An Application of Linear Programming and Bayesian Decision Models in Determining Least-Cost Rations and Optimal Rates of Gain

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An overview is presented of an economic decision model a) that incorporates linear programming and a Bayesian decision model and b) can be utilized by a cattle feeder in making decisions under conditions of livestock price uncertainty to maximize expected returns above variable costs [Williams]. The incomes from the Bayesian decision model strategies [Halter and Dean; Hays and Winkler; Raiffa and Schlaifer] are compared to the income derived from a "naive" model during the test period in 1972 and 1973. At the time cattle are placed on feed, the cattle feeder has some idea of what future livestock prices will be or at least what price is required to break even. Price changes up or down will increase or decrease profits accordingly. The decision model uses updated feed price information and livestock price expectations to determine the feeding and marketing actions that maximize expected income. The model provides answers to such questions as: a) Given feed prices and livestock price expectations, should cattle be fed or continued on feed; if so, for how long? b) What is the optimal rate of gain to achieve in order to maximize expected income? c) What is the composition and cost of the least-cost ration that provides the optimal rate of gain?

The naive model assumes that no forecast error exists in outlook market information and that the ration cattle are fed does not vary.

The cattle feeder who attempts to maximize net revenue will try to feed the ration (R) for which the difference between total revenue and total costs is the greatest.[1] Because rate of gain (g) depends upon R, the profit maximizing rate of gain is a function of expected beef prices and feed ingredient prices.

The time variable (T) affects profits in three ways. First, an increase in T increases total costs due to additional feed consumed, additional expenses attributed to labor, medical and veterinary bills, etc. Second, a higher proportion of the total feed consumed during the feeding period is required for body maintenance. This leaves a smaller proportion of total feed consumed available for growth, hence, the daily rate of gain decreases for an animal fed a constant ration. Third, assuming a homogeneous group of cattle started on feed at an equal age and weight and gaining an equal amount per day, then as T increases, so do age and weight and the proportion of cattle attaining a higher quality [Bullock and Logan; Dinkel and Busch].

The model illustrated in this paper provides a means for simultaneously determining the ration, daily rate of gain, and length of time to retain cattle on feed to maximize expected short-run profits when feed ingredient prices are known and livestock price expectations established.

Overview of the Model

The two variables R and T provide several feeding alternatives. Each combination represents one feeding alternative or strategy available to the cattle feeder. Cattle can be fed one ration through the entire feeding program, or various rations and for different rates of daily gain as feed prices and livestock price expectations fluctuate.

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[1] Ration defines proportion among feed ingredients and also total quantity of feed.

The planning horizon for any group of cattle can be divided into decision intervals or production periods. The planning horizon extends from the time cattle are purchased or planned to be purchased until the latest time that cattle can be retained in the feedlot or until maximum marketable weight is reached, whichever comes first. A decision interval (or production period) is the period of time that elapses between two successive points of time when the existing strategy or feeding alternative is re-evaluated. For illustrative purposes the conceptual decisions, decision intervals, and strategies available to the cattle feeder are illustrated in figure 1.

Figure 1 illustrates a planning horizon containing three decision intervals. Each decision interval contains elements for decision making, denoted by the rectangular boxes, and activities, denoted by ellipses. A_{abc} represents a specific combination of feeding activities during the decision interval denoted by the last nonzero a, b, or c subscript and during each proceding interval. A_{abc} is a selling activity corresponding to the A_{abc} -th feeding activities. The subscripts a, b, and c represent the feeding level or rate of gain expected during each decision interval. A_{230} is a feeding action during the second decision interval for cattle that have been fed at rate of gain two during the first decision interval and are fed at rate of gain three during the second decision interval. A_{230s} is the sell activity associated with A_{230}.

In figure 1 the subscripts a, b, and c can vary as follows:

$a = 0, 1, 2, 3$

$0 \leqslant b \leqslant 3; a - 1 \leqslant b \leqslant a + 1$

$0 \leqslant c \leqslant 3; b - 1 \leqslant c \leqslant b + 1$

An animal's rate of gain in any production period after the first cannot differ from his rate of gain in the previous period by more than one rate of gain.

