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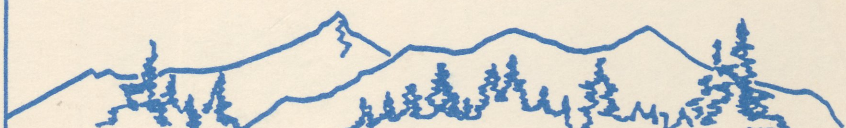

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Some Theoretical and Empirical Notes on the Armington Model Nancy H. Cottrell N.C. State Univ.

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This paper demonstrates that the current methods used to estimate an Armington Model in agricultural applications have not reflected the true theory of the model. Specific suggestions are made as to how the Armington estimation procedure can be amended to be more in line with the Armington theory.

Introduction

The Armington Model has been a popular approach to use in modeling trade flows for agricultural products which are differentiable by place of production since the application of the model to wheat by Grennes, Johnson and Thursby (1978). Recent empirical applications include studies by Figueroa and Webb (1986), Babula (1987), Duffy, Wohlgenant and Richardson (1990), Haniotis (1990), and Ito, Chen and Peterson (1990). However, the currently estimated form of the Armington Model is not equivalent to the true theoretical Armington Model. The first purpose of this paper is to point out why the estimated form of the model is not equivalent to the true theoretical form of the model. The second purpose is to suggest improvements that can be made in estimation to make the empirical model more in keeping with the true theoretical model. These purposes are achieved by working through the theoretical subtleties of the model and then relating these to the empirical application of the model. First, a brief review of the theory of the Armington Model is presented to facilitate the discussion. Further details on the model can be found in Armington (1969).

The Theory of the Armington Model

Armington used a two-stage budgeting procedure to design a model of a country's demand for a particular good, where the good can be obtained from various differentiable product sources, both domestic and import. The good of interest is defined as a group and the various sources of the good are defined as products within the group. Ultimately, Armington derives a set of demand equations for the product sources of the good. The set consists of a domestic demand equation, along with various import demand equations.

The specifics of Armington's two-stage budgeting procedure are as follows. At the first stage, a country allocates total expenditure over weakly separable groups of goods. The optimal allocation to each group is obtained through the following utility maximization:

$$\underset{Q_i}{MAX} \quad U = U(Q_1, Q_2, \ldots, Q_n) \quad s.t. \quad E = \sum_{i=1}^{n} P_i Q_i \qquad i = 1, \ldots, n. \tag{1}$$

where U = overall utility,
Q_i = quantity index for group i,
P_i = price index for group i,
E = total expenditure.

This maximization yields a system of first stage Marshallian demand equations with one equation for each group as:

$$Q_i = Q_i\,(E, P_1, \ldots, P_n). \tag{2}$$

The Armington Model focuses on one particular group. At the second stage, total expenditure on the group of interest is allocated to the various product sources that make up the group. For example, in Armington's original paper the group of interest was manufactured goods and the total demand for manufactured goods was allocated across the domestic and import sources at the second stage. The second stage allocation is done by minimizing group expenditure subject to the value of the quantity index, Q_i, obtained from the first stage.

Armington introduces a Constant Elasticity of Substitution (CES) function as the group quantity index at the second stage. This index is used because it allows the elasticity of substitution between import sources to be less than infinite and because it is a linearly homogeneous index. A linearly homogeneous index is a requirement in the theory of two stage budgeting for the consistency of two stage demand elasticities with one stage demand elasticities (Green, 1964). The minimization using the CES quantity index as the second stage utility function is:

$$\underset{q_{ij}}{MIN}\ E_i = \sum_{j=1}^{m} p_{ij}\, q_{ij} \qquad j = 1, \ldots, m.$$
$$s.t.\ Q_i = \left(\sum_{j=1}^{m} b_{ij}\, q_{ij}^{T_i}\right)^{-1/T_i}, \tag{3}$$

where q_{ij} = quantity of good i in country i from product source j,
p_{ij} = price of good i in country i from product source j,
E_i = total expenditure on group i in country i,
b_{ij} = a constant, $b_{ij}>0$, $\Sigma\, b_{ij} = 1$.

