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BENEFITS AND COSTS OF RESOURCES POLICIES AFFECTING
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INTRODUCTION

This volume contains the proceedings of the 1999 W-133 Western Regional Project Technical Meeting on "Benefits and Costs of Resource Policies Affecting Public and Private Land." Some papers from W-133 members and friends who could not attend the meeting are also included. The meeting took place February 24th - 26th at the Starr Pass Lodge in Tucson, Arizona. Approximately 50 participants attended the 1999 meeting, are listed on the following page, and came from as far away as Oslo, Norway.

The W-133 regional research project was rechartered in October, 1997. The current project objectives encourage members to address problems associated with: 1.) Benefits and Costs of Agro-environmental Policies; 2.) Benefits Transfer for Groundwater Quality Programs; 3.) Valuing Ecosystem Managment of Forests and Watersheds; and 4.) Valuing Changes in Recreational Access.

Experiment station members at most national land-grant academic institutions constitute the official W-133 project participants. North Dakota State, North Carolina State, and the University of Kentucky proposed joining the group at this year's meeting. W-133's list of academic and other "Friends" has grown, and the Universities of New Mexico and Colorado were particularly well represented at the 1999 W-133 Technical Meeting. The meeting also benefitted from the expertise and participation of scientists from many state and federal agencies including California Fish and Game, the U.S. Department of Agriculture's Economic Research and Forest Services, the U.S. Department of Interior's Fish and Wildlife Service, and the Bureau of Reclamation. In addition, a number of representatives from the nation's top environmental and resource consulting firms attended, some presenting papers at this year's meeting.

This volume is organized around the goals and objectives of the project, but organizing the papers is difficult because of overlapping themes. The last section includes papers that are very important to the methodological work done by W-133 participants, but do not exactly fit one of the objectives. -- I apologize for the lack of consistent pagination in this volume.

On A Personal Note... Any meeting or conference is successful (and fun!) only because of its participants, so I would first like to thank all the people who came and participated in 1999 - listed below. I also want to thank Jerry Fletcher for all his help at this meeting and prior to it, and John Loomis who passed on his knowledge of how to get a meeting like this to work, and who continues to have the funniest little comments to lighten the meetings up. I especially thank Paul Jakus, who helped me to organize this conference and have a lot of fun during it and afterward. Finally, I want to thank Nicki Wieseke for all her help in preparing this volume, and Billye French for administrative support on conference matters.

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June, 1999

P.S. P.F. and J.C. - As far as I can tell, that darn scorpion is still dead!

Empirical Specification Requirements for Two-Constraint Models of Recreation Demand

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Empirical Specification Requirements for Two-Constraint Models of Recreation Demand

The literature on recreation demand is gradually becoming more sophisticated as researchers respond to the myriad conceptual and empirical challenges that are associated with this particular area of demand analysis. One of the most challenging and important areas of research is how to consistently integrate the role of time into recreation choices. The importance of modeling time in recreation demand has been known by applied researchers since early in the development of the literature (e.g., Clawson; Knetsch). The empirical literature has followed a distinct progression, from ignoring time entirely, to assuming time has a value which is a researcher-chosen fraction of the wage rate (e.g., following suggestions by Cesario), to allowing the data to determine the fraction (McConnell and Strand), to recognizing the differences between the values of time for salaried and hourly workers (Bockstael *et al.*).

Interestingly, there has been relatively little formal guidance about how to specify recreation demand models where time is an important constraint, beyond the basic case originally analyzed by Becker where time can be converted to money according to an exogenous labor supply function. The intuition behind the Becker analyis is that all demands should be functions of "full prices" and "full budgets," where time valued at the wage rate is included in the price and budget terms. One of the contributions of the Bockstael *et al.* paper was to point out that not all recreationists have the opportunity to "reveal" their marginal wage rate through participation in a discretionary labor activity, and that for these individuals the relevant value of time is endogenous. However, their paper does not provide any guidance on how to specify the value of time in such "corner solution" cases where the individual offers zero discretionary labor supply.

Perhaps partly as a result of the paucity of theoretical guidance, the literature has focused almost exclusively on the role of time "prices" (travel costs, typically, in the recreation demand model), while the role of the time budget in demand has been largely ignored. No doubt this is because researchers are well aware (thanks to the work of Knetsch and Cesario, among others) that consumer's surplus estimates of the net economic benefits of recreational activities are heavily influenced by the own-price coefficient, which will be biased if a systematic part of the cost of a recreational activity (the opportunity cost of time spent) is ignored. However, the common practice of forming a full "price" of recreation, and including this variable in demand with money income alone (i.e., omitting the time budget) cannot be a correct procedure as it violates the requirements of theory.

This paper develops the implications of the two-constraint recreation demand model that give rise to this and other insights for empirical practice. We develop the theoretical restrictions implied by the two versions of Roy's Identity when any consumption choice is made subject to two binding constraints. These restrictions are analogous to the Slutsky-Hicks equations of standard (single-constraint) consumer choice problems, though derived from a different conceptual basis in the choice problem.

In the context of choice subject to money and time constraints, three sets of necessary conditions provide additional symmetry structure for estimation and testing of recreation demand models. One relates cross-equation money price and money budget terms alone, one relates cross-equation time price and time budget terms alone, and one set of restrictions relates time and money price and time and money budget coefficients and the marginal value of leisure time. The first two sets of restrictions are fully observable, which means they can be imposed or tested for in estimation. The third set can be used to "reveal" the marginal value of time from properly-specified empirical recreation demand models.

These results provide the structure necessary to correctly specify two-constraint recreation demand models. Two points about their applicability are worth noting. First, they hold for all recreationists, whether or not they are making marginal labor supply choices along with recreation choice. They are of particular use in identifying the demand structure for recreationists with endogenous marginal values of leisure time, where the literature does not generally advance any particular requirements for specification. They also suggest ways that one can specify a marginal value of leisure time function as part of the structure of the demand model, and estimate its parameters as part of the model. Workers making marginal labor leisure choices in response to exogenous marginal values of leisure time (the "interior solution" case of Bockstael *et al.*) can be seen to represent a special case of the general two-constraint choice theory.

