Classification of Risk Preferences with Elicited Utility Data: Does Functional Form Matter?

Wesley N. Musser, Michael E. Wetzstein, Susan Y. Reece, Lynn M. Musser, Philip E. Varca and Charlene C. J. Chou

Recently, several problems with elicited utility functions have emerged. This paper concerns a fundamental problem in risk preference classification with elicited data. For the sample in this research, different functional forms resulted in reversals in preference classifications. This paper suggests that preference classifications must be interpreted with caution.

Agricultural economists have estimated utility functions for both normative and positive research on risky behavior. Initial experience with estimating utility functions created optimism concerning the usefulness of this technique in research and extension—Anderson *et al.* is an example of this viewpoint. More recently several studies have dampened this optimism. Binswager concluded that direct elicitation procedures do not yield reliable data over time and among different interviewers. Whittaker and Winter supported this conclusion. They obtained significant regression coefficients of opposite sign from regressing risk aversion coefficients from utility functions elicited at two times on socioeconomic variables. In recent review articles of risk analysis, Young and Roumasset concluded that approaches other than direct elicitation appear to be more fruitful research approaches.

This paper considers choice of functional form, which is another problem in estimation of utility functions. Lin and Chang have previously noted that selection of appropriate functional form can improve specifications of optimal farm organization plans. The research reported in this paper concerns even a more fundamental problem—different functional forms can lead to different individual risk preference classifications. For the majority of the individuals in this study, a different functional form led to switches among the categories of risk preference, risk indifference, and risk aversion. This particular problem arose in the initial phases of a multidisciplinary study to evaluate elicitation methodologies. The research procedures are designed to accommodate problems found in previous studies and are discussed in the next section.

Design of Elicitation Procedure

Risk preferences are usually elicited in the context of generalized games (Ander-

Wesley N. Musser is Visiting Professor, Department of Agricultural and Resource Economics at Oregon State University, Michael E. Wetzstein is Associate Professor, Department of Agricultural Economics at University of Georgia, Susan Y. Reece is Graduate Student, Department of Psychology at University of Georgia, Lynn M. Musser is Assistant Professor, Department of Psychology at Oregon State University, Philip E. Varca is Assistant Professor, Department of Psychology at University of Georgia, and Charlene C. J. Chou is former Graduate Research Assistant, Department of Agricultural Economics at University of Georgia. Wesley N. Musser was Associate Professor, Department of Agricultural Economics at University of Georgia when the research in this paper was completed.

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son *et al.*). Recently, Young and Musser and Musser have utilized psychological literature to criticize this standard procedure. A major problem with the generalized games arises from the existence of goals in addition to income or wealth which are not encompassed in the expected utility model (Patrick and Kliebenstein). Since these other goals are likely to vary among situations, risk preferences estimated from generalized situations will not likely predict behavior in specific situations. The proposition is derived from conclusions in psychological research on attitudes—only specific attitudes reliably predict behavior (Fishbein and Ajzen).

Psychological concepts also suggest that use of generalized games could contribute to unstable measures of risk preferences. Tversky and Kahneman have noted that people are not very good intuitive statisticians and tend to use heuristics in their probability judgments. One of these heuristics, *availability*, refers to the tendency of individuals to use the most accessible rather than comprehensive information in their probability judgments. It is plausible that individuals would use an experience from a recent specific situation for a reference point in judgments in a generalized elicitation procedure. If a different situation is used for reference at a later date, the risk preferences would probably vary because other goal achievement may differ in the two situations. If the risk preferences are elicited within a particular context familiar to the individuals, this potential source of unstable preferences may be eliminated. Some evidence in support of this view is provided by Officer and Halter. Their research used a specific setting, fodder reserves, and found stable preferences over time.

The data used in this study were elicited from 13 graduate students in an agricultural finance class at the University of Georgia in winter quarter, 1981. The paradigm used for elicitation involved graduate research assistantships. The utility function was elicited for income for one academic quarter ranging from $0 to $3,500. Preliminary questionnaires revealed this range of budgetary levels for graduate students. Certainty equivalents were elicited for 15 pairs of risky incomes. Thus, the elicitation game concerned a specific standard decision context for a range of income within current experience. The modified Von Neumann-Morgenstern procedure was utilized in the elicitation. This approach was easier to explain to non-economists in the multidisciplinary context of this research than the more popular Ramsey approach. Officer and Halter provide some support for this decision by obtaining similar results with both procedures.