Note that there has been a slight change of subscript notation between the first and second stages. At the first stage i indexed weakly separable groups, whereas at the second stage i serves both as an index for the group of interest and as an index for the importing country. This simply alleviates the need for a third subscript indexing the importing country. (Although the Armington Model is laid out here as if there is only one country of interest, Armington actually considers multiple countries performing a two stage budgeting process in order to achieve his ultimate objective of constructing a trade flow matrix to be used in analyzing changes in market shares of competing exporting countries. The construction of the trade flow matrix is not stressed in this paper because it is not essential in understanding the thrust of the Armington theory.) The second stage minimization yields a system of second stage Hicksian demand equations for the products sources within group i for country i as:

$$q_{ij} = b_{ij}^{\sigma_i}\, Q_i \left(\sum_{k=1}^{m} b_{ik}^{\sigma_i}\, (p_{ij}/p_{ik})^{\sigma_i - 1}\right)^{\sigma_i/1-\sigma_i} \tag{4}$$

where $\sigma_i = \frac{1}{1+\tau_i}$ = elasticity of substitution between products of good i.

The second stage Hicksian demand equation for the quantity of good i from country j is a function of the optimal level of subgroup utility (or quantity index) and product prices within the group. Equation (4) is unwieldy so Armington simplifies it by substituting in the CES price index, the theoretically appropriate price index to be used in conjunction with the CES quantity index in two stage budgeting, to obtain an alternate form for the second stage Hicksian demand equation as:

$$q_{ij} = b_{ij}^{\sigma_i}\, Q_i\, (p_{ij}/P_i)^{-\sigma_i} \tag{5}$$

where P_i = the CES price index.

Equation (5) is often referred to as the "Armington equation". As a Hicksian demand equation, it is not observable and cannot be estimated. However since the subgroup utility function is represented by the quantity index, Q_i (the optimal level of which was chosen at the first stage), this Hicksian demand equation can be estimated and it is this equation that has been estimated in the empirical Armington literature in log-linear form.

Armington uses the demand equations from each stage to arrive at the "Armington elasticities" as:

$$\eta_{ijj} = s_{ij}\,\eta_i + (s_{ij} - 1)\,\sigma_i \qquad \eta_i < 0$$
$$\eta_{ijk} = s_{ik}\,\eta_i + s_{ik}\,\sigma_i \tag{6}$$

where

η_{ijj} = own price demand elasticity of country i for the good from product source j,

η_{ijk} = cross price demand elasticity of country i for the good from product source j with respect to the price of the good from product source k,

η_i = own price Marshallian demand elasticity for the total amount of the good over all product sources, obtained from the first stage,

σ_i = the elasticity of substitution between product sources, taken from the second stage.

s_{ik} = the expenditure share held by product source j for the good in country i, calculated from trade data for a particular year or average of years.

These elasticities are actually called synthesized own and cross price demand elasticities in the theory of two stage budgeting, but they are not called such in Armington's paper. Synthesized elasticities are found by combining information (elasticities) from both the first and second stage demand equations. Furthermore, synthesized elasticities are the elasticities of ultimate interest from the two stage budgeting procedure because they are the ones that are consistent with the elasticities from a traditional one step demand estimation procedure. Consistency of these elasticities is assured given that certain separability and aggregation criterion are met (Green, 1964), which Armington was careful in satisfying.

The Armington elasticities are the cornerstone of the Armington Model. Early applications of the model did not obtain these elasticities through the explicit estimation of the first and second stage demand equations. Instead expenditure shares for a particular year were chosen and the elasticity for total domestic demand for the good and the elasticity of substitution between product sources of the good were either surmised or borrowed from previous studies. In fact, this was the way that Armington intended the model to be used. His objective was to design a model that was theoretically sound, yet circumvented empirical problems caused by sparse trade data. Furthermore, he argued that the Armington elasticities and the results of the applications that the Armington elasticities are used for are not very sensitive to the magnitude of the values of the first and second stage elasticity estimates that are plugged into the elasticity formulas.

Although Armington never intended his two stage budgeting procedure to be estimated, preference among recent Armington Model users has shifted to directly estimating the two stage demand system to obtain the Armington elasticities, at least in the applications relating to agricultural commodities, such as wheat and cotton. Some of these applications only estimate the second stage demand and borrow first stage demand elasticities from previous studies.

