Crop Irrigation Scheduling via Simulation-Based Experimentation

Hovav Talpaz and James W. Mjelde

A method for optimizing the irrigation schedule is presented. When the response surface, generated by "experimenting" with the crop simulation models is concave (convex), an optimal solution can be found. The process is iteratively repeated till convergence is achieved. Corn irrigation scheduling is demonstrated, with soil moisture levels as control variables.

Key words: design matrix, experimentation, quadratic programming, response surface, simulation.

Intraseasonal water allocation to a single crop has been studied extensively. Previous studies have emphasized the need to coordinate irrigation scheduling with several factors including economic and physical constraints, amount of soil moisture available at the time of irrigations, growth stage of the plant, interactive effects from previous and/or subsequent irrigation, and the effect, timing, and stochastic nature of weather conditions (e.g., Hagan; Shipley, Regier, and Wehrly; Jensen and Musick). Experimental data incorporating these factors which can be used in addressing the irrigation scheduling question are limited. The typical approach in obtaining these data has been to perform expensive and time-consuming field experiments at a specific location.

Recent developments in reliable crop-growth simulation models have presented opportunities for advanced production and policy planning in macro- and microeconomics (Boggess, Musser and Tew). The use of simulation models was widely accepted in agriculture even a decade ago (e.g., Johnson and Rausser); yet, in practice, simulation models have been cautiously implemented. One reason was the lack of efficient methodology for validation and calibration which is needed for the introduction and adaptation of a model into a new location and its environment. Developments in nonlinear parameter estimation using nonlinear optimization techniques (Little and Sall, Murtagh and Saunders) have made it possible to perform model estimation and calibrations adding to the reliability and applicability of simulation models. Such an implementation was performed by Talpaz, da Roza, and Hearn in calibrating a cotton simulation model. These developments coupled with decreasing research funds are leading to an increased use of crop-growth simulation models for production and policy analysis.

The use of crop-growth simulation models in irrigation scheduling is not a new concept. Numerical search techniques to determine the optimal irrigation strategies have been applied by Harris and Mapp (1980) and Zavaleta, Lacewell, and Taylor. The results from these studies indicate that optimal irrigation strategies use less water and energy and provide greater net returns than conventional water intensive irrigation strategies. A limitation of these studies is that the decision rules are developed *ex post* fashion, and therefore are not implementable at the farm level. Economic simulation studies incorporating crop-growth simulation models were conducted by Ahmed, van Bavel, and Hiler and Harris and Mapp (1986). Such studies can not guarantee an op-

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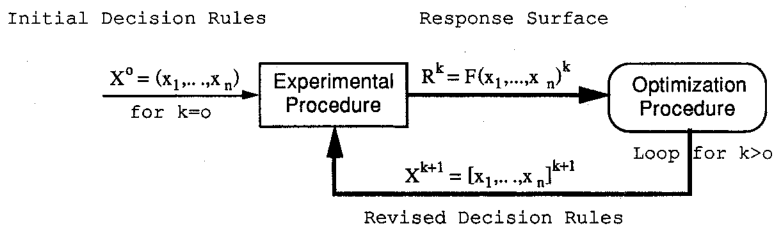

Figure 1. Interface between optimization and experimentation

timal irrigation strategy without simulating all possible irrigation strategies. Results reported by Harris and Mapp (1986) indicate that at least some water-conserving policies dominate by first-order stochastic dominance the more typical water-intensive irrigation strategy.

A methodological technique that has been employed to obtain *ex ante* decision rules is dynamic programming (DP). Burt and Stauber developed a DP model for irrigation investment and associated problems of scheduling for a sixty-day period for corn production. Stopping short of fully simulating crop response to water, Beilorai and Yaron developed an empirical response function of grain sorghum to soil moisture, which was later used in a sophisticated way by Yaron and Dinar in a DP-LP framework to allocate irrigation water optimally during peak seasons. However, the use of an estimated response to water in lieu of full-scale simulation tends to localize the results based on the empirical data used for the response function. In contrast, a theoretically oriented simulation model (i.e., where basic biological processes are determined functionally) has a wider spread of applicability, reliable response to dynamic changes of the environmental conditions and, most important, can respond functionally to the more realistic—stochastically behaved—environment. The Yaron and Dinar study was also limited by the deterministic setup governing the environment. McGuckin et al. state two especially important attributes for models designed to determine *ex ante* irrigation strategies: the model should account for stochastic weather conditions and should provide flexible decision rules.

Previous studies have indicated that there is a potential for increased water use efficiency in irrigation from developing optimal *ex ante* decision aids. This increased efficiency is in addition to that brought about by recent improvements in irrigation equipment (Mjelde et al.). When simulated production processes are optimized subject to real-world stochastic environmental conditions using stochastic experimentation, a powerful information base can be created to assist the decision-making process (Biles and Swain). The objective of this study is to illustrate the implementation of the methodology of optimization via experimentation for irrigation scheduling. This methodology combines the use of crop-growth simulation models and optimization techniques. Such a methodology may be applied beyond irrigation scheduling in agricultural economics, e.g., pesticide applications.