Besides the use of a specific decision context, the elicitation procedure involves two major operational differences from the standard procedure (Anderson *et al.*, pp. 71–75) as suggested by psychological methodology. First, the individuals were directly asked their certainty equivalents for risky prospects rather than asked to choose among alternative certainty equivalents suggested by the interviewer. The approach was utilized to avoid the potential anchoring bias in the standard approach (Roumasset, p. 10–11). The use of a specific decision context made this direct approach more feasible. The other difference is that a check question procedure to ensure consistent responses among the different decisions was omitted. Such practices can force the data to be consistent with a limited set of initial responses, which may be biased, rather than the subsequent responses. Furthermore, the check question may allow the preconceptions of the interviewer to bias the responses.

Functional Forms

Lin and Chang suggested use of Box-Cox transformations of linear and quadratic functions as a general approach to the problem of specification of functional form for utility functions. Subsequently, Buccola (1982b) demonstrated that Box-Cox transformations and other functional forms, such as the power function, are inappropriate utility functions since they do not include an intercept. Based on this reasoning, a quadratic, semilog, and modified power functional forms were utilized in this research. The reasoning for these particular choices is delineated in the following discussion.[1]

As Young noted, several different measures have been used to classify the risk preferences of individuals. The sign of the second derivative of the function, $U''(M)$, is a common measure for this purpose where $U''(M) > 0$, $U''(M) = 0$, and $U''(M) < 0$ imply risk preference, indifference and aversion, respectively. The absolute risk coefficient, $r(M) = -U''(M)/U'(M)$, can also be used for classifications where $r(M) > 0$, $r(M) = 0$, and $r(M) < 0$ imply risk aversion, indifference and preference, respectively. The latter measure is particularly useful for positive analysis with individual risk preferences because it is unique and therefore allows interpersonal comparisons (Pratt). These measures can be readily derived for the three functional forms used in this research.

The quadratic expresses utility (U) as a function of money (M) as follows:

$$U = a + bM + cM^2 \tag{1}$$

where a, b, and c are parameters. For risk classification, $U''(M) = 2c$ and $r(M) = -2c/(b + 2cM)$. The sign of c, subject to the requirement that $b + 2cM > 0$, allows classification. The quadratic utility function was popular in early applications of expected utility theory. Beginning with Pratt, however, the quadratic began to fall into theoretical disfavor because it requires the intuitively implausible assumption of increasing absolute risk aversion. As Anderson *et al.* (pp. 94–95) and Buccola (1982a) argue, this function may still approximate preferences relevant for many short-run decisions in agricultural economics. Since this research was a component of a longer study which required classification and comparisons of risk preferences and their relationship to short-run behavior, this function was included in the analysis. Another advantage of the quadratic is its ease of estimation, which is particularly important for a large sample.

The semilog function is another alternative also having linear parameters:

$$U = a + b \log M \tag{2}$$

where a and b are parameters, and log is the natural logarithm operator. For this function, $U''(M) = -b/M^2$ and $r(M) = 1/M$. This function has the desirable property of imposing decreasing absolute risk aversion. All risk averse individuals will have the same value of r(M) for any value of M, thus severely limiting this function for many positive analyses. In addition, the function imposes risk aversion on utility data since $U'(M) > 0$ requires $b > 0$. Therefore, the function is not useful for many applications concerning risk preference classification. The function does provide an interesting contrast with the other, more flexible function forms considered in this paper.

The third functional form used in this paper is a modified power function:

$$U = a + bM^c \tag{3}$$

where a, b, and c are parameters. For this function, $U''(M) = (c - 1)(c)bM^{c-2}$ and $r(M) = (1 - c)/M$. This function must have $b > 0$ and $c \geq 0$. The value of c can be used to classify risk preferences: an individual is risk averse, risk neutral or risk seeking if $c < 1$, $c = 1$, or $c > 1$, respectively. The function does embody decreasing absolute risk aversion for risk averse individuals. Its main disadvantage

[1] Hildreth has suggested several other more complex functional forms, which are theoretically appropriate. One of the more popular is the exponential form. Buccola (1982a) has recently contrasted the exponential and quadratic forms.

TABLE 1. Quadratic Regression Results: $U = a + bM + cM^2$.

