation (*precip* = 1) using the latest daily precipitation data for the township from the National Oceanic and Atmospheric Administration Climate Prediction Center over the period 2010-2018. In particular, for each township, for each of the 62 days from July 1 and August 31, the daily empirical probability of high precipitation for that day for that township is calculated as the fraction of years in that township over the period 2018-2018 for which *precip* = 1 on that day.

6. Results of Numerical Model

Our calibrated piecewise Chapman-Richards growth function for bamboo shoots, which combines a Chapman-Richards growth function for winter shoots with a separate Chapman-Richards growth function for spring shoots, is presented in Figure 1. Our calibrated Chapman-Richards growth function for bamboo stem growth is presented in Figure 2.

We run several specifications of our numerical model that vary the costs and prices for winter shoots, spring shoots, and bamboo stem. Winter shoots have higher prices and higher costs than spring shoots. Due to difficulties of locating and harvesting underground winter bamboo shoots, as well as popular preference over more tender taste, winter bamboo shoots have a higher market price than spring bamboo shoots. Winter shoots are more expensive and harder to find than spring shoots. Both winter shoots and spring shoots are more expensive than bamboo stem.

For each of our specifications for the costs and prices for winter shoots, spring shoots, and bamboo stem, we solve for the value function, the bamboo shoot harvest policy function, and the bamboo stem harvest policy function, each as a function of the number of bamboo shoots per hectare and the number of bamboo stem per hectare. Since our dynamic model nests an inner finite-horizon within-year daily dynamic programming problem within an outer finite-horizon between-year annual dynamic programming problem, there is a separate value function and policy function for each day of each year.

For the value functions, each graph represents a different year, and within each graph we plot the value function for each day of the year in a different color ranging from red for the first day in that year to blue for the last day in that year. Similarly, for the policy functions, each graph represents a different year, and within each graph we plot the policy function for each day of the year in a different color ranging from red for the first day in that year to blue for the last day in that year.

For each of our specifications for the costs and prices for winter shoots, spring shoots, and bamboo stem, we simulate 3 sets of optimal trajectories for each action and state variable over 11 years starting from the following 3 different initial values for number of bamboo shoots, respectively: 400 bamboo shoots per hectare ("initial state 6"), 240 bamboo shoots per hectare ("initial state 4"), and 80 bamboo shoots per hectare ("initial state 2").

For the stochastic model, we run the following versions of each of our specifications for the costs and prices for winter shoots and spring shoots:

Version A: *rain_high_prob* = 0. This version is equivalent to our deterministic model.

Version B: *rain_high_prob* is given by daily empirical probability for Sian

Version C: *rain_high_prob* is given by daily empirical probability for Shanchuan

We make 2 sets of all value function and policy function plots for Versions B and C:

Set 1: *precip* = 0

Set 2: *precip* = 1

Our numerical model yields several notable results.

First, for moderate values of the net prices of winter shoots, spring shoots, and bamboo stem (see for example Specification 3 of our deterministic model in Figure 3), the optimal strategy may be to harvest the bamboo shoots on the last days of winter shooting in each of the second to fourth bamboo growth years, and harvest all the bamboo stem at the beginning of the fourth bamboo growth year. The intuition is as follows. The number of bamboo shoots at the beginning of each year depends on the number of bamboo stem remaining at the beginning of each year. In the first year, when all the bamboo is in the form of bamboo shoots, it is optimal not to do any harvesting so that the bamboo shoots can grow into bamboo stem at the end of the first year, which would then result in there being both bamboo shoots and bamboo stem at the beginning of the second year. It is then optimal to harvest the bamboo shoots each year for which there are bamboo shoots, starting from the second year. If the net price of winter shoots is high enough relative to the net price of spring shoots, it may be optimal to harvest the shoots as winter shoots on the last days of winter shooting, when growers can still benefit from the higher winter shoots price and the winter shoots have the most biomass they will have (since it is during the last days of winter shooting), rather than as spring shoots. Since bamboo stem continue to grow each year until age 4-5 years, if the net price of bamboo stem is moderate to high, then it may be optimal to harvest the bamboo stem at the beginning of the fourth year, as soon as the bamboo stem growth begins to slow down, when the opportunity cost of waiting to harvest outweighs the benefits of waiting. It is optimal to harvest bamboo stem at the beginning of winter shooting at the beginning of the fourth year rather than at end of spring shooting at the end of the third year because waiting until the beginning of the fourth year enables there to be bamboo shoots to harvest in the fourth year as well. It is optimal to harvest bamboo stem at the beginning of winter shooting at the beginning of the year bamboo stem is being harvested, since the number of bamboo shoots that bamboo growth year only depends on the number of bamboo stem on the first day of the bamboo growth year, so keeping bamboo stem any longer will not affect profits from bamboo shoots. In addition, since the bamboo stem growth begins to slow down in the fourth year, any increase in bamboo stem biomass from delaying past the first day of the fourth year will be small.

Second, when we allow for uncertainty in rain, we find that if the probability of having a high-precipitation year is high enough, and if the net prices of winter shoots, spring shoots, and bamboo stem are moderate (see for example Specification 3, Version C of our stochastic model in Figure 4), then it may be optimal to harvest all the bamboo stem at the beginning of the fourth bamboo growth year and, if there is a lot of bamboo stem, also harvest some bamboo stem at stem at the beginning of the third bamboo growth year. The intuition is as follows. The number of bamboo shoots at the beginning of each year depends on the number of bamboo stem remaining at the beginning of each year. The number of bamboo shoots at the beginning of each year also increases if the preceding summer had high precipitation. Thus, if there is a lot of bamboo stem, a high probability of precipitation may enable one to harvest a little bamboo stem in an earlier year without forgoing too much winter shoot harvest.

Third, if the net price of winter shoots is low (see for example Specification 1 of our deterministic model in Figure 5), then then optimal strategy may be to harvest the bamboo shoots on the last days of winter shooting in each of the second and third years, and harvest all the bamboo stem at the beginning of winter shooting at the beginning of the third year. The intuition is as follows. If the net price of winter shoots is low, then it may be optimal to harvest all the bamboo stem earlier, at the beginning of the third year, well before the bamboo stem growth begins to slow down, since it might not be worthwhile to wait another year to get one additional year of winter shoot harvest and a very small increase in bamboo stem biomass when the net price of winter shoots is low.

Fourth, if the net price for winter shoots is high and the net price for bamboo stem is low (see for example Specification 13 of our deterministic model in Figure 6), then the optimal strategy may be to harvest the bamboo shoots on the last days of winter shooting in each of the second to seventh years (or, if there is enough bamboo stem, from the second to the ninth year), and harvest all the bamboo stem at the beginning of winter shooting at the beginning of the seventh year (or, if there is enough bamboo stem, at the beginning of winter shooting at the beginning of the ninth year). The intuition is as follows. If the net price for winter shoots is high and the net price for bamboo stem is low, then it may be optimal to wait to a later year to harvest all the bamboo stem since the benefits from additional years of winter shoot harvests are high when the net price of winter shoots is high, while the benefits of harvesting bamboo stem earlier are lower when the net price of bamboo stem is low.

Fifth, when we allow for uncertainty in rain, which increases the number of bamboo shoots in the following year, we find that if the probability of having a high-precipitation year is high enough, and if the net price for winter shoots is high and the net price for bamboo stem is low (see for example Specification 13, Version C of our stochastic model in Figure 7), then the optimal strategy may be to harvest all the bamboo shoots on the last days of winter shooting for each year (except the first year), harvest all the bamboo stem at the beginning of winter shooting at beginning of the last year, and, if there is a lot of bamboo stem, also harvest some bamboo stem at stem at the beginning of winter shooting at beginning of third year. The intuition is as follows. The number of bamboo shoots at the beginning of each year depends on the number of bamboo stem remaining at the beginning of each year. The number of bamboo shoots at the beginning of each year also increases if the preceding summer had high precipitation. So high precipitation may enable one to harvest a little bamboo stem in an earlier year without forgoing too much winter shoot harvest. Moreover, owing to the possibility of rain, there may more bamboo shoots each year, so if the net price for winter shoots is high and the net price for bamboo stem is low, then it may be optimal to wait until the last year to harvest all the bamboo stem since the benefits from additional years of winter shoot harvests are high when the net price of winter shoots is high, while the benefits of harvesting bamboo stem earlier are lower when the net price of bamboo stem is low.

Sixth, if the net price for winter shoots is very high and the net price for bamboo stem is low (see for example Specification 15 of our deterministic model in Figure 8), then the optimal strategy may be to harvest the bamboo shoots on the last days of winter shooting in each year, and harvest all the bamboo stem at the beginning of winter shooting at the beginning of the last year. The intuition is as follows. If the net price for winter shoots is very high and the net price for bamboo stem is low, then it may be optimal to wait until the last year to harvest all the bamboo stem since the benefits from additional years of winter shoot harvests are very high when the net price of winter shoots is very high, while the benefits of harvesting bamboo stem earlier are lower when the net price of bamboo stem is low.

Our solution for optimal bamboo forest management might also characterize the optimal forest management policy for other forests that produce products (such as fruits, nuts, sap, and maple syrup) that grow on trees that are renewable and can be harvested at more frequent intervals than the trees themselves.

7. Comparing Optimal Bamboo Management with Actual Harvest Decisions

We compare the optimal bamboo stem harvest and bamboo shoot thinning policy as given by our numerical dynamic model with actual data on bamboo shoot and bamboo stem harvests on multiple bamboo plots in multiple townships in Zhejiang province in China.

7.1. Data on actual bamboo shoot and bamboo stem harvest

We collect data on actual bamboo shoot harvest and bamboo stem harvest decisions on 20m by 20m plots in Shanchuan Township and Sian Township in Zhejiang province in China. Our data set includes 20 plots in Sian Township over the years 2017 and 2018; 15 plots in Shanchuan Township in 2017, and one plot in Shanchuan Township in 2018.

Our dataset includes the bamboo stem density for different sample plots in multiple years. However, number of shoots per hectare is seldomly reported unless under careful experimental design, whereas weights of shoots harvested is a commonly recorded variable. Our estimation of the number of shoots per hectare relies on converting the total weight of bamboo shoots into number of bamboo shoots per hectare and shoots harvest decisions, which we define to be proportion of shoots harvested of maximum possible shoots weights. The dilemma here is that we observe weight of shoots harvested, but do not observe either total amount of shoots or number of harvested shoots.

Ideally, we would like to convert the bamboo shoots harvested data and any estimated weight of bamboo shoots into the proportion of bamboo shoots harvested. Even though we can estimate the total possible weight of bamboo shoots, the actual weight of bamboo shoots would be different if some bamboo shoots were previously harvested that season. In addition, we cannot simply subtract the weight of bamboo shoots harvested earlier in the season from our estimate the total possible weight of bamboo shoots as a function of bamboo stems (culm), since those bamboo shoots that were harvested earlier in the season would have grown or changed in weight if they had not been harvested. So it would be ideal if we made the harvesting decision in terms of proportion of bamboo shoots harvested, so that we can model the weight and change in weight of the remaining bamboo shoots.

We estimate the unobserved bamboo shoot state and control variables as follows. First, for each plot and each day, we convert the weight of bamboo shoots harvest into the number of bamboo shoots harvested per hectare by dividing the weight of bamboo shoots harvest by the bamboo shoot biomass per bamboo shoot that day of the year from Chapman-Richard’s model for bamboo shoot growth, assuming that bamboo shoots start growing from the beginning of winter shooting; and then multiplying by 25 to convert from the 0.04 hectare plots to 1 hectare.

We then impute the maximum number of bamboo shoots in the ground in the absence of bamboo shoot harvest for each sample plot in each bamboo growth year. To do so, we apply the following model from Zheng (1998) to estimate the weight of bamboo shoots in the ground that remain after all the bamboo shoots have been harvested that season:

$$w_b = 0.0018 * d_b^{2.8637} ,$$

where w_b is weight of an individual bamboo shoot and d_b is its maximum diameter. As we do not have data on the maximum diameter of bamboo shoots, we use data on the diameter at breast height (DBH) of each newly grown bamboo stem that year to represent the diameter at breast height of bamboo shoots if they were to grow until the end of that season. For each sample plot and each year in our data set, we use data on the diameter at breast height (DBH) of newly grown bamboo stem, representing the diameter at breast height of bamboo shoots if they were to grow until the end of that season, to estimate the weight of a bamboo shoot if were to grow until the end of the season. Then, for each sample plot and each year, to calculate the weight of bamboo shoots per hectare on this sample plot that are not harvested, we take the sum over all the newly grown bamboo stems of the respective weights of a bamboo shoot if were to grow until the end of the season for that sample plot in that year. We convert the weight of bamboo shoots that are not harvested by the end of the season into the number of bamboo shoots that are not harvested per hectare by dividing the weight of bamboo shoots not harvested by the bamboo shoot biomass per bamboo shoot from Chapman-Richard's model for bamboo shoot growth, assuming that the unharvested bamboo shoots must have grown from the beginning of winter shooting until the last day of spring shooting, and then multiplying by 25 to convert from the 0.04 hectare plots to 1 hectare.

For each plot, to calculate the number n_s of bamboo shoots per hectare at the beginning of the season, in the absence of any bamboo shoots harvest, we add the total number of bamboo shoots harvested over the season to the total number of bamboo shoots that remain unharvested at the end of the season.

For each day on each sample plot, we calculate the bamboo shoots harvest action variable a_s as the number of shoots harvested per acre that day on that sample plot by the number n_s of bamboo shoots per hectare on that sample plot at the beginning of the season, in the absence of any bamboo shoots harvest.

We then calculate the number n_s of bamboo shoots per acre for each day on each sample plot as the number n_s of bamboo shoots per acre on that sample plot the previous day that season minus the number of bamboo shoots per acre harvested on that sample plot on the previous day that season.

7.2. Comparing actual with optimal

Figure 9 presents the actual data on the number of bamboo stem harvested on each sample plot in Zhejiang province in our data set. Time series plots of the number of bamboo stem harvested on each sample plot in Sian Township are in blue. Time series plots of the number of bamboo stem harvested on each sample plot in Shanchuan Township are in green. Vertical lines in red that go from the top to the bottom of the graph denote September 1 (first day of winter shooting) of each year. Dashed vertical lines in red that go from the top to the bottom of the graph denote March 1 (first day of spring shooting) and April 30 (last day of spring shooting) of each year.

As seen in Figure 9, most of the bamboo stem harvesting in Shanchuan Township (in green) takes place in the first few months of winter shooting and at the start of spring shooting of the second (and last) bamboo growth year of our data set. The harvesting of bamboo stem at the start of winter shooting is consistent with our dynamically optimal solution to harvest bamboo stem at the beginning of winter shooting at the beginning of the bamboo growth year that bamboo stem is being harvested, since the number of bamboo shoots that bamboo growth year only depends on the number of bamboo stem on the first day of the bamboo growth year, so keeping bamboo stem any longer will not affect profits from bamboo shoots.

In Sian Township, however, the bamboo stem harvesting (in blue) takes place at end of the 2017-2018 bamboo growth year. This is in contrast with our dynamically optimal solution to harvest bamboo stem at the beginning (rather than the end) of the bamboo growth year it is being harvested. Whether the bamboo stem is harvested at the beginning or end of the bamboo growth year would not affect the number or profits from bamboo shoots. Delaying the bamboo stem harvest in the bamboo growth year from the beginning to the end of the year would only by dynamically optimal if the growth in bamboo stem biomass and any increase bamboo stem price over the course of the bamboo growth year outweighed the time cost of delaying the profits from harvesting bamboo stem at the beginning of the bamboo growth year.

Figure 10 presents time series plots of the cumulative fraction of bamboo stem harvested by age of bamboo on each sample plot. No bamboo stems aged 0-2 are harvested, which is consistent with the dynamically optimally policy of waiting until the third or later bamboo growth year to harvest bamboo stem. On sample plots in Shanchuan Township, the majority of the bamboo stem of age 5 are harvested in the 2017-2018 bamboo growth year, either during the first third of winter shooting or at the beginning of spring shooting, which is consistent with the dynamically optimally policy of waiting until the third or later bamboo growth years to harvest bamboo stem. In Sian Township, all the bamboo stems of age 6 are harvested during the 2017-2018 bamboo growth year, which is consistent with the dynamically optimally policy of waiting until the third or later bamboo growth years to harvest bamboo stem, but on some plots the bamboo stem harvest takes place at the end, rather than the beginning, of the bamboo growth year.

Figure 11 presents the actual data on weight of bamboo shoots harvested on each sample plot in Zhejiang province in our data set. Time series plots of the weight of bamboo shoots harvested on each sample plot in Sian Township are in blue. Time series plots of the weight of bamboo shoots harvested on each sample plot in Shanchuan Township are in green. Vertical lines in red that go from the top to the bottom of the graph denote September 1 (first day of winter shooting) of each year. Dashed vertical lines in red that go from the top to the bottom of the graph denote March 1 (first day of spring shooting) and April 30 (last day of spring shooting) of each year.

As seen in Figure 11, bamboo shoots are harvested as both winter shoots and spring shoots. Winter shoots are harvested about a third to half of the way into the winter shooting season, and spring shoots are harvested near the beginning of the spring shooting season. In contrast, our dynamic model suggests that it is optimal to harvest all shoots as winter shoots at the end of the winter shooting season. It is possible that the bamboo managers are weighing the benefits of an increased bamboo shoot biomass from waiting to harvest with the costs of delay. It is also possible that bamboo managers worry that flooding the winter bamboo shoot market at the end of the winter shoot season may lead to a lower bamboo shoot price than the price they would receive from spreading their harvest over a longer interval of time.

Figure 12 presents time series plots of the cumulative fraction of bamboo shoots harvested, as imputed above, on each sample plot. On each of the sample plots, less than 50 percent of the bamboo shoots are harvested by the end of each bamboo growth year. In contrast, our dynamic model suggests that it is optimal to harvest all the bamboo shoots each year of bamboo shoot harvest.

Thus, results of our comparison between the optimal bamboo stem harvest and bamboo shoot harvest given by our dynamic model with the data on actual bamboo stem harvests and bamboo shoots harvest is that actual bamboo stem and bamboo shoot harvests come close to approximating the optimal harvesting strategy, but have some features that differ from what our model suggests to be optimal.

8. Dynamic Structural Econometric Model

To understand the beliefs and perceptions of bamboo farmers that underlie and rationalize their bamboo shoot and bamboo stem harvesting decisions as revealed in the data, we use our nested dynamic bioeconomic model to develop a dynamic structural econometric model adapted from Rust (1987). Our dynamic structural econometric model builds upon our numerical bioeconomic model, and additionally accounts for unobservable state variables that bamboo farmers observe (but we do not observe) when they make their bamboo shoot and bamboo stem harvesting decisions.

The vector of structural parameters θ we estimate include parameters in the Chapman-Richards growth function for winter shoots, and parameters regarding prices and costs. In particular, the structural parameters θ we estimate are: the growth rate α_{s_w} for winter shoots, the maximum possible winter shoots biomass $Y_{s_w}^{\max}$ at the end of winter shooting, the winter day of inflection $t_{s_w}^{\text{infl}}$ for winter shoots, the net price $(p_{s_w} - c_{s_w})$ for winter shoots, the net price $(p_{s_s} - c_{s_s})$ for spring shoots, the net price $(p_b - c_b)$ for bamboo stem, and the daily fixed cost c_0.

To account for unobservable state variables that bamboo farmers observe (but we do not observe) when they make their spraying and harvesting decisions, we next expand the per-period payoff to each choice a to include both a deterministic component $\pi_0(s,a,d,y;\theta)$ and a stochastic component $\varepsilon(a)$. The stochastic component to the per-period payoff to each action is an unobserved shock $\varepsilon(a)$ associated with that action choice a that is assumed to be distributed i.i.d. extreme value across days *d*, years *t*, farmers *i*, and actions a.The value function incorporating these unobserved shocks $\varepsilon(a)$ is now given by:

$$V(s,d,y;\theta) = \max_{a=(a_b,a_s)} \pi_0(s,a,d,y;\theta) + \varepsilon(a) + \beta E[V(s',d',y';\theta) \mid s,a,d,y],$$

where the deterministic component of the per-period payoff is given by:

$$\pi_0(s,a,d,y;\theta) = R_b(s,a,d,y;\theta) - C_b(s,a,d,y;\theta) + R_s(s,a,d,y;\theta) - C_s(s,a,d,y;\theta) - c_0 .$$

The conditional choice probabilities $\Pr(a \mid s,d,y;\theta)$ are given by:

$$\Pr(a \mid s,d,y;\theta) = \frac{\exp(\pi_0(s,a,d,y;\theta) + \beta V^c(s,a,d,y;\theta)}{\sum_{\tilde{a}} \exp(\pi_0(s,\tilde{a},d,y;\theta) + \beta V^c(s,\tilde{a},d,y;\theta)},$$

where $V^c(s,a,d,y;\theta)$ is the continuation value, which is the expected value of the value function next period given the states and actions this period:

$$V^c(s,a,d,y;\theta) = E[V(s',d',y';\theta) \mid s,a,d,y].$$

We use maximum likelihood estimation to find the parameters θ that maximize the log-likelihood function $L(\theta)$, which is the following function of the conditional choice probabilities $\Pr(a \mid s,d,y;\theta)$:

$$L(\theta) = \sum_i \sum_d \sum_y \Pr(a_{idy} \mid s_{idy}, d, y;\theta).$$

Building on the nested fixed point maximum likelihood estimation technique developed by Rust (1987), our maximum likelihood estimation methodology nests an inner finite-horizon within-year daily dynamic programming problem within an outer finite-horizon between-year annual dynamic programming problem to solve for the continuation values and conditional choice probabilities for each day *d* in each year *y* at each evaluation of the likelihood function.

Identification of the parameters θ comes from the differences between per-period payoffs across different action choices, which in finite-horizon dynamic discrete choice models are identified when the discount factor β, the distribution of the choice-specific shocks $\varepsilon(a)$, and the final period continuation value are fixed (Rust, 1994; Magnac and Thesmar, 2002; Abbring, 2010). In particular, because the discount factor β and the distribution of the choice-specific shocks $\varepsilon(a)$ are fixed and the final period continuation value is zero, the parameters in our model are identified because each term in the deterministic component $\pi_0(s,a,d,y;\theta)$ of the per-period payoff depends on the action a being taken in day *d* in year *y*, and therefore varies based on the action taken; as a consequence, the parameters do not cancel out in the differences between per-period payoffs across different action choices and are therefore identified. For example, net price $(p_b - c_b)$ for bamboo stem is identified in the difference between the per-period payoff from choosing to harvest bamboo stem and the per-period payoff from any action choice a that does not involve harvesting bamboo stem.

Standard errors are formed by a non-parametric bootstrap. Sample plots are randomly drawn from the data set with replacement to generate 100 independent panels each with the same number of sample plots as in the original data set. The structural model is run on each of the new panels. The standard errors are then formed by taking the standard deviation of the parameter estimates from each of the panels.

We run each specification of our dynamic structural model using the entire sample (“All”), Sian Township only (“Sian”), and Shanchuan Township only (“Shanchuan”). According to preliminary parameter estimates of our dynamic structural model, the most notable difference between our structural parameter estimates and the parameter values we use in our numerical model is that our structural parameter estimate for the growth rate α_{s_w} for winter shoots, 0.000, is much smaller than the growth rate α_{s_w} for winter shoots we use in our numerical model, which we calibrated based on data and information on winter shoots to be 0.03. Thus, the harvesting behavior of bamboo farmers in China is rationalized by a very low and negligible growth rate for winter shoots. In other words, bamboo farmers in China are acting as if they perceive or believe the growth rate for winter shoots to be very low and negligible, and lower than may actually be the case based on data and information on winter shoots. Figure 13 plots bamboo farmers’ perceived Chapman-Richards growth function for winter shooting and spring shooting based on the preliminary parameter estimates from our dynamic structural model. Thus, by harvesting more of their shoots as spring shoots rather than harvesting all of their shoots as winter shoots, China bamboo farmers are acting as if winter shoots have little or no biomass.

9. Discussion and Conclusion

Moso bamboo forest management involves making decisions about the timing and quantity of bamboo stem harvests and bamboo shoot harvests. In this paper, we solve for the optimal bamboo stem harvest and bamboo shoot harvest policy using a numerical dynamic model that nests an inner finite-horizon within-year daily dynamic programming problem within an outer finite-horizon between-year annual dynamic programming problem. We use a Chapman-Richards growth function as our model for bamboo biomass accumulation.

The results of our numerical dynamic model suggest that since the number of bamboo shoots at the beginning of each year depend on the number of bamboo stem remaining at the beginning of each year and on whether the previous year was a high-precipitation year, and since bamboo stem continue to grow each year until age 4-5 years, while bamboo shoots only grow within a year, it is generally optimal to harvest bamboo shoots as winter shoots at the end of winter shooting each year that there are bamboo shoots starting the second year; and to wait to harvest all the bamboo stem until the third year or later, after their growth has begun to slow down. If the net price of winter shoots is high enough relative to the net price of spring shoots, it may be optimal to harvest the shoots as winter shoots on the last days of winter shooting, when growers can still benefit from the higher winter shoots price and the winter shoots have the most biomass they will have (since it is during the last days of winter shooting), rather than as spring shoots.

When we allow for uncertainty in rain, which increases the number of bamboo shoots in the following year, we find that when the probability of having a high-precipitation year is high enough, then if there is enough bamboo stem, it may be optimal to harvest a little bamboo stem on the first day of the third year, and harvest all the bamboo stem on the first day of a later year. Since the number of bamboo shoots at the beginning of each year depends on the number of bamboo stem remaining at the beginning of each year, and increases if the preceding summer had high precipitation, high precipitation may enable one to harvest a little bamboo stem in an earlier year without forgoing too much shoot harvest.

We compare the optimal bamboo stem harvest and bamboo shoot thinning policy with actual data on bamboo shoot and bamboo stem harvests on multiple bamboo plots in multiple townships in Zhejiang province in China. We find that the actual bamboo stem and bamboo shoot harvests come close to approximating the optimal harvesting strategy, but have some features that differ from what our model suggests to be optimal. Our results have important implications for bamboo forest management and, to the extent that some of the differences between actual harvests and optimal bamboo harvests reflect possible sub-optimal behavior on the part of Moso bamboo forest managers, for ways to improve Moso bamboo forest management and policy.

The remaining differences between actual harvests and optimal bamboo harvests may reflect features that we do not capture in our model, including variation in bamboo shoot price and/or bamboo stem price over time, particularly within a season; concerns that harvesting all the bamboo shoots at the same time may flood the market and lower the bamboo shoot price they receive; concerns that harvesting all the bamboo stem at the same time may flood the market and lower the bamboo stem price they receive; capacity and/or labor constraints on the number of bamboo stems that are feasible to harvest in one day; capacity and/or labor constraints on the number of bamboo shoots that are feasible to harvest in one day; liquidity constraints during the season that may lead bamboo managers to harvest some bamboo shoots or bamboo stem earlier or later; variation in age of bamboo stem in a bamboo forest; possible missing markets or market failures in the winter shoots, spring shoots, or bamboo stem markets; carbon sequestration motives; alternative crops or uses of the land; environmental benefits of a bamboo forest; and/or actual parameter values that differ from the ones we use in the model.

We account for some of these considerations by estimating a dynamic structural econometric model using data on the actual bamboo shoot harvest, bamboo stem harvest, bamboo shoot price, and bamboo stem harvest, which will enable us to estimate the parameters econometrically. Preliminary parameter estimates from our dynamic structural model suggest that bamboo farmers in China are acting as if they perceive or believe the growth rate for winter shoots to be very low and negligible, and lower than may actually be the case based on data and information on winter shoots.

If some of the differences between actual harvests and optimal harvests arise because of economic constraints such as liquidity constraints and/or labor constraints, it is possible that some of these constraints can be ameliorated by well-designed institutions or policies.

Our results have important implications for Moso bamboo forest management in particular, and forest management more generally. Our novel nested dynamic bioeconomic model has important implications for the sustainable management of forests worldwide, particularly when the forests produce products (such as fruits, nuts, and maple syrup) that grow on trees, that are renewable, and can be harvested at more frequent intervals than the trees themselves.

10. Ongoing and Next Steps

In ongoing work, we will continue to work on our dynamic structural econometric model, including estimating standard errors, estimating different specifications, and estimating different sets of parameters. We also hope to incorporate the environmental benefits of bamboo forest into a social welfare function, and conduct welfare analysis.

We will use our dynamic model to simulate the effects of various alternative forest conservation policies and forest management approaches on bamboo shoot harvest, bamboo stem harvest, forest conservation, and welfare. Our novel dynamic bioeconomic model has important implications for the sustainable management of forests worldwide, particularly when the forests produce products that can be harvested at more frequent intervals than the trees themselves. Our model and results will enable us to design sustainable and effective forest management policies that maximize net benefits to society.

References

Abbring, J. (2010). Identification of dynamic discrete choice models. Annual Review of Economics, 2, 367-394.

Agarwal, N., I. Ashlagi, M. Rees, P. Somaini, and D. Waldinger. (2021). Equilibrium allocations under alternative waitlist designs: Evidence from deceased donor kidneys kidneys. Econometrica, 89, 37-76.

Aguirregabiria, V., and A. Luengo. (2016). A microeconometric dynamic structural model of copper mining decisions. Working paper, University of Toronto.

Amidon, E.L., and G.S. Akin. (1968). Dynamic programming to determine optimum levels of growing stock. Forest Science, 14(3), 287-291.

Anderson, A., J. Rosen, J. Rust, and K.-P. Wong. (2021). Disequilibrium play in tennis. Working paper, Georgetown University.

Araujo, R., F.J.M. Costa, and M. Sant'Anna. (2020). Efficient forestation in the Brazilian Amazon: Evidence from a dynamic model. Working paper, FGV EPGE and University of Delaware.

Arimizu, T. (1958). Regulation of the cut by dynamic programming. Journal of the Operations Research Society of Japan, 1(4), 175-182.

Bare, B.B., and T.R. Waggener. (1980). Forest land values and return on investment. Forest Science, 26(1), 91-96.

Bellman, R. (1954). The theory of dynamic programming. Bulletin of the American Mathematical Society, 60(6), 503-515.

Berck, P. (1981). Optimal management of renewable resources with growing demand and stock externalities. Journal of Environmental Economics and Management, 8(2), 105-117.

Blundell, W., G. Gowrisankaran, and A. Langer. (2020). Escalation of scrutiny: The gains from dynamic enforcement of environmental regulations. American Economic Review, 110, 2558-2585.

Bowes, M.D. (1983). Economic foundations of public forestland management. Resources for the Future.

Brodie, J.D., and C. Kao. (1979). Optimizing thinning in Douglas-fir with three-descriptor dynamic programming to account for accelerated diameter growth. Forest Science, 25(4), 665-672.

Brodie, J.D., D.M. Adams, and C. Kao. (1978). Analysis of economic impacts on thinning and rotation for Douglas-fir, using dynamic programming. Forest Science, 24(4), 513-522.