Optimization via Simulated Experimentation

Obtaining optimal irrigation scheduling under stochastic environmental behavior is fundamentally different from that under a deterministic environment (Bertsekas). In the former case we have to deal with uncontrollable and uncertain variables. The errors encountered under stochastic conditions reflect both the model's bias and the unforeseen environmental behavior. Therefore, the need for experimentation arises. The early foundation for optimization via experimentation was rigorously developed by Box and Hunter, following the initial conceptual work by Box and Wilson. Optimization via experimentation is a methodological procedure that combines the seemingly disjoint domains of experimentation and optimization. This procedure has been used in different, primarily nonagricultural industries (e.g., Biles; Farrell, McCall, and Russell). The interfacing of experimentation and optimization is illustrated in figure 1.

The objective of the optimization procedure is to take the current (kth) experiment responses (R^k) into account to provide new values, X^{k+1}, on the controllable inputs to be used in the next set of trials with the experimental procedure. This iterative process involves experimentation yielding R^k, and optimization yielding X^{k+1}, which is initiated at a selected set of starting values, X^0. The process is repeated until an optimal solution, X^*, is obtained.

In this study the experimental procedure employs a crop-growth simulation model. The response variables are the per-acre net returns generated from the simulation model over different weather patterns, while the control variables are soil moisture levels, where soil moisture is measured as a fraction of the moisture level at field capacity.

Many methods of optimization have been proposed in conjunction with experimentation (for a detailed discussion of these methods, see Biles and Swain). Our proposed combined technique is an extension of the quadratic response surface methodology (e.g., Myers).

The quadratic response surface methodology can be summarized as follows. First, an interpolating polynomial (second degree in our case) is fit, using least squares regression and is used as a local approximation to the function in the region of interest. In this regression, the response, R, is the dependent variable, while the decision vector, X^k, is composed of the decision independent variables. The approximation given by the estimated response is the basis for the second step of the overall methodology; the maximum of the fitted polynomial is solved for with respect to the decision vector, X^{k+1}. These values of X become the new independent variables of the first step, and so on. This iterative process continues until convergence is achieved.

Mathematically, forsaking rigor, this methodology is summarized by the following procedure: For the first iteration: Set the iteration index $k = 0$. For all other iterations: Set $k = k + 1$. Let X^k be a row vector containing the n elements of the decision set and $F(X^k)$ be the corresponding crop simulation objective function response (i.e., per-acre net income). If m such trials are considered, the corresponding information is held by $X^k[m \times n]$ matrix, and $F(X^k)$ m-element vector, respectively.

For the neighborhood of $X^k + \Delta X$, obtain a second-order approximation using Taylor's series:

$$(1) \qquad F(X^k + \Delta X) = F(X^k) + \nabla F(X^k)'X + \Delta X'\nabla^2 F(X^k)\Delta X,$$

and let

$$R^k = F(X^k + \Delta X),$$

where ΔX is a perturbation $[m \times n]$ design matrix (see an example in the appendix), ∇ is the vector of the gradient estimates with respect to X, and ∇^2 is composed of estimates of the hessian matrix for the function F.

Consider ΔX to be the hexagonal experimental design matrix (Box and Hunter), which is known to possess the orthogonality and rotatability characteristics and shown to be the most efficient of those considered by Montgomery and Evans.[1] Performing "experiments" amounts to the evaluation of the simulation model m times, each with a different set of irrigation triggers, or "treatment," specified by the corresponding row in the $(X^k + \delta\Delta X)$ matrix to obtain the response vector $R[m \times 1]$. From this response, a full quadratic function of the type $R = \mu'X + X'\Omega X$ (the quadratic response surface of the objective function) can be estimated using OLS regression. R is the estimated response; δ is a scaling unit of increment; μ and Ω are estimates of the vector of linear and quadratic coefficients, respectively. If Ω is negative (semi)definite, then concavity is guaranteed and the conditional global maximum can be found by solving the quadratic programming (QP) problem.

$$(2) \qquad \text{Maximize } Y = \mu'X + X'\Omega X$$
$$\text{S.T.}$$
$$(3) \qquad AX \geq B$$
$$(4) \qquad X \geq 0,$$

where A is a matrix of technological restrictions, X_{opt} are the values which maximize (2), and B is a vector of constraint levels. In the case study discussed below, equation (3) simply represents bounds on the x_j's. Note that additional iterations are necessary since μ and Ω are conditional on the original first guess $X^{k=0}$. Repeat using the newly obtained optimal values of X, namely, setting $X^{k+1} = X^k_{opt}$. The process ends if convergence is achieved or a prespecified maximum number of iterations has been performed. Only a few such iterations were necessary for the case study discussed below.

[1] For details, see Biles and Swain (1980), pp. 136–55.

