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The Effects of Including Bankruptcy on Dynamic Investment Decisions

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Abstract
This paper evaluates the effects of including the costs of bankruptcy in a dynamic model of off-farm investment decisions using a stochastic dynamic programming model (SDP) which incorporates the stochastic dynamic nature of investment returns and the interrelationships between financial structure and investment decisions when the costs of bankruptcy are considered. Results suggest that ignoring bankruptcy costs in determining investment decisions results in large bankruptcy probabilities.

1. Introduction

The agricultural economics literature contains many examples of research into the profitability and riskiness of different investment and growth strategies (Young and Barry, and Moss et al.). A potential limitation of these studies is the failure to include the effects of financial structure on investment decisions. For example, a common result is that at low levels of risk aversion, the optimal portfolio is highly leveraged. Many studies which have considered the growth and investment problem have found that high debt levels may put the business at great risk of bankruptcy. Models which specifically incorporate bankruptcy may produce different results. Studies which have explicitly considered the linkage between financial structure and investment decisions (Schnitkey, Taylor and Barry) have illustrated the suboptimal nature of more naive decision models.

The purpose of this study is to determine optimal off-farm investment decisions for a Central Illinois hog finisher. These decisions are investigated using a stochastic dynamic programming model (SDP) which incorporates information on the stochastic dynamic nature of investment returns (both farm and nonfarm) and the interrelationships between financial structure and investment decisions when the costs of bankruptcy are considered. The objective used here is the maximization of expected terminal wealth in the presence of bankruptcy. This objective represents a departure from the traditional form of risk averse objective functions, following Antle's notion of risk. Further motivation for this approach is provided by the fact that a direct application of von Neumann Morgenstern utility functions to dynamic choice problems may not yield consistent results (Zacharias). The implications of this formulation are tested below by comparing results with and without the bankruptcy constraint. Differences in optimal behavior and resulting wealth may provide an indication of the costs of ignoring bankruptcy in this type of decision model.
The objectives of this research are:

1. To contribute to the understanding of how the dynamic interrelationships discussed above affect investment decisions and the financial condition of farm businesses.
2. By solving for the optimal decision rules under bankruptcy versus no recognition of bankruptcy it will be possible to determine the effects of these different risk formulations on optimal decisions, financial structure and risk exposure.

The following section contains a conceptual model of an Illinois hog finishing operation which is then incorporated into the SDP model to determine optimal stock investment decisions. Numerical solution of the SDP model and a graphical analysis of the optimal stock investment decision rules. These decision rules describe how stock investment decisions are affected by different levels of farm returns, stock prices, interest rates and financial structure. Conditional probability analysis is used to illustrate the effects of optimal stock investment decisions on expected financial structure and risk and to determine the effects of including bankruptcy costs on optimal investment decisions. The paper concludes with a discussion of limitations and implications for further research.

1.1 Farm Model With Stock Investment

This section formulates a model of an Illinois hog finishing operation which is then expressed as a stochastic dynamic programming model which will be used to determine optimal stock investment decisions.

Suppose that a manager is considering the investment of funds in nonagricultural opportunities such as common stocks and money market instruments. Each month, net revenue (gross margin per hog (R_t) times the number of hogs sold) is earned by the marketing of finished hogs. The manager considers two uses for the funds generated by the farm operation. First, stock market investment which is achieved through a stock market index mutual fund. The value of stock holdings then is the number of fund units held (S_t) times the value of the index or stock price (P_t).

Returns to stock investment are the changes in the price of stock from period to period plus any dividend income. The second use of funds is investment in financial instruments (F_t) which earn a return equivelant to the short term rate of interest (I_t). Holdings of financial instruments may represent an asset when held in positive amounts or a liability when sold short (i.e., operating credit). Purchases or sales of financial instruments can occur each month. Any excess cash flows from farm operations or stock dividends and sales are used to purchase financial assets and represent a positive investment. Short term cash requirements for farm operations, family consumption, debt payment or stock investment are provided by sale of financial instruments (i.e., borrowing) and represent a negative investment.