Subjects	a	b	c	Absolute Risk Coefficient[a]
A	1,077.33 (0.528)	7.642 (1.286)	−0.00186 (−0.488)	0.00325
B	1,039.46 (1.543)*	3.448 (2.638)***	−0.00026 (0.569)	0.00020
C	−13,002.7 (−3.078)***	16.7682 (3.564)***	−0.00319 (−2.703)***	0.00113
D	−11,697.3 (−1.029)	13.150 (1.168)	−0.00209 (−0.810)	0.00071
E	−2,452.34 (−1.893)*	7.164 (4.237)***	−0.00113 (−2.494)***	0.00070
F	−9,442.38 (−2.043)**	9.934 (2.165)**	−0.00127 (1.187)	0.00046
G	−42,327.3 (3.298)***	35.634 (3.446)***	−0.00626 (−3.244)***	0.00091
H	−2,948.68 (−1.126)	6.193 (1.820)**	−0.00059 (−0.557)	0.00028
I	−7,441.51 (−1.351)	6.024 (1.266)	−0.0034 (−0.350)	0.00014
J	−70,726.7 (−1.473)	46.604 (1.417)*	−0.00687 (−1.249)	0.00061
K	−770.32 (−0.693)	4.519 (3.037)***	−0.00044 (−1.048)	0.00029
L	−510.06 (−0.264)	2.623 (1.060)	0.00024 (0.329)	−0.00014

[a] Absolute risk coefficients are estimated at the mean, M = 1,750.
* Indicates a significance level of 0.10.
** Indicates a significance level of 0.05.
*** Indicates a significance level of 0.025.

is that nonlinear least squares procedures must be used for estimation which may be undesirable for a large sample of individuals.

After regression results are obtained, two approaches can be utilized to classify risk preferences for individuals. The simplest method uses the regression coefficients as point estimates of the appropriate parameters for classification with the different functional forms discussed above. The other approach employs student's t-tests to test hypotheses concerning the relation of the regression coefficients to the theoretical values of these parameters. Following King, the standard hypothesis that the regression coefficients equal zero may not be of the most theoretical interest. The above discussion implies that $c = 0$ and $b = 0$ are appropriate tests for the quadratic and semilog respectively. However, $c = 1$ is the appropriate test for the modified power function. These hypotheses tests are employed in the next section.

Results

The regression results for the three functional forms for the individuals in the sample are presented in Tables 1–3. Standard t-statistics for the hypothesis that the coefficient equals zero are included in parentheses. Absolute risk aversion coefficients estimated at the mean level of

TABLE 2. Semilog Regression Results: U = a + b log M.

Subjects	a	b	Absolute Risk Coefficient[a]
A	−8,390.8 (−1.690)*	2,152.24 (2.742)***	0.00057
B	−5,111.49 (−2.671)***	1,554.36 (5.498)***	0.00057
C	−54,345.2 (−6.155)***	8,056.27 (6.740)***	0.00057
D	−59,554.7 (−3.960)***	8,607.85 (4.302)***	0.00057
E	−27,959.0 (−7.223)***	4,605.28 (8.573)***	0.00057
F	−63,447.8 (−6.877)***	9,048.27 (7.436)***	0.00057
G	−43,531.8 (−2.365)***	6,130.28 (2.644)***	0.00057
H	−35,564.6 (−4.358)***	5,992.52 (5.349)***	0.00057
I	−73,136.7 (−7.701)***	10,075.6 (8.244)***	0.00057
J	−127,178.0 (−4.959)***	16,640.2 (5.159)***	0.00057
K	−19,933.7 (−5.266)***	3,488.21 (6.653)***	0.00057
L	−23,549.8 (−3.656)***	3,895.21 (4.446)***	0.00057

[a] Absolute risk coefficients are estimated at the mean, M = 1,750.
* Indicates a significance level of 0.10.
** Indicates a significance level of 0.05.
*** Indicates a significance level of 0.025.

$1,750 in the elicitation procedure are also presented. For the quadratic form, only C, E, and G have coefficients on the income squared that differ significantly from zero at the 95 percent confidence level (Table 1). These three coefficients are negative implying that these individuals are risk averse. All other individuals would be classified as risk neutral. However, the quadratic did not fit well for subjects A, D, I and L, as none of the coefficients were significant at 90 percent level. In contrast, the semilog results in Table 2 are more satisfactory statistically with all the b coefficients significant at the 97.5 percent confidence level, so that all individuals are classified as risk averse. Only individual L had a significant coefficient for the modified power function results in Table 3.