At D-1 the decision maker must determine which of the 28 feeding alternatives maximize expected profits. A Bayesian decision model was used to select the feeding strategy that maximized expected discounted returns. If none of the feeding activities seem profitable, then A_{000} is selected; this is a decision not to feed cattle. Suppose that for the feed prices and livestock price expectations existing at D-1, A_{100s}, A_{230s}, and A_{333s} maximize expected discounted returns for the first, second, and third decision intervals, respectively, and returns from A_{100s} < returns from A_{333s} < returns from A_{230s}. Then the decision maker selects A_{200} for the first production period. Activity A_{200} is a prerequisite for A_{230}.

At the end of the first decision interval or production period, at node D-2, the decision maker re-evaluates earlier decisions in the light of new price information and expectations. The decision is made to either sell or continue feeding the cattle.

All strategies to be considered at D-2 are enclosed in the outer boxed area (dashed line). If total profits cannot be increased or losses decreased by continued feeding, the cattle will be sold (A_{200s}).

Assuming that A_{210s} and A_{223s} increase profits for the next two decision intervals, respectively, and expected returns from A_{223s} exceed those from A_{210s}, then A_{220} is the selected feeding action for decision interval two because A_{220} is the prerequisite for A_{223s} .

At the end of the second decision interval, at node D-3, the decision again must be made to either sell the cattle or continue feeding. Updated price expectations and feed prices can be included. Assuming the cattle must be sold at the end of the third decision interval, the four strategies available are contained in the inner box (dotted line) of figure 1 (A_{220s}, A_{221s}, A_{222s}, A_{223s}). If total returns can be increased or losses minimized, the cattle will be continued on feed, otherwise the sell action (A_{220s}) is implemented.

The economic decision model used to select the optimal feeding and marketing strategy at each decision node in figure 1 incorporated several production and marketing variables. A summary of required input data, analyses, and output information are shown in figure 2. This flow chart presents the analysis carried out at each decision node. A linear programming model [Brokken] was developed to calculate the least cost ration for each feeding alternative as a function of average weight of the animal, expected daily rate of gain, and feed ingredient prices.

Return above variable costs for each feeding alternative was calculated by subtracting expense of the least-cost ration, purchase cost of the animal and other nonfeed variable costs from total revenue. Total revenue was computed as a function

Fig. 1. A decision tree for a feeding program involving three decision intervals and three rates of gain