Beginning wealth is defined as the sum of the values of farm assets (FA) and nonfarm assets. Nonfarm assets are described by the state variables for stocks (P_t,S_t) and financial instruments (F_t). Thus,

$$(1) \quad Wealth_t = FA + P_t S_t + F_t$$

Farm assets include those assets dedicated to the production of finished hogs such as buildings, equipment, land and inventory. It is assumed here that hog finishing is the only farming activity being undertaken and that the land base is minimal.

At the beginning of each period the investor makes stock purchase/sell decisions[1]. The decision variable DS_t is the number of units of stock purchased or sold. The holdings of stocks (S_t) is equal to:

$$(2) \quad S_t = S_{t-1} + DS_t$$

The stock investment decision results in a cash flow equal to the value of stocks bought or sold, $DS_t P_t$. The investment decision also affects the level of financial holdings (F_t) since stock investment represents a cash outflow and sales produce a cash inflow. Once portfolio decisions are made the investor realizes income.

$$(3) \quad Inc_t = (R_t - FC)NH + (F_{t-1} - DS_t P_t - With) \cdot (I_t + BLD(F_{t-1}, DS_t, P_t)) + Div P_t S_t$$

where

(a) $(R_t - FC)NH$ is net revenue from farm operations and is defined as gross margin minus fixed costs per hog (FC) times the number of hogs marketed (NH).

(b) $(F_{t-1} - DS_t P_t - With) \cdot (I_t + BLD(\cdot))$ is interest revenue (cost) if the first expression in brackets, which represents the effects of stock purchase/sell decisions and consumption withdrawals on financial holdings, is greater than (less than) zero. *With* represents monthly consumption withdrawals, and $BLD(\cdot)$ is a borrowing/lending differential on interest rates equal to 0 for savings (financial holdings greater than 0) and 3 percent annually on operating credit (financial holdings less than 0).

(c) $Div P_t S_t$ is dividend income from stock holdings calculated as a percentage of stock value.

End of period financial holdings are:

$$(4) \quad F_t = F_{t-1} + Inc_t - DS_t P_t - With$$

Changes in wealth, or retained earnings, equals

$$(5) \quad RE_t = F_t - F_{t-1} + (P_t - P_{t-1})S_t$$

The above equations represent the main elements of the dynamic programming model of portfolio choice for the hog feeder operation. Given the firm's wealth level at any point in time, and armed with information on past returns earned by the farm (R_t) and investments (P_t, I_t) and expectations of their future earnings, the manager makes decisions on the levels of investment (DS_t). Following investment decisions, income (Inc_t) is realized, withdrawals for consumption are made, and asset values change, all of which contribute to changes in wealth. The resulting wealth level represents the starting point for investment decisions in the following period. This recursive process is developed formally as a dynamic programming problem in the next section.

1.2 The Dynamic Programming Model

Empirical formulation of the dynamic programming model requires specification of the relevant state and decision variables, state transition equations and a recursive objective function. The stock investment model described above contains three stochastic state variables: hog returns (R_t), stock prices (P_t), and return on other financial instruments (I_t), and two deterministic state variables: holdings of other financial instruments (F_t), and stock holdings (S_t). The decision variable is the number of stocks to purchase or sell (DS_t).

1 It is assumed here that there are no transactions costs for purchases and sales of stocks or financial instruments. There are currently a wide range of mutual fund investment groups or families where investors can move capital between stock market and money market accounts without transactions costs. Transactions costs are no longer a significant factor in these types of investments.

The stock investment model is formulated as an expected terminal wealth maximization problem. Denoting the terminal year as T, terminal wealth can be written as a function of the state variables:

(6) $V_T(R_T,\ P_T,\ I_T,\ F_T,\ S_T) \ = \ (P_T S_T) \ + \ F_T \ + \ FA_T$

where $V_T(\cdot)$ is the recursive objective function for year T and *FA* are farm assets devoted to the production of finished hogs. This function leads to the following general recursive equation:

(7) $$V_{t-1}(R_{t-1}, P_{t-1}, I_{t-1}, F_{t-1}, S_{t-1}) = \underset{DS_t}{\text{Max}}\, E[V_t(R_t, P_t, I_t, F_t, S_t)]$$

where $E[\cdot]$ is the expectations operator and $V_t(\cdot)$ is the value of wealth assuming that optimal decisions are made.