The risk classifications for the subjects from these regressions based on $U''(M)$ are summarized in Table 4. As indicated above, the semilog relationships impose a risk averse classification on all subjects. The nonlinear relationship exhibits the same risk classification for all subjects, risk indifference. Using the value of $(c - 1)$ to classify subjects, none of the c parameters are significantly different from one based on t-tests at a 90 percent confidence level. Thus, the quadratic function is the only relationship which does not classify all the subjects into the same risk preference classification. Subjects C, E, and G are classified as risk averse while all the other subjects are risk indifferent. The differences in classification are dramatic: not one subject is classified the same with all three functions.

Risk preferences can also be classified with the absolute risk aversion coefficient. Since the classifications are the same as with the second derivative, these are not explicitly discussed in this paper. The absolute risk aversion coefficient does allow comparison of the level of risk aversion among individuals except for the semilog function. The coefficients vary from −0.00014 for subject L, the least risk averse, to 0.00325 for subject A, the most risk averse subject for the quadratic function (Table 1). For the modified power function, the same individuals exhibit the extreme values ranging from −0.00014 for subject L to 0.00026 for subject A (Table 3). A Spearman rank coefficient test indicated that the risk aversion coefficients were associated at a 90 percent confidence level. These results suggest that the use of different functional forms may not alter the rankings of risk aversion of individuals for positive analysis. However, this con-

TABLE 3. Nonlinear Regression Results: U = a + bM^c.

Subjects	a	b	c	Absolute Risk Coefficient[a]
A	−222.902	152.746	0.553	0.00026
	(−0.035)	(0.161)	(0.715)	
B	649.033	21.173	0.748	0.00014
	(0.536)	(0.382)	(2.345)	
C	50.000	1.000	1.100	0.000006
	(0.004)	(0.030)	(0.281)	
D	50.0000	1.000	1.100	−0.00006
	(0.002)	(0.018)	(0.169)	
E	−748.299	11.177	0.843	0.00009
	(−0.188)	(0.180)	(1.301)	
F	27.711	1.036	1.096	−0.00006
	(0.002)	(0.040)	(0.374)	
G	−570.253	1.939	0.993	0.000004
	(−0.015)	(0.014)	(0.121)	
H	−807.105	2.006	1.076	−0.00004
	(−0.171)	(0.119)	(1.051)	
I	−785.702	1.842	1.031	−0.00002
	(−0.050)	(0.038)	(0.343)	
J	−715.579	2.168	0.994	0.000004
	(−0.005)	(0.005)	(0.041)	
K	−226.071	3.193	0.960	0.00002
	(−0.073)	(0.130)	(1.046)	
L	−169.800	0.461	1.244	−0.00014
	(−0.069)	(0.164)	(1.666)*	

[a] Absolute risk coefficients are estimated at the mean, M = 1,750.
* Indicates a significance level of 0.10.
** Indicates a significance level of 0.05.
*** Indicates a significance level of 0.025.

clusion is only tentative because many of the coefficients are calculated from insignificant regression coefficients.

Conclusion

This paper presents initial analysis of a potentially fundamental problem in the use of elicited utility data. Different functional forms can result in different risk preference classifications based on the same data. Two of the functions in this analysis classified most of the individuals as risk neutral while the semilog imposed risk aversion on all the individuals. The quadratic and the modified power function also indicated different preference

TABLE 4. Risk Preference Classification for Student Subjects.[a]

Subject	Quadratic $U''(M) = 2c$	Semilog $U''(M) = -b/M^2$	Nonlinear $U''(M) = (c-1)cbM^{c-2}$
A	I	A	I
B	I	A	I
C	A	A	I
D	I	A	I
E	A	A	I
F	I	A	I
G	A	A	I
H	I	A	I
I	I	A	I
J	I	A	I
K	I	A	I
L	I	A	I

[a] I stands for risk indifference, A stands for risk averse.

classifications for several individuals. These results indicate that caution must be observed in analyzing utility functions derived from a specified functional form. Past studies of risk preference classification, such as reviewed by Young, therefore, are not necessarily definitive. Estimation of several functional forms and comparison of their results is one method of increasing confidence in the risk preference classification. For example, most of the subjects in the analysis in this paper appear to be risk neutral when function forms that do not impose risk aversion are utilized. The lack of statistical significance in the quadratic and modified power function suggest utilization of more complex forms, especially for normative analysis. However, the computational disadvantages of nonlinear least squares may limit the usefulness of some of these alternative functions for positive analysis involving large samples.

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