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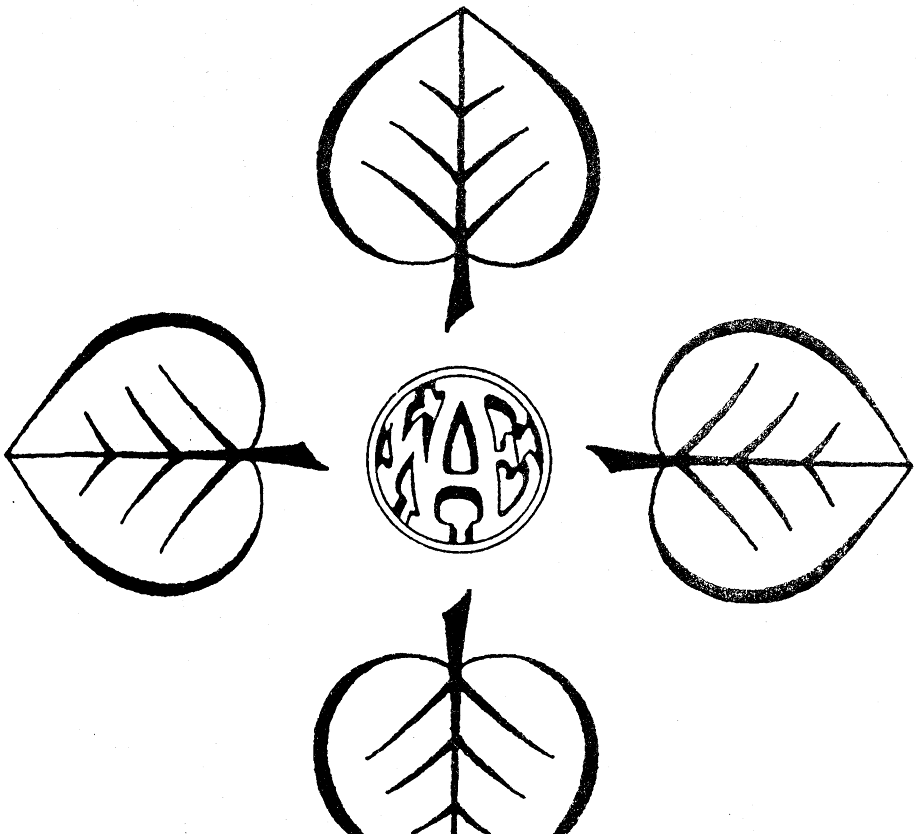

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FUNCTIONAL FORM AND ANALYTIC SIMPLIFICATION IN AGRICULTURAL ANALYSIS

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Abstract

Parametric tests of production nonjointness and separability were conducted in four geographically diverse states. Using three locally flexible functional forms, results indicate that some model simplification is clearly justifiable in all states. The extent of justified simplification, however, is affected both by state and choice of functional form.

Introduction

Neither economic theory nor standard differentiability assumptions is sufficient to determine a priori whether outputs can be consistently aggregated or whether their supplies can be examined independently of other output supplies without adversely affecting the reliability of the statistical estimates. Production independence or consistent aggregation is possible when production is either nonjoint or homothetically separable, respectively. These are structural properties of technology which, when valid, justify considerable analytic simplification in modeling the technology and/or economic relationships, thus conserving degrees of freedom and often reducing collinearity. Therefore, formal hypothesis testing is required to determine whether the analytical simplification of independent production or consistent aggregation is justified.

Despite their practical analytical importance, these structural properties of the technology are seldom submitted to formal testing. In some studies, parametric testing has been conducted, but the testing for a given property has been limited to the use of a single functional form (e.g., Ball; Moschini; Pope and Hallam). Empirical evidence (Berndt, Darrough, and Diewert; Chalfant) shows that a common limitation for the generalization of such results is the unknown, and often high, sensitivity of important model implications to the functional form used in modeling.

This paper will examine statistical support, or lack thereof, for independent modeling of individual agricultural output categories and for consistent aggregation. Parametric tests of necessary and sufficient conditions for independent short-run output relationships and consistent aggregation will be performed using three locally flexible functional forms (translog, generalized Leontief, and normalized quadratic) in each of four geographically dispersed major agricultural states: Texas, California, Iowa, and Florida. The objectives will be to determine which simplifying test conclusions are independent of functional form choice and to determine whether test conclusions are broadly generalizable.