This maximization is subject to the following state transition equations:

(8*a*) $R_t = f_1(R_{t-1})$

(8*b*) $P_t = f_2(P_{t-1}, I_{t-1})$

(8*c*) $I_t = f_3(I_{t-1})$

(8*d*) $S_t = S_{t-1} + DS_t$

(8*e*) $F_t \ = \ F_{t-1} \ + \ Inc_t \ - \ DS_t P_t \ - \ With$

Rather than discounting returns, as in a present value maximization, returns in the terminal wealth maximization problem are compounded. Within the stock investment model, compounding is achieved through the retained earnings equation (5) which is affected by beginning financial structure, stock investment decisions, consumption withdrawals, farm returns, stock investment returns and interest revenue or costs.

1.3 Estimation of Transition Probabilities

Numerical solution of the investment model requires state transition probabilities which, in this study, are derived from estimated state transition equations. The estimated equations describe a continuous Markovian probability density function which gives the future probability distribution for a state variable conditional on the current level of that variable

Monthly hog returns were based on budgets reported in the Livestock Meat Situation and Outlook Report published by USDA. Data were adjusted to reflect Illinois costs of production as closely as possible.

Monthly stock prices (S&P 500 index) were collected from the Standard and Poor's Statistical Reporting Service and dividend data were based on information provided by Ibbotson and Associates. Short term interest rate data were from the Economic Report of the President. All series covered the period from the beginning of 1974 to the third quarter of 1987.

Time series and regression techniques were used to identify and estimate the interrelationships among hog returns, interest rates and stock prices. The final forms of these relationships are:[2]

$R_t \ = \ 1.688 \ + \ .8177R_{t-1} \qquad \sigma \ = 8.058$

$P_t \ = \ P_{t-1} \ - \ .0064I_{t-1} \qquad \sigma_e \ = .0381$

$I_t \ = \ I_{t-1} \qquad \sigma_e \ = .0717$

$\sigma_{SI} \ = -.0035 \qquad \Rightarrow \rho_{SI} \ = -.1267$

1.4 Optimal Stock Investment Decision Rule

The previous sections describe the conceptual model of an Illinois hog finishing operation and the stock investment problem it faces, as well as describing the statistical nature of the stochastic state transition equations. The model determines optimal stock investment decisions toward an objective of maximizing expected terminal wealth in the presence of bankruptcy. Five state variables describe the economic characteristics of the world in which these decisions are made. Three stochastic state variables, hog returns (R_t), stock price (P_t) and interest rate (I_t), whose Markovian structures were given above, describe the earnings possibilities faced by the firm. Two deterministic state variables track the effects of optimal decisions on stock (S_t) and financial (F_t) holdings.

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2 Details of the estimation procedure are available from the senior author.

An optimal decision rule identifies the effects of changes in state variables on stock investment decisions. The optimal stock investment decision rule was derived using a value-iteration dynamic programming algorithm. Numerical solution required specification of discrete state and decision variable levels. A hog return range of -$20 to $40 was chosen to match historical variation. Five hog return intervals produced state levels of -20, -5, 10, 25, and 40 dollars respectively. The stock price range was chosen to be wide enough to allow for sufficient growth potential given a 5 year planning horizon and historical stock index growth values. Stock prices covered 10 intervals ranging from 100 to 325 and stock holdings ranged from 0 to 2,000 units in increments of 200. Financial instrument holdings covered the range -$350,000 to $350,000 in $70,000 increments. Interest rates ranged from 6 percent to 16 percent, matching the range over the time period for which data were collected, in two percentage point increments. This formulation resulted in 36,300 states. The stock purchase decision was allowed to take on values of -200 (sell), 0, or 200 (buy). Specifications for the model farm are chosen to match a large commercial finishing operation in Illinois. Farm assets are valued at $350,000, monthly production is 750 hogs, monthly consumption is $2000, fixed costs are $5 per hog, the borrowing differential is 3 percent annually and monthly stock dividends are .083 percent.[3]