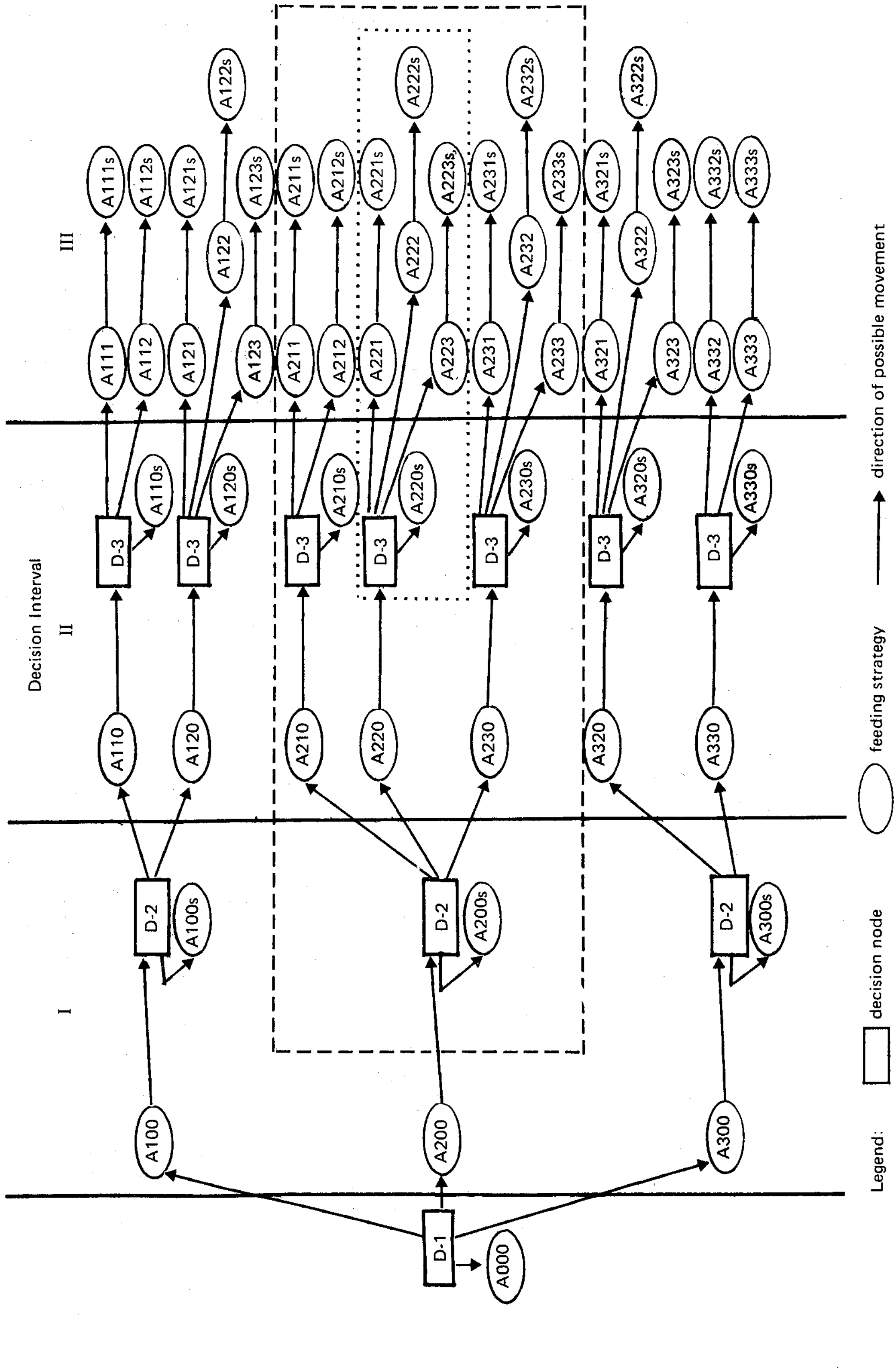
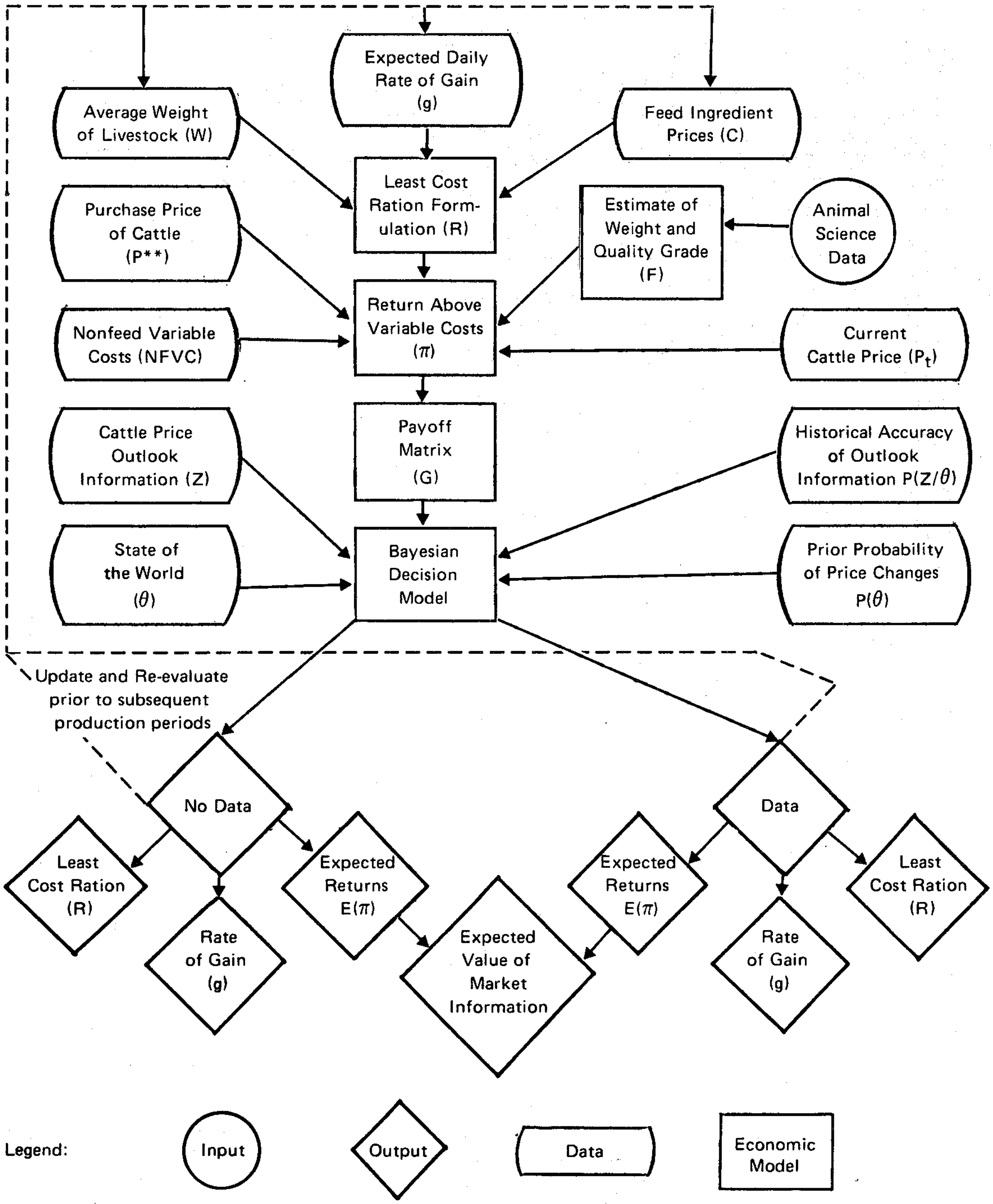