Buongiorno, J., and J.K. Gilless. (2003). Decision Methods for Forest Resource Management. Academic Press, San Diego.

Bystriakova, N., V. Kapos, I. Lysenko, and C.M.A Stapleton. (2003). Distribution and conservation status of forest bamboo biodiversity in the Asia-Pacific Region. Biodiversity & Conservation, 12(9), 1833-1841.

Carroll, C.L., C.A. Carter, R.E. Goodhue, and C.-Y.C. Lin Lawell. (2019). Crop disease and agricultural productivity: Evidence from a dynamic structural model of Verticillium wilt management. In W. Schlenker (Ed), Agricultural Productivity and Producer Behavior (pp. 217-249). Chicago: University of Chicago Press.

Carroll, C.L., C.A. Carter, R.E. Goodhue, and C.-Y.C. Lin Lawell. (2022a). Supply chain externalities and agricultural disease. Working paper, Cornell University.

Carroll, C.L., C.A. Carter, R.E. Goodhue, and C.-Y.C. Lin Lawell. (2022b). The economics of decision-making for crop disease control. Working paper, Cornell University.

Calish, S., R.D. Fight, and D.E. Teeguarden. (1978). How do nontimber values affect Douglas-fir rotations?. Journal of Forestry, 76(4), 217-221.

Chang, S. (1982). An economic analysis of forest taxation's impact on optimal rotation age. Land Economics, 58(3), 310-323.

Chang, S.J. (1981). Determination of the optimal growing stock and cutting cycle for an uneven-aged stand. Forest Science, 27(4), 739-744.

Chang, S.J. (1983). Rotation age, management intensity, and the economic factors of timber production: Do changes in stumpage price, interest rate, regeneration cost, and forest taxation matter? Forest Science, 29(2), 267-277.

Chen, A., and S. Scott. (2014). Rural development strategies and government roles in the development of farmers' cooperatives in China. Journal of Agriculture, Food Systems, and Community Development, 4(4), 35-55.

Chen, C.M., D.W. Rose, and R.A. Leary. (1980). Derivation of optimal stand density over time—a discrete stage, continuous state dynamic programming solution. Forest Science, 26(2), 217-227.

China Forestry and Grassland Administration. (2018). "Forest Resources in China."

Ching, A.T., and M. Osborne. (2020). Identification and estimation of forward-looking behavior: The case of consumer stockpiling. Marketing Science, 39, 669-848.

Cook, J.A., and C.-Y.C. Lin Lawell. (2020). Wind turbine shutdowns and upgrades in Denmark: Timing decisions and the impact of government policy. Energy Journal, 41, 81-118.

Deegen, P., M. Hostettler, and G.A. Navarro. (2011). The Faustmann model as a model for a forestry of prices. European Journal of Forest Research, 130(3), 353-368.

Démurger, S., Y. Hou, and W. Yang. (2009). Forest management policies and resource balance in China: an assessment of the current situation. Journal of Environment and Development, 18(1), 17-41.

Dixit, A.K., and R.S. Pindyck. (1994). Investment under uncertainty. Princeton University Press.

Dong, D., Y. Tong, X. Yi., C. Zhang, and M. Gu. (2015). Harvesting cost and economics efficiency analysis of bamboo forests in various slope hill lands. Journal of Bamboo Research, 34(4), 23-27. [in Chinese]

Donna, J.D. (2019). Measuring long-run price elasticities in urban travel demand. Working paper, Ohio State University

Du, H., G. Zhou, W. Fan, H. Ge, X. Xu, Y. Shi, and W. Fan. (2010). Spatial heterogeneity and carbon contribution of aboveground biomass of Moso bamboo by using geostatistical theory. Plant Ecology, 207(1), 131-139.

Fang, J.Q., and Z. Xie. (1994). Deforestation in preindustrial China: The Loess Plateau region as an example. Chemosphere, 29(5), 983-999.

Faustmann, M. (1849). Calculation of the value which forest land and immature stands possess for forestry. General Journal of Forestry and Hunting. [in German]

Feger, F., N. Pavanini, and D. Radulescu. (2020). Welfare and redistribution in residential electricity markets with solar power. Working paper.

Fekedulegn, D., M.P. Mac Siurtain, and J.J. Colbert. (1999). Parameter estimation of nonlinear growth models in forestry. Silva Fennica, 33 (4), 653.

Feng, J., Q. Gong, and H. Wu. (2012). Research on the standing bamboo density in Phyllostachys pubescens forest." Hunan Forestry Science and Technology, 39(4), 37-40. [in Chinese]

Food and Agriculture Organization [FAO]. (2005). Global Forest Resources Assessment 2005.

Food and Agriculture Organization [FAO]. (2015). Global Forest Resources Assessment 2015.

Forestry Department of Hunan Province. (2008). Moso Bamboo Cultivation Techniques. 25 April 2008.

Fu, J. (2001). Chinese Moso bamboo: its importance. Bamboo, 22(5), 5-7.

Fu, S., X. Xu, and W. Wu. (2015). Analysis on the determinants of Phyllostachys edulis's input-output efficiency. Journal of Zhejiang A&F University, 32(4), 596-602. [in Chinese]

Gillingham, K., F. Iskhakov, A. Munk-Nielsen, J. Rust, and B. Schjerning. (2016). Equilibrium trade in automobile markets. NBER Working Paper No. 25840.

Gowrisankaran, G., and M. Rysman. (2012). Dynamics of consumer demand for new durable goods. Journal of Political Economy, 120, 1173-1219.

Gregersen, H.M. (1975). Effect of inflation on evaluation of forestry investments. Journal of Forestry, 73(9), 570-572.

Haight, R.G. (1985). A comparison of dynamic and static economic models of uneven-aged stand management. Forest Science 31(4), 957-974.

Hall, D.O. (1983). Financial maturity for even-aged and all-aged stands. Forest Science, 29(4), 833-836.

Hardie, I.W., J.N. Daberkow, and K.E. McConnell. (1984). A timber harvesting model with variable rotation lengths. Forest Science, 30(2), 511-523.

Hartman, R. (1976). The harvesting decision when a standing forest has value. Economic Inquiry, 14(1), 52-58.

Hu, Y., Y. Zhang, J. Zhou, G. Wang, and Q. Guo. (2020). Transcriptome reveals the specificity of Phyllostachys edulis "Pachyloen" shoots at different developmental stages. Forests, 11(8), 861.

International Bamboo and Rattan Organisation [INBAR]. (2012). China Bamboo Industry Plans at National and Provincial Levels. INBAR. Accessed 15 February 2022. URL: https://www.inbar.int/resources/inbar_publications/china-bamboo-industry-plans-at-national-and-provincial-levels/

Iskhakov, F. (2010). Structural dynamic model of retirement with latent health indicator. Econometrics Journal, 13, S126-S161.

Jackson, D. H. (1980). The Microeconomics of the Timber Industry. Westview Press.

Jiang, Y. (2020). How to achieve one-on-one efficient reduction of poverty. JYB.cn, 25 May 2020. Retrieved February 18, 2022. URL: http://www.jyb.cn/rmtzgjyb/202005/t20200525_330371.html [in Chinese]

Jiang, Z. (2007). Bamboo and Rattan in the World. Beijing: China Forestry Pub. House.

Kant, S., and J.R.R. Alavalapati. (2014). Handbook of Forest Resource Economics. Abingdon: Routledge.

Katsigris, E., G.Q. Bull, A. White, C. Barr, K. Barney, Y. Bun,, . . ., and H. Weyerhaeuser. (2004). The china forest products trade: Overview of Asia-Pacific supplying countries, impacts and implications. International Forestry Review, 6(3/4), 237-253.

Kilkki, P., and U. Väisänen. (1969). Determination of the optimum cutting policy for the forest stand by means of dynamic programming. Acta Forestalia Fennica, 102.

Klemperer, W.D. (1979). Inflation and present value of timber income after taxes. Journal of Forestry, 77(2), 94-96.

Kling, C.L., R.W. Arritt, G. Calhoun, and D.A. Keiser. (2017). Integrated assessment models of the food, energy, and water nexus: a review and an outline of research needs. Annual Review of Resource Economics, 9(1), 143-163.

Kusters, K., and B. Belcher. (2004). Forest Products, Livelihoods and Conservation: Case studies of non-timber forest product systems. Volume 1 - Asia. Center for International Forestry Research (CIFOR).

Langer, A., and D. Lemoine. (2018). Designing dynamic subsidies to spur adoption of new technologies. NBER Working Paper No. 24310.

LeBeau Bamboo Nursery. (2015). Bamboo as a Vegetable—Harvesting Edible Bamboo Shoots. 15 June 2014. Retrieved April 10, 2021. URL: https://lebeaubamboo.com/article-bamboo-vegetable-harvesting-bamboo-shoots.php

Li, C., G.M. Zhou, Y.J. Shi, Y.F. Zhou, X.J. Xu, Y.P. Zhang, Y.Q. Fan, and Z.M. Shen,. (2016). Effects of old bamboo forests and relevant management measures on growth of new bamboo forests. Acta Ecologica Sinica, 36(8), 2243-2254. [in Chinese]

Li, S., Y. Liu, and C.D. Wei. (2021). The cost of greening stimulus: A dynamic analysis of vehicle scrappage programs. Working paper, Cornell University.

Liu, Z.-G., and F.-R. Li. (2003). The generalized Chapman-Richards function and applications to tree and stand growth. Journal of Forestry Research, 14(1), 19-26.

Magnac, T., and D. Thesmar. (2002). Identifying dynamic discrete decision processes. Econometrica, 70, 801-816.

Matthews, R. (2012). The economic and employment benefits of forests. Eco-Business, 7 March 2012.

McConnell, K.E., J.N. Daberkow, and I.W. Hardie. (1983). Planning timber production with evolving prices and costs. Land Economics, 59(3), 292-299.

Meng, H., Q. Liu, and W. Wu. (2014). Study on the cost-benefit and carbon sequestration capacities of Phyllostachys edulis plantations under different types of management. Journal of Zhejiang A&F University, 31(6): 959-964. [in Chinese]

Nath, A.J., G. Das, and A.K. Das. (2009). Above ground standing biomass and carbon storage in village bamboos in North East India. Biomass and Bioenergy, 33(9), 1188-1196.

National Agricultural Products Business Information Public Service Platform, People's Republic of China Ministry of Commerce. (2018, May 28). Retrieved from http://nc.mofcom.gov.cn/ [in Chinese].

Nautiyal, J.C., and J.S. Williams. (1990). Response of optimal stand rotation and management intensity to one-time changes in stumpage price, management cost, and discount rate. Forest Science, 36(2), 212-223.

Newman, D.H., C.B. Gilbert, and W.F. Hyde. (1985). The optimal forest rotation with evolving prices. Land Economics, 61(4), 347-353.

Newman, D H. (1988). The optimal forest rotation: A discussion and annotated bibliography. Gen. Tech. Rep. SE-48. Asheville, NC: US Department of Agriculture, Forest Service, Southeastern Forest Experiment Station.

Nguyen, D. (1979). Environmental services and the optimum rotation problem in forest management Journal of Environmental Management, 8(2), 127-136

Oliva, P., B.K. Jack, S. Bell, E. Mettetal, and C. Severen. (2020). Technology adoption under uncertainty: Take-up and subsequent investment in Zambia. Review of Economics and Statistics, 102, 617-632.

Ollikainen, M. (1991). The effect of nontimber taxes on the harvest timing—the case of private nonindustrial forest owners: A note. Forest Science, 37(1), 356-363.

Pearse, P.H. (1967). The optimum forest rotation. The Forestry Chronicle, 43(2), 178-195.

People.cn. (2014). One hundred tons of daily winter shoot consumption in Hangzhou: Why winter shoots are more tender and valuable? (2014). People.cn, 26 November 2014. Retrieved February 16, 2022. URL http://travel.people.com.cn/n/2014/1126/c41570-26095502.html [in Chinese]

Pérez, M.R., M. Zhong, B. Belcher, C. Xie, M. Fu, J. Xie. (1999). The role of bamboo plantations in rural development: The case of Anji County, Zhejiang, China. World Development, 27(1), 101-114.

Rapson, D. (2014). Durable goods and long-run electricity demand: Evidence from air conditioner purchase behavior. Journal of Environmental Economics and Management, 68, 141-160.

Ren, G., S.S. Young, L. Wang, W. Wang, Y. Long, R. Wu, ... , and D.W. Yu. (2015). Effectiveness of China's national forest protection program and nature reserves. Conservation Biology, 29(5), 1368-1377.

Richards, F. (1959). A flexible growth function for empirical use. Journal of Experimental Botany, 10(2), 290-301.

Rideout, D.B. (1982). Micro-Economic Properties of Four Local Forest Tax Methods. (doctoral dissertation). University of Washington, Seattle, United States.

Riitters, K.H.; J.D. Brodie, and D.W. Hann. (1982). Dynamic programming for optimization of timber production and grazing in ponderosa pine. Forest Science, 28(3), 517-526.

Rust, J. (1987). Optimal replacement of GMC bus engines: An empirical model of Harold Zurcher. Econometrica, 55 (5), 999-1033.

Rust, J. (1994). Structural estimation of Markov decision processes. In Robert F. Engle and Daniel L. McFadden (Eds.), Handbook of Econometrics, Volume IV (pp. 3081-3143). Elsevier.

Sambucci, O., C.-Y.C. Lin Lawell, and T.J. Lybbert. (2022). Pesticide spraying and disease forecasts: A dynamic structural econometric model of grape growers in California. Working paper, Cornell University.

Samuelson, P.A. (1976). Economics of forestry in an evolving society. Economic Inquiry, 14(4), 466-492.

Scott, P. (2013). Dynamic discrete choice estimation of agricultural land use. Working paper, New York University.

Scurlock, J.M.O., D.C. Dayton, and B. Hames. (2000). Bamboo: an overlooked biomass resource? Biomass and Bioenergy, 19(4), 229-244.

Sears, L., C.-Y.C. Lin Lawell, and M.T. Walter. (2022). Groundwater under open access: A structural model of the dynamic common pool extraction game. Working paper, Cornell University.

SFA. (2014b). Report of forest resources in China (2009–2013). China Forestry Press, Beijing, p 86.

Shen, Y., Y. Cheng, X. Xu, X. Cai, B. Zhang. (1998). Analysis prediction of market supply and demand for bamboo shoot products. Journal of Zhejiang A&F University, 15(4), 333-339. [in Chinese]

Shi, Y., E. Liu, G. Zhou, Z. Shen, and S. Yu. (2013). bamboo shoot growth model based on the stochastic process and its application. Scientia Silvae Sinicae, 49(9), 89-93. [in Chinese]

Song, X., H. Gu, M. Wang, G. Zhou, and Q. Li. (2016). Management practices regulate the response of Moso bamboo foliar stoichiometry to nitrogen deposition. Scientific Reports, 6, 24107.

Song, X., C. Peng, G. Zhou, H. Gu, Q. Li, and C. Zhang. (2016). Dynamic allocation and transfer of non-structural carbohydrates, a possible mechanism for the explosive growth of Moso bamboo (*Phyllostachys heterocycla*). Scientific Reports, 6, 25908.

Song, X., G. Zhou, H. Jiang, S. Yu, J. Fu, W. Li, and C. Peng. (2011). Carbon sequestration by Chinese bamboo forests and their ecological benefits: assessment of potential, problems, and future challenges. Environmental Reviews, 19, 418-428.

Strang, W.J. (1983). On the optimal forest harvesting decision. Economic Inquiry, 21(4), 576-583.

Su, W. (2012). Fertilization theory and practice for Phyllostachys edulis stand based on growth and nutrient accumulation rules. (doctoral dissertation). Chinese Academy of Forestry, Beijing, China.

Sun, H., Y. Lou, L. Li, H. Zhao, and Z. Gao. (2017). Research advances in the growth and development of bamboo culm. World Forestry Research, 30(4), 18-23.

Tao, G.-Y., M. Ramakrishnan, K.K. Vinod, K. Yrjala, V. Satheesh, J. Cho, Y. Fu, and M. Zhou. (2020). Multi-omics analysis of cellular pathways involved in different rapid growth stages of moso bamboo. Tree Physiology, 40 (11), 1487-1508.

Timmins, C. (2002). Measuring the dynamic efficiency costs of regulators' preferences: Municipal water utilities in the arid west. Econometrica, 70, 603-629.

Tyler, A.L., D.C. MacMillan, and J. Dutch. (1996). Models to predict the General Yield Class of Douglas fir, Japanese larch and Scots pine on better quality land in Scotland. Forestry: An International Journal of Forest Research, 69(1), 13-24.

Wang, G., J.L. Innes, S. Dai, and G. He. (2008). Achieving sustainable rural development in Southern China: the contribution of bamboo forestry. The International Journal of Sustainable Development & World Ecology, 15(5), 484-495.

Wang, G., J.L. Innes, J. Lei, S. Dai, and S.W. Wu. (2007). China's forestry reforms. Science, 318(5856), 1556-1557.

Wang, S., G.C. Van Kooten, and B. Wilson, B. (2004). Mosaic of reform: Forest policy in post-1978 China. Forest Policy and Economics, 6(1), 71-83.

Wang, Y., X. Sun, Y. Ding, Zhangjun Fei, C. Jiao, M. Fan, B. Yao, P. Xin, J. Chu, and Q. Wei. (2019). Cellular and molecular characterization of a thick wall variant reveal a pivotal role of shoot apical meristem in bamboo culm transversal evolution. Journal of Experimental Botany, 70, 3911-3926.

Wei, Q., C. Jiao, Y. Ding, S. Gao, L. Guo, M. Chen, P. Hu, S. Xia, G. Ren, and Z. Fei. (2018). Cellular and molecular characterizations of a slow-growth variant provide insights into the fast growth of bamboo. Tree Physiology, 38, 641-654.

Wei, Q., C. Jiao, L. Guo, Y. Ding, J. Cao, J. Feng, X. Dong, L. Mao, H. Sun, F. Yu, G. Yang, P. Shi, G. Ren, and Z. Fei. (2017). Exploring key cellular processes and candidate genes regulating the primary thickening growth of Moso underground shoots. New Phytologist, 214, 81-96.

Wu, W., and X. Cao. (2016) Study on the relationship between input, output and economic benefit of Moso bamboo plantation. Journal of Nanjing Forestry University (Natural Sciences Edition), 40(3), 108-114. [in Chinese]

Wu, B., H. Hu, X. Lin, S. Huang, J. Lai, and W. Li. (2016). A study of bamboo shoot market in Longyou County, Zhejiang. World Bamboo and Rattan, 14(6). [in Chinese]

Xi, W., F. Wang, P. Shi, E. Dai, A.O. Anoruo, H. Bi, ..., and W. Li. (2014). Challenges to sustainable development in China: A review of six large-scale forest restoration and land conservation programs. Journal of Sustainable Forestry, 33(5), 435-453.

Xiao, X., Y. Luo, and C. Ni. (1997). Study on the bambooo shoot yield of Phyllostachys pubescens. Journal of Fujian Forestry and Technology, 24(3), 24-27. [in Chinese]

Xinhua Finance. (2020). Guard the mountain, follows the fortune: Green Insurance in Anji Zhejiang. Xinhua Finance, 28 April 2020. Retrieved February 17, 2022. URL: http://greenfinance.xinhua08.com/a/20200428/1933450.shtml [in Chinese]

Xu, L., Y. Shi, G. Zhou, X. Xu, E. Liu, Y. Zhou, ... , and X. Deng. (2017). Temporal change in aboveground culms carbon stocks in the Moso bamboo forests and its driving factors in Zhejiang Province, China. Forests, 8(10), 371.

Xu, Y., M. Wong, J. Yang, Z. Ye, P. Jiang, and S. Zheng. (2011). Dynamics of carbon accumulation during the fast growth period of bamboo plant. Botanical Review, 77(3), 287-295.

Yeh, D.A., M.I. Gómez, and C.Y.C. Lin Lawell. (2022). Sustainable pest management, beliefs, and behavior: A dynamic bioeconomic analysis. Working paper, Cornell University.

Yen, T.-M. (2015). Comparing aboveground structure and aboveground carbon storage of an age series of Moso bamboo forests subjected to different management strategies. Journal of Forest Research, 20(1), 1-8.

Yen, T.-M. (2016). Culm height development, biomass accumulation and carbon storage in an initial growth stage for a fast-growing Moso bamboo (Phyllostachy pubescens). Botanical Studies, 57(1), 10.

Yen, T.-M., and J.-S. Lee. (2011). Comparing aboveground carbon sequestration between moso bamboo (Phyllostachys heterocycla) and China fir (Cunninghamia lanceolata) forests based on the allometric model. Forest Ecology and Management, 261(6), 995-1002.

Yousefpour, R., and M. Hanewinkel. (2009). Modelling of forest conversion planning with an adaptive simulation-optimization approach and simultaneous consideration of the values of timber, carbon and biodiversity. Ecological Economics, 68(6), 1711-1722.

Yu, L. (2011). Study on density effect of Phyllostachys edulis stands in Southern Anhui Province. (doctoral dissertation). Chinese Academy of Forestry, Beijing, China.

Zeng, X., & Peng, L. (2013). Abnormal climate adversely affecting Zhejiang Winter Shoots Production and Doubled Previous Year-to-year Price. People.cn, 26 December 2013. [in Chinese]. URL: http://shipin.people.com.cn/n/2013/1226/c85914-23952769.html.

Zhang, C., and X. Ding. (1997). Studies on yield of Phyllostachys Heterocycla var. Pubescens stands. Journal of Bamboo Research, 16(3), 31-36. [in Chinese]

Zhang, H., S. Zhuang, B. Sun, H. Ji, C. Li, and S. Zhou. (2014). Estimation of biomass and carbon storage of moso bamboo (*Phyllostachys pubescens* Mazel ex Houz.) in southern China using a diameter–age bivariate distribution model. Forestry, 87(5), 674–682.

Zhang, Q.-S. (2003). Attaching importance to science and innovation in the processing and utilization of bamboo timber in China. Journal of Zhejiang A&F University, 20(1), 1-4.

Zhang, S., A. Wolz, and Y. Ding. (2020). Is there a role for agricultural production cooperatives in agricultural development?: Evidence from China. Outlook on Agriculture, 49(3), 256-263.

Zhejiang Local Winter Shoots Trading on Site: Demonstrating Winter Shoots Price in Late November for Reference. (2020). bilibili.com/. [Video]. URL: https://www.bilibili.com/video/BV1Ui4y1V7dy. [in Chinese]

Zheng, Y., W. Hong, and W. Zhang. (1998). Study on prediction of bamboo shoot weight and yield. Scientia Silvae Sinicae, 34(1), 69-72. [in Chinese]

Zhou, B.-Z., M.-Y. Fu, J.-Z. Xie, X.-S. Yang, and Z.-C. Li. (2005). Ecological functions of bamboo forest: Research and application. Journal of Forestry Research, 16(2), 143-147.

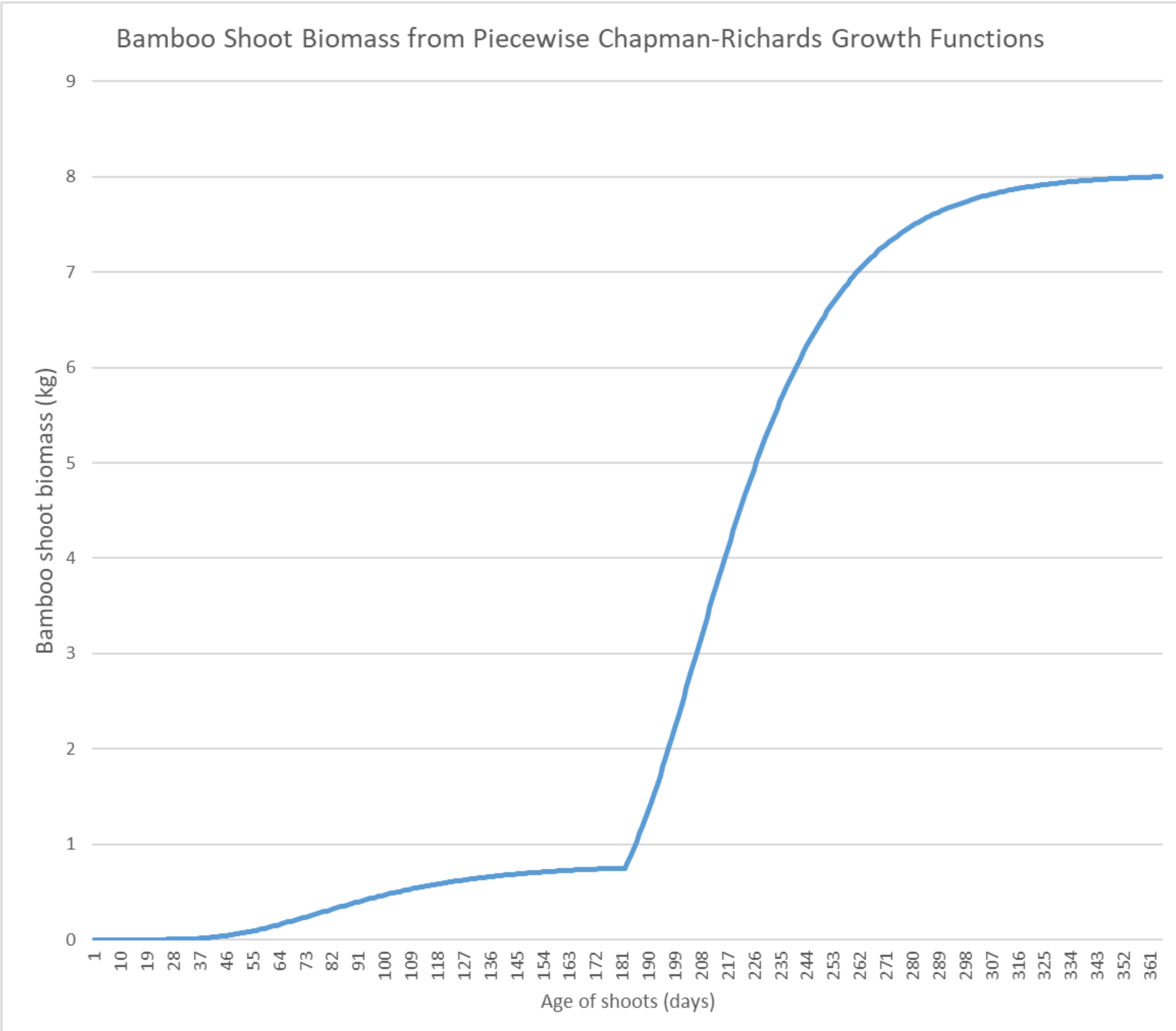
Zhou, F. (1998). Mathematical model for bamboo growth. Bamboo Research, 19(1), 74-78. [In Chinese]

Zhou, G.M. (2006). Carbon storage, fixation, and distribution in Moso bamboo forest ecological system. (doctoral dissertation). Zhejiang University, Hangzhou, China.

Zhu, Z. (2001). Sustainable development of the bamboo and rattan sectors in tropical China. China Forestry Publishing House.

Zhuang, S., H. Ji, H. Zhang, and B. Sun. (2015). Carbon storage estimation of Moso bamboo (Phyllostachys pubescens) forest stands in Fujian, China. Tropical Ecology, 56(3), 383-391.

Figure 1. Piecewise Chapman-Richards Growth Function for Bamboo Shoots



Notes: We use separate Chapman-Richards growth function for winter shooting and spring shooting. The first day of winter shooting is September 1. Winter shooting is from September 1 until February 28. The number of winter shooting days is therefore 181 days. The spring shooting period starts on March 1 and ends on August 31, the last day of the bamboo growth year. The number of spring shooting days is 184 days.