For both models, calculation of the recursive objective function begins in the final year of the planning horizon, denoted here as year T. The final year's recursive objective function $\{V_T(\cdot)\}$ contains the wealth level for each state increment and represents the expected wealth maximizing objective. The optimal investment decisions and resulting recursive objective functions are calculated recursively. In any year t, the decisions that maximize the expected value of $\{V_{t+1}(\cdot)\}$ are found for all state increments associated with solvency. In the case of the bankruptcy model, optimal decisions were not calculated for state intervals which defined technical bankruptcy[4].

The large size of the optimal decision rule (a matrix with 36,300 elements for each stage) prevents a complete description within the paper. Decision rules produced by both models, beginning with the model which incorporates bankruptcy, are summarized below.

1.4.1 Optimal Stock Investment Decisions in the Presence of Bankruptcy

Figure 1 illustrates a portion the optimal stock purchase decision rule. Two economic effects can be identified in the decision rules graphed in Figure 1. The first is the effect of financial structure which can be described using panel A. For all stock price combinations, few stock purchases occur when financial holdings are negative and there are no purchases for financial holdings at or below -$280,000. For a stock price of 200, purchases begin at financial holdings levels above -$280,000, occurring at interest rates up to 10 percent by the time financial holdings reach $-140,000. Purchases occur up to the 12 percent level for financial holdings levels betweeen $-140,000 and $0. Purchases then climb up to the 14 percent region for the $70,000 to $350,000. This pattern is consistent across stock price levels. The financial structure effect described above can be explained by recalling that this formulation incorporates bankruptcy. In this model the investor is presumed to be striving to maximize expected terminal wealth, recognizing the wealth losses associated with bankruptcy. When the investor does not have sufficient funds in savings, borrowing occurs which explains the increased sensitivity of decisions to interest rates at low financial holdings levels. At very low wealth levels borrowing does not occur because the investor is close to technical bankruptcy (ie. negative wealth resulting from debts exceeding the value of assets), and the combination of interest payments on debt, and any drops in stock prices or farm losses would result in bankruptcy. Thus, stock purchases do not occur at low wealth levels.

Profits from the farm operation provide cash which may be used for investment or paying off debt. As hog returns improve, there is more cash available which results in an increase in investment. Space limitations prevent the graphical presentation of the effect of hog returns on optimal decisions. This returns effect is summarized by stating that, ceteris paribus, higher hog returns imply greater stock purchases.

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3 Asset values, production levels and fixed costs were based on budgets prepared for hog finishing operations in the Midwest. Stock dividends are based on historical averages and the borrowing/lending differential is based on banking industry averages.

4 Optimal decision rules were generated until the optimal decision rules converged; which occurred by month six of year three. Thus, the converged decision rule was applicable to all years up to the thirty months before the end of the planning horizon. For example, if the planning horizon is ten years long, the converged decision rule would be applicable from year one through to month six of year seven.

The optimal stock investment decision rule illustrates the effects of changes in levels of state variables on stock purchases. It is clear that changes in market conditions, hog returns and wealth all affect stock investment. The decision to purchase stocks as part of the farm portfolio must be made while recognizing the firm's current balance sheet and cash flow situations as well as expectations of future farm and financial market conditions. This suggests that static models which do not account for these effects are providing suboptimal results. The wealth effects of ignoring financial structure are evaluated below.