Second, it is important to emphasize that this paper is about the relationships between the covariates in the systematic part of recreation demand models. Because of this, they are applicable to all empirical recreation demands where time plays a role, whether single-equation or multiple-equation, whether continuous or discrete.

The basic results are developed in the context of a demand systems approach to recreation, because it is within this framework that much of the literature of how to treat recreation time has been developed. However, because many recent analyses have used count data or random utility formulations of the recreation choice model, we also show how the theoretical two-constraint requirements apply to these models.

Two-Constraint Recreation Choice Models

The standard consumer choice problem with two binding constraints provides the appropriate theoretical foundation for developing the specification requirements for recreation demand models when time has an opportunity cost.[1] Let $\mathbf{x} \equiv (x_1,\ldots,x_n)$ be consumption goods with corresponding non-negative money prices $\mathbf{p} \equiv (p_1,\ldots,p_n)$ and time prices $\mathbf{t} \equiv (t_1,\ldots,t_n)$, and choices are made subject to a money budget constraint $M = \mathbf{px}$ and a time constraint $T = \mathbf{tx}$, both of which are strictly binding. The money and time budgets M and T can be thought of as resulting from a labor supply decision by the individual, which results in discretionary income and time to be allocated to leisure time activities and goods consumption.

Note that binding time and money constraints must characterize the model used whenever researchers argue that time spent in recreation has a "value" or opportunity cost. If the time constraint is non-binding, the marginal value of time is zero, the standard consumer choice problem results, and there is no bias to recreation benefit estimates from ignoring time. Intuitively, though, time must always be "spent" in some activity, so binding time constraints are highly plausible. Nonsatiation and the presence of numeraire activities with only one price (i.e, a positive money price and zero time price, or vice versa)[2] are sufficient for both constraints to bind.

Consider a consumer with utility function u(**x**,**s**), with **s** a vector of shift parameters. The primal version of the choice problem is solved by the Marshallian demands $x_i = x_i(\mathbf{p},\mathbf{t},\mathbf{s},M,T)$ which are functions of both time and money prices and time and money budgets. The indirect utility function V(**p**,**t**,**s**,M,T) for this problem is

$$V(\mathbf{p},\mathbf{t},\mathbf{s},M,T) \equiv \max_{\mathbf{x}} u(\mathbf{x}) + \lambda\{M - \mathbf{px}\} + \mu\{T - \mathbf{tx}\} \qquad (1)$$

where, with both constraints binding, the ratio of the Lagrange multipliers on the time and money constraints, $\mu/\lambda = V_T(\cdot)/V_M(\cdot)$[3], is the money value of time.

Much of the recreation demand literature based on utility-theoretic foundations for the value of time notes that individuals observed at "interior" solutions with respect to labor supply effectively reveal their marginal value of time through their observed trades of time for money at a marginal or discretionary wage rate. This exogenous parameter can be used to collapse the two-constraint choice problem into a single-constraint problem of maximizing utility subject to full prices and full budgets, with the wage acting as the terms of trade between time and money (e.g., Becker). On the other hand, individuals at "corner solutions" work fixed hours and do not (or are not observed to) trade time for money marginally. Their marginal value of leisure time is endogenous, not observable as an exogenous parameter.

Empirical Implications of the Two Roy's Identities

The presence of an additional binding (time) constraint implies additional structure on the consumer choice problem. This structure can be developed by noting that with two constraints on choice, there are two versions of Roy's Identity, relating the price and budget slopes within each constraint.

Empirical recreation demand analysis is often based on incomplete demand systems estimated on a subset of consumption goods. In the two-constraint case, following Bockstael *et al.* we assume that the incomplete demand system estimated by the researcher is augmented by a time numeraire good which has a positive time price and a zero money price, and a money numeraire good with zero time price and a positive money price. As LaFrance and Hanemann have shown, welfare analysis can be conducted on incomplete demand systems conditional on the prices of goods excluded from the estimated system remaining unchanged. This is generally not true of partial demand systems, where separability of preferences leads to demand systems based on group budget, unless one also explains the allocation of overall income to group budgets (Hanemann and Morey).

Let goods 1,...,n (where $n \geq 1$) be the goods in the estimated incomplete demand system, with all having strictly positive time and money prices, and let good n+1 be the money numeraire and n+2 be the time numeraire good. The symmetry conditions for price and budget coefficients which follow apply to the n goods in the estimated demand system.[4]

From the envelope theorem applied to (1), we can see that $V_{p_j} = -\lambda x_j$, $V_{t_j} = -\mu x_j$, $V_M = \lambda$, and $V_T = \mu$, so that for all goods in the estimated incomplete demand system one can write

$$x_j(\mathbf{p},\mathbf{t},\mathbf{s},M,T) \equiv -V_{p_j}/V_M \equiv -V_{t_j}/V_T, \qquad \text{for } j=1,\ldots,n. \tag{2}$$

The two Roy's Identities in equation (2) are a source of parameter restrictions in the empirical demand system and prove useful for specification and identification of the marginal value of leisure time from demand system coefficients.[5]

Cross-Price Restrictions

Differentiating (2) with respect to p_i, one obtains two expressions for the Marshallian cross-money price slope $\partial x_j/\partial p_i$,

$$\partial x_j/\partial p_i = -[V_T \cdot V_{t_j p_i} - V_{t_j} \cdot V_{T p_i}]/V_T^2 = -[V_M \cdot V_{p_j p_i} - V_{p_j} \cdot V_{M p_i}]/V_M^2.$$

Noting that $V_{Tp_i} \equiv \mu_{p_i}$ and $V_{Mp_i} \equiv \lambda_{p_i}$, replacing the partial derivatives V_M and V_T with their respective shadow values λ and μ from (1), and using (2), this can be simplified to

$$\partial x_j/\partial p_i = (V_{t_j p_i} - x_j \cdot \mu_{p_i})/\mu = (V_{p_j p_i} - x_j \cdot \lambda_{p_i})/\lambda. \tag{3}$$