Fig. 2. Schema of data, analyses, and output of an economic livestock feeding-marketing decision model



of current cattle prices and expected weight and quality grade of the cattle at marketing. Each feeding alternative represents one action in the Bayesian decision model.

States of the world are changes in choice slaughter cattle prices from current levels. The prior probability distribution of states of the world was based on the historical frequency that actual price changes occurred. Cattle price forecasts were obtained from the Iowa Farm Outlook Letter. The historical accuracy of the forecasts were used to compute posterior probabilities for the states of the world.

The results of the decision model provide both the no-data and data Bayesian strategies. In addition, ration, profit maximizing daily rate of gain, time to retain cattle on feed, and maximum expected returns are determined for each Bayesian strategy. The difference between expected income from following the no-data strategy and expected weighted average income from following the data strategy is the expected value of slaughter cattle market information in the Iowa Farm Outlook Letter.

The Bayesian strategies are re-determined before each subsequent decision interval or production period.

Results

The accumulated net return for the two year test period derived from following strategies associated with the Bayesian decision model was greater than the return from the naive model. The feeding strategies associated with the data and no-data models were identical; therefore the realized net returns were the same. The Accumulated net returns for the Bayesian decision model were $228.11 per head of feedlot capacity. Two daily rates of gain were fed to maximize expected returns. A 2.5 pound daily rate was attained for the first three groups of cattle fed during the test period, and a 2.0 pound daily gain for the last group fed. The accumulated net return was $118.25 per head of feedlot capacity following the naive model.

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