Figure 2. Chapman-Richards Growth Function for Bamboo Stem

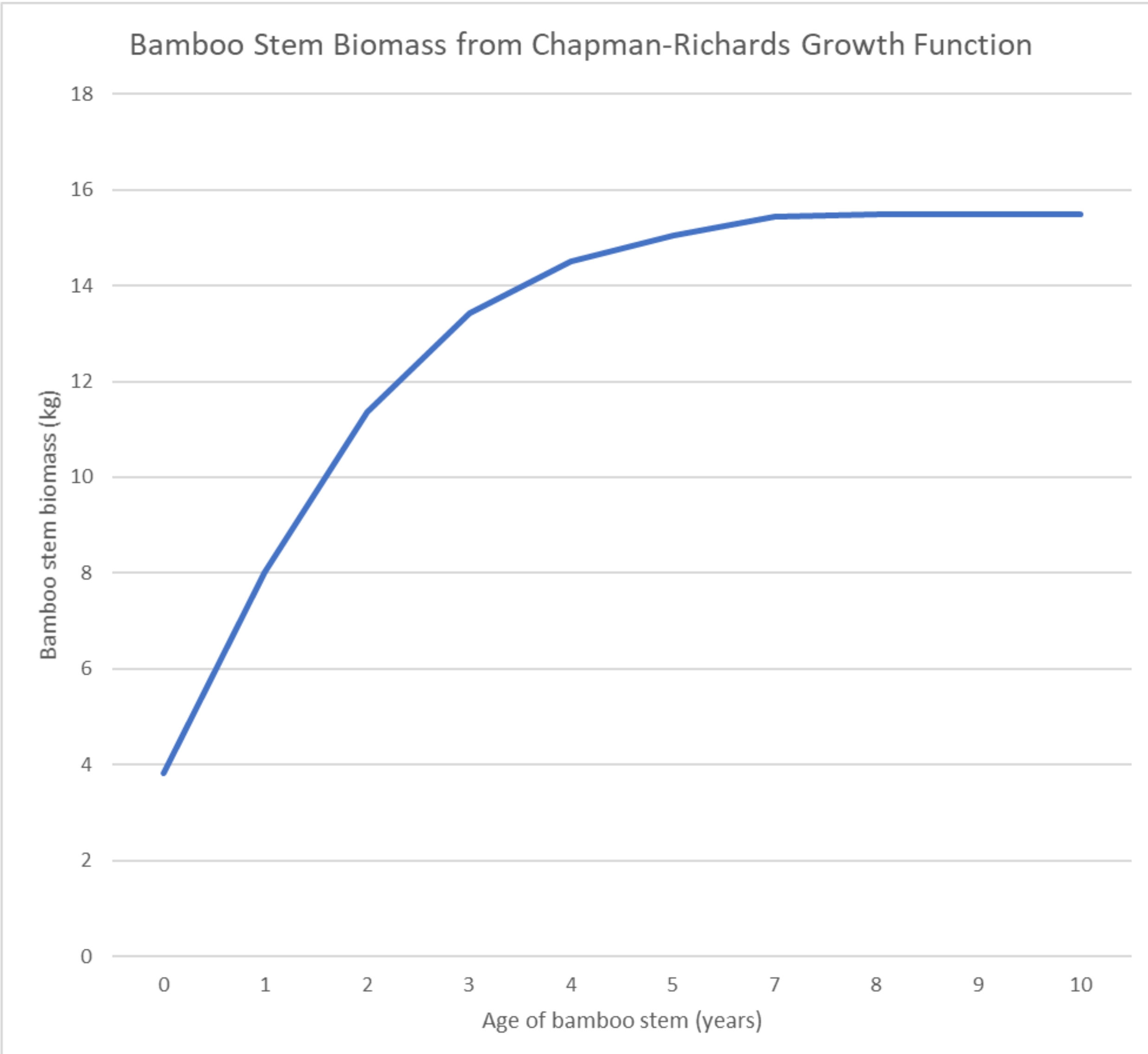
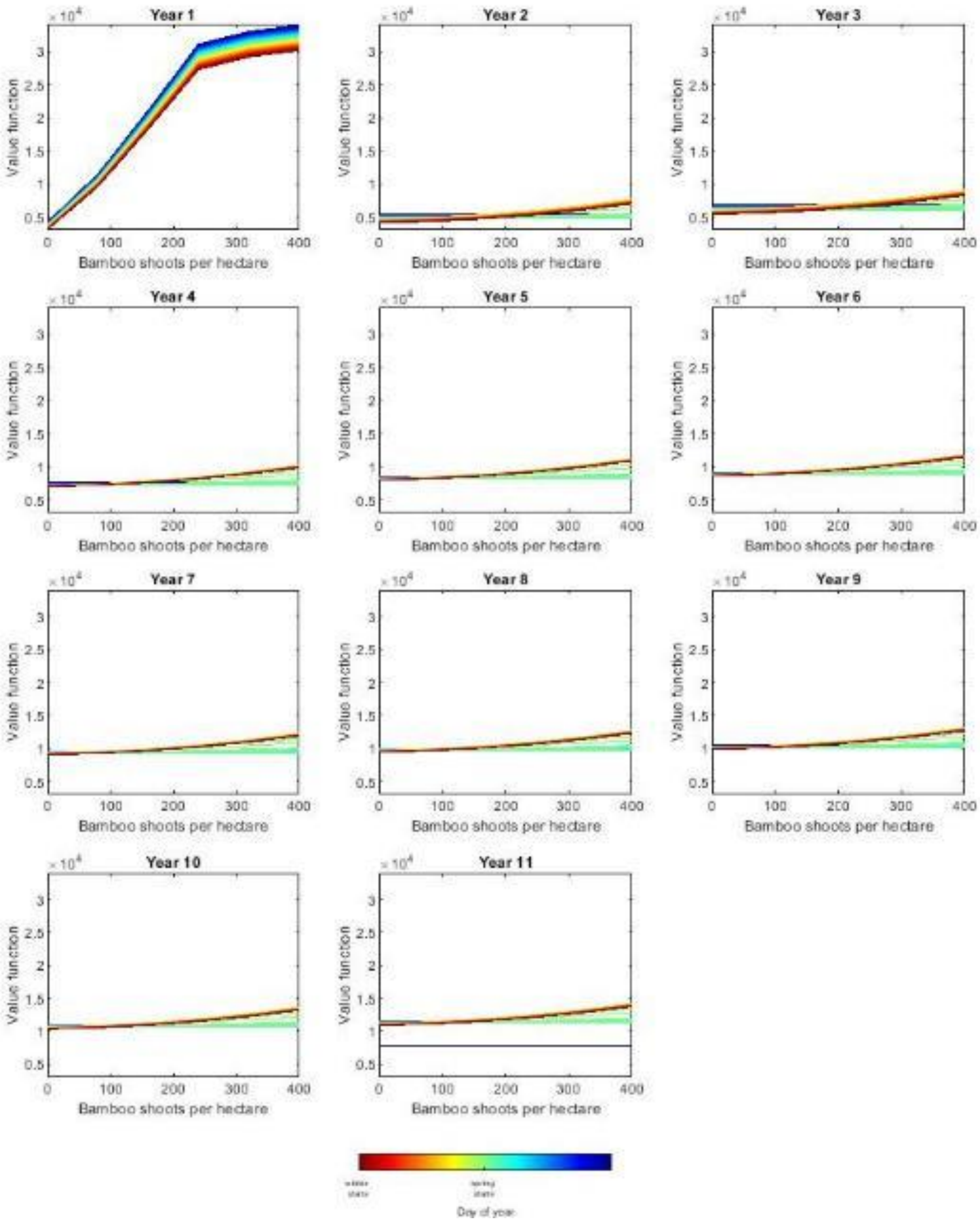


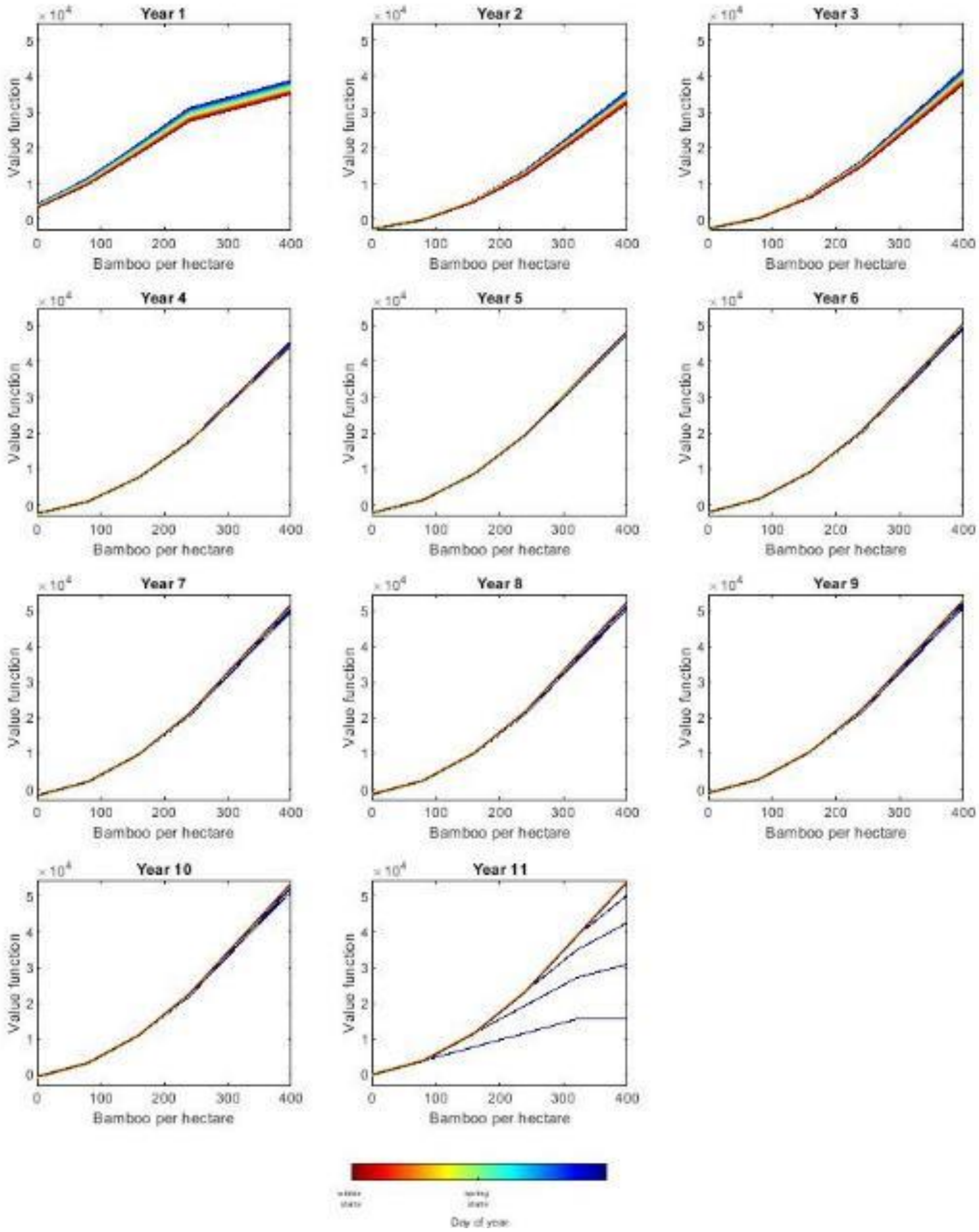
Figure 3. Deterministic Model, Specification 3

$$p_b = 1.4,\ p_{s,winter} = 25,\ p_{s,spring} = 3;$$
$$c_b = 300/2000,\ c_{s,winter} = 300/20,\ c_{s,spring} = 150/100;$$

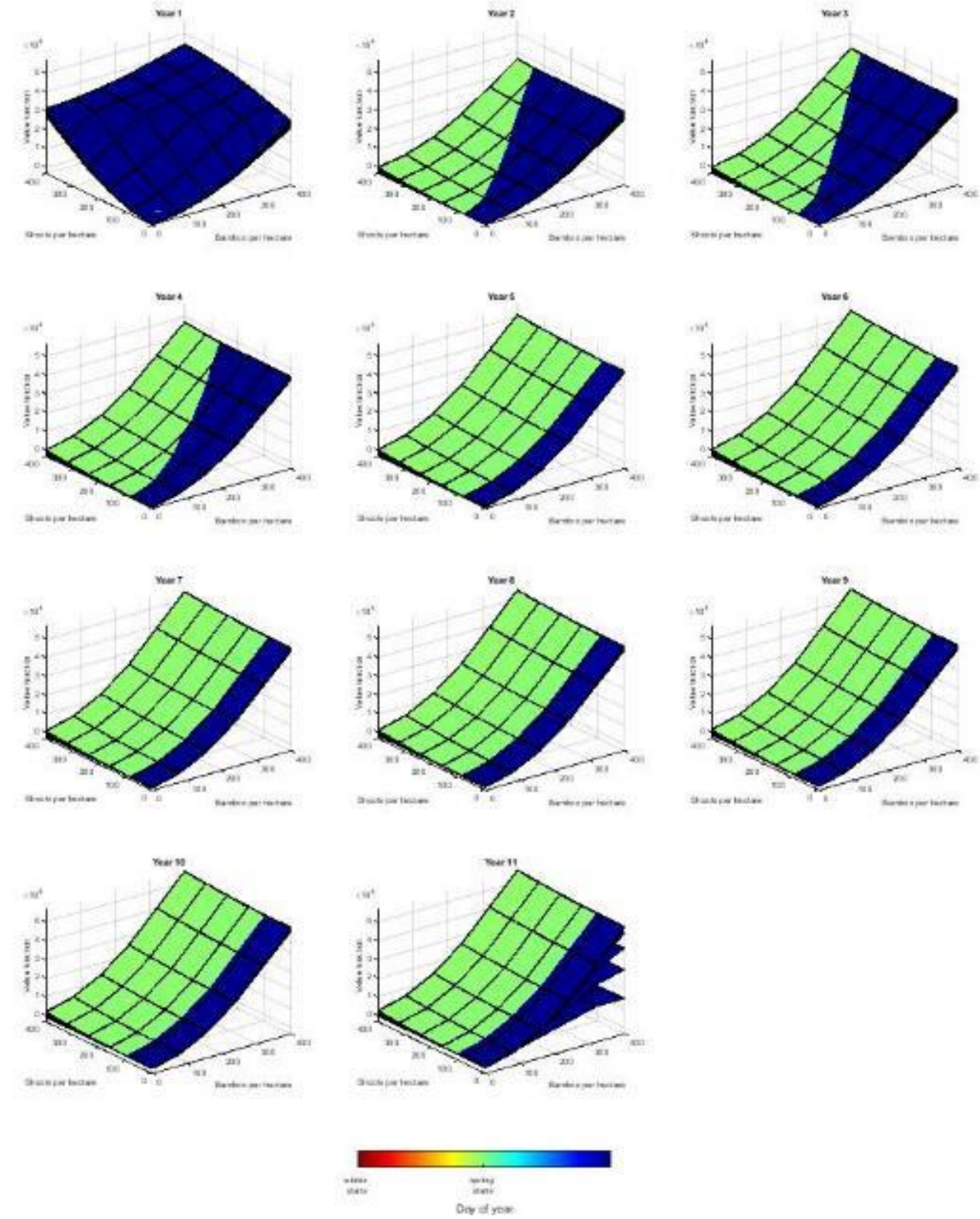
a) Value function as function of bamboo shoots per hectare



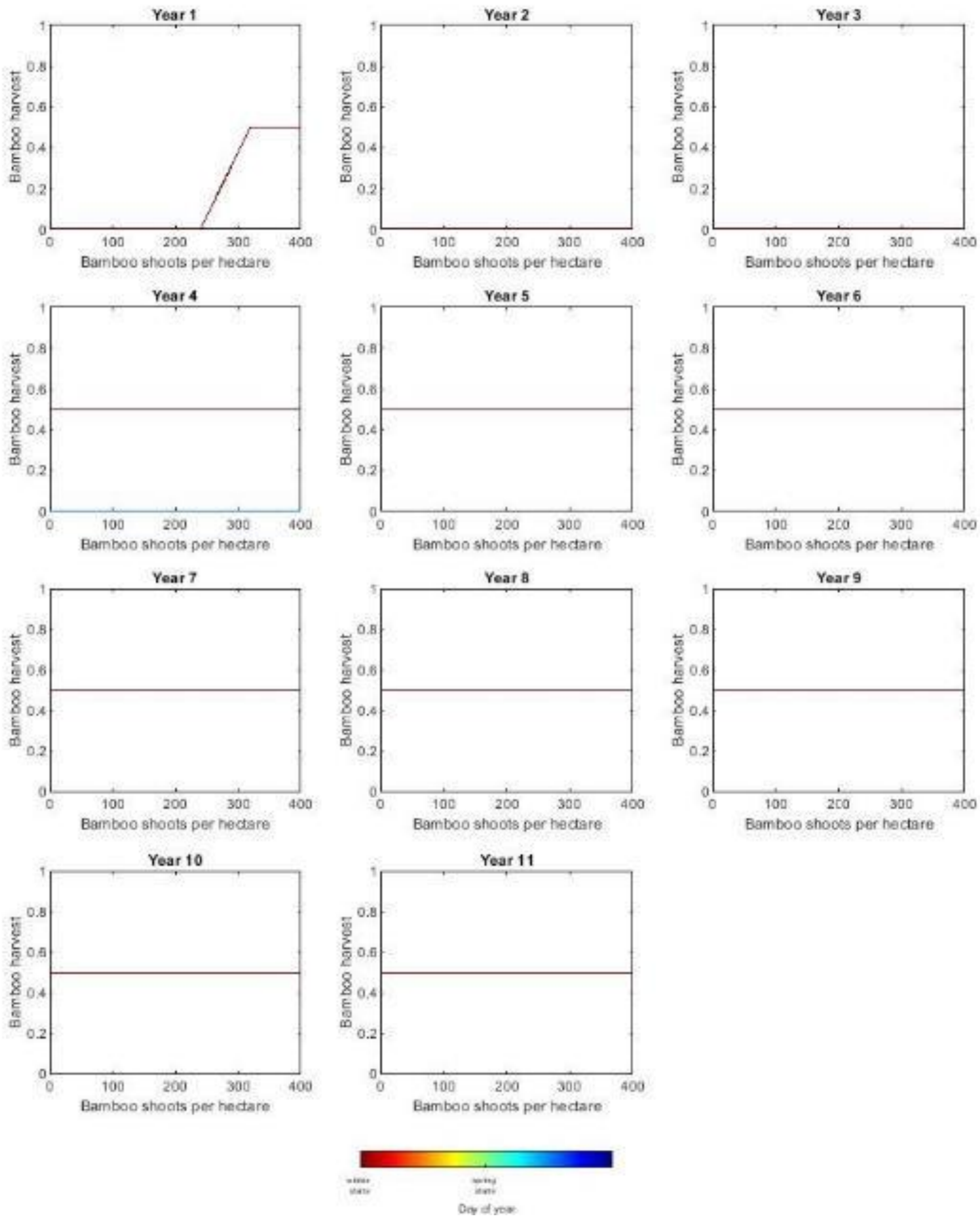
b) Value function as function of bamboo stem per hectare



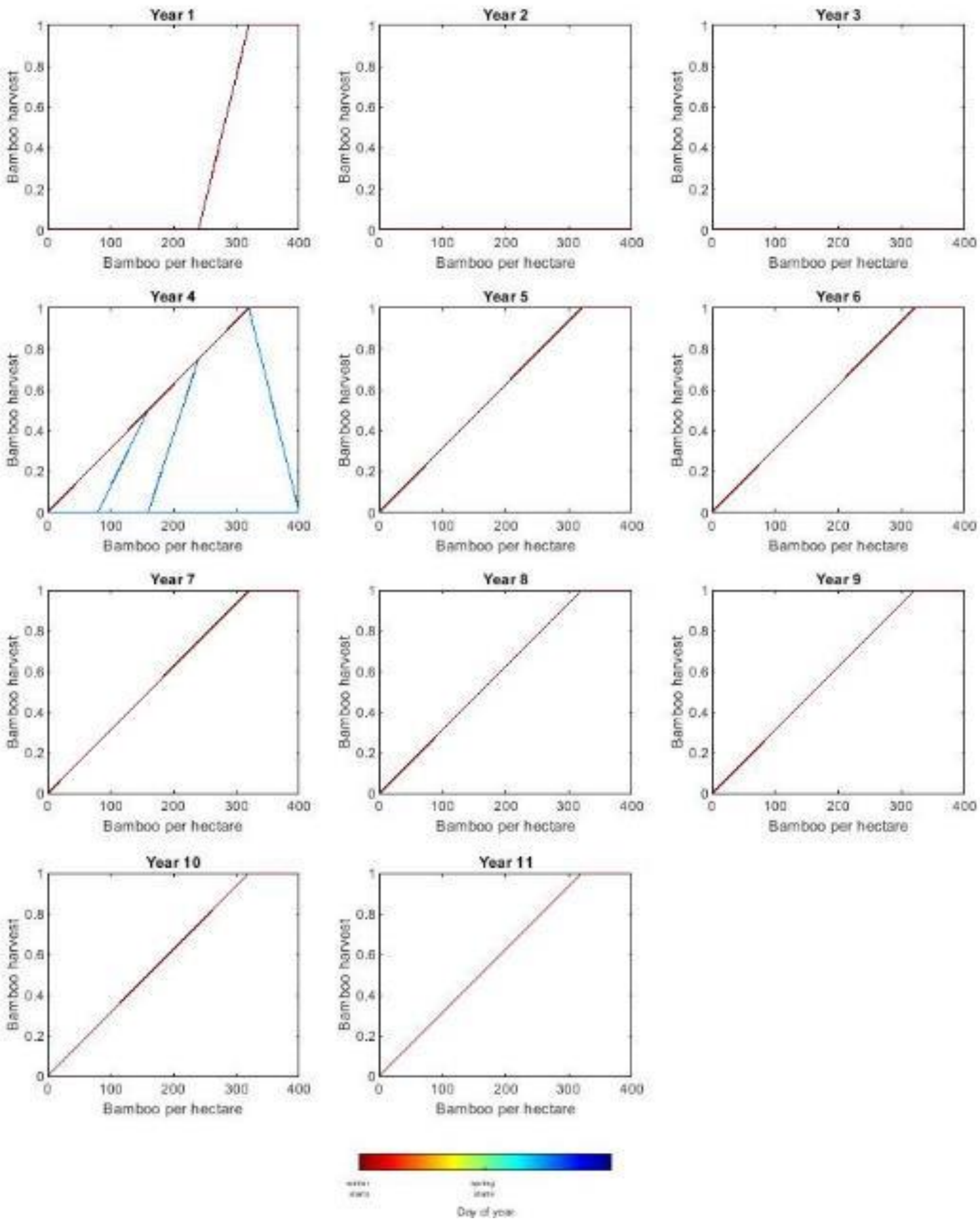
c) Value function as function of bamboo shoots per hectare and bamboo stem per hectare



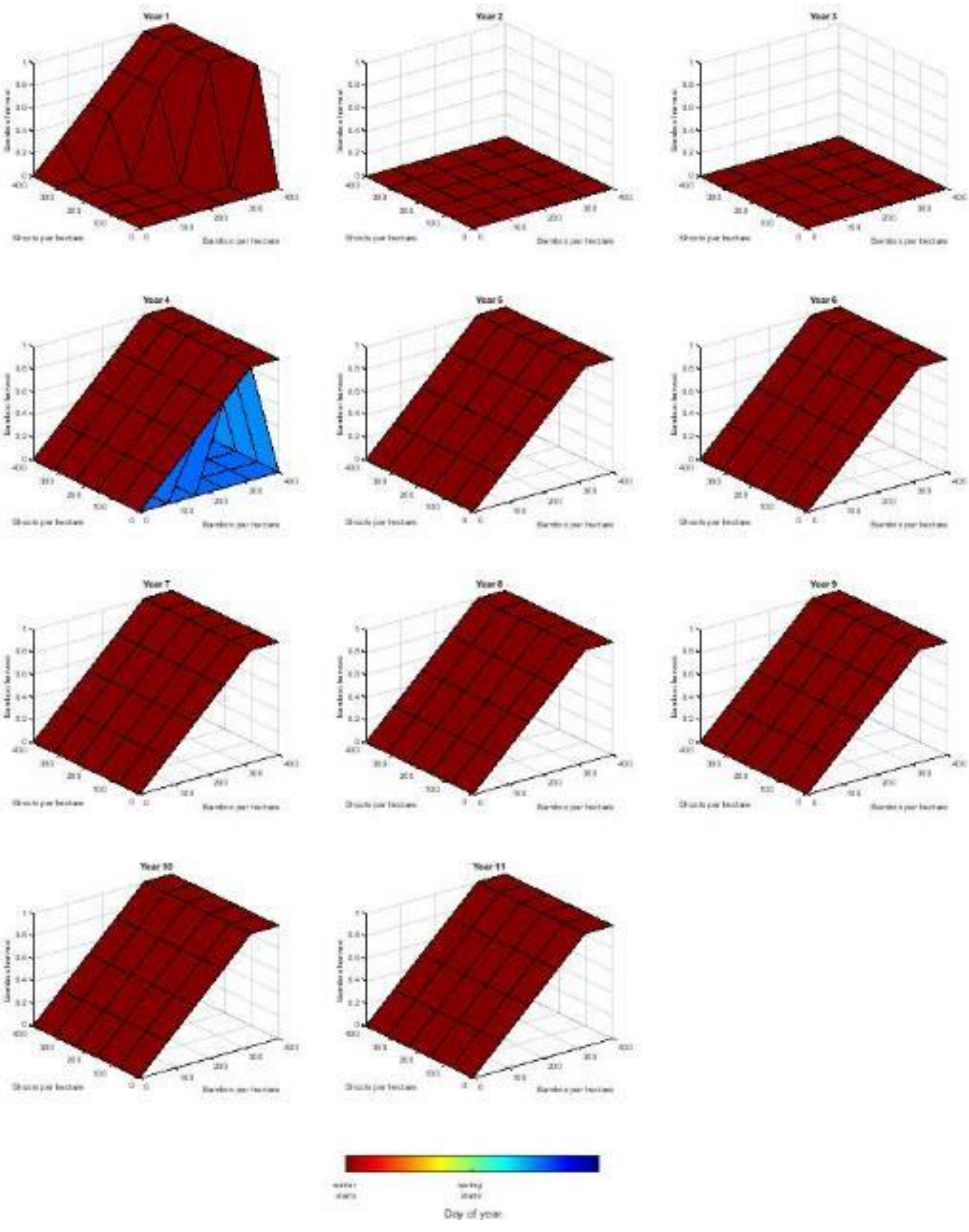
d) Bamboo stem harvest policy function as function of bamboo shoots per hectare



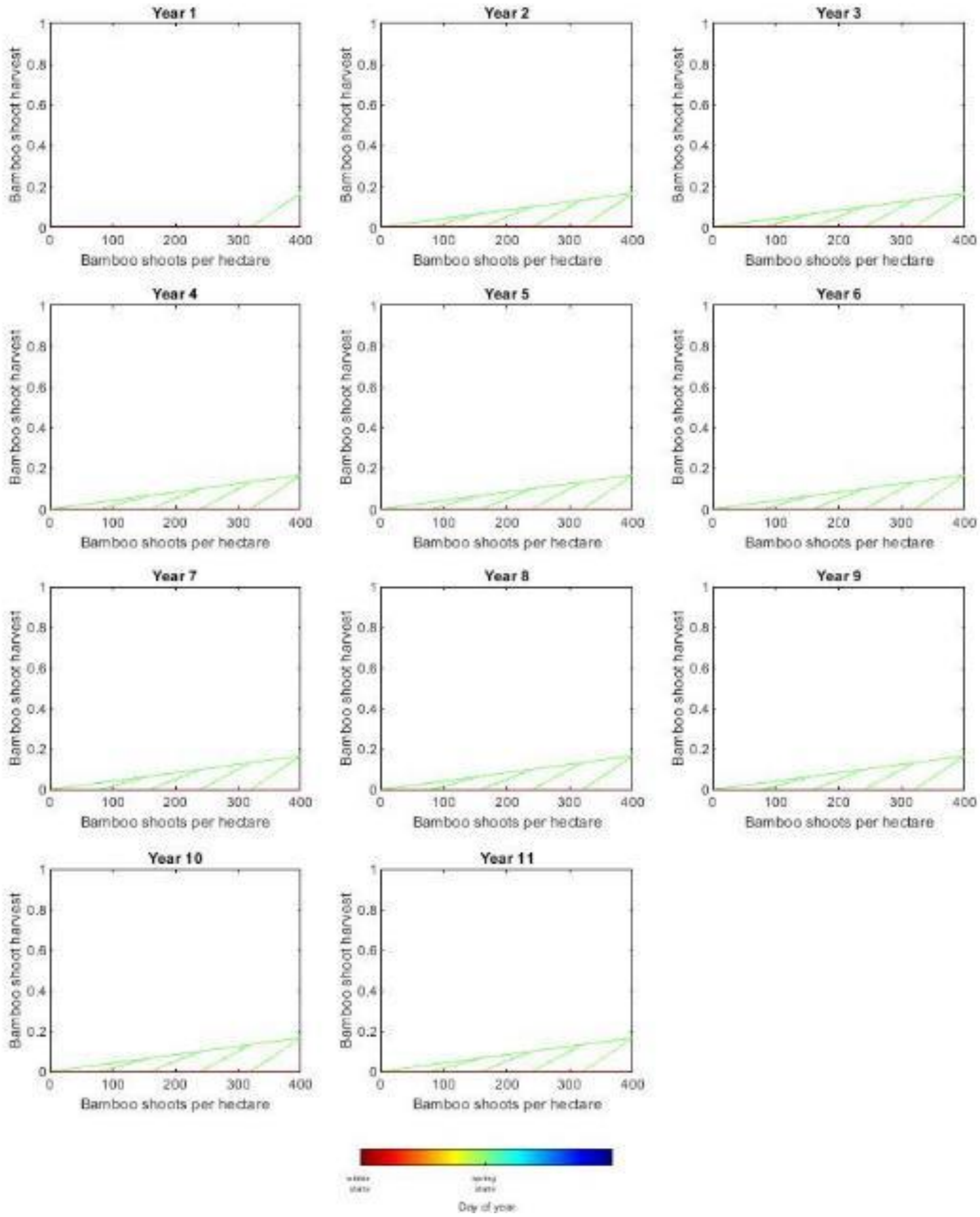
e) Bamboo stem harvest policy function as function of bamboo stem per hectare



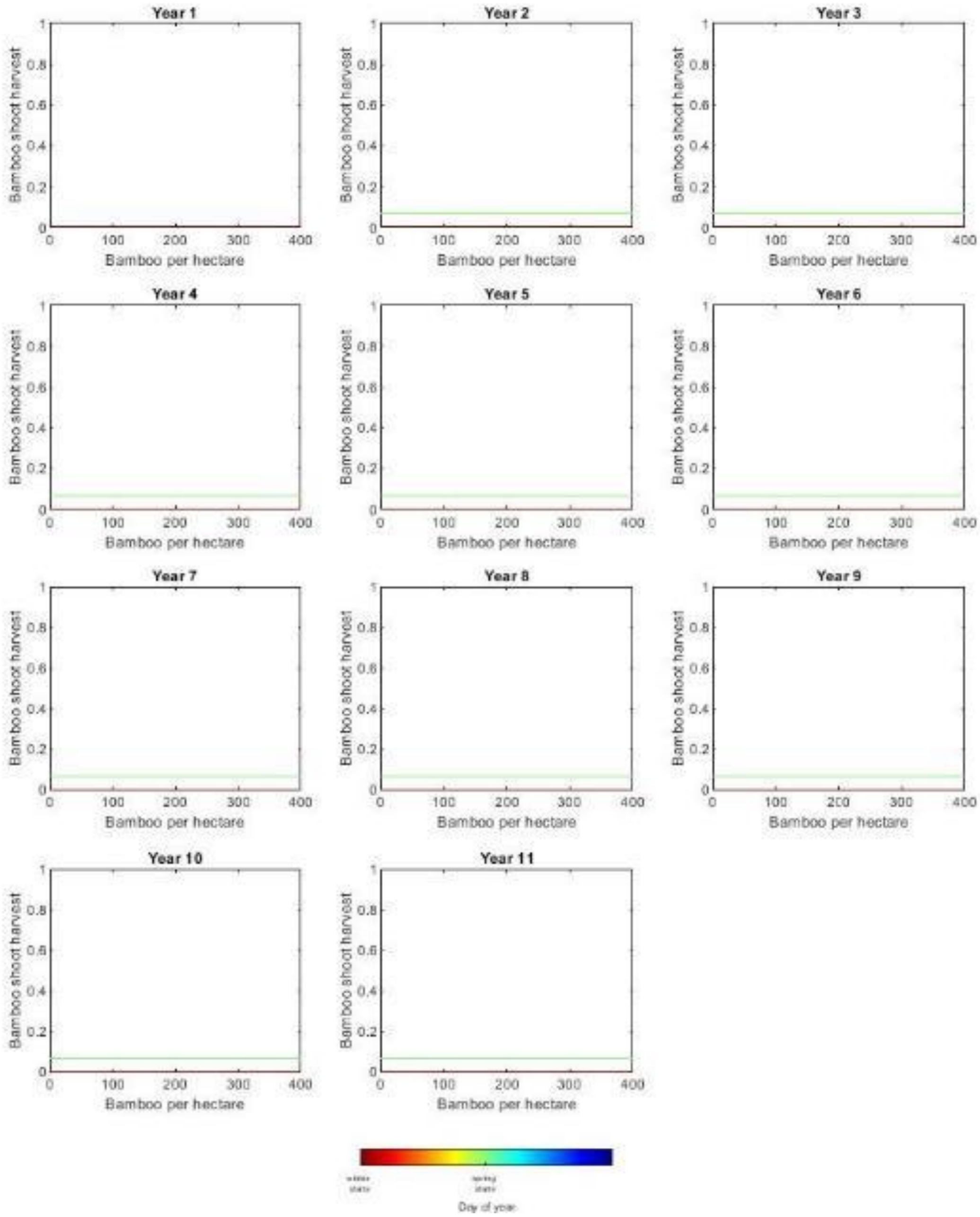
f) Bamboo stem harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare



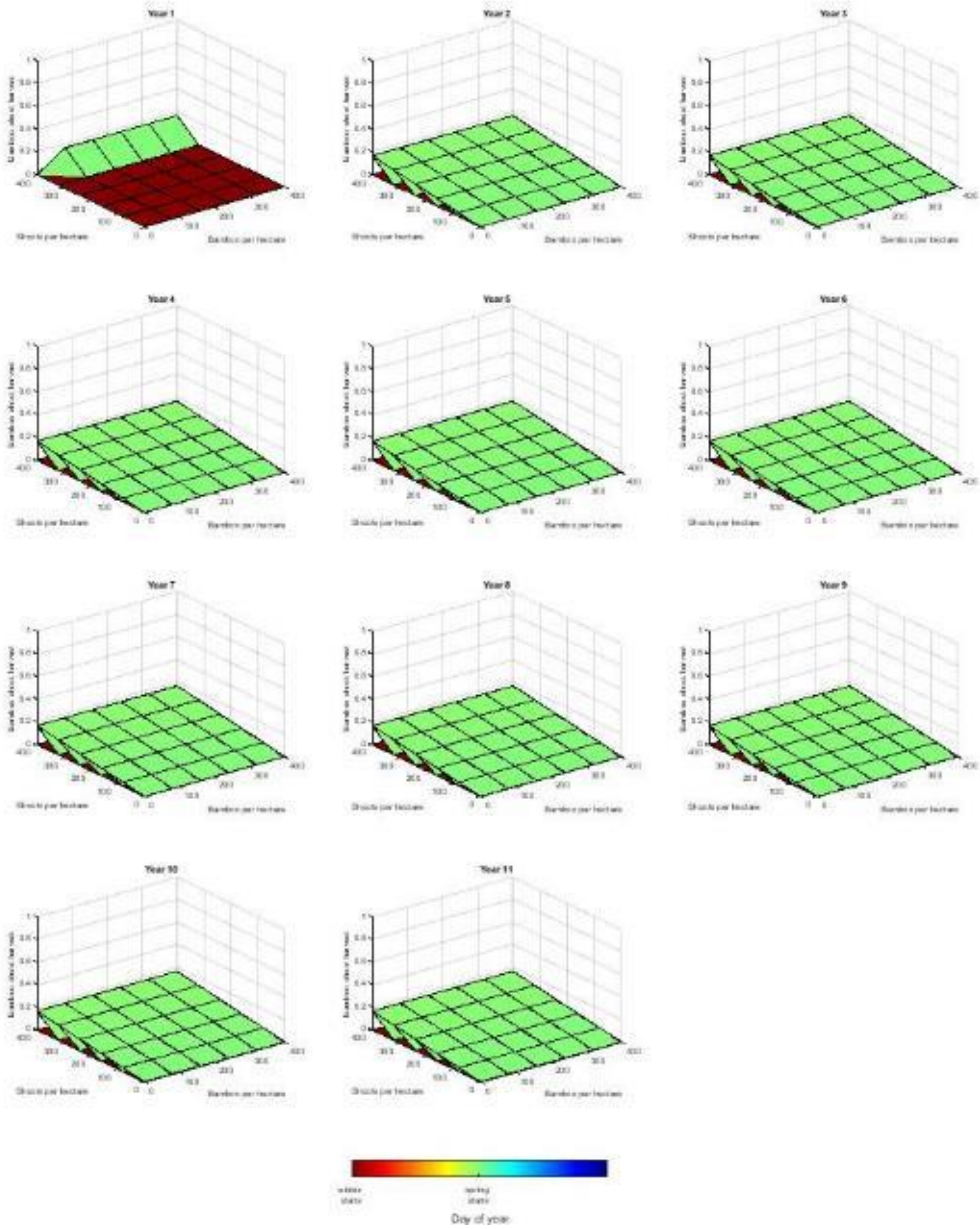
g) Bamboo shoot harvest policy function as function of bamboo shoots per hectare



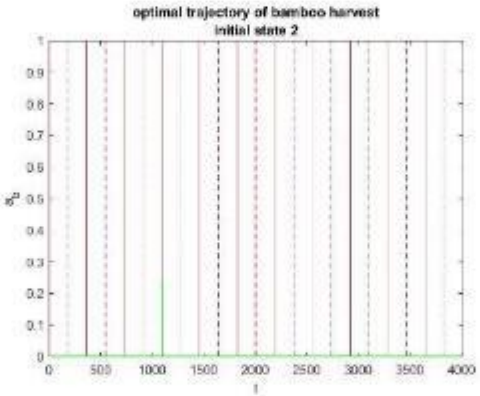
h) Bamboo shoot harvest policy function as function of bamboo stem per hectare

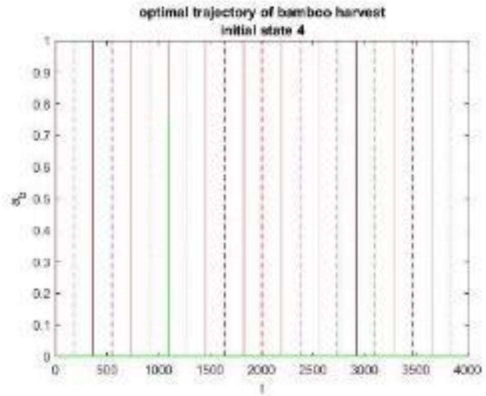


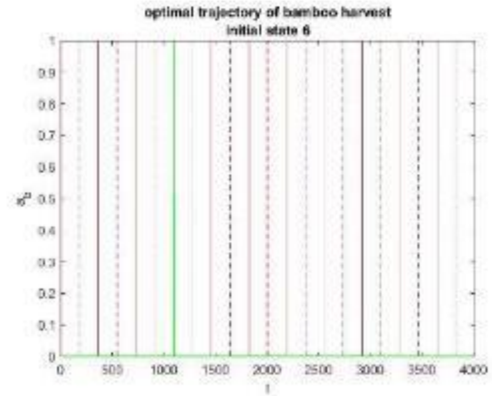
i) Bamboo shoot harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare

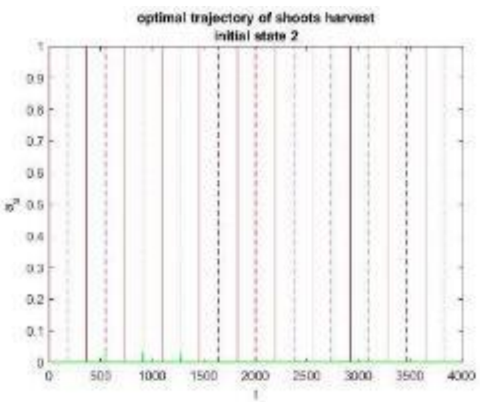


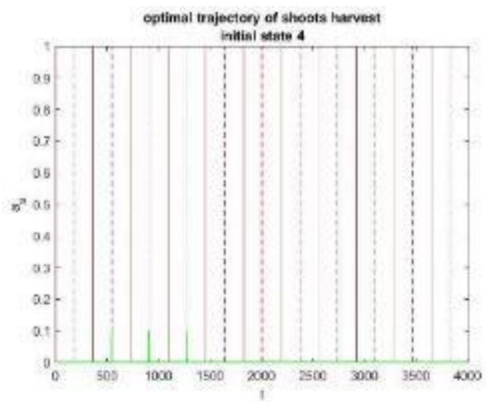
j) Optimal trajectories for each action and state variable over 11 years

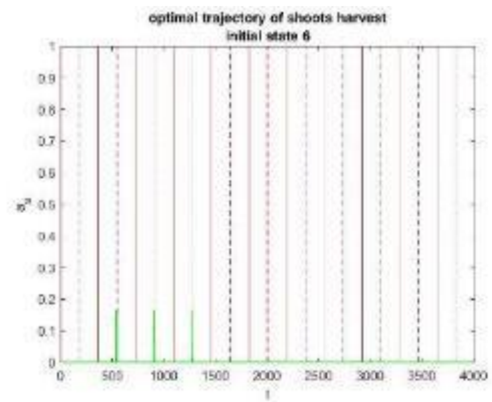


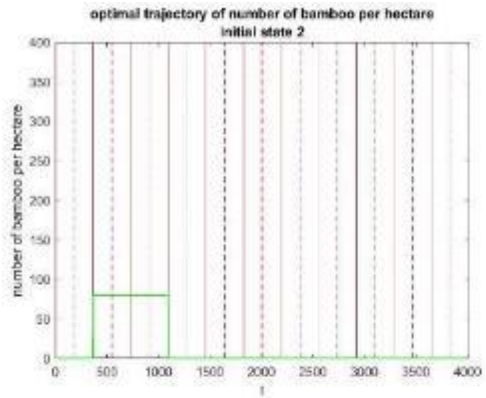


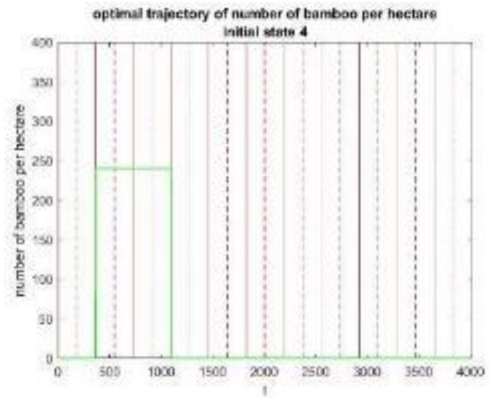


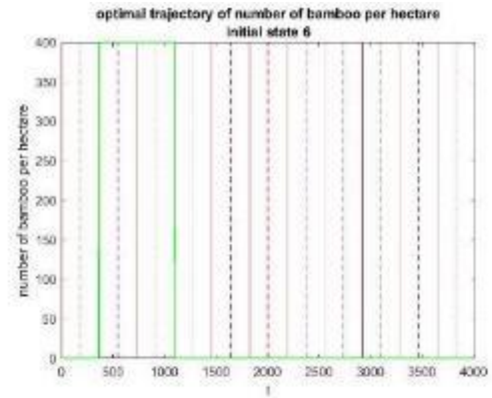


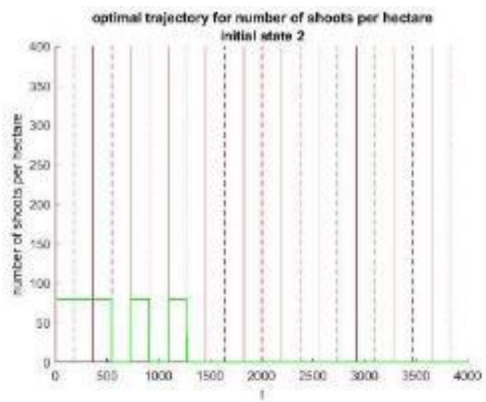


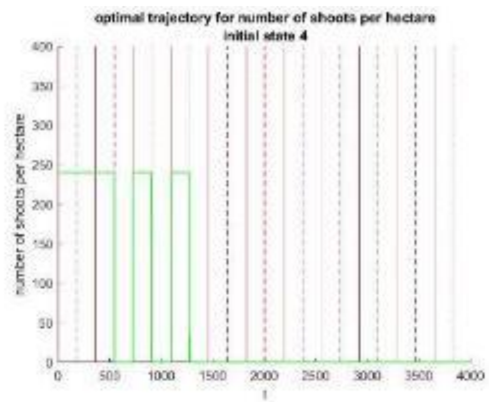



optimal trajectory of number of bamboo per hectare
initial state 2
number of bamboo per hectare
0 50 100 150 200 250 300 350 400
0 500 1000 1500 2000 2500 3000 3500 4000
t


optimal trajectory of number of bamboo per hectare
initial state 4
number of bamboo per hectare
0 50 100 150 200 250 300 350 400
0 500 1000 1500 2000 2500 3000 3500 4000
t


optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
0 50 100 150 200 250 300 350 400
0 500 1000 1500 2000 2500 3000 3500 4000
t


optimal trajectory for number of shoots per hectare
initial state 2
number of shoots per hectare
0 50 100 150 200 250 300 350 400
0 500 1000 1500 2000 2500 3000 3500 4000
t


optimal trajectory for number of shoots per hectare
initial state 4
number of shoots per hectare
0 50 100 150 200 250 300 350 400
0 500 1000 1500 2000 2500 3000 3500 4000
t

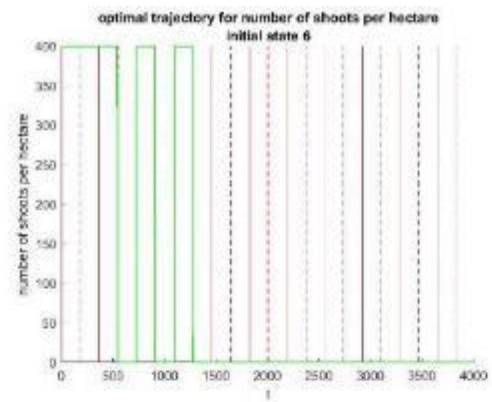

optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
0 50 100 150 200 250 300 350 400
0 500 1000 1500 2000 2500 3000 3500 4000
t

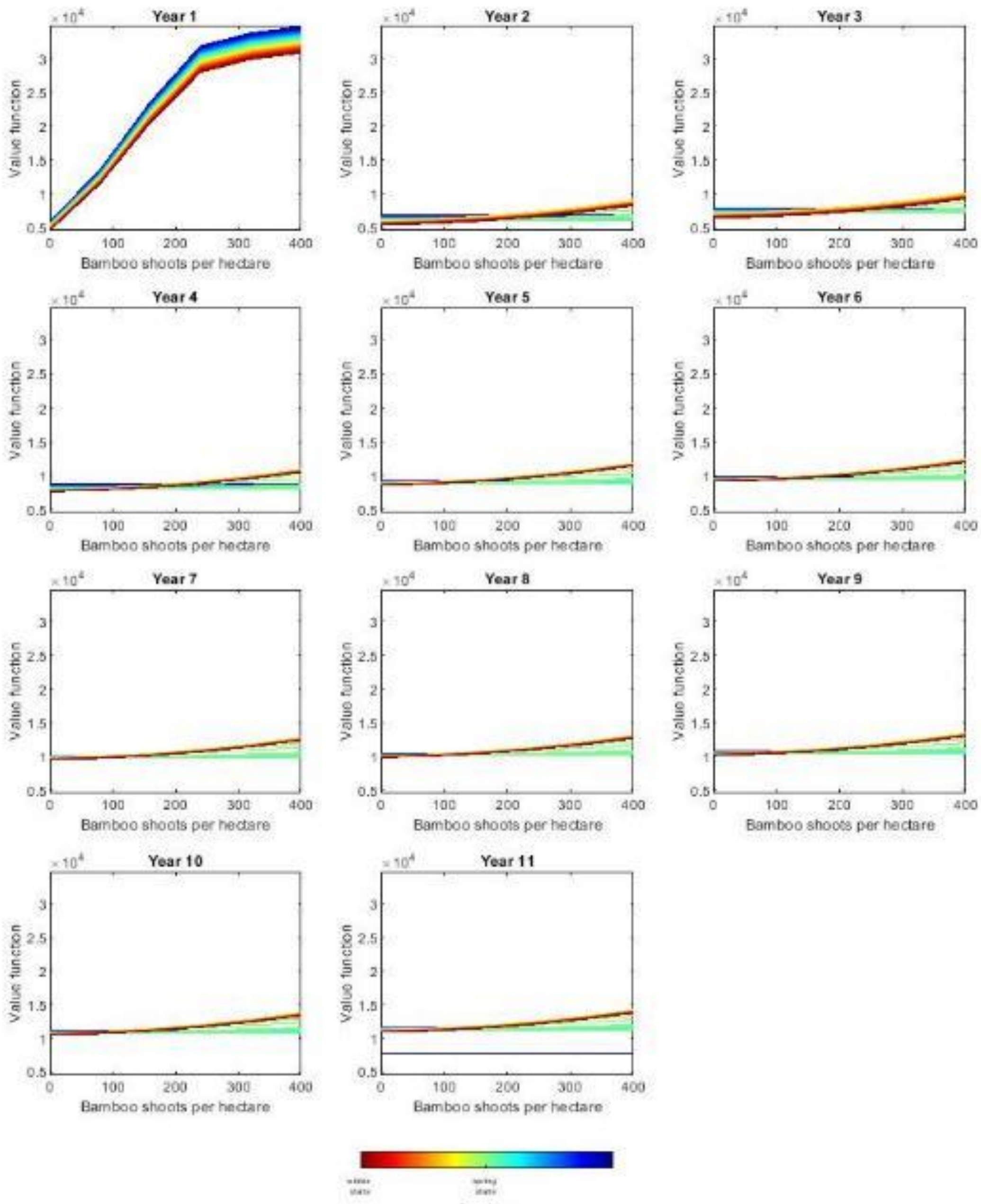
Figure 4: Stochastic Model, Specification 3, Version C

$p_b = 1.4,\ p_{s,winter} = 25,\ p_{s,spring} = 3;$

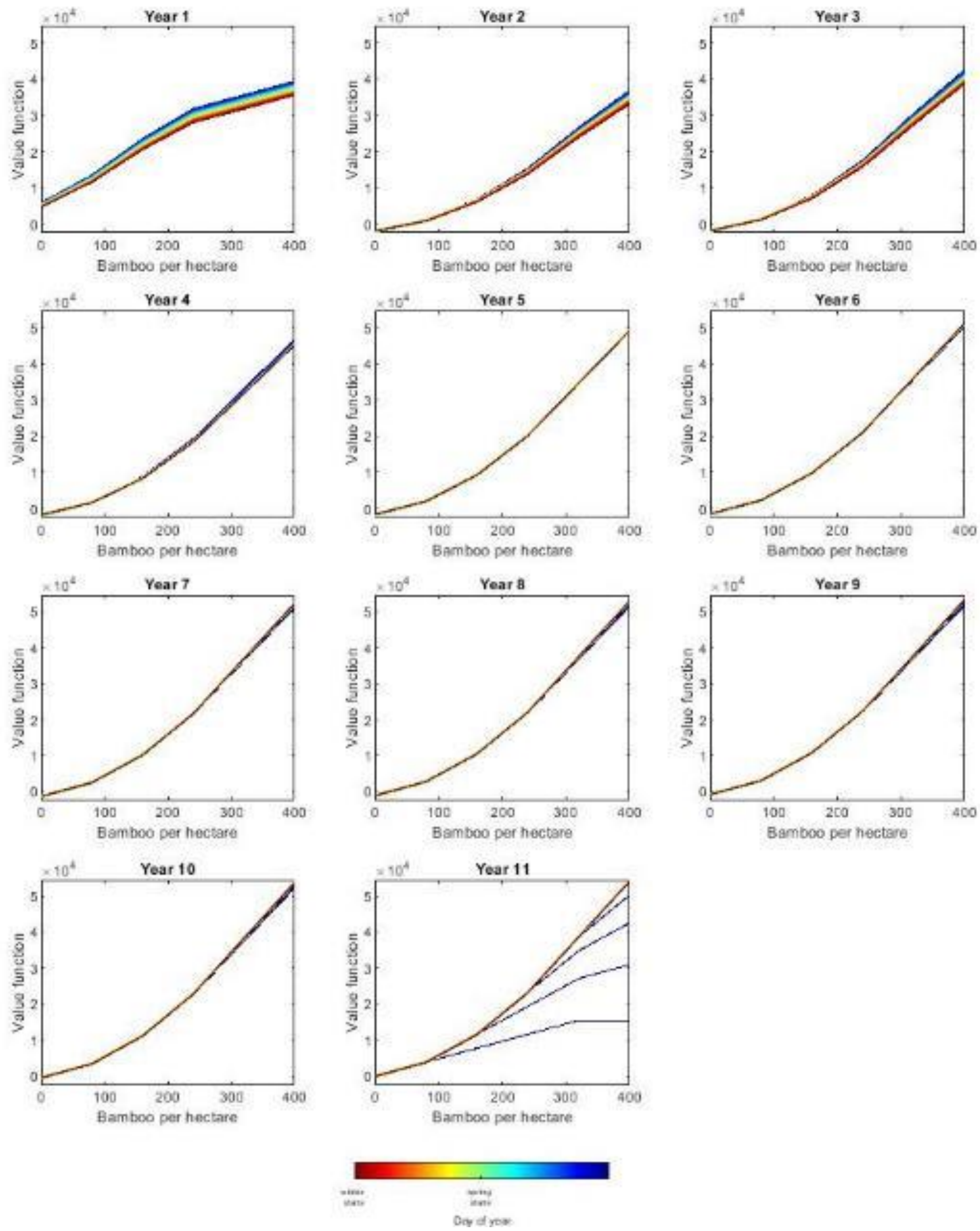
$c_b = 300/2000,\ c_{s,winter} = 300/20,\ c_{s,spring} = 150/100;$

$rain\_high\_prob$ is given by daily empirical probability for Shanchuan

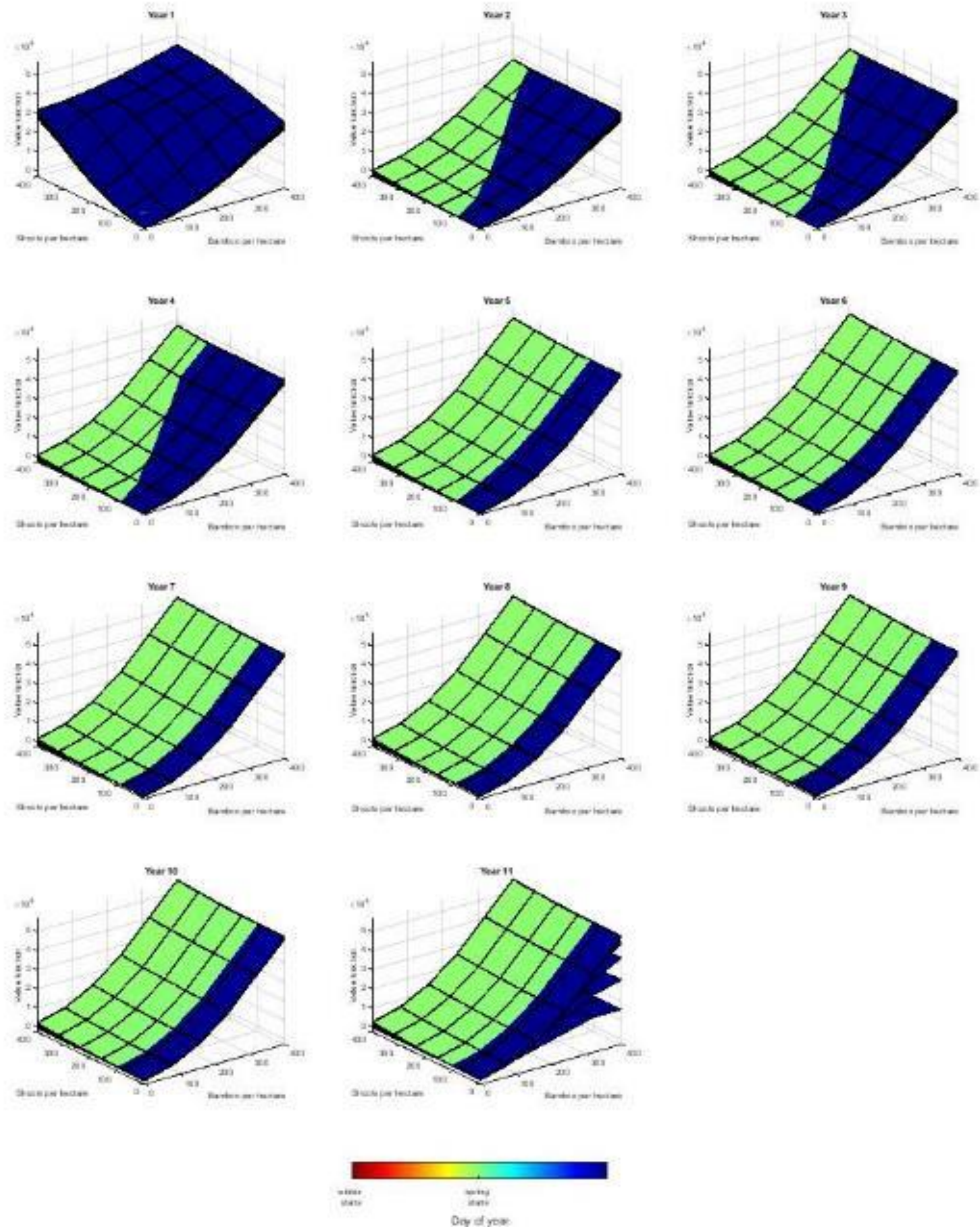
a) Value function as function of bamboo shoots per hectare



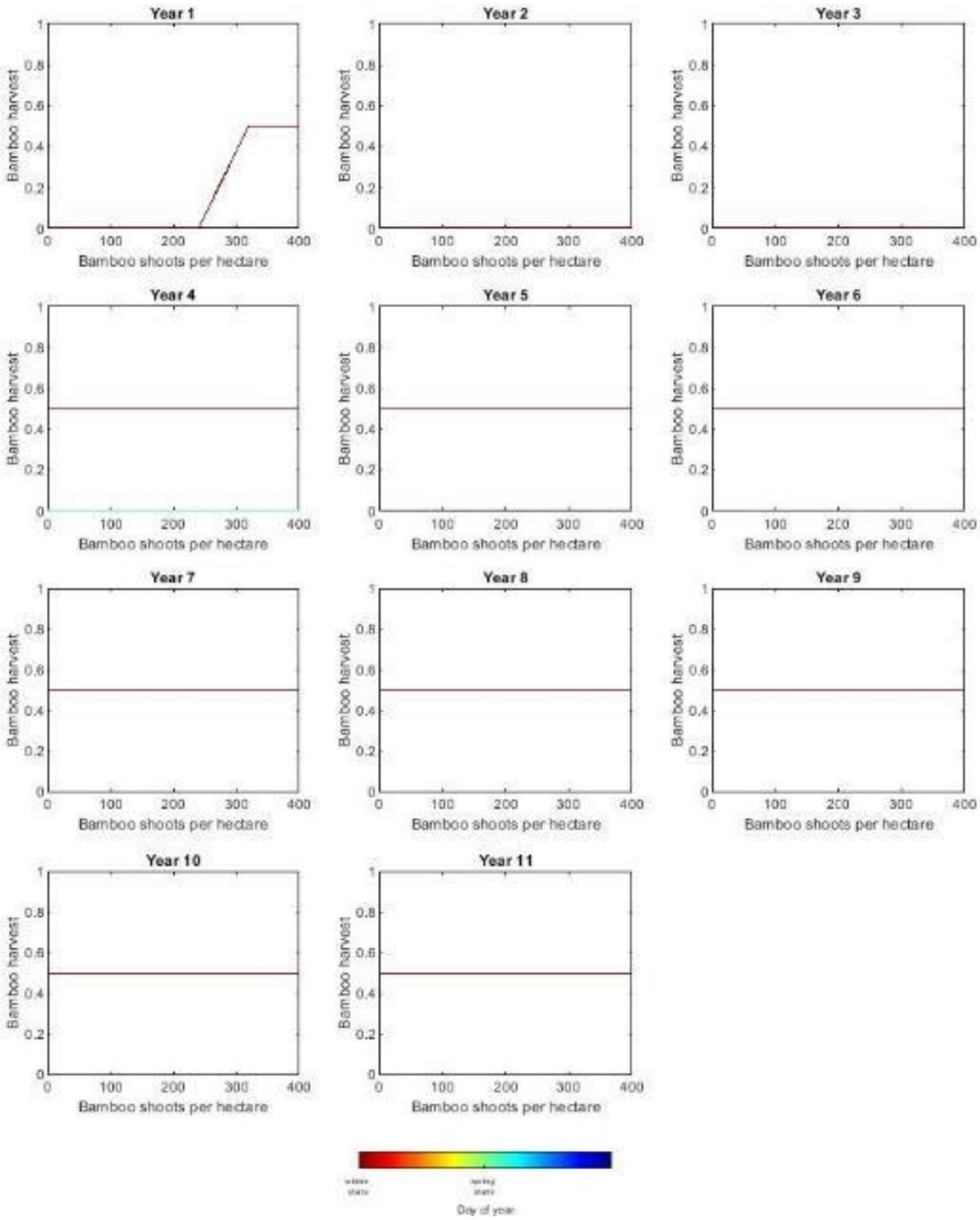
b) Value function as function of bamboo stem per hectare



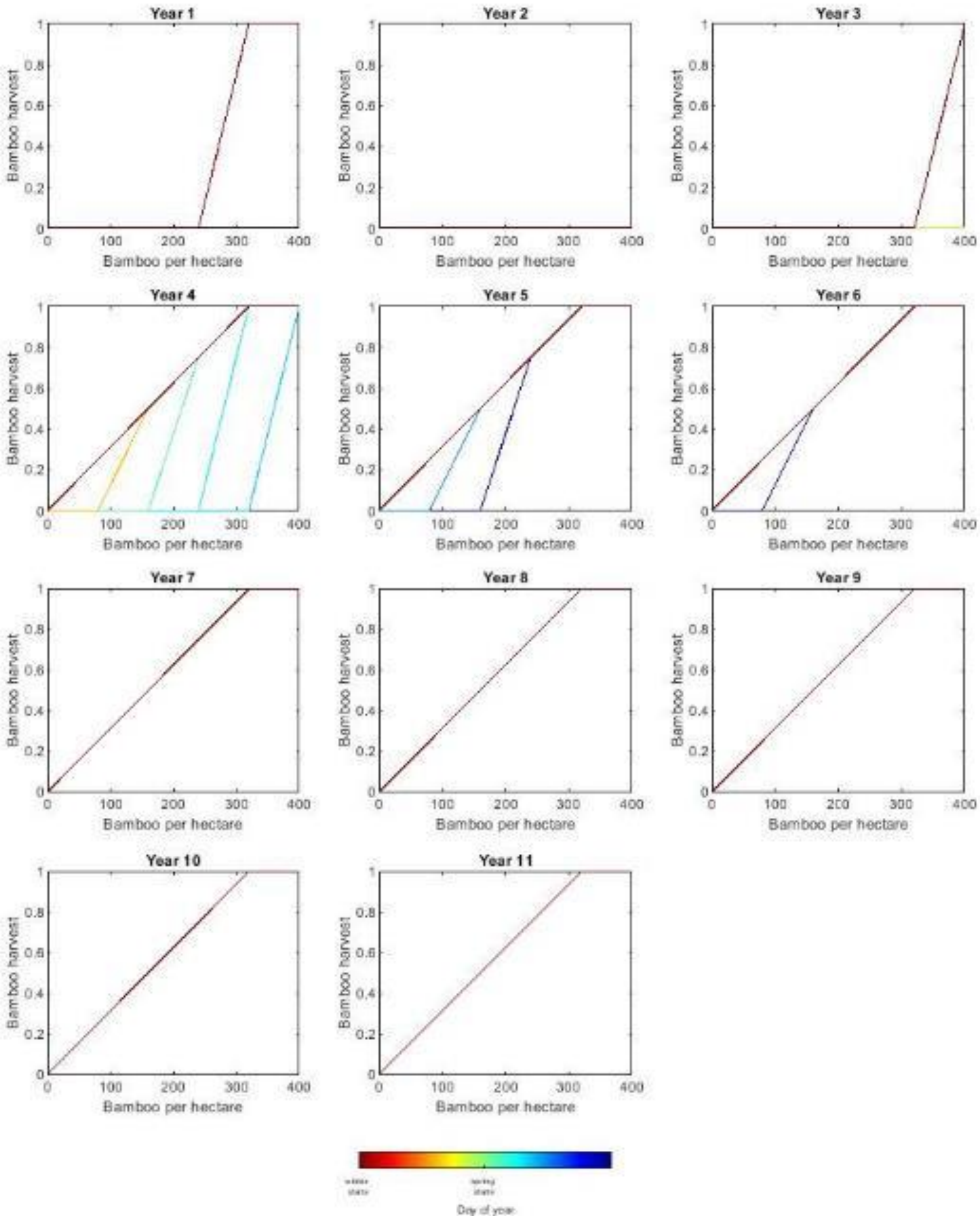
c) Value function as function of bamboo shoots per hectare and bamboo stem per hectare