Analysis of the Theory of the Armington Model

A source of confusion regarding the Armington Model is the specific two stage budgeting procedure Armington employs. In demand theory the two stage budgeting procedure is usually expressed in terms of a two stage utility maximization framework, whereby a utility maximization is performed at both the first and second stages. In contrast, the Armington budgeting procedure is expressed in terms of a utility maximization at the first stage and an expenditure minimization at the second stage. In general the MAX/MAX two stage procedure does not yield the same results as the MAX/MIN two stage procedure in terms of synthesized elasticities.

Table 1 shows the four possible optimization procedures that can be used for two stage budgeting and the synthesized elasticities produced by each. It shows that when a two stage maximization procedure is used, synthesized Marshallian elasticities are obtained. Alternately when a two stage minimization is used, synthesized Hicksian elasticities are obtained. However, in general when a MAX/MIN or MIN/MAX procedure is used the synthesized elasticities are neither strictly Marshallian or Hicksian but a mixture of the two.

Table 1: Two Stage Budgeting Alternate Optimization Procedures

	Case 1	Case 2	Case 3	Case 4
First Stage Elasticities	Maximization Marshallian	Minimization Hicksian	Maximization Marshallian	Minimization Hicksian
Second Stage Elasticities	Maximization Marshallian	Minimization Hicksian	Minimization Hicksian	Maximization Marshallian
Synthesized Elasticities	Marshallian	Hicksian	?	?

* Marshallian and Hicksian elasticities are linked through the Slutsky equation within each stage and after being synthesized.

Since Armington uses a MAX/MIN procedure what do the Armington synthesized elasticities represent: Marshallian, Hicksian or a combination of both? In order to explore what these elasticities mean the Armington two stage budgeting was reoptimized using a MAX/MAX optimization and the results were compared with those of the Armington MAX/MIN optimization. It was found that the two procedures did not yield equivalent results within the second stage, but yielded identical synthesized elasticities.

More specifically, the results of the two procedures differ at the second stage because the MAX/MIN approach used a minimization at the second stage yielding second stage Hicksian elasticities and the MAX/MAX approach used a maximization at the second stage yielding second stage Marshallian elasticities. Slutsky's equation readily verified the relationship between the second stage Hicksian and Marshallian elasticities. Interestingly enough, however, when the second stage Hicksian elasticities were synthesized with the first stage Marshallian elasticities, the results were the same as when the second stage Marshallian elasticities were synthesized with the first stage Marshallian elasticities. It was uncovered that the synthesized elasticities were identical due to some aberration of the CES functional form used by Armington at the second stage. In general, the MAX/MAX and MAX/MIN procedures do not lead to identical synthesized elasticities, as is easily verified by working through the optimizations for these general cases.

Implications from Theory for the Estimation of the Armington Model

Now that it has been established that the Armington elasticities are synthesized Marshallian elasticities, it may seem that there is no difference, in the Armington case, between estimating the two stage budgeting process as a MAX/MIN or as a MAX/MAX procedure. And since the model was originally set up under the MAX/MIN procedure it also seems natural that the empirical applications of the model to date have followed the same framework. But, in the demand literature, the two stage budgeting procedure has only been outlined as a two stage maximization. This is because the empirical estimation of a two stage budgeting procedure is carried out in from the bottom up in a block recursive fashion as outlined by Barnett (1980). More explicitly, the second stage is estimated first. Then the second stage results are used to construct the group quantity and price indices and these indices are used in the first stage estimation.

In the Armington context this would mean that a utility maximization would be performed at the second stage and a system of second stage Marshallian equations would be estimated as follows:

$$q_{ij} = E_i \, (p_{ij}/b_{ij})^{-\sigma_i} \, (\sum_{k=1}^{m} b_{ik}^{\sigma_i} \, p_{ik}^{1-\sigma_i})^{-1} \tag{7}$$

The estimates from this system of σ_i and the b_{ij}s would be used to construct the CES quantity and price indices to be used in the first stage estimation. However, the problem using this method with the Armington Model is that there is no simple simplification of the second stage Marshallian demand equation as there was for the second stage Hicksian equation shown above in (5). Notice that all of the product source prices are in the second stage Marshallian demand equation. Estimation by the authors in applying the Armington Model to traded wheat has shown that wheat import prices tend to be too collinear for the second stage Marshallian estimation to produce a nonsingular variance/covariance matrix. This renders parameter estimates impossible to obtain. For other agricultural commodities it may also be the case that import prices are too collinear to enable the second stage Marshallian demand system of the Armington Model to be estimated. Thus, the procedure outlined by Barnett may not be feasible when applying the Armington Model to agricultural commodities.