Similarly, the two expressions for the cross-time price derivative $\partial x_i/\partial t_j$ that follow from (2) are

$$\partial x_i/\partial t_j = (V_{t_i t_j} - x_i \cdot \mu_{t_j})/\mu = (V_{p_i t_j} - x_i \cdot \lambda_{t_j})/\lambda. \tag{4}$$

Since the middle term of (3) and the right term of (4) have the common term $V_{p_i t_j}$ ($\equiv V_{t_j p_i}$ by Young's Theorem), each can be solved for this term and equated, yielding a restriction on the cross-time and cross-money prices,

$$\partial x_i/\partial t_j = (\mu/\lambda) \cdot \partial x_j/\partial p_i + (x_j \cdot \mu_{p_i} - x_i \cdot \lambda_{t_j})/\lambda. \tag{5}$$

As a special case of (5), when $i = j$, the own-time and money price slopes are related by

$$\partial x_i/\partial t_i = (\mu/\lambda) \cdot \partial x_i/\partial p_i + x_i \cdot (\mu_{p_i} - \lambda_{t_i})/\lambda. \tag{6}$$

Equations (5) and (6) show how the marginal value of leisure time relates the time price slopes $\partial x_i/\partial t_j$ and the money price slopes $\partial x_j/\partial p_i$. This is not, in general, a simple relationship, as it is affected by the difference in quantity-weighted effects of each price change on the shadow value of the other constraint.

Because of the unobservables, (5) and (6) are not directly useful as sources of empirical restrictions on two-constraint demand models. However, by comparing with cross-budget effects, it becomes possible to derive such restrictions.

Cross-budget Restrictions

The Marshallian cross-budget effects are also derived by differentiating both versions of Roy's Identity in (2) with respect to M and T, yielding

$$\partial x_j/\partial M = -(\lambda_{t_j} + x_j \cdot \mu_M)/\mu = -(\lambda_{p_j} + x_j \cdot \lambda_M)/\lambda \tag{7}$$

and

$$\partial x_i/\partial T = -(\mu_{t_i} + x_i \cdot \mu_T)/\mu = -(\mu_{p_i} + x_i \cdot \lambda_T)/\lambda. \tag{8}$$

Because the cross-derivatives $\mu_M \equiv \lambda_T \equiv V_{MT}$, when (7) is solved for μ_M and (8) for λ_T, the two expressions can be equated. When this equality is simplified, the result can be written as

$$\partial x_i/\partial T = (\mu/\lambda) \cdot (x_i/x_j) \cdot \partial x_j/\partial M - (1/x_j) \cdot (x_j \cdot \mu_{p_i} - x_i \cdot \lambda_{t_j})/\lambda. \qquad (9)$$

Parameter Restrictions On Two-Constraint Demands

When (9) and (5) are compared, the general form of the Marshallian cross-equation restrictions in the two-constraint problem emerges as

$$\partial x_i/\partial t_j + x_j \cdot \partial x_i/\partial T = (\mu/\lambda) \cdot [\partial x_j/\partial p_i + x_i \cdot \partial x_j/\partial M], \qquad (10)$$

and again as a special case where $i = j$, the own-price and own-budget slopes must be related by

$$\partial x_i/\partial t_i + x_i \cdot \partial x_i/\partial T = (\mu/\lambda) \cdot [\partial x_i/\partial p_i + x_i \cdot \partial x_i/\partial M]. \qquad (11)$$

Equations (10) and (11) take a form comparable to the Slutsky-Hicks equations from standard consumer theory, and express necessary conditions which follow from utility maximization subject to two binding constraints. They are conceptually distinct from, though closely related to, the two sets of Slutsky-Hicks equations that result from the two expenditure minimization problems dual to the two-constraint utility maximization problem. The advantage of casting the requirements of theory in a form such as (10), though, is that all quantities $x_i(\mathbf{p},\mathbf{t},\mathbf{s},M,T)$ and $x_j(\mathbf{p},\mathbf{t},\mathbf{s},M,T)$ in (10) and (11) are Marshallian, not Hicksian, so they represent directly observable levels and slopes of ordinary demand.

To complete the comparative statics, when cross-money price slopes are compared to cross-money budget slopes, and cross-time price slopes are compared with cross-time budget slopes, the cross-equation restrictions are

$$\partial x_i/\partial p_j + x_j \cdot \partial x_i/\partial M = \partial x_j/\partial p_i + x_i \cdot \partial x_j/\partial M \tag{12}$$

and

$$\partial x_i/\partial t_j + x_j \cdot \partial x_i/\partial T = \partial x_j/\partial t_i + x_i \cdot \partial x_j/\partial T. \tag{13}$$

The necessary conditions represented in (12) and (13) further illustrate the empirical advantages of developing the symmetry requirements of two-constraint choice theory from Roy's Identities. All terms are observable, so these conditions can be directly tested for or imposed in estimating empirical recreation demand models.

Equations (10), (12), and (13) provide the general symmetry structure which empirical two-constraint consumer models must follow.[6] This has several implications for how leisure time enters the specification of recreation demand models. The analysis of these implications begins with the simplest case, most familiar in the literature, of exogenous values of time revealed through auxiliary choices recreationists make regarding labor supply. One finding is that the linear-in-parameters demand equation used by Bockstael *et al.* does not generalize readily to multiple equation systems, though the two-constraint theory helps identify alternative empirical functional forms, involving symmetric full prices and multiplicative full budgets, that do work.

Next the general "corner solution" case is considered, where there are no auxiliary labor supply decisions that reveal an exogenous value of time for the individual. We show that specifications involving symmetric full prices and multiplicative full budgets also satisfy the two-constraint restrictions, even though the marginal value of time in this case is endogenous and is, itself, a function of all parameters of the problem. The power of this result is that it shows how researchers can estimate value of time functions jointly with the recreation demand equations in models that satisfy the requirements of utility theory.