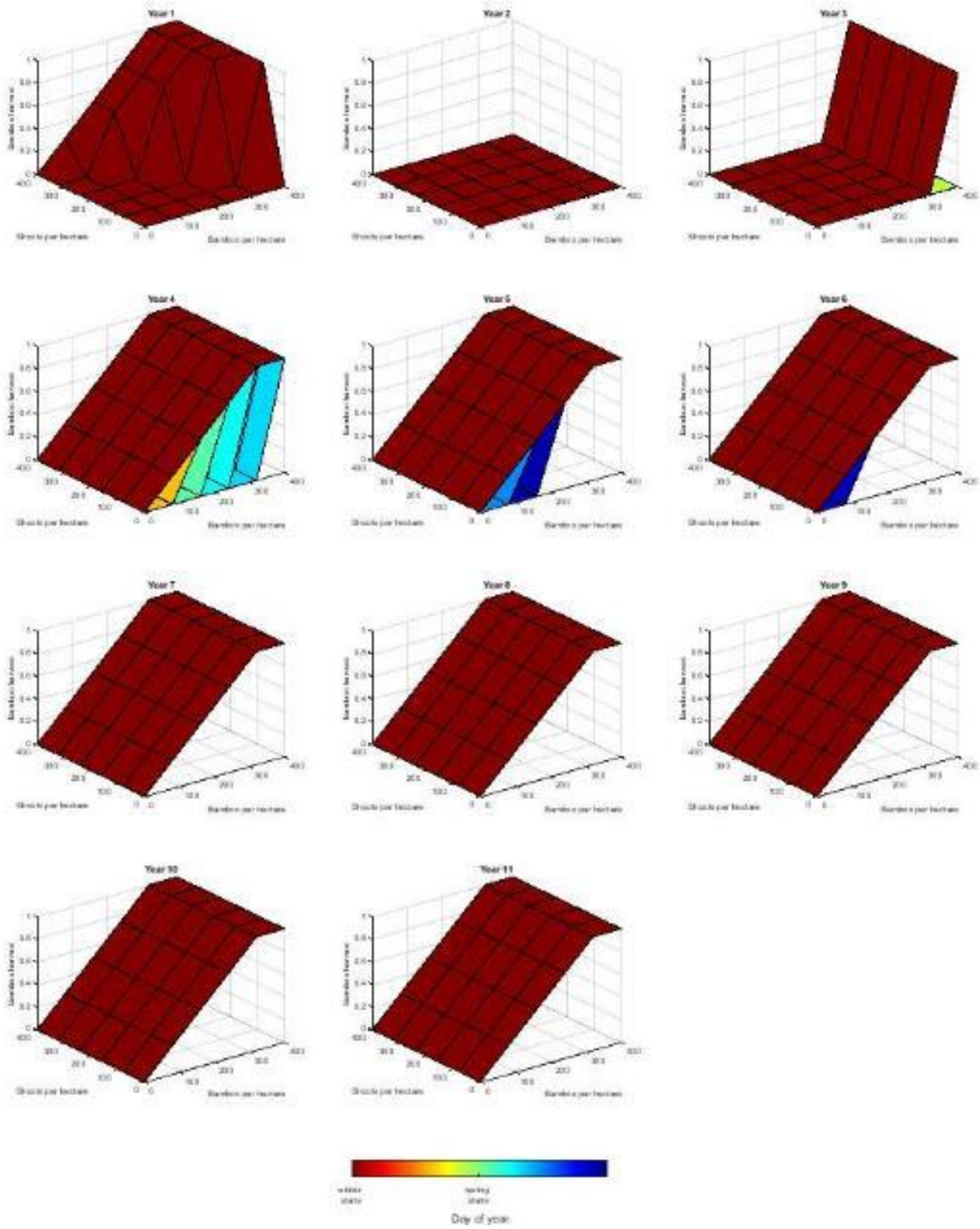
d) Bamboo stem harvest policy function as function of bamboo shoots per hectare



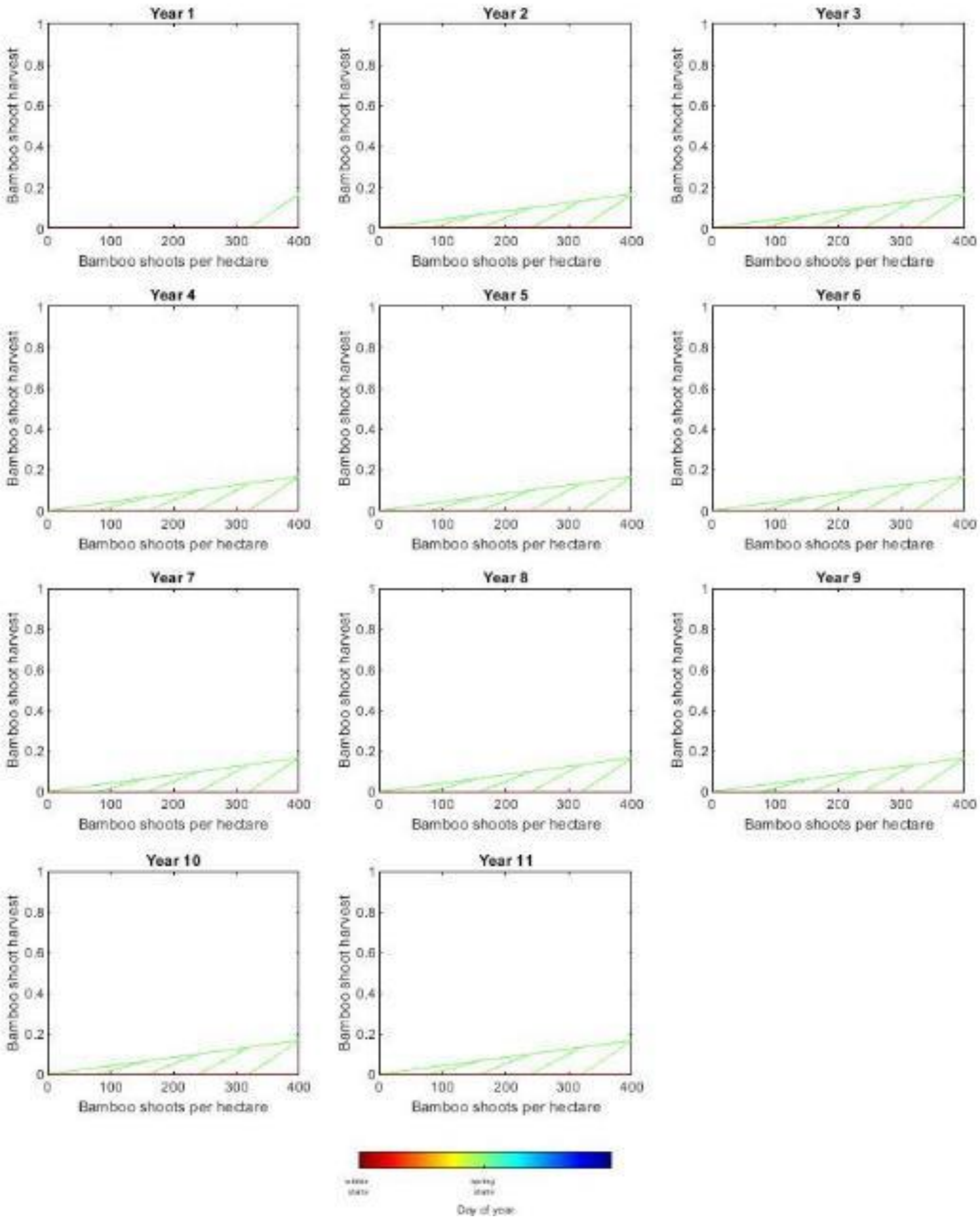
e) Bamboo stem harvest policy function as function of bamboo stem per hectare



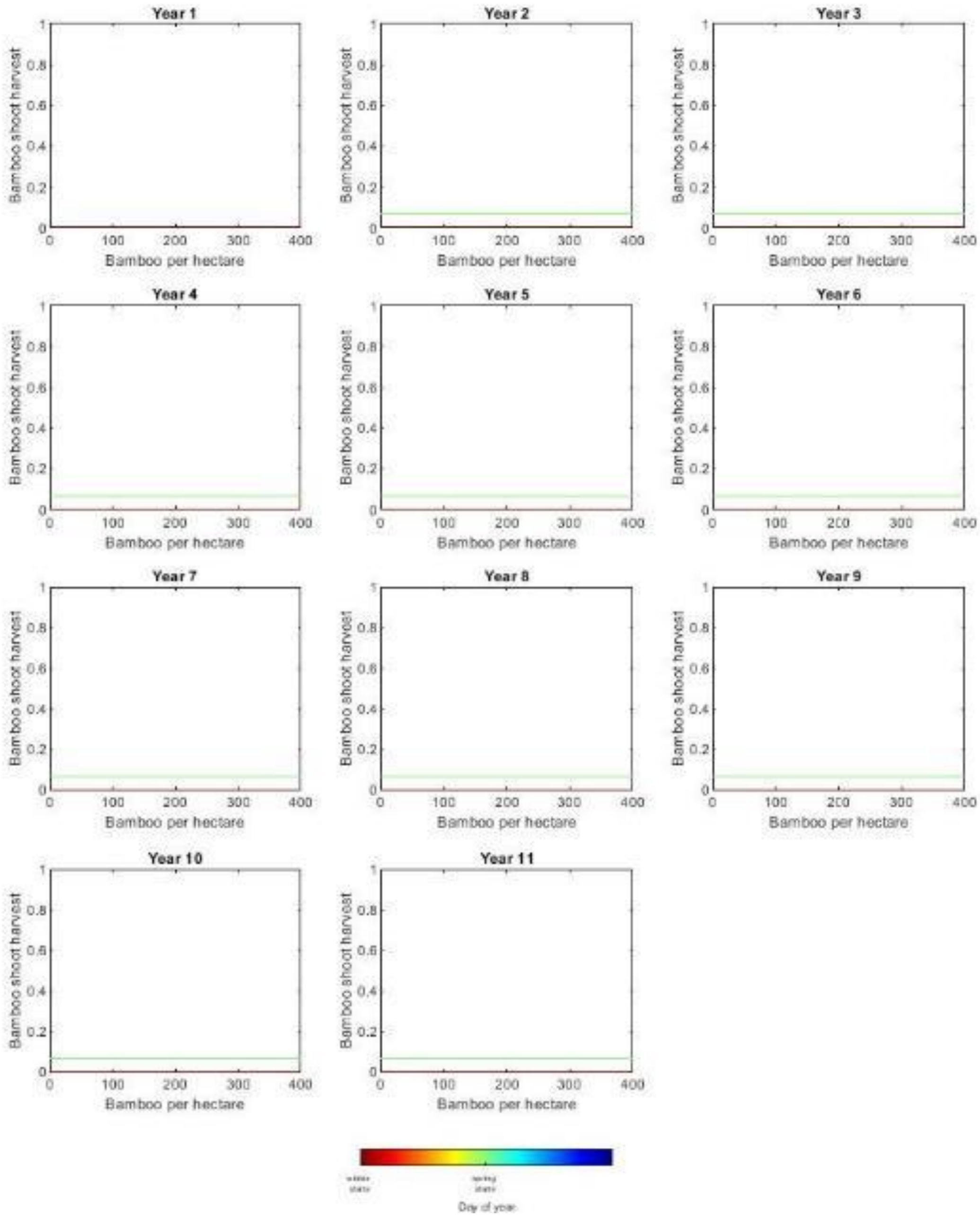
f) Bamboo stem harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare



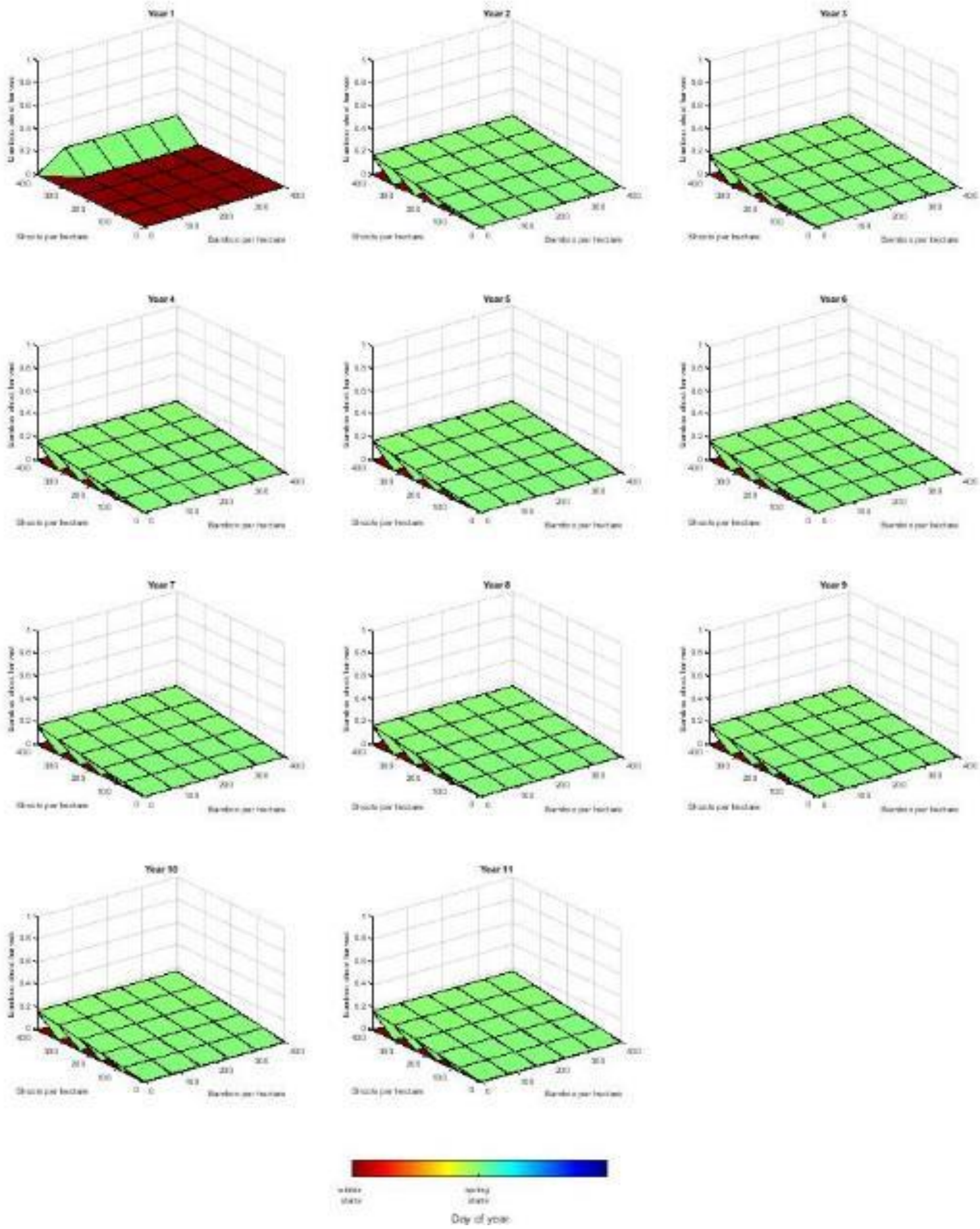
g) Bamboo shoot harvest policy function as function of bamboo shoots per hectare



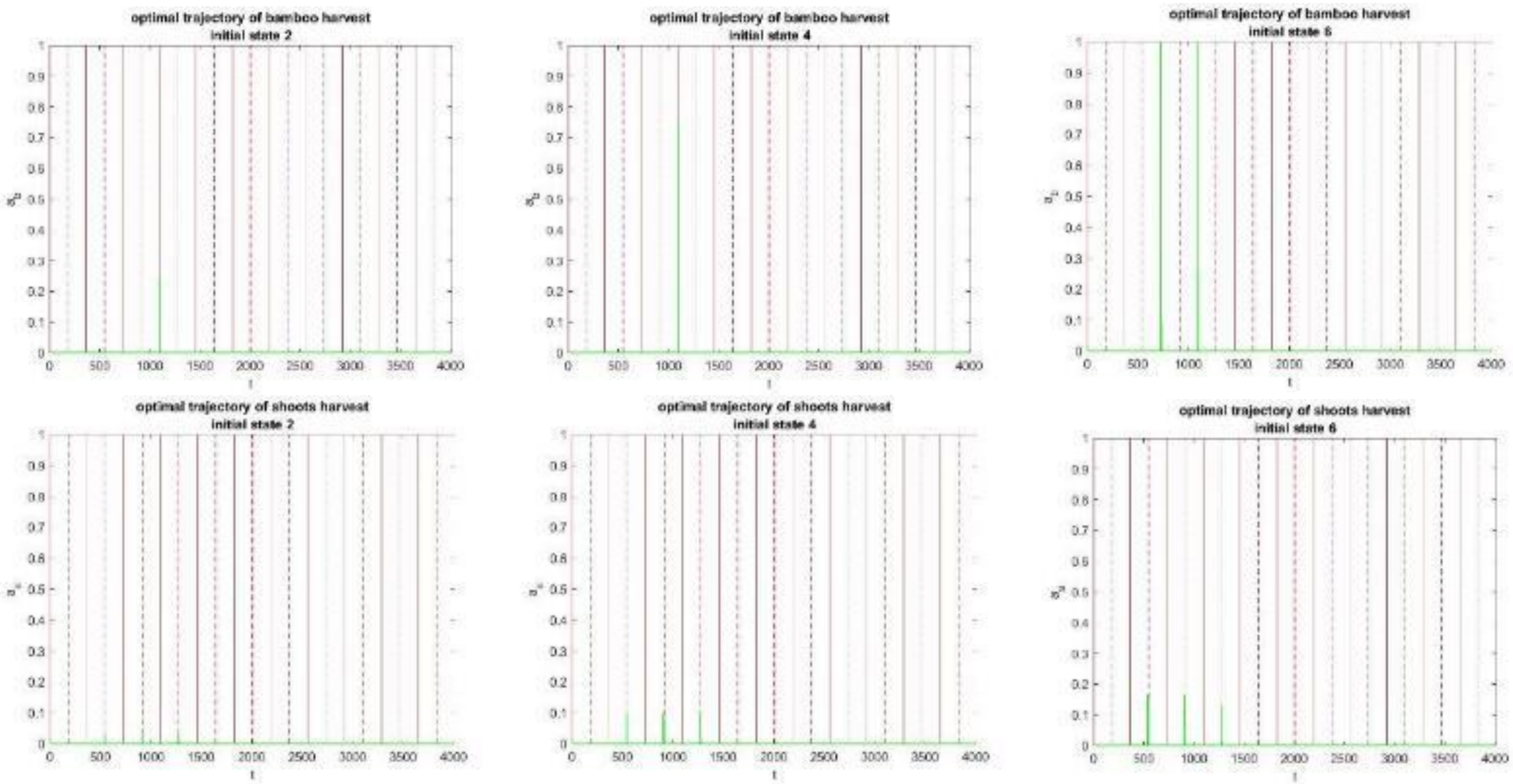
h) Bamboo shoot harvest policy function as function of bamboo stem per hectare



i) Bamboo shoot harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare



j) Optimal trajectories for each action and state variable over 11 years



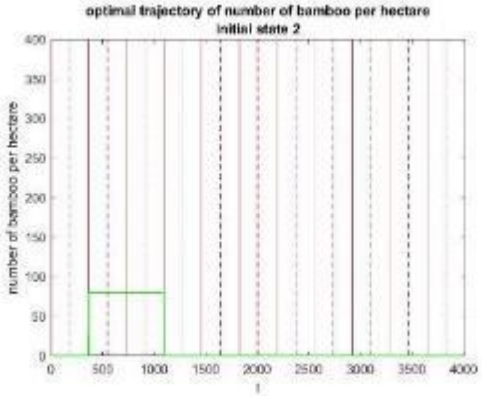
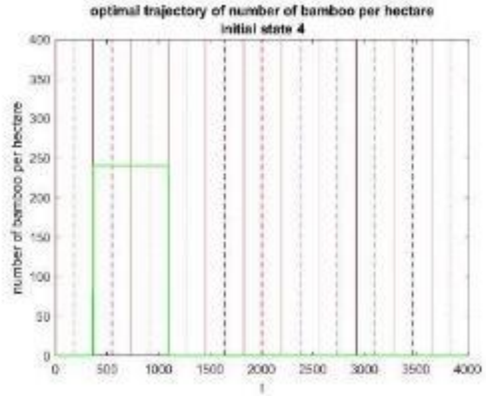
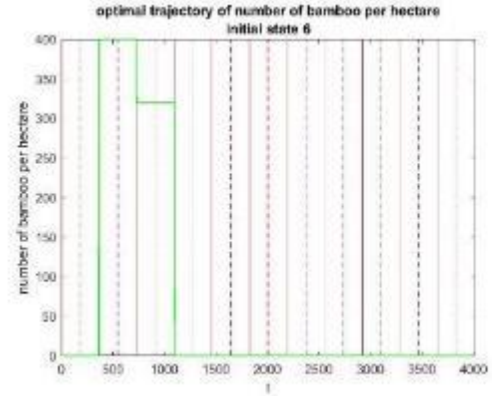
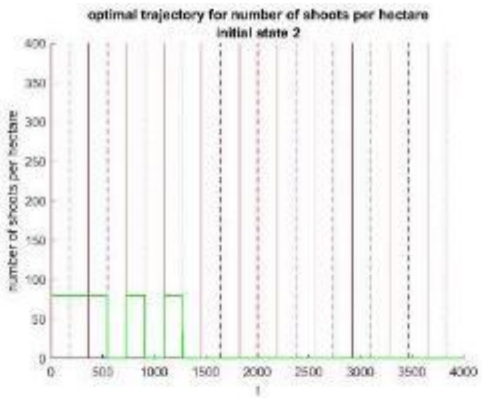
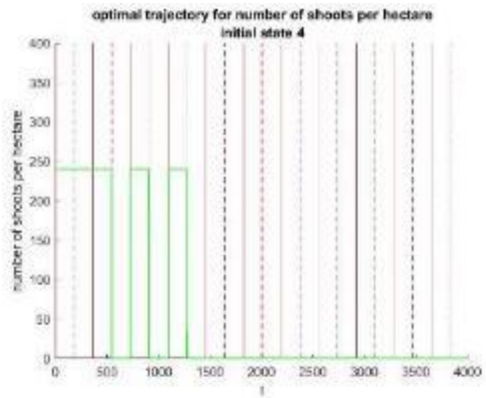
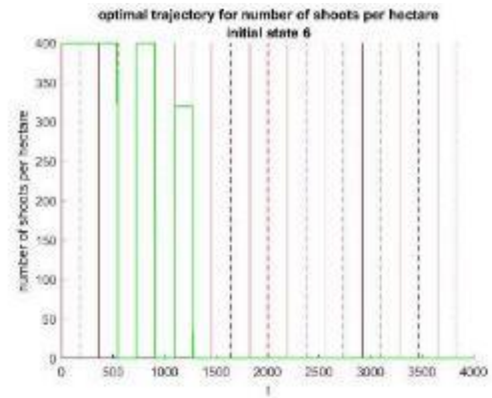
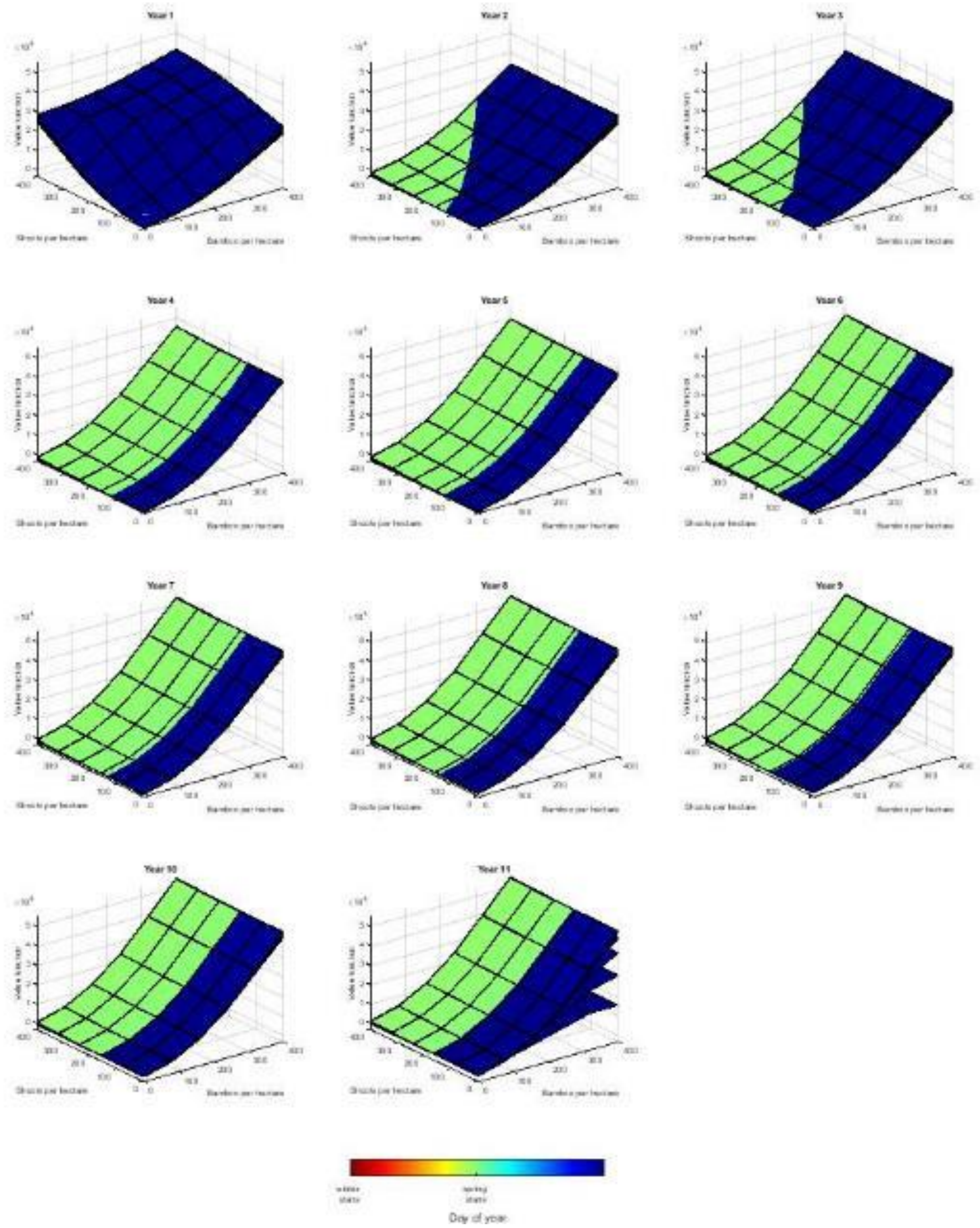

optimal trajectory of number of bamboo per hectare
initial state 2
number of bamboo per hectare
t

optimal trajectory of number of bamboo per hectare
initial state 4
number of bamboo per hectare
t

optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t

optimal trajectory for number of shoots per hectare
initial state 2
number of shoots per hectare
t

optimal trajectory for number of shoots per hectare
initial state 4
number of shoots per hectare
t

optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t

Figure 5. Deterministic Model, Specification 1

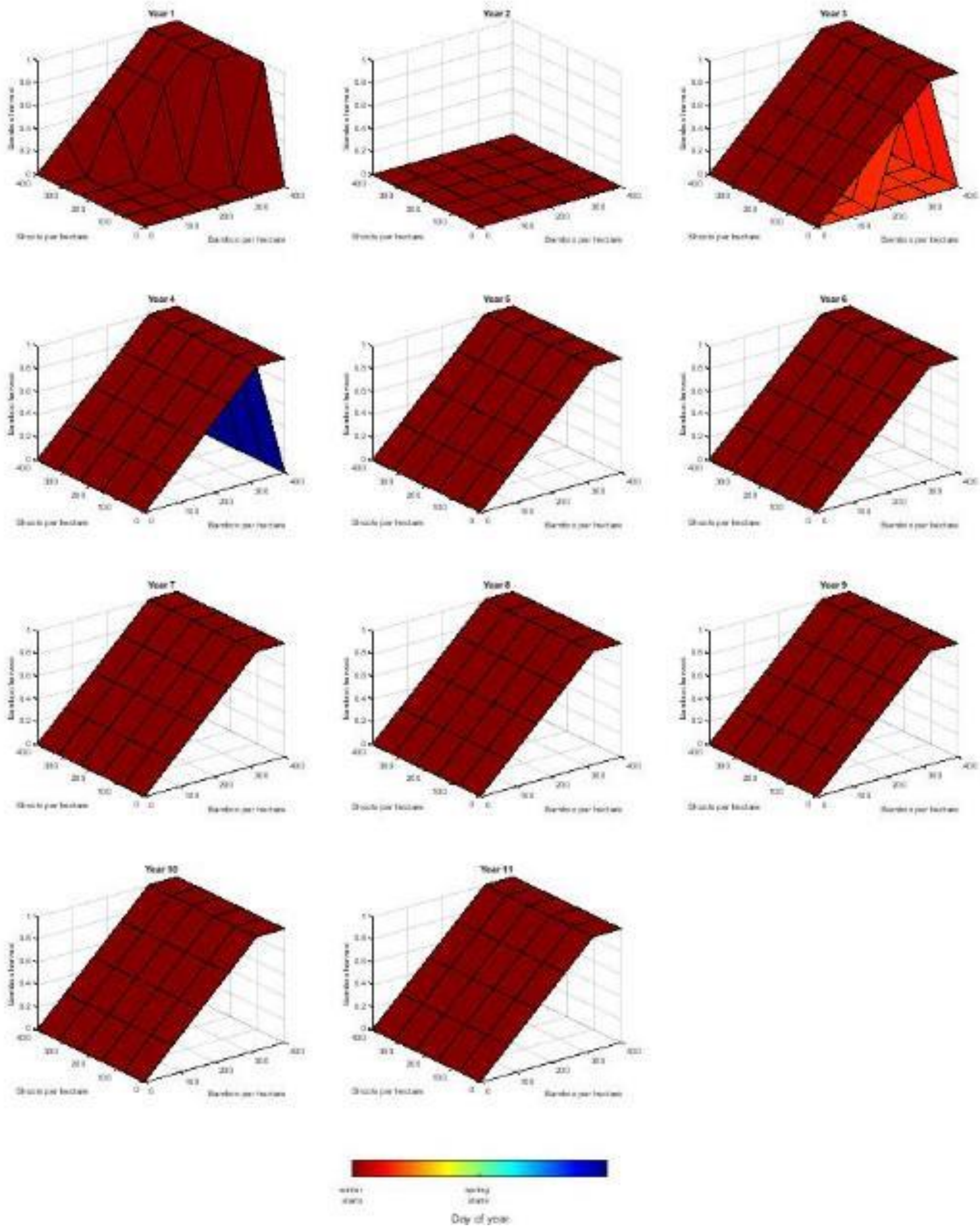
$p_b = 1.4,\ p_{s,winter} = 25,\ p_{s,spring} = 3;$