Given this reality, estimating the second stage Armington Model as a Hicksian demand system is reconsidered. The true functional form of the second stage Hicksian system as shown in (4) suffers from the same collinear price problem that the second stage Marshallian system does when considering agricultural commodities. However, the modified functional form, the "Armington equation", for the second stage Hicksian system as shown in (5) alleviates this collinearity problem by substituting in the CES price index for the product source prices and the functional form simplifies dramatically. However, while the simplified form of (5) is appealing and is the form used in all existing empirical work, it is the use of this form that causes a discrepancy between the empirical Armington Model and the true theoretical Armington Model. The remainder of the paper will discuss this discrepancy and suggest improvements that can be made in estimation to make the empirical model more in line with the true theoretical model.

The first issue relating to the Armington equation of (5) deals with the price and quantity indices used in the estimation of the equation. Theoretically in the Armington Model these are the CES price and quantity indices. The CES quantity index has been shown above in (3) and the CES price index is a similarly structured function of product source prices and the b_{ij} and σ_i parameters. But, using the appropriate CES indices in the Armington equation is not possible because the parameters necessary to construct them are part of the equation to be estimated and, hence, the block recursive estimation procedure for the indices is not possible. Because of this constraint, what has been done to date in the empirical Armington literature is that proxy indices have been used; an expenditure share weighted price index has been used as a price index proxy and a simple straight sum quantity index has been used as a quantity index proxy. These may be good proxies, but it is conceivable that others could perform better, particularly in regard to the quantity index. A straight sum quantity index implies perfect substitutability between the product sources. The very foundation of the Armington Model is that the products sources are not perfect substitutes. An approximating index that incorporates product differentiation would be more appropriate than a simple straight sum quantity index. By experimenting with different indices it may be found the quantity index used does not matter much to the Armington elasticity estimates obtained, but this in itself would be information for Armington users. Thus, a comparison of Armington parameter estimates using different approximating indices at the first and second stages is an area that warrants further study.

Another procedure that is suggested here to be an improvement in the handling of the price and quantity indices in the Armington equation is to use an iterative procedure to arrive at estimates of the CES price and quantity indices. The procedure begins by estimating the system of Armington equations in the manner currently done in the literature using the share weighted price index and the straight sum quantity index. The estimates of σ_i and the b_{ij}s obtained from this initial estimation are used to construct the CES price and quantity indices. These CES indices are plugged back into the system of Armington equations and the system is then re-estimated. The procedure is iterated until the σ_i and b_{ij} parameter estimates converge. The final CES price and quantity indices are then constructed and used in the first stage estimation. It is hypothesized that this procedure may yield results that are more in keeping with the theory of the Armington Model and the theory of two stage budgeting than the methods that are currently being used in estimating the Armington Model.

A final adjustment that is suggested here to make the estimation of the Armington equations more true to the Armington theory would be to restrict the b_{ij} parameters in the Armington equation in the manner that the CES function dictates. As shown above in (3) a characteristic of the CES is that the b_{ij} parameters are between zero and one and the sum of the b_{ij}s in the system sum to one. These functional restrictions have not been imposed in the existing empirical Armington literature. Since the restrictions have not been imposed, a true CES function has not been estimated in previous work. The actual equation estimated is an unknown functional form, rendering questionable the value of the parameter estimates.

Summary and Implications

This paper makes two basic points. The first is that the methods used to estimate the Armington Model for agricultural applications have not reflected the true theory of the Armington Model. This has obvious implications for how the results of the previous applications are evaluated. This also has implications regarding the recent testing and rejection of the assumptions of the Armington Model conducted by Winters (1984) and Alston, et. al. (1990). Since the true Armington Model was not estimated in these latter cases, rejecting the assumptions of the model based on the results of this estimation seems unjustified.

The second point of the paper is that some concrete steps can be taken to make the empirical estimation of the Armington Model more of a true reflection of the Armington theory. The suggested steps warrant further investigation and may lead to improved empirical modeling procedures for bilateral trade flows.

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