Theoretical Framework

Assuming that each state's collection of producers behaves like a price-taking, profit maximizing firm with a state-level aggregate production function, each state was modeled as though it were a perfectly competitive firm. Homothetic separability of the technology is necessary for consistent aggregation of both quantity and price indices (Pope and Hallam). With perfect competition in output and variable input markets, homothetic separability of the technology in a partition of quantity variables implies and is implied by homothetic separability in the corresponding partition of price variables in the dual restricted profit function. Nonjointness implies that off-diagonal elements in the output submatrix of the profit function's Hessian matrix are zero. Both short-run nonjointness and homothetic separability tests were conducted using the system of first derivative equations of the restricted profit function.

(1) $\pi = x_0^*(P,Z) + P'X^*(P,Z) = \pi^*(P,Z)$,

where π is profit (receipts less variable costs) divided by the price of netput 0, $P = (p_1,\ldots,p_m)$ is the vector of output and variable input prices divided by the price of netput 0, $Z = (z_{m+1},\ldots,z_n)$ is the vector of fixed input quantities and other non-price exogenous variables, x_0^* is the profit-maximizing quantity of netput 0, and $X^* = (x_1^*,\ldots,x_m^*)$ is the vector of profit-maximizing netput quantities (positively measured for outputs and negatively measured for inputs) and are functions of the exogenous variables P and Z.

Since the researcher never knows the true functional form, tests were conducted with three forms -- translog, generalized Leontief, and normalized quadratic. Each is a second-order Taylor series expansion, is linear in parameters, and is appropriately labeled a "locally flexible" functional form. For consistency with the competitive theory and a twice-continuously-differentiable technology, linear homogeneity of the profit function in prices was maintained through normalization, and symmetry conditions among the system of first derivative equations were imposed via linear parameter restrictions.[1]

For the translog functional form, the estimation system was the share equations for the variable netputs (exclusive of netput 0):

(2) $p_i x_i/\pi = s_i = b_i + \Sigma_{j=1}^{m} b_{ij} \ln p_j + \Sigma_{j=m+1}^{n} b_{ij} \ln z_j$, for $i=1,\ldots, m$,

where s_i is the i^{th} netput share of profits. For the generalized Leontief the estimation system consisted of the output supply and input demand equations,

(3) $x_i = c_i/p_i^{.5} + c_{ii} + \Sigma_{j=1,\, j\neq i}^{m} c_{ij} p_j^{.5}/p_i^{.5} + \Sigma_{j=m+1}^{n} c_{ij} z_j^{.5}/p_i^{.5}$, for $i=1,\ldots, m$.

And, for the normalized quadratic, it was the supply and demand equations,

(4) $x_i = d_i + \Sigma_{j=1}^{m} d_{ij} p_j + \Sigma_{j=m+1}^{n} d_{ij} z_j$, for $i=1,\ldots,m$.

Global short-run nonjointness required that the following linear restrictions be satisfied:

(5) $c_{ij} = 0, \forall i \in P^S; j = 1,\ldots,\ell; i \neq j$,

for the generalized Leontief functional form, and

(6) $d_{ij} = 0, \forall i \in P^S; j = 1,\ldots,\ell; i \neq j$,

for the normalized quadratic. Testing nonjointness using the translog functional form could only be done locally. At the point of approximation, i.e., $\ln p_i = 0, \forall i$, short-run nonjointness implied:

(7) $b_{ij} = -b_i b_j, \forall i \in P^S; j = 1,\ldots,\ell; i \neq j$.

The restricted profit function was homothetically separable in P^S if any one of three sets of parametric restrictions were satisfied for a given functional form (Shumway). Necessary and sufficient conditions for homothetic separability include both linear and nonlinear test restrictions. For the translog, sufficient conditions were either:

(8a) $b_i/b_j = b_{ik}/b_{jk}, \forall i,j \in P^S, \forall k$; or

(8b) $\Sigma_{k\in s} b_{ik} = 0, \forall i\in P^s$, and $b_{ik} = 0, \forall i\in P^s, \forall k\notin P^s$; or

(8c) $b_i/b_j = b_{ik}/b_{jk}, \forall i,j,k\in P^s$, and $b_{ik} = 0, \forall i\in P^s, \forall k\notin P^s$.