Implications for Models with Exogenous Marginal Values of Leisure Time

First it is shown how the general two-constraint restrictions in (10)-(13) encompass as a special case the most common formulation of time in the literature, where individuals are at interior solutions in the labor market, optimizing with regard to an exogenous discretionary wage w^D and offering a positive hours supply. If one of the goods in (1) is taken to have money price $-w^D$, time price 1, and zero marginal utility, that good corresponds to the hours supplied variable in the Bockstael *et al.* "interior solution case." As they show, its first order condition relates the two constraint shadow values as

$$\mu(\mathbf{p} + w^D \cdot \mathbf{t},s,M + w^D \cdot T) \equiv w^D \cdot \lambda(\mathbf{p} + w^D \cdot \mathbf{t},s,M + w^D \cdot T). \tag{14}$$

where all optimized choice variables are functions of full prices and full budget. From (14), it is clear that

$$\mu_{p_i} = w^D \cdot \lambda_{p_i} = \lambda_{t_i}, \tag{15}$$

and in light of (15), the term $(\mu_{p_i} - \lambda_{t_i})/\lambda = 0$ in (6) and $\partial x_i/\partial t_i = w^D \cdot \partial x_i/\partial p_i$; that is, as Bockstael *et al.* point out, all the Marshallian demands $h_i^I(\mathbf{p}+w^D\mathbf{t},s,M+w^D T)$, i=1,...,n, are functions of full prices and full budget. For this special case, (14) implies that (10) collapses to either (12) or (13), which are equivalent statements, depending on whether one wishes to characterize the demand restrictions in money terms or time terms.

This empirical model provides a useful illustration of the principles. For individuals at interior solutions, they estimated a single-equation model of the form

$$x_1 = \alpha + \gamma_1 \cdot (M + w^D \cdot T) + \beta_1 \cdot (p_1 + w^D \cdot t_1) + \gamma_2 \cdot q + \epsilon \qquad (16)$$

where γ_1 and β_1 are the full budget and full price coefficients, respectively, and q is a quality argument. Clearly their model satisfies the own-price version of the two-constraint restrictions, given in equation (11), because $\partial x_1/\partial t_1 = w^D \cdot \beta_1$, $\partial x_1/\partial T = w^D \cdot \gamma_1$, $\partial x_1/\partial p_1 = \beta_1$, $\partial x_i/\partial M = \gamma_1$, and $\mu/\lambda = w^D$. For this model, (11) is then

$$w^D \cdot \beta_1 + x_1 \cdot w^D \cdot \gamma_1 = (w^D) \cdot [\beta_1 + x_1 \cdot \gamma_1],$$

which always satisfies the two-constraint requirement.

Multiple-Equation Interior Solution Models

Equation (10) goes beyond the single-equation incomplete demand case empirically estimated by Bockstael *et al.* to indicate the cross-equation restrictions on Marshallian demand coefficients required in multiple-equation systems of recreation demands where time is a constraint on choice. The linear-in-parameters specification does not work in the multiple-equation context because the cross-equation restrictions in (10) are violated, unless consumption quantities are constrained or there are no income effects. To see this, define a two-good incomplete demand system as

$$x_1 = \alpha_1 + \gamma_1 \cdot (M{+}w^D T) + \beta_{11} \cdot (p_1{+}w^D t_1) + \beta_{12} \cdot (p_2{+}w^D t_2) + \gamma_{12} \cdot q + \epsilon$$

$$x_2 = \alpha_2 + \gamma_2 \cdot (M{+}w^D T) + \beta_{21} \cdot (p_1{+}w^D t_1) + \beta_{22} \cdot (p_2{+}w^D t_2) + \gamma_{22} \cdot q + \epsilon$$

and for this system, equation (10) is

$$\beta_{12} \cdot w^D + x_2 \cdot (\gamma_1 \cdot w^D) = w^D \cdot [\beta_{21} + x_1 \cdot \gamma_2]$$

which defines a linear dependence between x_1 and x_2. If budget terms are zero ($\gamma_1 = \gamma_2 = 0$) and Marshallian cross-price effects are symmetric ($\beta_{12} = \beta_{21}$), equation (10) can hold without a linear dependence of consumption quantities.

It is no surprise that a linear Marshallian demand system in general fails to satisfy the two-constraint requirements, especially in light of LaFrance's work on integrability of single-constraint linear demand systems, which found that cross-price and income coefficients must be highly linearly dependent for integrability to be satisfied. The interesting thing about the result here is that the failure comes from a different facet of the integrability problem, namely satisfying the maintained hypothesis of two binding constraints on choice.

Satisfying the Two-Constraint Requirements in Multiple Equation Demand Systems

One can devise empirical interior-solution demand systems that satisfy the two-constraint requirements of (10)-(13), as for example with the system

$$x_i = h_i(\mathbf{p} + w^D \cdot \mathbf{t},\mathbf{s}) \cdot g(M + w^D \cdot T,\mathbf{s}), \qquad \text{for } i = 1,...,n \qquad (17)$$

where the cross-partial price slopes are symmetric (i.e., $\partial h_j/\partial p_i = \partial h_i/\partial p_j$). The demand functions in this system have individual full-price effects $[h_i(\mathbf{p} + w^D \cdot \mathbf{t},\mathbf{s})]$ and a common full budget effect $[g(M + w^D \cdot T,\mathbf{s})]$. The price and budget slopes are

$$\partial x_j/\partial p_i = \frac{\partial h_j}{\partial p_i} \cdot g \qquad (18)$$

$$\partial x_i/\partial t_j = w^D \cdot \frac{\partial h_i}{\partial p_j} \cdot g \tag{19}$$

$$\partial x_j/\partial M = h_j \cdot g_M \tag{20}$$

$$\partial x_i/\partial T = w^D \cdot h_i \cdot g_M, \tag{21}$$

where $g_M \equiv \partial g(\cdot)/\partial M$. Using (17) to substitute out the $h_i(\cdot)$ and $h_j(\cdot)$ terms, (20) and (21) can be written as $\partial x_j/\partial M = x_j \cdot g_M/g$ and $\partial x_i/\partial T = x_i \cdot w^D \cdot g_M/g$, respectively. Using these with the price slopes in (18) and (19) and the fact that $\mu/\lambda = w^D$, equation (10) for this model is

$$w^D \cdot \partial h_i/\partial p_j \cdot g + x_j \cdot (x_i \cdot w^D \cdot g_M/g) = w^D \cdot [\partial h_j/\partial p_i \cdot g + x_i \cdot (x_j \cdot g_M/g)]$$

which holds given the symmetric Marshallian cross-price effects $\partial h_i/\partial p_j \equiv \partial h_j/\partial p_i$.