1.4.2 Optimal Stock Investment Decisions in the Absence of Bankruptcy

The no bankruptcy model produces markedly different decision rules. The stock purchase decisions in this model are fairly simple and may be described by the phrase, buy stock. For example, when stock holdings are zero and stock price is 200, the optimal decision rule indicates that stock should be purchased at almost all combinations of financial holdings and hog returns. The only time that stock purchases are not optimal is for some combinations of financial holdings and hog return levels when interest rates exceed 12 percent. The wealth maximizing decision maker who does not consider the cost of bankruptcy simply chooses the highest yielding investment which in this case is almost always the stock market. The financial structure effect discussed above is not evident because the decision maker is allowed to continue borrowing money as long as the expected return warrants it. The hog return effect is also not evident because the investor has a ready source of investment capital and need not rely on farm operations as a source of capital.

The inclusion of bankruptcy yields different optimal decisions from a model which ignores this constraint. We should now ask whether or not this difference is important to the investor's financial well being. The following chapter contains a conditional probability analysis to determine how stock investment affects expected financial progress of the farm business and compares the profitability and risk levels arising from the two decision rules.

1.5 CONDITIONAL PROBABILITY ANALYSIS

The optimal decision rules described in the previous chapter show how stock investment decisions change with different combinations of state variable values. The optimal decisions for the firm will change through time because values of the state variables change. Market conditions will vary over time and the financial structure of the firm will change due to past decisions and realized returns. In order to evaluate the effects of optimal decisions on the financial status of the business, it is important to probabilistically track the changes to the firm as it follows the optimal investment strategy through time.

In this section, discrete conditional probability methods (Howard) are used to determine ex ante probability forecasts of investor wealth assuming that optimal investment decisions are made each month. This analysis is conducted to investigate the following issues: 1) What are the effects of stock investment on the level and riskiness of wealth compared to a farm with no stock investment opportunities and how do these results change given different beginning wealth levels? 2) How do the different assumptions about bankruptcy affect the level and riskiness of wealth?

1.5.1 Financial Effects of Stock Investment

Consider the case of a beginning state with no stock holdings, hog returns of $10, a stock price of 150, and a 10 percent interest rate. Conditional probabilities were calculated for different initial financial holdings levels ranging from -$140,000 to $140,000.

The relative financial performance of the farm firm with and without stock purchases are summarized in table 1. As expected, greater initial financial holdings result in higher levels of terminal wealth and lower probabilities of bankruptcy. The riskiness of hog finishing, especially in the case of low financial reserves, is illustrated by the high bankruptcy probabilities.

The effects of stock purchases are quantified by comparing stock and no stock model results. At $0 beginning financial holdings, following the optimal stock purchase decision rule over a five year period resulted in an expected increase in wealth of $35,784. In all cases, expected terminal wealth is higher when stock purchases are made compared to the no stock model. The differences between stock and no stock are larger for higher beginning financial instruments holdings. Stock purchases also resulted in slight increases in the probability of bankruptcy, suggesting an increase in risk.

The results also suggest that off-farm investments provide positive returns and should be considered further as potential uses of capital. This study only considered one possible investment, "the stock market". It may be possible to find more suitable investments in portions of the stock market which provide wealth enhancing and/or risk reducing effects. The choice would be determined in part by the objective function of the decision maker. This could be another subject for future study.

1.5.2 A Comparison of Model Formulations

The previous section describes the results of the conditional probability analysis of the effects of stock investment decisions on firm wealth in the presence of bankruptcy costs. The optimal decisons for the no bankruptcy case did not show the sensitivity to financial structure and farm returns that was evident in the bankruptcy model results. The effects of this difference in decisions is evaluated below in terms of expected wealth and probability of bankruptcy. A comparison of these results clearly shows the costs associated with ignoring bankruptcy costs in making stock investment decisions. At the end of year 1, the probability of bankruptcy for the bankruptcy included model is only .003 while the no bankruptcy model has a bankruptcy probability of .907. By the end of year 5 these probabilities are .195 and .999 respectively. Optimal decisions in the nobankruptcy model were not sensitive to financial structure. By not considering bankruptcy costs, investment decisions result in such high debt levels that bankruptcy is almost assured. These results are useful in illustrating the importance of recognizing the effects of investment decisions on financial structure and the costs which may result.