$c_b = 300/2000,\ c_{s,winter} = 300/15,\ c_{s,spring} = 150/100;$

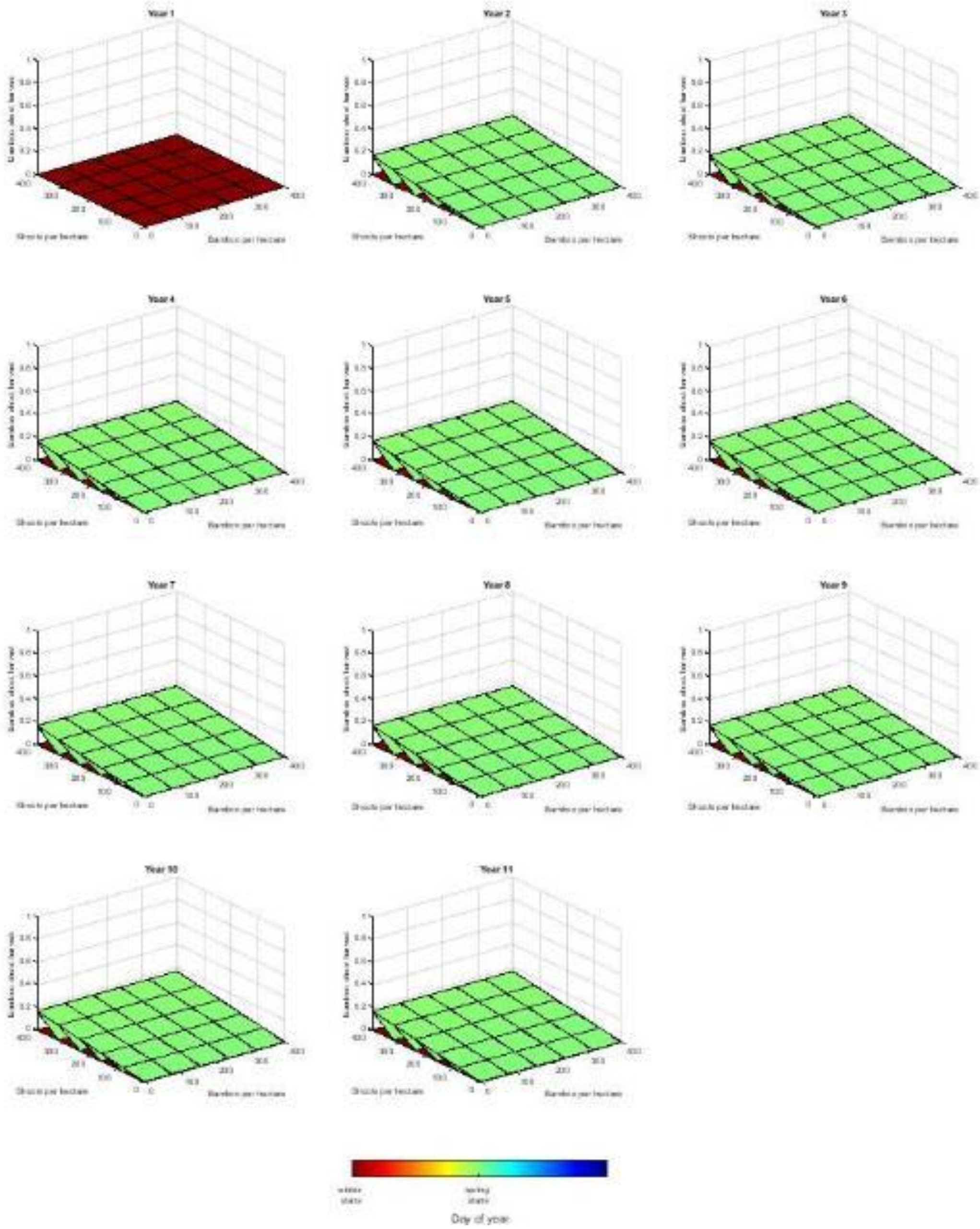
a) Value function as function of bamboo shoots per hectare and bamboo stem per hectare



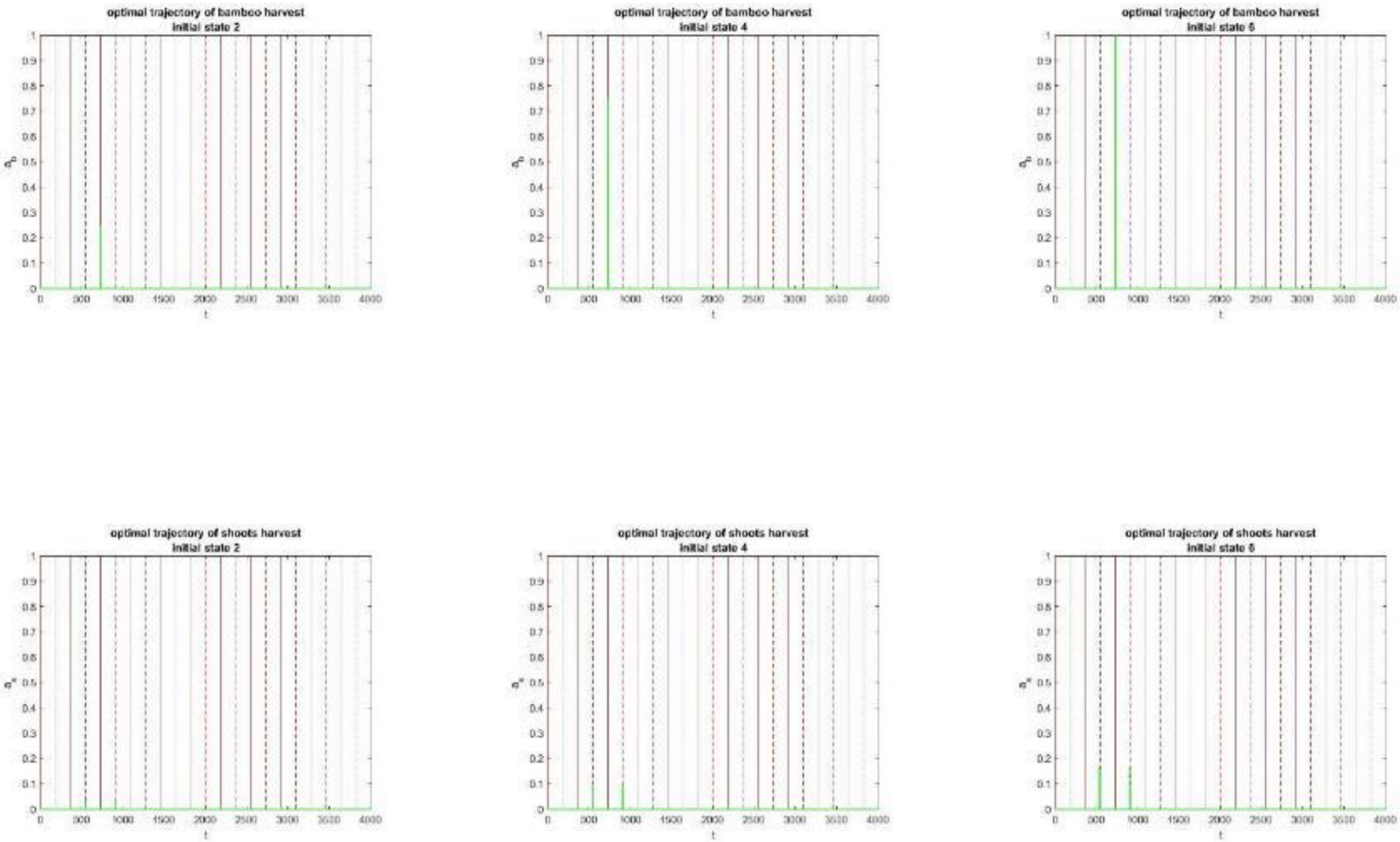
b) Bamboo stem harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare

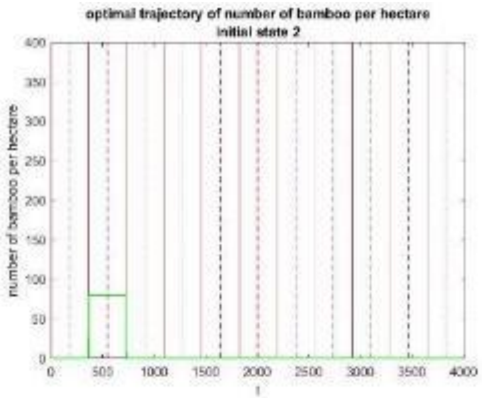


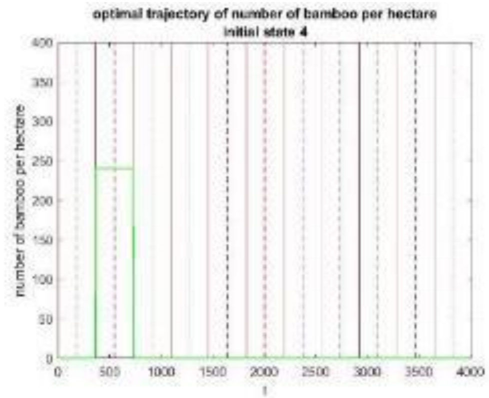
c) Bamboo shoot harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare

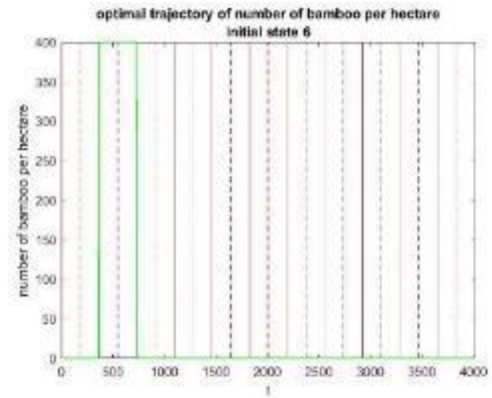


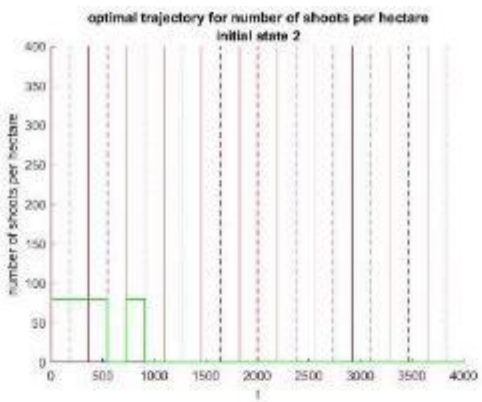
d) Optimal trajectories for each action and state variable over 11 years

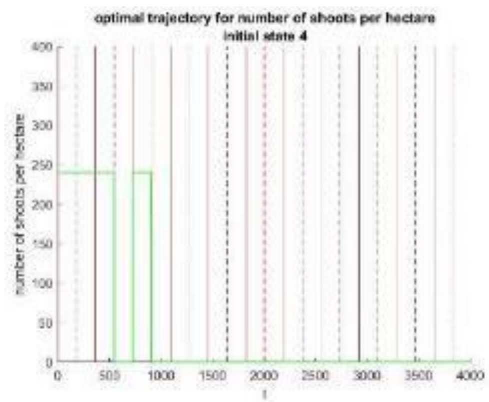



optimal trajectory of number of bamboo per hectare
initial state 2
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 4
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t


optimal trajectory for number of shoots per hectare
initial state 2
number of shoots per hectare
t


optimal trajectory for number of shoots per hectare
initial state 4
number of shoots per hectare
t

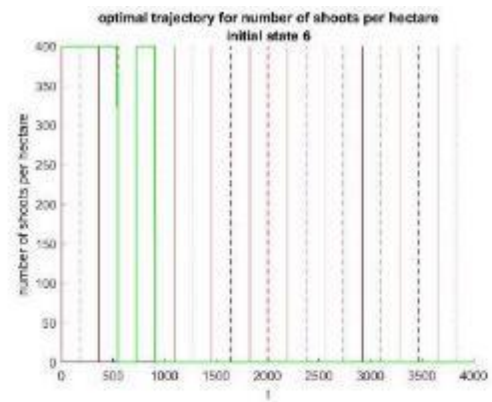
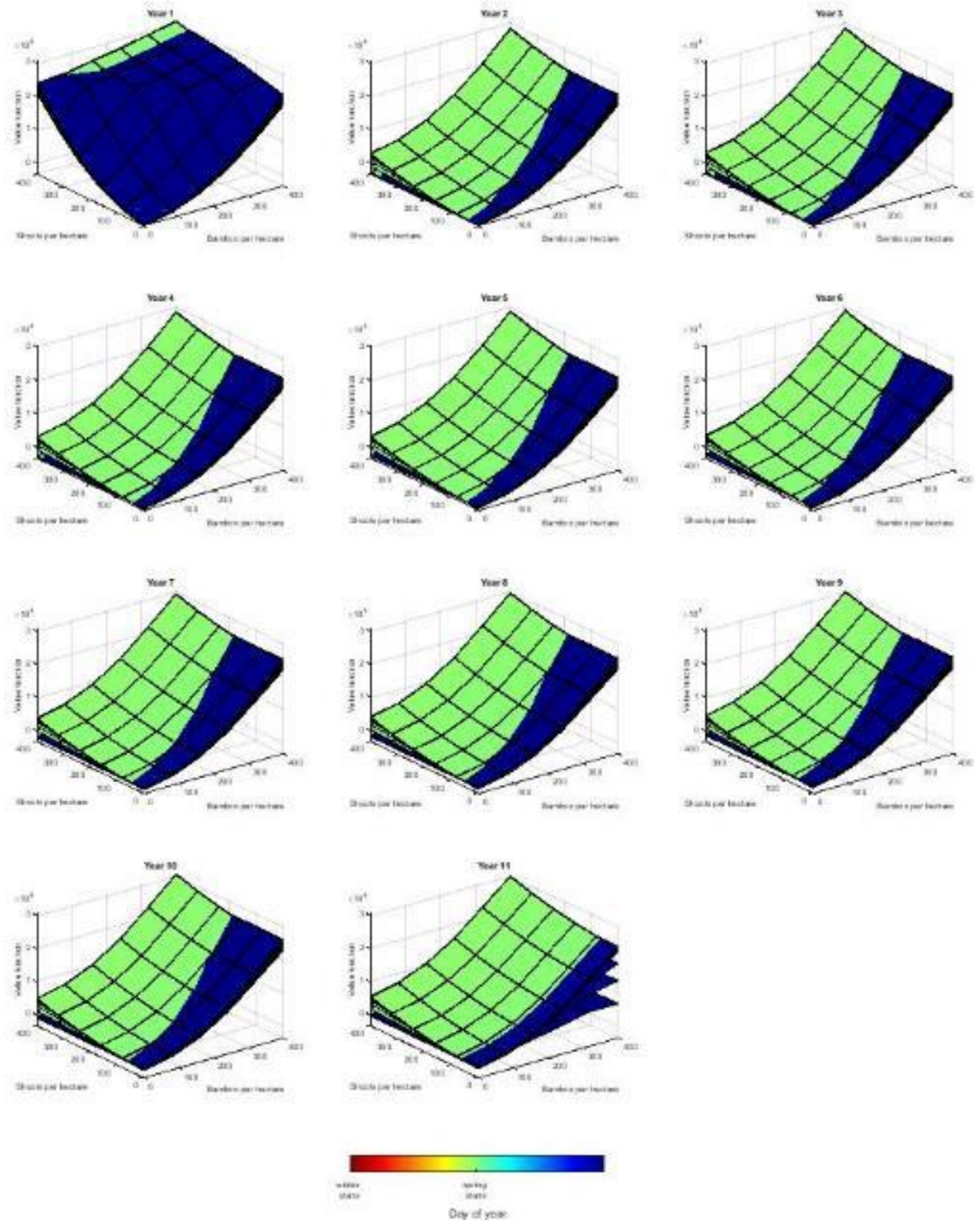

optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t

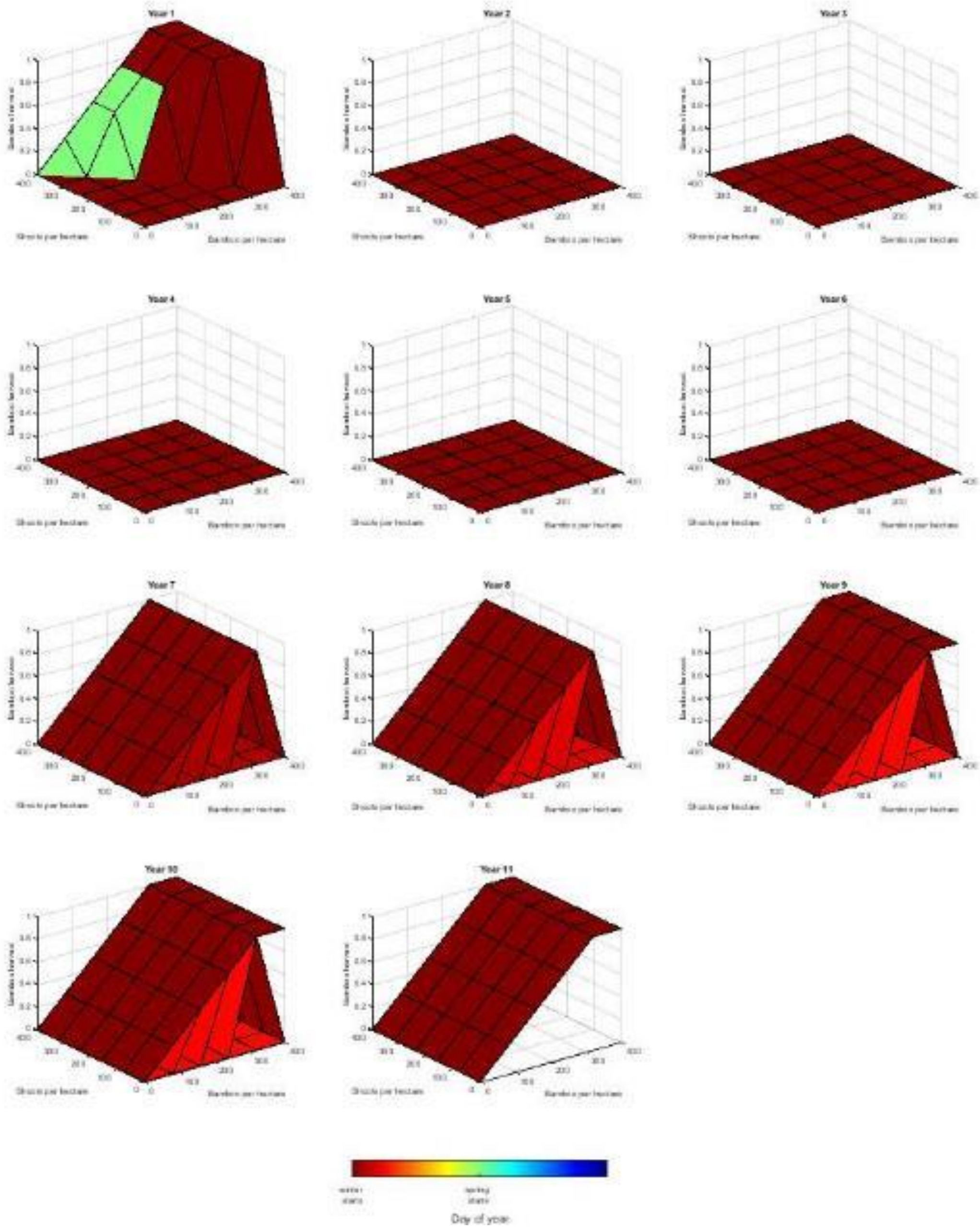
Figure 6: Deterministic Model, Specification 13

$$p_b = 0.8,\ p_{s,winter} = 40,\ p_{s,spring} = 2;$$
$$c_b = 300/1250,\ c_{s,winter} = 300/15,\ c_{s,spring} = 150/100;$$

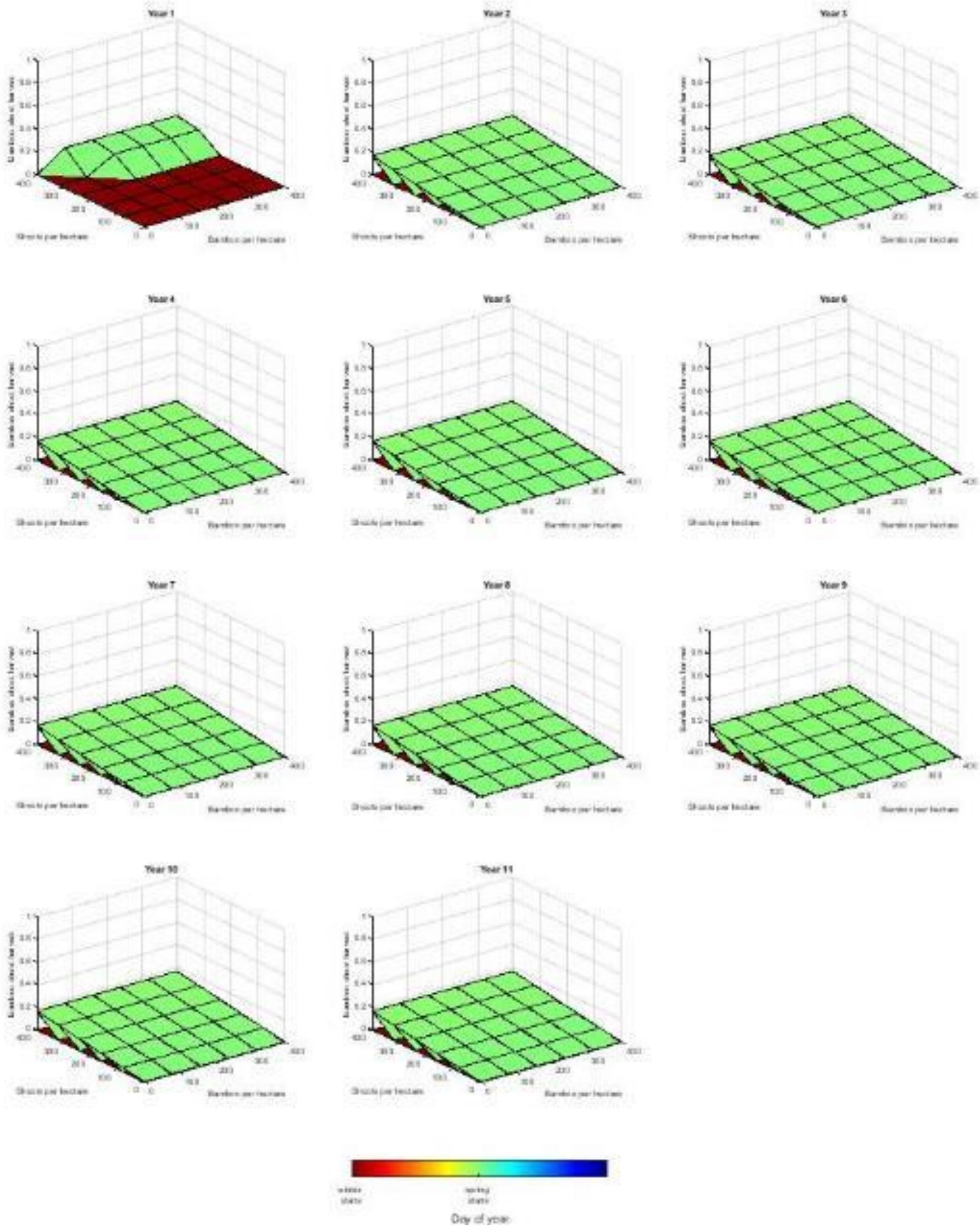
a) Value function as function of bamboo shoots per hectare and bamboo stem per hectare



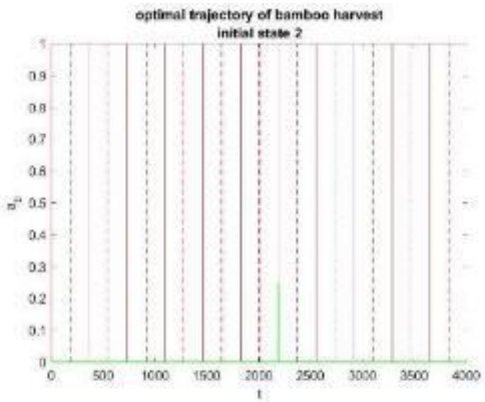
b) Bamboo stem harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare

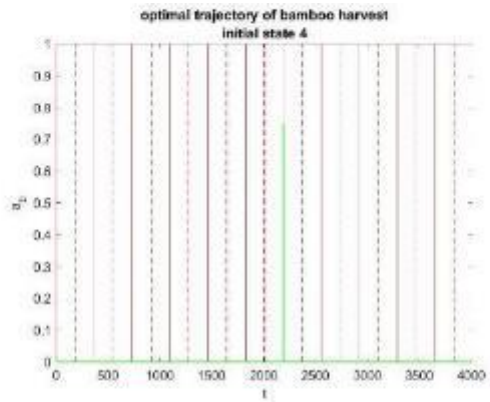


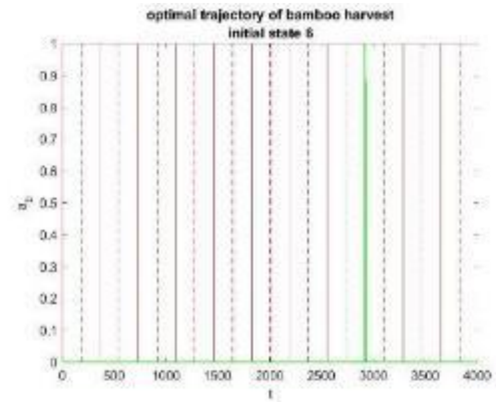
c) Bamboo shoot harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare

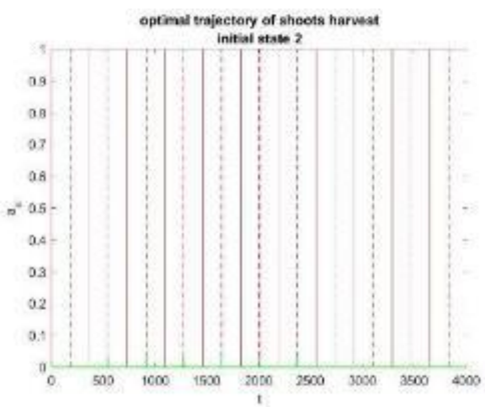


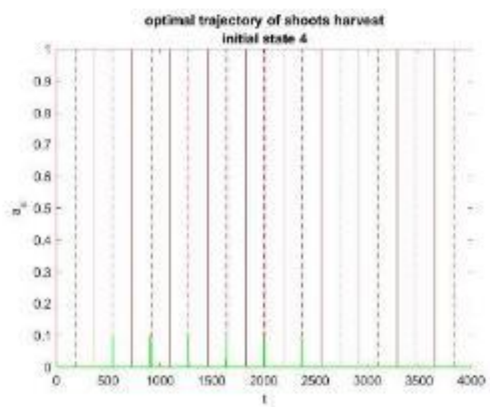
d) Optimal trajectories for each action and state variable over 11 years

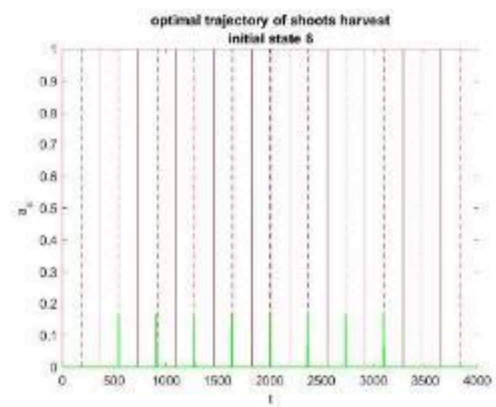


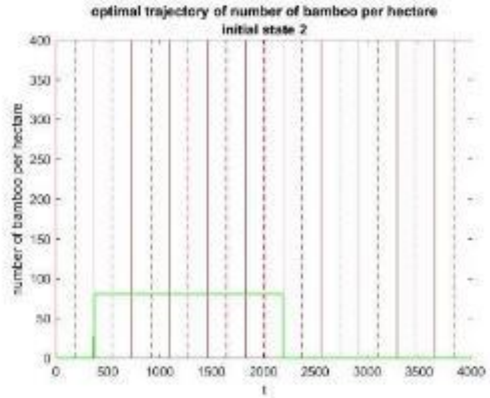


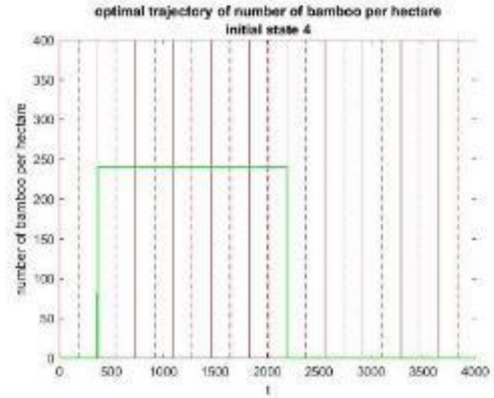


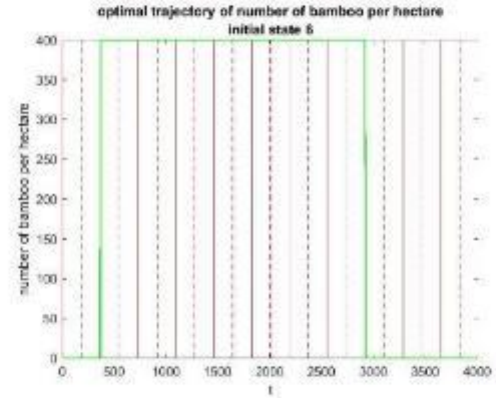


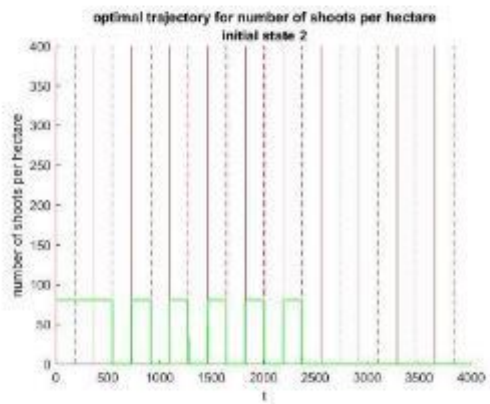


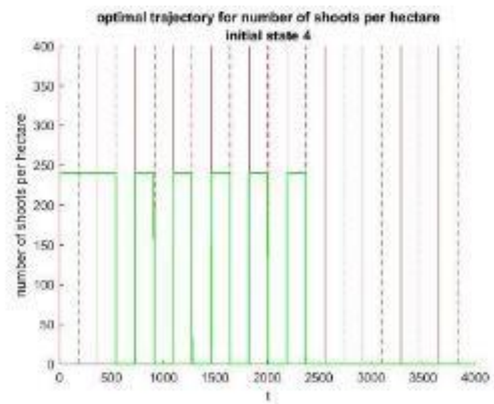



optimal trajectory of number of bamboo per hectare
initial state 2
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 4
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t


optimal trajectory for number of shoots per hectare
initial state 2
number of shoots per hectare
t


optimal trajectory for number of shoots per hectare
initial state 4
number of shoots per hectare
t