Clearly it is possible to design multiple-equation empirical demand systems to satisfy the two-constraint hypothesis implicit in models of recreation demand where the value of time plays an important role. An important question for further work is which forms of $h(\cdot)$ and $g(\cdot)$ are consistent with the other aspects of integrability (i.e., the negative definiteness and rank conditions identified by Partovi and Caputo).

Implications for Models with Endogenous Marginal Values of Leisure Time

The previous sections discussed the implications of the two-constraint choice structure for the special "interior solution" case where individuals reveal their marginal value of leisure time by making a discretionary labor supply choice. Because equations (10)-(13) hold for general marginal value of leisure time functions μ/λ, they describe the structure that must also apply to the system of demands $x_i = h_i^C(\mathbf{p},\mathbf{t},\mathbf{s},M,T)$ for those at corner solutions rather than interior solutions in the labor market. In this case, the marginal value of time (μ/λ) is an endogenous variable, which in general is a function of all parameters of the problem. What problems does the endogeneity of the marginal value of leisure time cause for specification of two-constraint demand systems?

Denoting this marginal value of leisure time function as $\mu/\lambda \equiv \rho(\mathbf{p},\mathbf{t},\mathbf{s},M,T)$, a set of sufficient conditions for (10)-(13) to hold is for the price and budget slopes to be related as

$$\partial x_i/\partial t_j = \rho(\mathbf{p},\mathbf{t},\mathbf{s},M,T) \cdot \partial x_j/\partial p_i \qquad \text{for all i, j} \qquad (22)$$

and

$$\partial \log(x_i)/\partial T = \rho(\mathbf{p},\mathbf{t},\mathbf{s},M,T) \cdot \partial \log(x_j)/\partial M \qquad \text{for all i, j.} \qquad (23)$$

One might anticipate problems with models using full prices $[p_i + \rho(\mathbf{p},\mathbf{t},\mathbf{s},M,T) \cdot t_i]$ and full budget $[M + \rho(\mathbf{p},\mathbf{t},\mathbf{s},M,T) \cdot T]$, because of the dependence of $\rho(\cdot)$ on prices and budgets. In deriving the price and budget slopes in (22) and (23), terms involving changes in $\rho(\cdot)$ with those prices and budgets must be accounted for.

For the case of endogenous marginal value of leisure time, equation (17) is

$$x_i = h_i(p_1 + \rho(\cdot) \cdot t_1,\ldots,p_n + \rho(\cdot) \cdot t_n) \cdot g(M + \rho(\cdot) \cdot T,\mathbf{s}), \quad \text{for i} = 1,\ldots,n. \quad (24)$$

Demand equations of this form satisfy (22) and (23), which are sufficient conditions for (10)-(13) to hold, despite the dependence of $\rho(\mathbf{p},\mathbf{t},\mathbf{s},M,T)$ on the full set of prices and budgets. For this demand system, again assuming symmetric cross-partial price derivatives ($\partial h_j/\partial p_i = \partial h_i/\partial p_j$), the price slopes are

$$\partial x_j/\partial p_i = \frac{\partial h_j}{\partial p_i} \cdot g + \frac{\partial \rho}{\partial p_i} \cdot (\sum_k t_k \cdot \frac{\partial h_j}{\partial p_k} \cdot g + h_j \cdot g_M \cdot T) \qquad (25)$$

and

$$\partial x_i/\partial t_j = \rho \cdot \frac{\partial h_i}{\partial p_j} \cdot g + \frac{\partial \rho}{\partial t_j} \cdot (\sum_k t_k \cdot \frac{\partial h_j}{\partial p_k} \cdot g + h_j \cdot g_M \cdot T) \qquad (26)$$

while the budget slopes are

$$\partial x_j/\partial M = h_j \cdot g_M + \frac{\partial \rho}{\partial M} \cdot (\sum_k t_k \cdot \frac{\partial h_j}{\partial p_k} \cdot g + h_j \cdot g_M \cdot T) \tag{27}$$

and

$$\partial x_i/\partial T = \rho \cdot h_i \cdot g_M + \frac{\partial \rho}{\partial M} \cdot (\sum_k t_k \cdot \frac{\partial h_j}{\partial p_k} \cdot g + h_j \cdot g_M \cdot T). \tag{28}$$

Homogeneity of degree zero of Marshallian demands in the price and budget arguments of each constraint imply that the term in parentheses in each of (25)-(28) is identically zero. The terms $h_i \cdot g_M$ are the specific form of the income budget slope $\partial x_i/\partial M$ (for i=1,...,n) for the multiplicative demand given in (24), while the terms $(\partial h_i/\partial p_k) \cdot g$ are the money price slopes $\partial x_i/\partial p_k$ for all i,k=1,...,n. The term in parentheses is then

$$(\sum_k t_k \cdot \partial x_i/\partial p_k + \partial x_i/\partial M \cdot T) \equiv 0$$

by homogeneity.[7] Thus, for general value of time functions, (25)-(28) simplify to

$$\partial x_j/\partial p_i = \frac{\partial h_i}{\partial p_i} \cdot g \tag{29}$$

$$\partial x_i/\partial t_j = \rho \cdot \frac{\partial h_i}{\partial p_j} \cdot g \tag{30}$$

$$\partial x_j/\partial M = h_j \cdot g_M \tag{31}$$

$$\partial x_i/\partial T = \rho \cdot h_i \cdot g_M, \tag{32}$$

and as with (18)-(21), these slopes satisfy (22) and (23) and, hence, the two-constraint choice restriction in equation (10).