1.6 SUMMARY AND CONCLUSIONS

The effects of stock purchases are quantified by comparing stock and no stock model results over a range of beginning wealth levels. In all cases, expected terminal wealth is higher when stock purchases are made with the differences widening for higher beginning financial instruments holdings. Stock purchases also resulted in slight increases in the probability of bankruptcy suggesting an increase in risk. As expected, higher initial wealth implies greater ending wealth and a lower probability of bankruptcy.

The costs associated with ignoring bankruptcy costs in making stock investment decisions are also identified. Failure to account for bankruptcy in determining the optimal decision rule results in very high probabilities of bankruptcy. By not considering bankruptcy costs, investment decisions result in such high debt levels that bankruptcy is almost assured. These results are useful in illustrating the importance of recognizing the effects of investment decisions on financial structure and the costs which may result.

A dynamic programming model was chosen for this analysis because it provides more information than is available from static investment models. The dynamic investment model provides an operational investment strategy which identifies the effects of stock prices, interest rates, financial structure and hog returns on stock investment decisions. Static models do not provide this information. Future research may be directed at determining the relative costs of using static investment models compared to models which incorporate more information. The dynamic approach also provides ex ante forecasts of future financial structure and investment mix when the optimal decision rule is followed. The extra information obtained from the dynamic model should be of interest to decision makers and researchers alike.

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Note: The lines on these graphs identify the regions over which a particular investment action would occur.

Figure 2 Portions of the Optimal Decision Rule for Stock Holdings of 400 and Hog Returns of $10

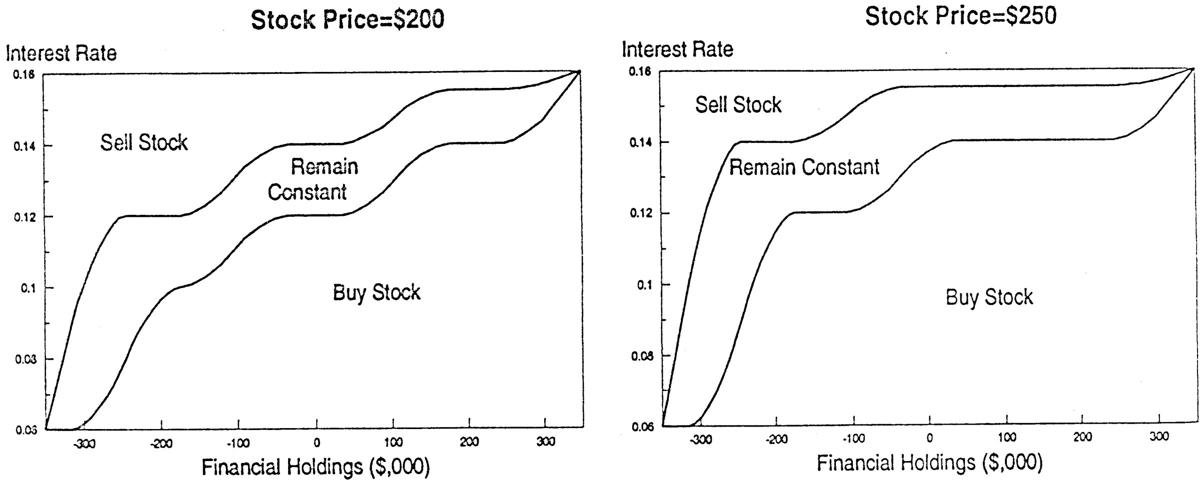

Table 1: Expected wealth and probability of bankruptcy in year five for stock and no stock models.

	Expected Wealth	Probability of Bankruptcy
Stock		
Beginning Financial Holdings		
-$140,000	338,062	.443
$0	583,115	.208
$140,000	830,373	.087
No Stock		
Beginning Financial Holdings		
-$140,000	316,523	.421
$0	547,331	.179
$140,000	782,212	.059