For the generalized Leontief, they were either:

(9a) $c_i/c_j = c_{ik}/c_{jk}, \forall i,j\in P^s, \forall k$; or

(9b) $c_i = 2\Sigma_{k=1}^{n} c_{ik}, \forall i\in P^s$, and $c_{ik} = 0, \forall i\in P^s, \forall k\notin P^s$; or

(9c) $c_{ik} = 0, \forall i\in P^s, \forall k$.

For the normalized quadratic, they were either:

(10a) $d_i/d_j = d_{ik}/d_{jk}, \forall i,j\in P^s, \forall k$; or

(10b) $d_i = \Sigma_{k=1}^{n} d_{ik}, \forall i\in P^s$, and $d_{ik} = 0, \forall i\in P^s, \forall k\notin P^s$; or

(10c) $d_{ik} = 0, \forall i\in P^s, \forall k$.

Data and Model Specification

Annual state-level data for Texas, California, Iowa, and Florida for the period 1951-1982 were used in this study. Output and input prices and quantities were obtained from the data set compiled by Evenson and associates at Yale University. Pesticide price and quantity data were obtained from McGath at the Economic Research Service. Sources of government policy and weather data were McIntosh, and Teigen and Singer, respectively.

Because of the large number of individual commercial outputs (as many as 25 in some states) and input categories (8), it was necessary to initially aggregate the data. Based on common nonrejected deterministic and stochastic nonparametric tests of separability using 1956-1982 data for each of these states (Lim), the data were aggregated into four output categories (crops, meat animals, milk-poultry, and other livestock) and three variable input categories (labor-capital, materials, and pesticides).

Effective diversion payments and effective support prices were specified following Houck and Ryan. Guided by Lim's findings, one-year lagged prices were used as anticipated output market prices. Following Romain, expected prices of farm program commodities were specified as weighted averages of the anticipated market price and effective support price. Weather variables were monthly averages of temperature and precipitation for critical growing months, weighted by cropland. Therefore, exogenous variables included in the models were expected output prices, current variable input prices, quantities of the fixed inputs (family labor and land), time (included as a proxy for disembodied technical change), temperature, precipitation, and effective diversion payments.

Estimation and Tests

A system of four output supply (or share) equations, and two input demand (or share) equations was estimated for each state and functional form as specified in equations (2), (3), and (4), for the translog, generalized Leontief, and normalized quadratic, respectively. The capital-labor input price was used to normalize all other output and variable input prices.

Error terms associated with each supply and demand (or share) equation were assumed to be normally and independently distributed but contemporaneously correlated across equations. Since nonlinear parameter restrictions were required for some of the structural tests, efficient estimation was accomplished by performing nonlinear estimation utilizing the iterative version of Zellner's seemingly unrelated regression. Following Gallant and Jorgenson, an asymptotically valid chi-square test at the 0.01 level of significance was used for all tests.

Short-run nonjointness in inputs was tested for all outputs, for each pair of outputs, and for individual outputs by sequentially imposing the restrictions for the respective subset outlined in equations (5)-(7) for the various functional forms.[2]

Guided by Hall's impossibility theorem of nonjointness and weak separability for a linear homogeneous production function, tests of the hypothesis of homothetic separability were conducted in partitions of outputs for which nonjointness was rejected by one or more tests. These separability tests were performed exhaustively by utilizing each of the three sufficient tests. For a given functional form, nonrejection of any of the three sufficient tests implied that the technology was homothetically separable in that partition for that state. To determine whether the conclusion was dependent on choice of functional form, the tests were conducted for each functional form.

Empirical Results

Short-Run Nonjointness

The results of all short-run nonjointness tests conducted for each of the four states using each of the three functional forms are reported in table 1. Short-run nonjointness of all four output categories was not rejected in either Texas and California using any functional form. For Iowa this hypothesis was not rejected using two functional forms, but it was rejected using the third (translog). For Florida, findings were the opposite of Iowa.

A similar pattern was found for pairs of outputs. Except for the meat animals and milk-poultry pair using the translog[3] and normalized quadratic in Texas, nonjointness was not rejected for any pair of output categories using any functional form in either Texas or California. For Iowa, short-run nonjointness was not rejected only for crops and other livestock and for milk-poultry and other livestock using any functional form. In Florida, short-run nonjointness in pairs of output categories was not rejected with any functional form only for crops and meat animals.