Thus the endogeneity of the marginal value of leisure time in the general corner solution case causes no additional problems beyond those raised in the interior solution case. The two-constraint restrictions must hold, and equations (17) and (24) are examples of how these restrictions can be satisfied with Marshallian recreation demand functions. Equation (24) further suggests how researchers can incorporate hypotheses about the structure of the marginal value of leisure time, as it may depend on prices, budgets, and other shifters **s**, directly into the demand model and estimate the marginal value of leisure time directly as part of the model.

This can be useful in the "interior solution" case as a validity check on the maintained hypothesis of the marginal value of leisure time (which is assumed to be w^D). It can often be difficult to measure the discretionary wage accurately even when people indicate they are trading time for money at the margin. For interior solution models, the researcher can specify w^D as one of the elements of **s** [*viz.*, $\rho(\mathbf{p},\mathbf{t},w^D,\mathbf{s},M,T)$] and test whether or not the empirically-measured discretionary wage is the best explainer of the marginal value of time and recreation demanded.

Implications for Current Practice

The two-constraint requirements have significant implications for current practice. One concerns the acceptability of formulating recreation demand models with full prices of travel and money income alone, which is common in the literature, both in conceptual and empirical models. The practice occurs in a wide variety of models, from standard recreation demand models (e.g., McConnell and Strand; Smith *et al.*) to count data models (e.g., Creel and Loomis, 1990; Englin and Shonkwiler; Hellerstein) to random utility models (e.g., Adamowicz *et al.*; Creel and Loomis 1992; Feather *et al.;* Morey *et al.*). Such formulations are inconsistent with the two-constraint requirements.

A second implication is that the value of time is "revealed" from coefficient estimates of correctly-specified models. This point is illustrated using empirical estimates from the Bockstael *et al.* model.

A Problem with Common Practice in Modeling Time

It is common in the literature to find recreation demand models that include a time price of recreation but no corresponding time budget variable. That is, full price (money cost plus time cost) and money income are included in the specification. This may be a concession to the difficulty of determining what the relevant time budget is for a recreation choice occasion,[8] or to data limitations. And it may be based on an assumption that the major specification issue is to avoid bias in the full price coefficient, on which welfare calculations are based. However, the point which may not be fully appreciated is that omission of the time budget variable invalidates the use of full prices in the model. This can be seen by considering each of the major types of models (continuous demand models, count data models, and random utility models) in light of the two-constraint requirements in (10)-(13).

Continuous Demand Models

The inconsistency of using full prices and money budget alone can be seen by recalling equation (11) for the single-equation demand model with exogenous marginal value of leisure time. This equation must hold in the empirical model if the researcher includes a time price (thereby invoking the maintained hypothesis of two constraints on choice). The rationale for omitting time budget must be an assumption that $\partial x_i/\partial T=0$, and when this is imposed on (11) the two-constraint restriction for the interior solution case is

$$\partial x_i/\partial t_i = w^D \cdot [\partial x_i/\partial p_i + x_i \cdot \partial x_i/\partial M]. \tag{33}$$

If the money income effect on demand is nonzero, then a demand model based on full prices and budgets, such as (16) or (17), would not satisfy (33). An obvious problem is the dependence on a consumption quantity (x_i), but any term beyond $\partial x_i/\partial p_i$ on the right side invalidates the use of full prices.

Time budgets play an integral role in the two-constraint recreation demand model, in maintaining the theoretical justification for the use of full prices. To avoid estimating incorrect models based on full prices and full budgets, they must be included in the empirical specification.

Count Data Models

Count data models are often used for single-equation demand models, to more realistically depict the distribution of the dependent variable, which is non-negative integer-valued. The principal difference from standard demand models is in the choice of the stochastic term of the model, which is usually assumed to be either Poisson or negative binomial (e.g., Greene). For example, the Poisson count model of recreation trips assumes that for individual i, the random trips variable X_i takes on the value x_i with $\text{Prob}(X_i = x_i) = e^{-\lambda_i}\lambda_i^{x_i}/x_i!$, with λ_i most commonly specified as $\lambda_i = e^{\mathbf{z}_i\boldsymbol{\alpha}}$, where $\mathbf{z} \equiv [\mathbf{p},\mathbf{t},\mathbf{s},M,T]$ is the vector of all demand covariates (time and money prices, time and money budgets, and shifters $\mathbf{s}$) and α is the corresponding parameter vector.[9] The systematic part of this demand model is

$$E[X_i|\mathbf{z}_i] = \lambda_i = e^{\mathbf{z}_i\alpha}.$$

The analysis that develops equations (2)-(13) for this model is the same as for the demand systems case; equation (11) for the single count model (with general value of time function $\rho(\mathbf{z})$) is

$$\partial E[X_i|\mathbf{z}_i]/\partial t_i + E[X_i|\mathbf{z}_i] \cdot \partial E[X_i|\mathbf{z}_i]/\partial M = \rho(\mathbf{z}) \cdot \{\partial E[X_i|\mathbf{z}_i]/\partial p_i$$

$$+ E[X_i|\mathbf{z}_i] \cdot \partial E[X_i|\mathbf{z}_i]/\partial M\}.$$

This will always be satisfied if the systematic part of the demand model is a function of full prices $(p_j + \rho(\mathbf{z}) \cdot t_j)$ and full budgets $(M + \rho(\mathbf{z}) \cdot T)$. This can be seen from the fact that (denoting the full price i coefficient α_{p_i}) the relevant derivatives are $\partial E[X_i|\mathbf{z}_i]/\partial t_i = \alpha_{p_i} \cdot \rho(\mathbf{z}) \cdot E[X_i|\mathbf{z}_i]$ and $\partial E[X_i|\mathbf{z}_i]/\partial p_i = \alpha_{p_i} \cdot E[X_i|\mathbf{z}_i]$. Similarly, denoting the full budget coefficient as α_b, the money and time budget slopes are $\partial E[X_i|\mathbf{z}_i]/\partial T = \alpha_b \cdot \rho(\mathbf{z}) \cdot E[X_i|\mathbf{z}_i]$ and $\partial E[X_i|\mathbf{z}_i]/\partial M = \alpha_b \cdot E[X_i|\mathbf{z}_i]$.