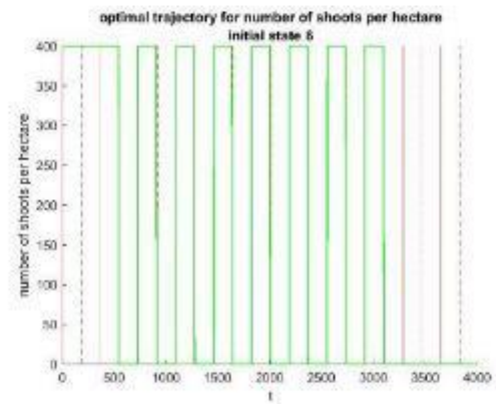

optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t

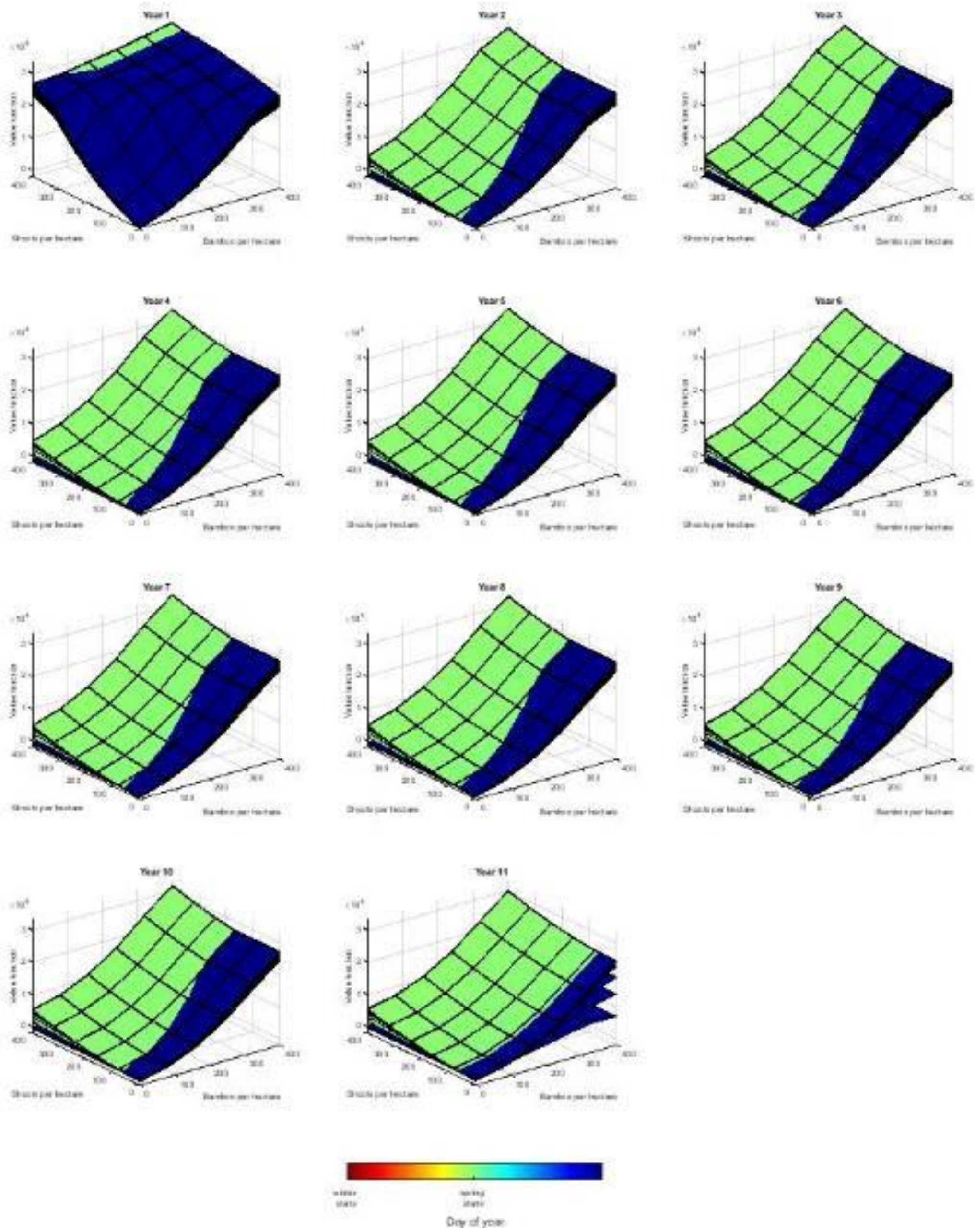
Figure 7: Stochastic Model, Specification 13, Version C

$p_b = 0.8,\ p_{s,winter} = 40,\ p_{s,spring} = 2;$

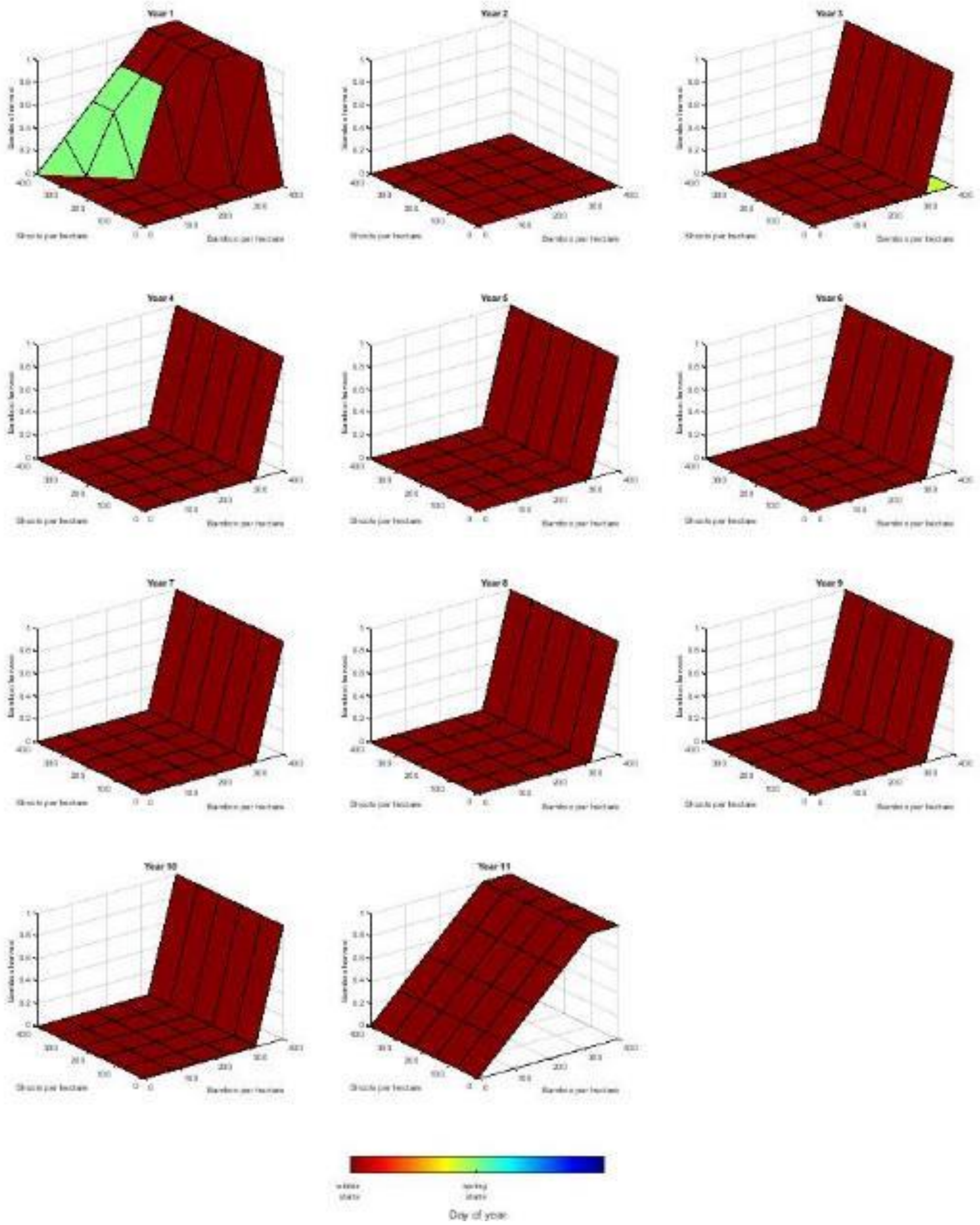
$c_b = 300/1250,\ c_{s,winter} = 300/15,\ c_{s,spring} = 150/100;$

$rain\_high\_prob$ is given by daily empirical probability for Shanchuan

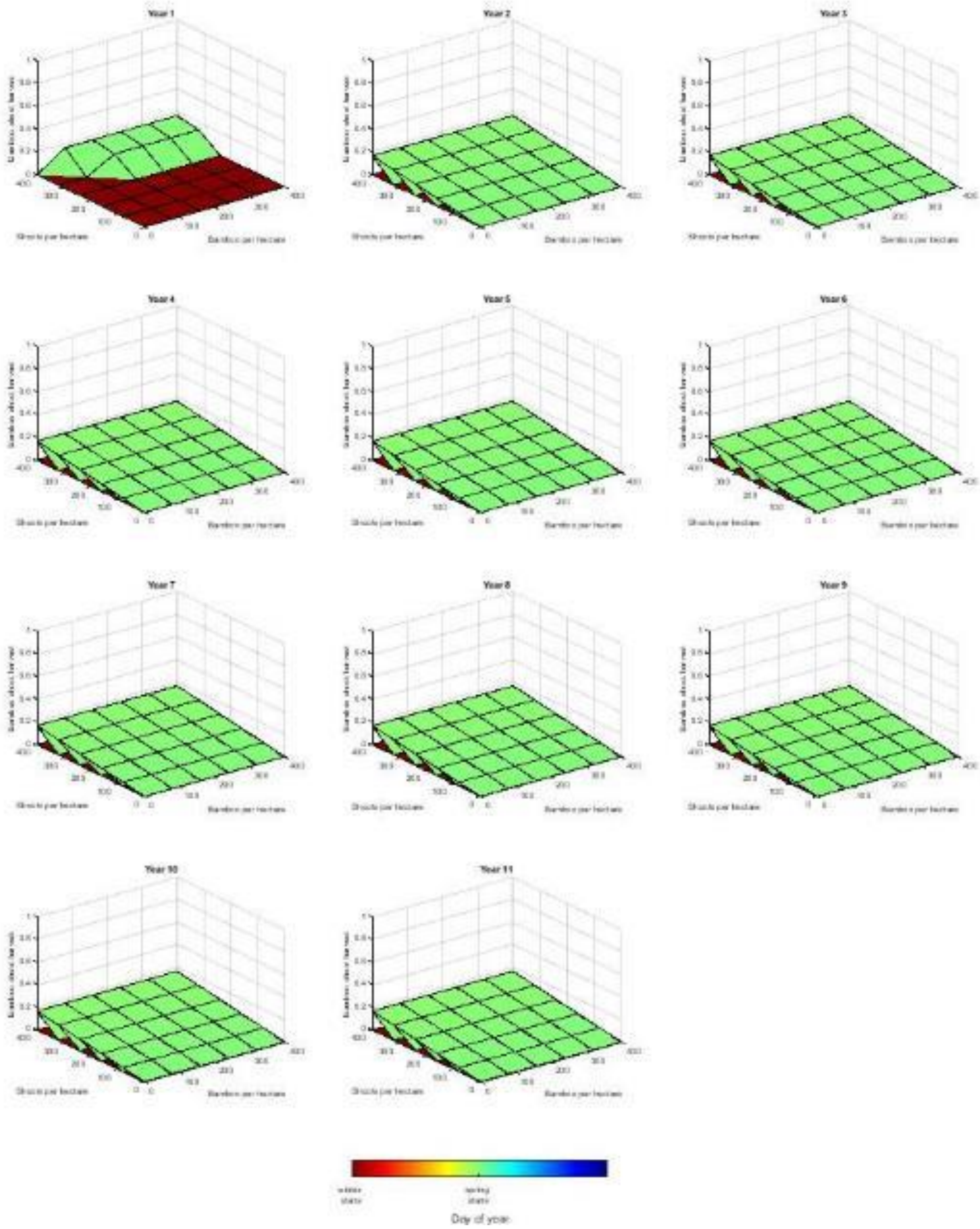
a) Value function as function of bamboo shoots per hectare and bamboo stem per hectare



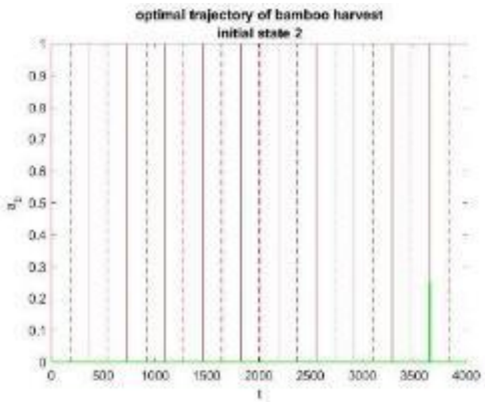
b) Bamboo stem harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare

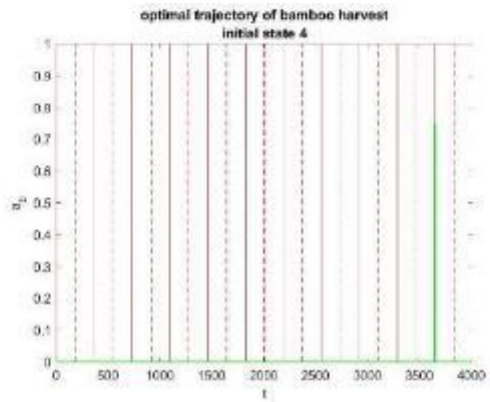


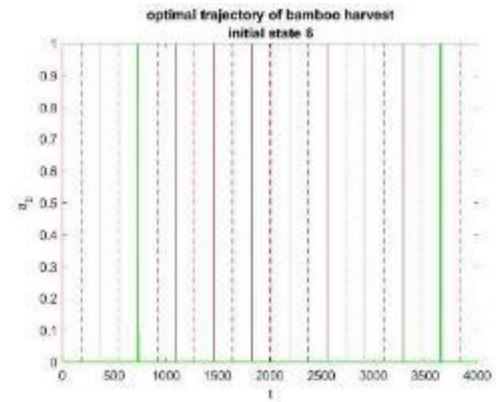
c) Bamboo shoot harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare

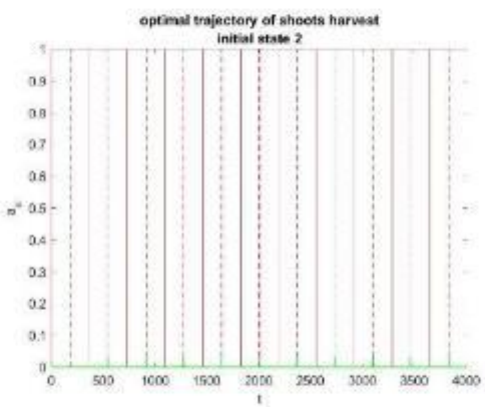


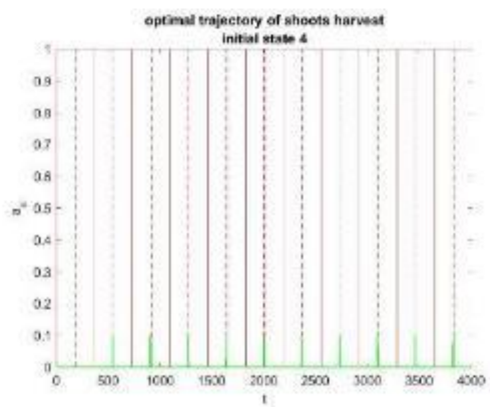
d) Optimal trajectories for each action and state variable over 11 years

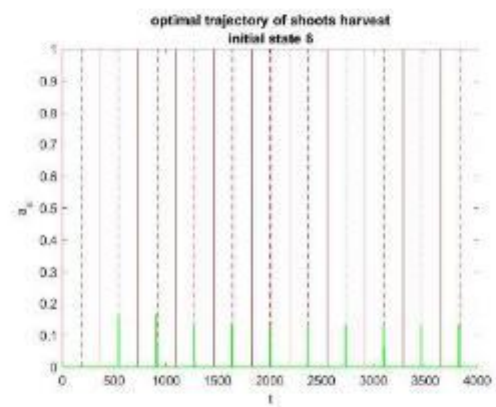


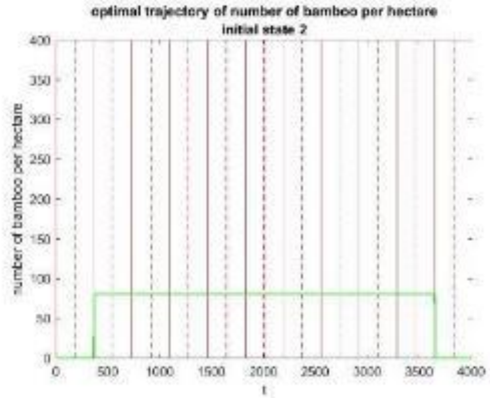


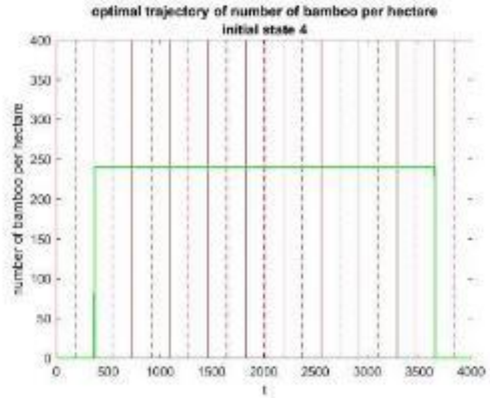


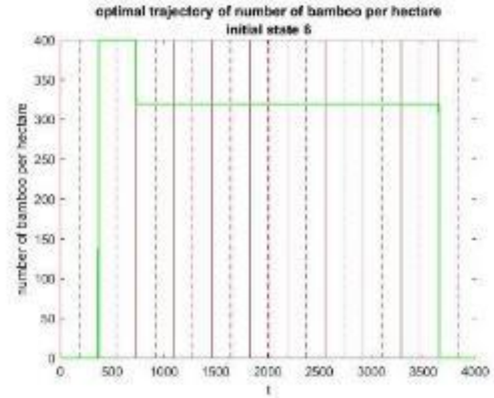


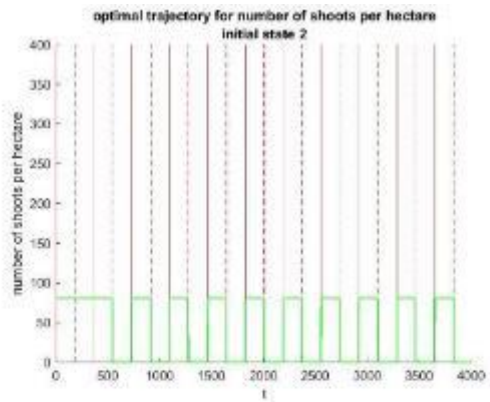


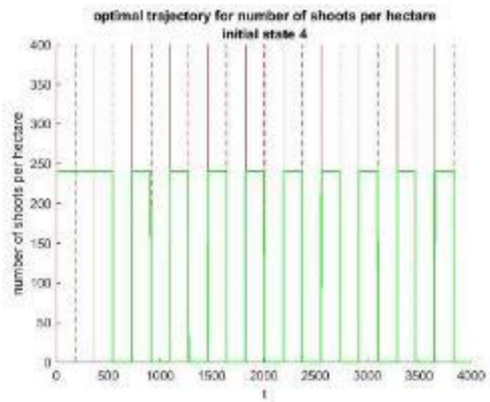



optimal trajectory of number of bamboo per hectare
initial state 2
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 4
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t


optimal trajectory for number of shoots per hectare
initial state 2
number of shoots per hectare
t


optimal trajectory for number of shoots per hectare
initial state 4
number of shoots per hectare
t

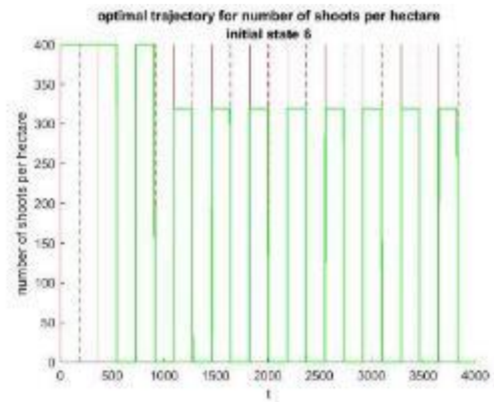
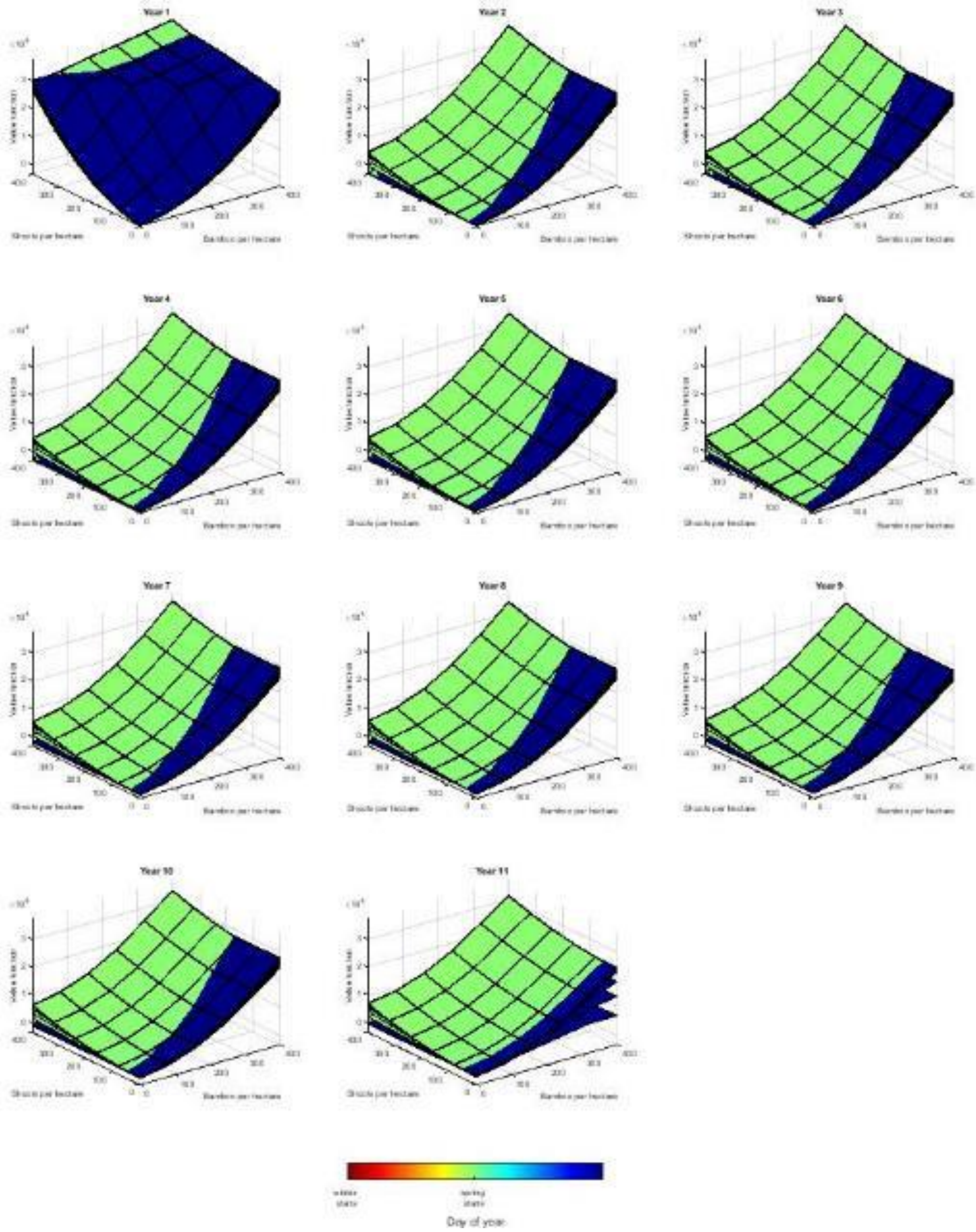

optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t

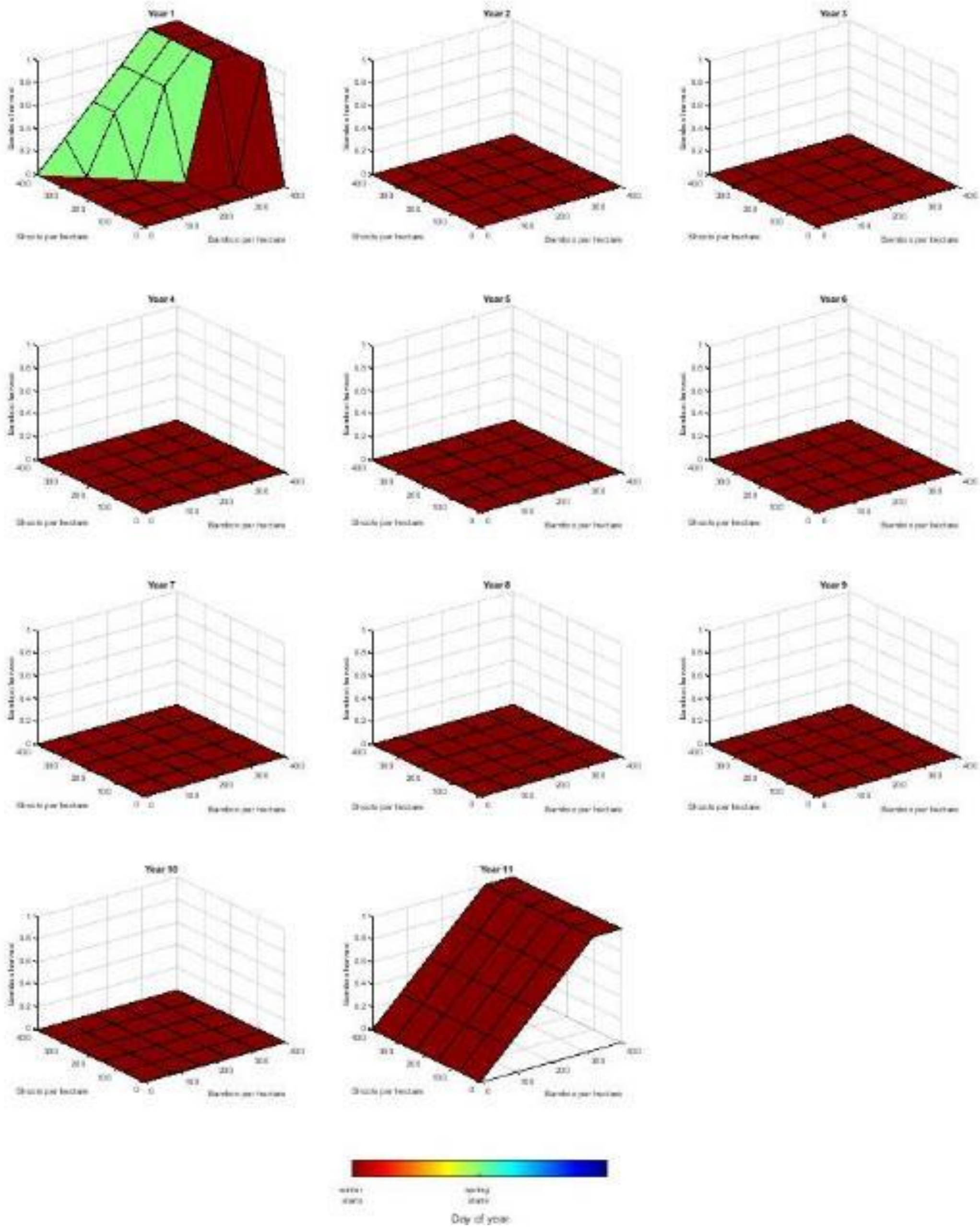
Figure 8: Deterministic Model, Specification 15

$$p_b = 0.8,\ p_{s,winter} = 40,\ p_{s,spring} = 2;$$
$$c_b = 300/1250,\ c_{s,winter} = 300/20,\ c_{s,spring} = 150/100;$$