With the exception of meat animals, short-run nonjointness also could not be rejected for any individual output in Texas or California. For this output category, the hypothesis was rejected in both states using the normalized quadratic but not using either of the other two functional forms. In Iowa, short-run nonjointness was not rejected for any individual output using either the generalized Leontief or normalized quadratic functional form. When using the translog, nonjointness could not be rejected for only two individual output categories, crops and other livestock. In Florida, short-run nonjointness was not rejected for any individual output category using the translog. Only for crops and meat animals was the hypothesis not rejected using any functional form.

Short-run nonjointness of all outputs can be justifiably maintained in subsequent model design only where it was not rejected by any of the three functional forms at any level. With one exception, the same logic applies when considering whether to maintain nonjointness for a given pair of outputs. Because it is not possible to have joint production of only one output, rejection of the nonjointness hypothesis for a single output (when nonjointness was not rejected for any other output, any pair of outputs, or for all outputs) is not a meaningful rejection.

Short-run nonjoint production was not rejected using any functional form for all outputs or for any pair of outputs in California. Short-run nonjoint production was not rejected in Texas for all outputs using any functional form. However, since it was rejected using the normalized quadratic both for the meat animals and milk-poultry subset and also for meat animals as an individual output, clear justification for maintaining short-run nonjointness in Texas applies only to crops and other livestock. Although specific test results differ between states, the same conclusion applies to Iowa. In Florida, only the crops and meat animals subset can be treated as nonjoint in the short run.

These results provide justification for modeling short-run supplies for each of the four output categories in California without considering changes in any other output category price. Texas and Iowa output supplies for crops and other livestock can be modeled without considering any output prices other than the own-category price. The same model simplification is implied for crops and meat animals supplies in Florida.

Homothetic separability

Following the logic of Hall's impossibility theorem, homothetic separability was tested for all output partitions for which nonjointness was not clearly justified. These partitions included the meat animals and milk-poultry subset in Texas and Iowa and the milk-poultry and other livestock subset in Florida. Test results using all three functional forms are reported in table 2.

Homothetic separability test results provided no further justification for model simplification. With the exception of Florida, each sufficient test was rejected for each state with each functional form. The exception in Florida occurred because convergence was not obtained for the normalized quadratic and generalized Leontief for one of the three sufficient tests.

Conclusions

Dual models of agricultural production for Texas, California, Iowa, and Florida using the translog, generalized Leontief, and normalized quadratic locally flexible functional forms were specified. Each state was modeled as a competitive industry with a twice-continuously-differentiable multiproduct transformation function and facing exogenous output and variable input prices. The initial model specification included four output and three variable input categories based on separability hypotheses not rejected by prior nonparametric tests. Exhaustive dual tests of short-run nonjointness (production independence) and homothetic separability (consistent aggregation and two-stage choice) of outputs were conducted to determine potential for analytic simplification and to determine whether conclusions were dependent on choice of functional form.

Short-run nonjointness was not rejected by any functional form for some or all of the four output categories in each state. However, homothetic separability was rejected by all functional forms in all tested partitions of outputs in each state. Given the empirical models designed for this study and the use of three equally plausible functional forms, justification for legitimate analytic simplification was provided only in the form of imposing nullity restrictions on the matrix of independent parameters requiring estimation. Although additional possibilities for consistent aggregation were consistently rejected, degrees of freedom can be conserved by maintaining short-run nonjointness in final model design. The importance of empirical testing with data sets of concern using a variety of plausible functional forms is clearly documented by the sensitivity of conclusions both to state and functional form.

Footnotes

1. Two other properties of the profit function, convexity and monotonicity, are implied by the competitive theory but were not maintained in this study. Jorgenson and Lau, pp. 71-72, have shown that the assymptotic properties of the structural tests are the same with and without convexity being maintained. Monotonicity was not imposed since prior empirical work (e.g., Moschini) has shown this property to be rarely violated.

2. There are no independent nonjointness tests for a subset that consists of three output categories. Nonjointness of any subset of three outputs implies nonjointness of all four.