Note that, as with the demand systems model above, specifications with full price and money budget alone are not consistent with these requirements. For multiple-equation count models with time constraints, the specification for the λ_j for each good j can be formulated along the lines of equation (24).

Random Utility Models

Random utility models are becoming very common in the literature, to explain the choice of which site, or recreation alternative, is chosen on a given choice occasion. The model is usually motivated based on a comparison of (indirect) utilities of the different alternatives, with the highest-valued alternative being chosen.

To see how the results on including time variables extend to this class of models, we can re-motivate equation (1) to describe the optimization of a continuous choice x_j

associated with discrete alternative j, for j=0,...,J.[10] The $J+1$ indirect utilities $V_j \equiv V(p_j, t_j, s_j, M, T)$ in (1) then describe the optimal utility derivable from each alternative, based on its own prices (p_j and t_j) and characteristics s_j, and on the consumer's money and time budgets. Incomplete observation by the researcher leads to an error ϵ_j for each alternative, and the optimal choice i is such that

$$V_i + \epsilon_i > \max_{j \neq i} V_j + \epsilon_j.$$

Given functional forms for the V_j and a distributional assumption for the ϵ_j (commonly, as iid extreme value or Generalized extreme value variates), the model can be estimated.

For the random utility model to validly represent economic behavior, it must be consistent with the requirements of theory, including the two-constraint requirements when time and money variables both enter the specification. For this model, the requirements can be seen most clearly from the two Roy's Identities in equation (2), since the indirect utility functions V_j are specified directly to motivate estimation of this model. Rearranging (2) slightly, for each alternative j, it must be true that

$$V_{t_j}/V_{p_j} \equiv V_T/V_M \tag{34}$$

and, therefore, the indirect utility functions of the different alternatives are linked as well; it must be true that

$$V_{t_i}/V_{p_i} = V_{t_j}/V_{p_j} \tag{35}$$

for all $i \neq j$.

A specification using full prices and full budgets again is sufficient to satisfy the two-constraint requirements in (34) and (35), regardless of whether the value of time is endogenous or exogenous. For example, writing

$$V_j = V_j[(p_j + \rho t_j), s_j, (M + \rho T)] \qquad \text{for all j,} \tag{36}$$

it is easy to see that[11]

$$V_{t_i}/V_{p_i} = V_{t_j}/V_{p_j} = V_T/V_M = \rho.$$

Two points should be noted. First, since the most common specifications of the V_j are linear in parameters, e.g.,

$$V_j = \alpha_{jp} \cdot (p_j + \rho(\mathbf{z}) \cdot t_j) + \alpha_{jM} \cdot (p_j + \rho(\mathbf{z}) \cdot t_j) + \boldsymbol{\alpha}_{jp} \cdot s_j,$$

it is well-known that the budget terms drop out of the choice probability for alternative j, since they do not vary across alternatives. Thus a model with full prices of the alternatives, but no budget terms (e.g., Jakus *et al.*, Parsons and Hauber), is consistent with the two-constraint requirements.

A number of recent random utility formulations have postulated that the indirect utility of an alternative is the difference between the available budget and the cost of the alternative itself; e.g., using full prices and budgets,

$$V_j = V_j[(M + \rho T) - (p_j + \rho t_j), s_j] \qquad \text{for all j.} \tag{37}$$

It is easy to see that (37) satisfies the two-constraint requirements, because it is a special case of (36), which satisfies them.

However, in a number of cases in the literature, the income remaining after cost is calculated not like (37), but instead as a difference between money income and full cost, so that $V_j = V_j[(M - (p_j + \rho t_j), s_j]$ (e.g., Feather *et al.*; Parsons and Kealy; Kaoru *et al.;* Herriges, Kling, and Phaneuf; Montgomery and Needelman). Here, $\partial V_j/\partial M = \partial V_j/\partial p_j$ and $\partial V_j/\partial t_j = \rho \cdot \partial V_j/\partial p_j$, but $\partial V_j/\partial T = 0$, so the two-constraint requirements (34) and (35) cannot be satisfied in this conceptual formulation. Whether this raises a problem empirically depends on how the budget enters indirect utility: in the linear-in-parameters specification, the individual's budget (whether full or just money budget) cancels from the choice probabilities. With increasingly-sophisticated specifications being made possible by advances in computation speed and estimation techniques, including the development of linked participation-site choices, the specification of the individual's budget becomes more important to achieve consistency with utility theory.

Inferring the Marginal Value of Leisure Time from Utility-Theoretic Demands

A second empirical point is that the marginal value of leisure time can be measured from the demand coefficients of a properly-specified system. Perhaps the easiest way to make this point is to return to the empirical model of Bockstael *et al.*, this time using instead their corner solutions model, which was

$$x_1 = \alpha + \gamma_1 \cdot M + \gamma_2 \cdot T + \beta'\gamma_1 \cdot p_1 + \beta'\gamma_2 \cdot t_1 + \gamma_3 \cdot q + \epsilon$$

where q is an exogenous quality variable and $\beta' \equiv \beta/(\gamma_1+\gamma_2)$. Because this system is utility-theoretic, it satisfies (29-32) and, therefore, the two-constraint choice restriction in (11). From (22) and (23), it can be seen that the marginal value of time can be measured directly from the demand coefficients, as

$$\rho = (\partial x_1/\partial t_1)/(\partial x_1/\partial p_1) = (\partial \log(x_1)/\partial T)/(\partial \log(x_1)/\partial M).$$

For this model, $\partial x_1/\partial p_1 = \beta'\gamma_1$, $\partial x_1/\partial t_1 = \beta'\gamma_2$, $\partial \log(x_1)/\partial M = \gamma_1/x_1$, and $\partial \log(x_1)/\partial T = \gamma_2/x_1$, so (34) becomes

$$\rho = \beta'\gamma_2/\beta'\gamma_1 = \gamma_2/\gamma_1.$$

Bockstael *et al.* estimated the money price slope to be $\hat{\gamma}_1 = .024$, with a time price slope of $\hat{\gamma}_2 = 2.982$. Thus the marginal value of time in this model is a constant, $\rho \approx$ (2.982 units x/hour)/(.024 units x/\$) $\approx$ \$124/hour.[12] This contrasts with the estimate of the authors, who infer an estimate of \$60/hour for the marginal value of leisure time by comparing compensating variation estimates of welfare loss from eliminating the resource, denominated in dollar and time units.[13]