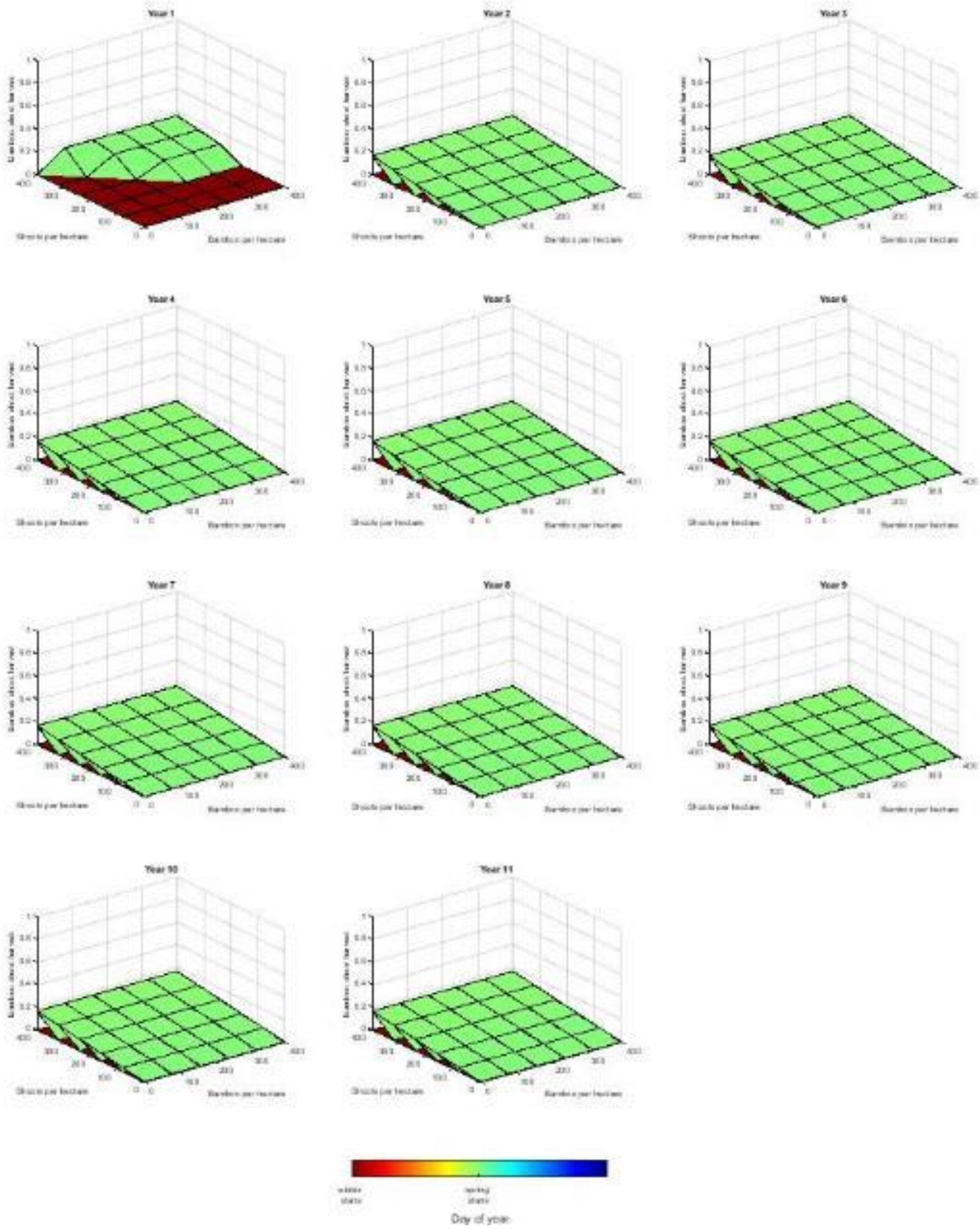
a) Value function as function of bamboo shoots per hectare and bamboo stem per hectare



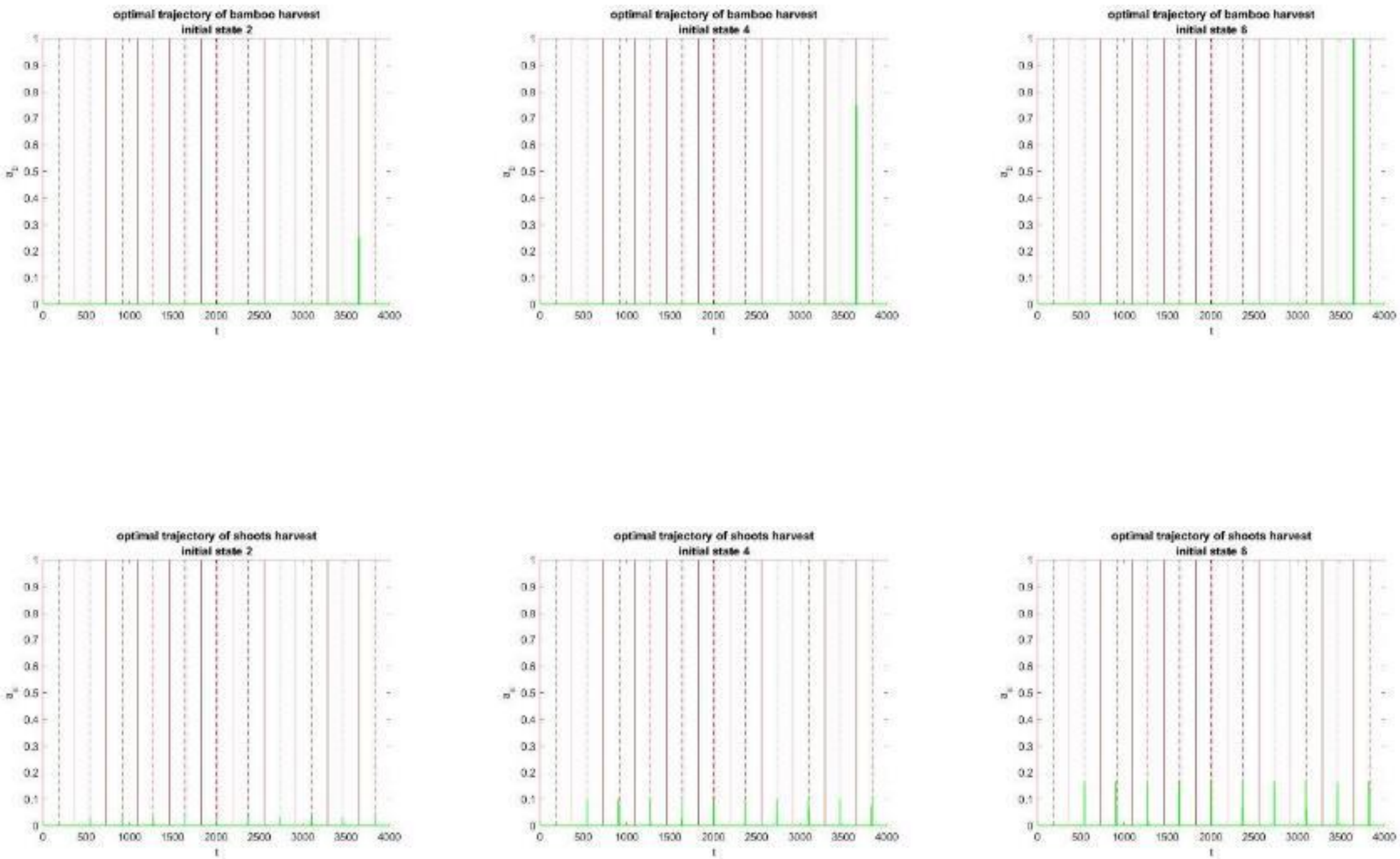
b) Bamboo stem harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare

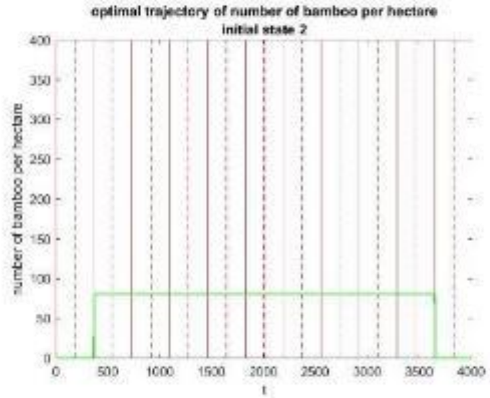


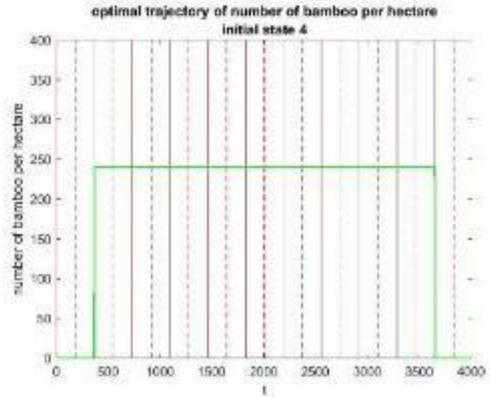
c) Bamboo shoot harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare

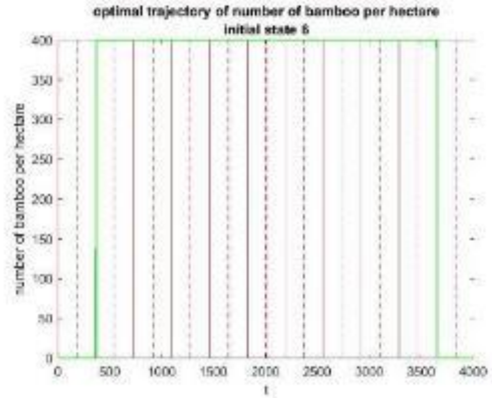


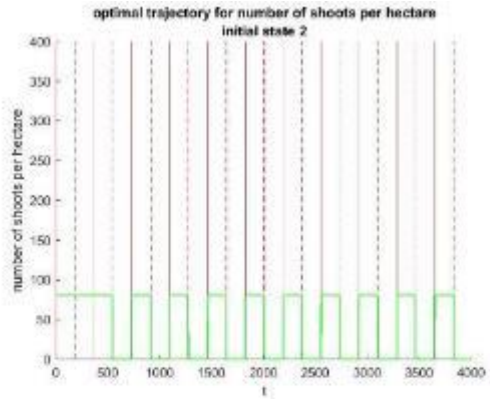
d) Optimal trajectories for each action and state variable over 11 years

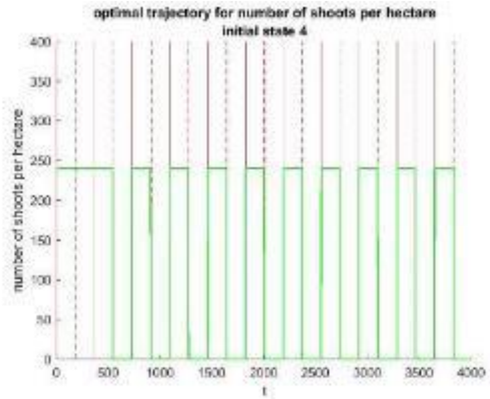



optimal trajectory of number of bamboo per hectare
initial state 2
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 4
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t


optimal trajectory for number of shoots per hectare
initial state 2
number of shoots per hectare
t


optimal trajectory for number of shoots per hectare
initial state 4
number of shoots per hectare
t

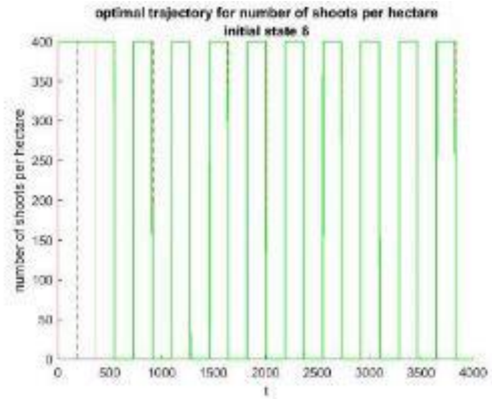
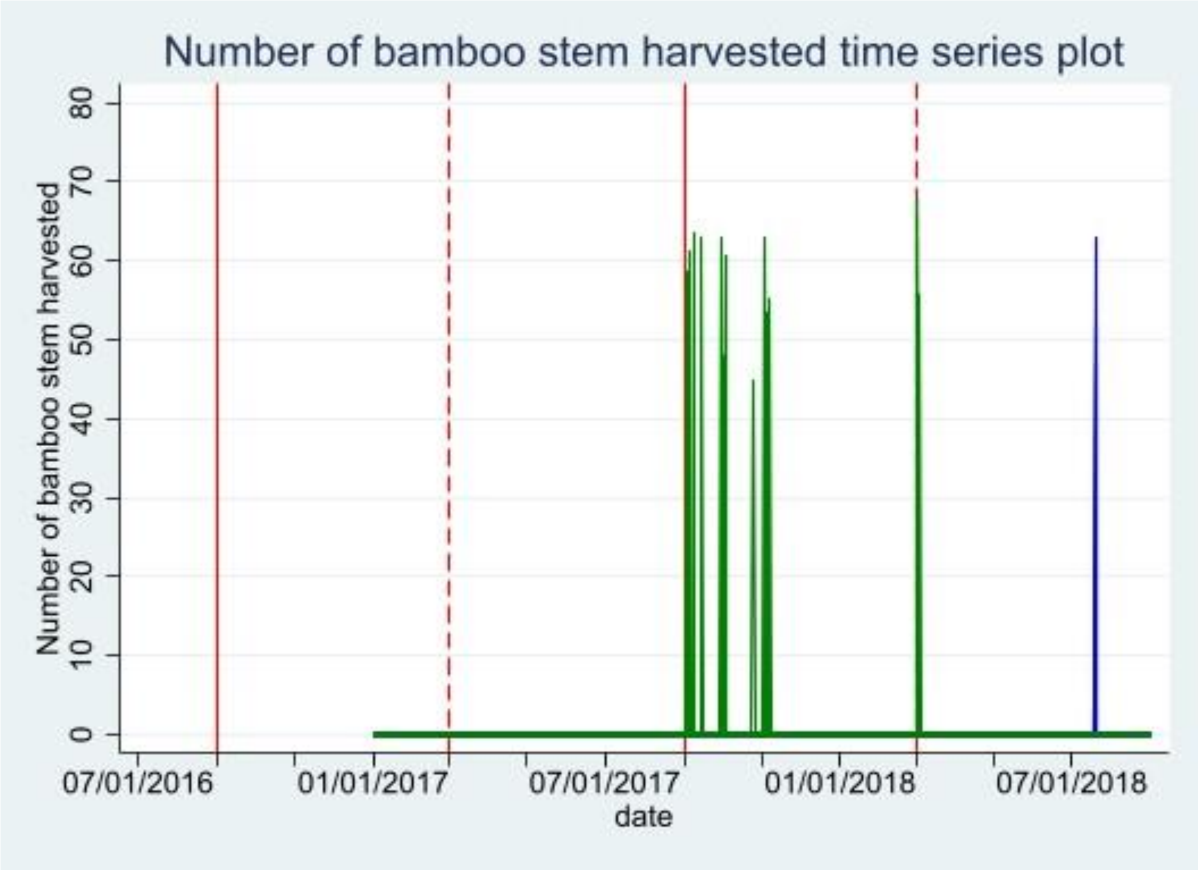
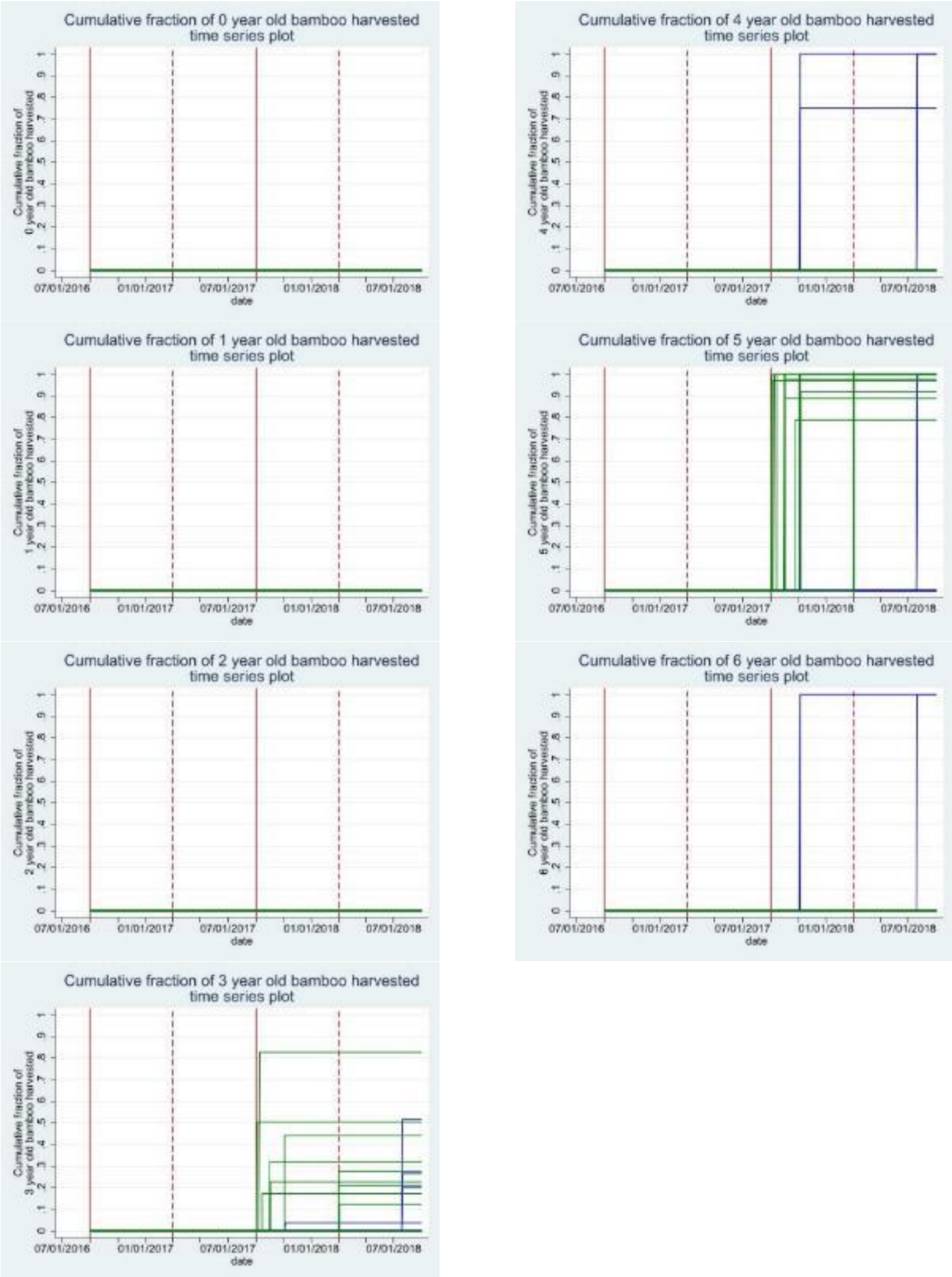

optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t

Figure 9: Number of Bamboo Stem Harvested



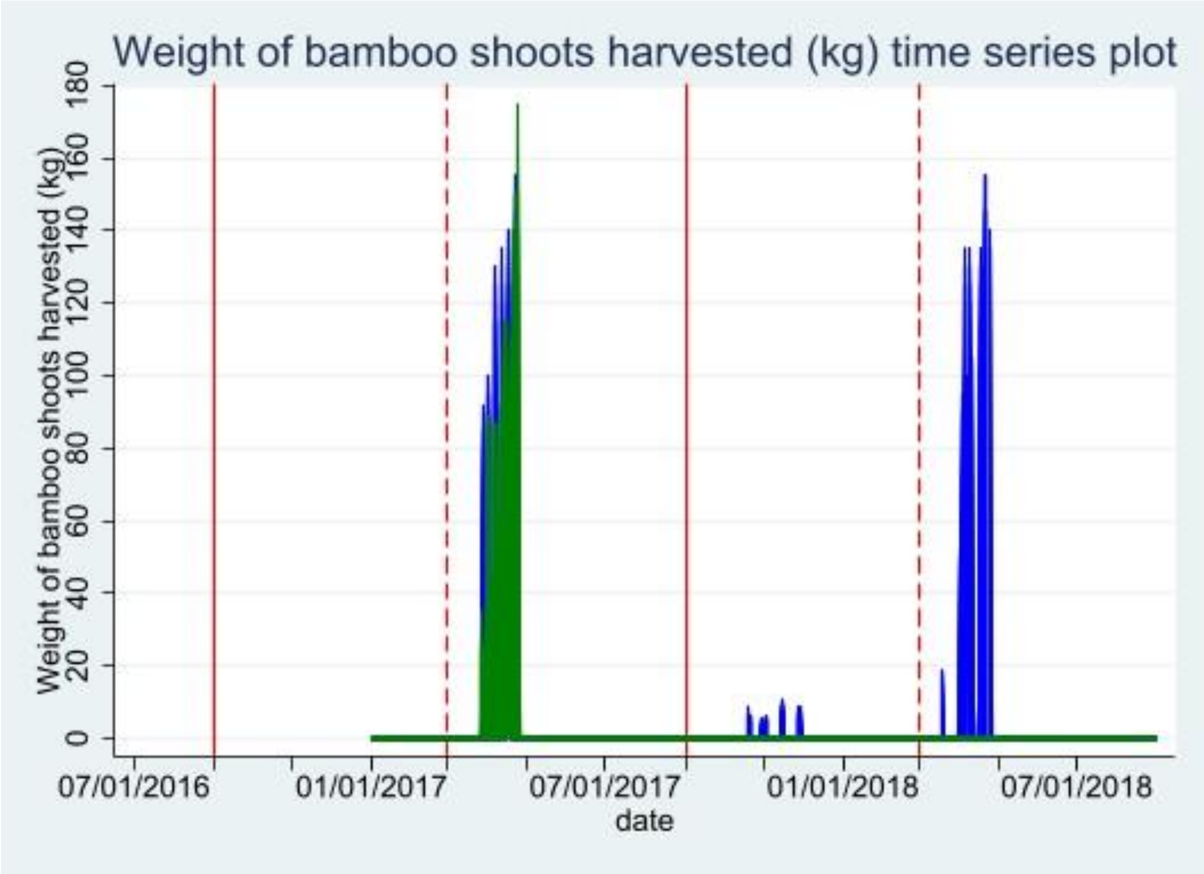
Notes: Time series plots of the number of bamboo stem harvested on each sample plot in Sian Township are in blue. Time series plots of the number of bamboo stem harvested on each sample plot in Shanchuan Township are in green. Vertical lines in red that go from the top to the bottom of the graph denote September 1 (first day of winter shooting) of each year. Dashed vertical lines in red that go from the top to the bottom of the graph denote March 1 (first day of spring shooting) of each year.

Figure 10: Cumulative fraction of bamboo stem harvested by age



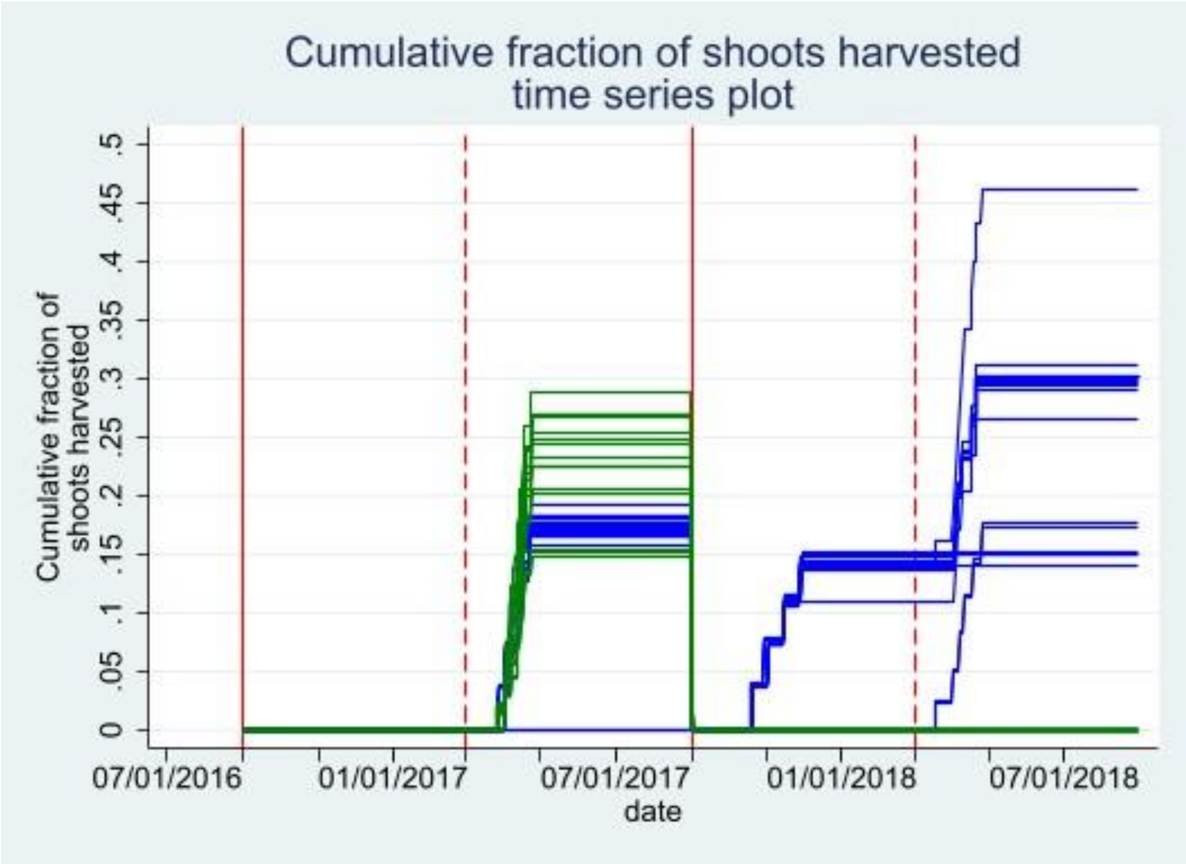
Notes: Time series plots of the cumulative fraction of bamboo stem harvested by age of bamboo on each sample plot in Sian Township are in blue. Time series plots of the cumulative fraction of bamboo stem harvested by age of bamboo on each sample plot in Shanchuan Township are in green. Vertical lines in red that go from the top to the bottom of the graph denote September 1 (first day of winter shooting) of each year. Dashed vertical lines in red that go from the top to the bottom of the graph denote March 1 (first day of spring shooting) of each year.

Figure 11: Actual Weight of Bamboo Shoots Harvested



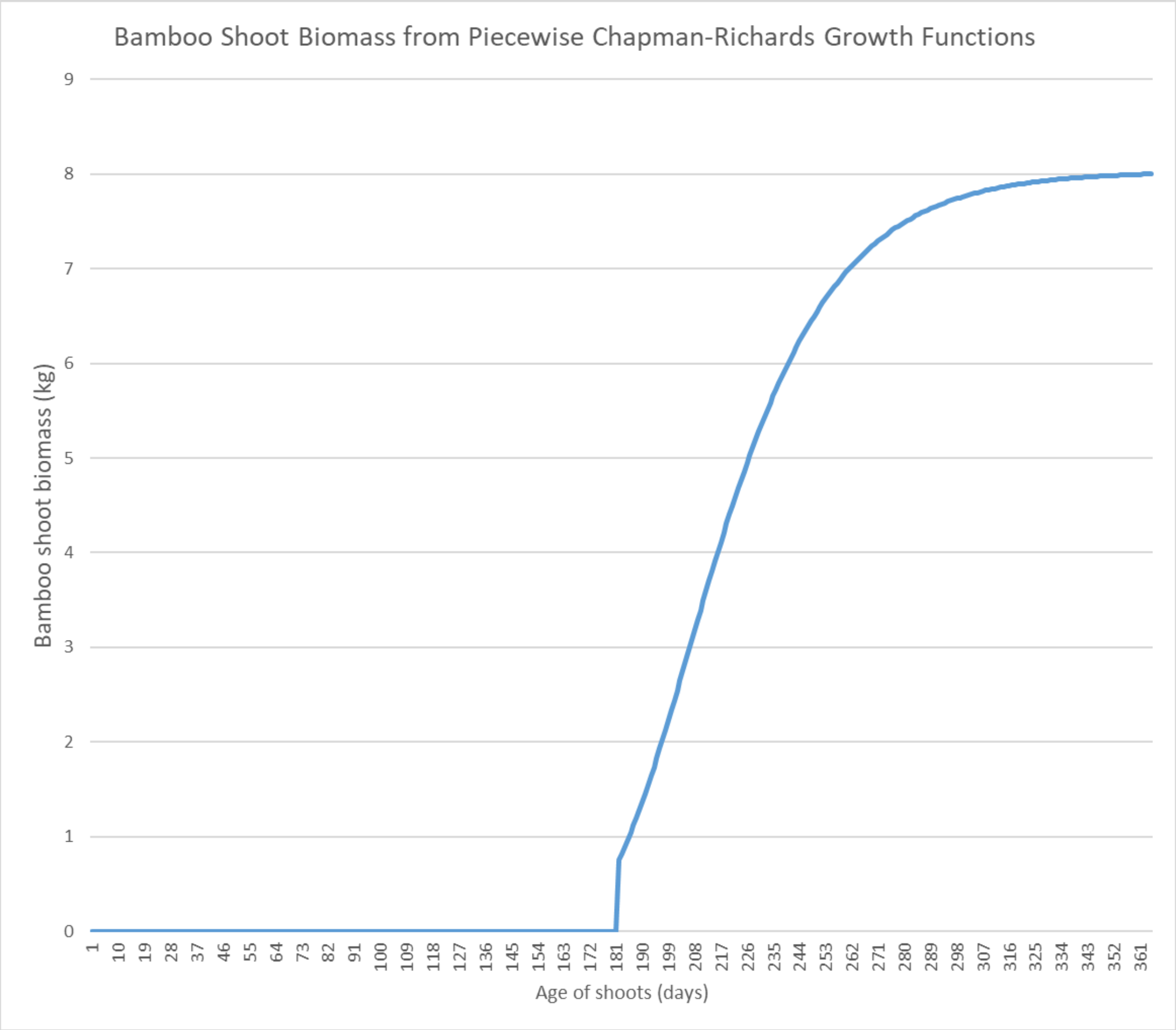
Notes: Time series plots of the weight of bamboo shoots harvested on each sample plot in Sian Township are in blue. Time series plots of the weight of bamboo shoots harvested on each sample plot in Shanchuan Township are in green. Vertical lines in red that go from the top to the bottom of the graph denote September 1 (first day of winter shooting) of each year. Dashed vertical lines in red that go from the top to the bottom of the graph denote March 1 (first day of spring shooting) of each year.

Figure 12: Cumulative fraction of bamboo shoots harvested



Notes: Time series plots of the cumulative fraction of bamboo shoots harvested on each sample plot in Sian Township are in blue. Time series plots of the cumulative fraction of bamboo shoots harvested on each sample plot in Shanchuan Township are in green. Vertical lines in red that go from the top to the bottom of the graph denote September 1 (first day of winter shooting) of each year. Dashed vertical lines in red that go from the top to the bottom of the graph denote March 1 (first day of spring shooting) of each year.

Figure 13. Perceived Chapman-Richards Growth Function for Bamboo Shoots



Notes: Figure plots bamboo farmers' perceived Chapman-Richards growth function for winter shooting and spring shooting based on the preliminary parameter estimates from our dynamic structural model. We use separate Chapman-Richards growth function for winter shooting and spring shooting. The first day of winter shooting is September 1. Winter shooting is from September 1 until February 28. The number of winter shooting days is therefore 181 days. The spring shooting period starts on March 1 and ends on August 31, the last day of the bamboo growth year. The number of spring shooting days is 184 days.