3. No conclusion is available for this test with the translog since convergence was not obtained.

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Table 1. Short-Run Nonjointness Test Results Using Three Locally Flexible Functional Forms

OUTPUT	χ^2 Statistic												Initial Value $\chi^2_{.01}$[b]
	TEXAS			CALIFORNIA			IOWA			FLORIDA			
	TL	GL	NQ[a]	TL	GL	NQ	TL	GL	NQ	TL	GL	NQ	
All Outputs	10.07	13.08	15.36	7.07	5.21	14.91	17.33[e]	8.61	10.62	11.37	18.00[e]	47.37[e]	16.81
Crops, Meat Animals	7.85	13.74	14.66	7.17	4.64	14.03	16.33[e]	8.61	10.24	0.91	9.86	10.14	15.09
Crops, Milk-Poultry	10.05	9.59	8.31	7.05	4.51	11.09	17.33[e]	8.50	10.06	9.47	18.09[e]	45.58[e]	15.09
Crops, Other Livestock	10.11	6.14	11.39	2.19	5.20	12.18	8.15	4.10	5.01	11.37	10.71	24.16[e]	15.09
Meat An., Milk-Poultry	--[c]	12.99	15.31[e]	7.61	5.19	14.62	24.70[e]	8.40	10.24	10.31	17.95[e]	47.00[e]	15.09
Meat An., Other. Live.	9.97	11.70	11.97	2.30	4.09	14.91	24.37[e]	8.54	10.49	9.85	18.88[e]	46.73[e]	15.09
Milk-Pou., Other Live.	6.18	11.56	14.17	1.21	5.17	10.50	14.17	6.32	7.65	9.85	16.36[e]	44.05[e]	15.09
Crops	4.12	1.29	4.00	1.54	2.03	4.52	7.11	3.11	4.57	6.74	6.15	4.37	11.35
Meat Animals	0.96	11.25	11.40[e]	1.88	3.13	13.65[e]	23.92[e]	8.41	9.73	2.57	7.19	7.97	11.35
Milk-Poultry	0.47	8.69	7.97	0.63	4.49	6.23	14.11[e]	6.12	6.73	3.47	13.63[e]	42.07[e]	11.35
Other Livestock	1.13	4.54	7.26	0.58	3.44	8.24	0.78	0.94	0.64	7.78	8.46	18.58[e]	11.35

[a] Functional form codes: TL is translog; GL is generalized Leontief; NQ is normalized quadratic. [b] $\chi^2_{.01,6}$ for nonjointness test; $\chi^2_{.01,5}$ for test of pair; $\chi^2_{.01,3}$ for test of individual output category. [c] Convergence not obtained by nonlinear ITSUR procedure. [e] Means hypothesis rejected at .01 level of significance.

Table 2. Homothetic Separability Test Results for Texas, Iowa, and Florida Using Three Locally Flexible Functional Forms[a]

Separable Group	State	Functional Form[b]	TEST A[c]		TEST B[d]		TEST C[e]	
			χ^2statistic	Critical Value $\chi^2_{.01,8}$	χ^2statistic	Critical Value $\chi^2_{.01}$[f]	χ^2statistic	Critical Value $\chi^2_{.01}$
Meat Animals and Milk-Poultry	Texas	Translog	88.84	20.09	153.29	29.14	173.31	29.14
		Gen. Leont.	91.03	20.09	584.79	30.58	577.46	30.58
		Norm. Quad.	25.05	20.09	661.58	30.58	73.60	30.58
	Iowa	Translog	24.76	20.09	100.43	29.14	97.87	29.14
		Gen. Leont.	47.19	20.09	357.87	30.58	621.88	30.58
		Norm. Quad.	59.24	20.09	294.05	30.58	95.61	30.58
Milk-Poultry and Other Livestock	Florida	Translog	25.28	20.09	46.44	29.14	46.39	29.14
		Gen. Leont.	NO CONVERGENCE		280.58	30.58	278.52	30.58
		Norm. Quad.	NO CONVERGENCE		254.35	30.58	64.50	30.58

[a] All hypotheses were rejected at the .01 level of significance. [b] See equations (8a), (9a), (10a). [c] See equations (8b), (9b), (10b). [d] See equations (8c), (9c), (10c). [e] TL: $\chi^2_{.01,14}$; GL and NQ: $\chi^2_{.01,15}$.