Conclusions

This paper develops a number of the structural requirements for the specification of recreation demand models where time is thought to be an important choice constraint. Coefficient restrictions take a form similar to the Slutsky-Hicks equations from standard consumer theory of choice subject to a single constraint, but arise from a different facet of the consumer choice problem when multiple constraints bind. The Slutsky-Hicks equations arise from the identity of Hicksian and Marshallian demands when income or utility is chosen appropriately, where the two-constraint restrictions arise from the equivalence of the two Roy's Identities that govern the response of Marshallian demands to parameter changes. Thus the two constraint restrictions relate observable Marshallian demand slopes and the generally-unobservable marginal value of leisure time. The restrictions relating cross-money price and money budget effects are fully observable, as are the restrictions relating cross-time price and time budget effects, so they can be implemented and tested for easily in practice. They provide guidance in two important areas not addressed by the existing literature: specification of how time should enter systems of demand equations, and how to deal with endogenous marginal values of leisure time. The two-constraint requirements apply to all types of empirical demand models where time is a second constraint on choice, whether motivated as systems of continuous demands, count data models, or random utility models. We show how these requirements can be applied to the specification of each of these classes of models.

An important finding is that the basic intuition of the simple model where time is an exogenous function, and the resulting demand is a function of full prices and full budgets, carries through to models where the value of time is endogenous. This should enable researchers to estimate value of leisure time functions auxiliary to the recreation demand model of interest. Individuals with exogenous values of time (those at "interior solutions" in the labor market) represent a special case where the marginal value of time is a constant or a known exogenous function.

Use of the structure required by the hypothesis of choice subject to two binding constraints is also helpful in empirical practice. We show that the approach used by much of the current literature on valuing time, to include full price of the activity but only money income, cannot be consistent with the requirements of consumer theory. We also show how the theory can also be used to infer the marginal value of time from properly specified two-constraint models. Thus the empirical two-constraint restrictions should be of considerable use in specifying theoretically-consistent demand systems and in inferring marginal values of leisure time from their empirical implementation.

Footnotes

1. This formulation is common in the recreation demand literature with utility-theoretic formulations for the value of time, such as Bockstael *et al.* Smith, in particular, examines some of the primal and dual properties of the two-constraint problem.

2. Examples of such goods include taking walks on the beach (positive time price but no money price) and making charitable contributions (positive money price but no–or nearly no–time price).

3. Parameters appearing as subscripts refer to partial derivatives; e.g., $V_{Tp_i} \equiv \partial^2 V(\mathbf{p},\mathbf{t},z,M,T)/\partial T \partial p_i$. The subscripts i and j index the consumption goods and their corresponding prices.

4. Thus, for example, a single-equation empirical demand function has n=1 and implies a three good world, with only the own-price and own-budget restrictions holding.

5. To minimize notational clutter, it is noted here that all restrictions developed below hold for goods i, j = 1,...,n; that is, they are restrictions which must be accounted for in the estimated incomplete demand system.

6. The results we develop here have also been derived by Partovi and Caputo, who examine the implications of the general K-constraint consumer choice problem. They also prove the negative semidefiniteness and rank conditions for the matrix of cross-equation restrictions for the general K-constraint problem.

7. It is well-known that the two-constraint Marshallian demand functions are homogeneous of degree zero in the parameters of each constraint (Partovi and Caputo; Smith). For general two-constraint demands, zero-degree homogeneity implies $\mathbf{x}(\theta\mathbf{p},\mathbf{t},\mathbf{s},\theta M,T) = \mathbf{x}(\mathbf{p},\mathbf{t},\mathbf{s},M,T)$, and differentiation with respect to θ yields $(\sum_k p_k \cdot \partial x_i/\partial p_k + \partial x_i/\partial M \cdot M)=0$. For the two-constraint model with full prices and full budgets [which has, as a special case, equation (24)], scale both money and time prices and budgets by θ (which leaves the ratio of Lagrange

multipliers, ρ, unchanged). Then homogeneity of degree zero implies $\mathbf{x}(\theta\mathbf{p}+\rho \cdot \theta\mathbf{t}, \mathbf{s},\theta M+\rho \cdot \theta T) = \mathbf{x}(\mathbf{p}+\rho \cdot \mathbf{t},\mathbf{s},M+\rho \cdot T)$, which upon differentiation with respect to θ yields $(\sum_k p_k \cdot \partial x_i/\partial p_k + \partial x_i/\partial M \cdot M) + \rho \cdot (\sum_k p_k \cdot \partial x_i/\partial t_k + \partial x_i/\partial M \cdot T) = 0$. Since the first term in parentheses must be zero by homogeneity in the money budget alone, the second term in parentheses must be zero also.

8. In reality, it may not be too difficult to assess the time budget with at least as much accuracy as the relevant money budget variable, which is complicated by tax, credit, and household size differences.

9. Applications to recreation demand include Hellerstein, Creel and Loomis, and Englin and Shonkwiler.

10. The following arguments generalize readily to a set of continuous choices x_{ij}, $i=1,...,I_j$ made to optimize the utility derived from the j^{th} alternative.

11. As with the demand systems case, homogeneity of degree zero of each alternative's indirect utility in the parameters of each constraint leads the terms involving ρ in the partial derivatives to cancel. There are fewer cross-equation restrictions in the random utility setting because typically researchers specify the utility of an alternative as a function of own price but no other prices.

12. Bockstael *et al.* note (p.298) that one of the undesirable features of the utility function they use for their illustration is that it implies a constant money-time tradeoff for the corner solution case.

13. The empirical magnitude of the difference is a secondary issue, as the denominator (money budget coefficient) is statistically insignificant anyway; the empirical estimate would also be affected if, for instance, separate parameters were estimated for people at corner solutions versus those at interior solutions. The main point is how knowing the structure of two-constraint models makes the value of time immediately available from demand coefficients for this model.

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