A few technical remarks are necessary. Solving the QP problem equations (2) through (4) provides a fast convergence and can be performed using the QP solver designed by Harpaz and Talpaz (1986). Difficulties arise when Ω is not negative (semi)definite. In such cases an alternative approach can be taken, such as the linear (setting $\Omega = 0$) gradient projection algorithm as modified by Zoutendijk (see Bard). However, in such cases a local rather than global optimum may be achieved! Note that because of the stochastic setting, whenever convergence is achieved, it only converges in probability to the optimal strategy, local or global, respectively. A major consideration must be given to the specification of X. These decision variables must be carefully designed to provide meaningful decision space and be dimensionally low. Examples of possible decision variables for irrigation scheduling are dates of irrigation applications and quantities. However, a much better choice is the fraction of available soil moisture threshold. It can be shown that moving away from absolutely defined decision criteria like dates or quantities toward policy-natured definitions like those based on relative water stress or their proxies (i.e., soil moisture level), not only makes more sense economically but also increases convergence efficiency. Equation (3) usually determines the feasible region for the decision variable in X. For example, lower limits on soil moisture triggering irrigation.

Important as they are, the optimal strategies described above are not the only interesting information generated by the optimization process. Competing suboptimal strategies should be collected along the optimization process. Such information could provide useful data under real-world conditions where some aspect may not be quantifiable.

A Case Study: Irrigated Corn

The above methodology has been applied to the corn simulation model developed by Stapper and Arkin under Texas High Plains conditions. The model simulates daily corn growth and development. The growth process is sensitive to environmental factors such as temperature, rainfall, and soil moisture deficit. Soil moisture over the root zone is computed dynamically, accounting for water uptake (evapotranspiration and evaporation) and rainfall plus irrigation. It was calibrated for various genotypes at six different locations across North America. The code used here was extended and updated beyond the 1980 publication date (Jackson and Arkin).

Daily data on temperature, and rainfall for thirty years for the High Plains region of Texas comprise the environmental data set. The simulation model's parameters associated with soil characteristics have been adjusted to reflect the High Plains area. Prices reflecting current conditions for water and corn and other variable costs are employed in calculating the net income per acre. The common practice in the High Plains regions is to plant corn at near field capacity soil moisture level. Hence, a preplanting irrigation is assumed. To demonstrate the methodology, it is further assumed here that a maximum of three irrigations are to be allocated beyond preplanting. Examination of unpublished experimental data for the High Plains area indicates that the number of postplant irrigations varied between three and eight, with four and five irrigations being the most common (Onken). Furthermore, previous researchers have indicated that most fields are over irrigated (e.g., Harris and Mapp 1986; Mjelde et al.). Assuming three post-plant irrigations is not unrealistic with the preceding two observations in mind. Also, assuming three irrigations allows for easier demonstration of the optimization via experimentation methodology. The quantity of water applied each time is such as to bring the soil profile at the root zone depth to field capacity. This is the common practice in the real world, although one finds other application practices too. Filling to soil capacity along with only three possible irrigations is taken here for ease of exposition. The methodology is flexible enough to enable other modified strategies. Note, though, that whenever the per-irrigation setup cost is relatively high, optimal policy will be shifted in favor of the filling to soil capacity strategy anyway.

The decision criterion used here is the available soil moisture level in the root zone. The problem is to find the soil moisture threshold levels (in fraction of field capacity), or x_j (j = 1, 2, 3), which trigger corresponding irrigations. That is, irrigate when soil moisture $\leq x_j$. Note that these thresholds, $0 \leq x_j \leq 1$, may or may not be equal to each other.

Experimentation aimed at estimating the response surface is handled through the evalu-

Table 1. Net Returns ($/acre) and the Corresponding Decision Variables for a Single Set of Ten Randomly Selected Years for the Simulated Corn Crop at Each Iteration C^k

Year	C^0	C^1	C^2	C^3	C^4
1954	18.79	−23.73	198.66	183.57	191.42
1955	100.74	76.38	267.11	270.37	248.91
1958	−10.76	23.58	215.21	205.74	217.54
1961	205.90	37.20	161.98	193.41	200.82
1962	248.40	115.49	175.14	247.40	251.00
1963	69.13	−29.98	240.71	232.93	229.26
1968	121.17	32.17	225.90	229.47	211.62
1969	150.05	82.96	237.81	240.86	222.04
1970	32.38	−37.99	196.81	169.21	198.80
1977	132.04	28.36	221.76	224.26	215.50
Mean	106.78	30.44	214.11	219.72	218.69
Standard deviation	82.43	51.07	31.71	31.30	20.00
x_1	.50	.015	.142	.182	.195
x_2	.50	.260	.371	.356	.256
x_3	.50	.107	.333	.331	.295

Note: Net returns are calculated as gross returns minus irrigation and harvesting costs. Other costs, common to all considered alternatives, like planting, seeds, cultivation, fertilizers, and land, are not accounted for. The price of corn was $1.75/bu.; the cost of irrigation water was $2.66 per acre-inch.

ation of the simulation model's various combination sets of x_j (j = 1, 2, 3) defined by a uniform precision central composite design matrix (see the appendix and Montgomery and Evans) with α = 1.684, a unit increment of δ = 0.15 (Biles and Swain, pp. 264–69), and a single central point.[2] The initial guess is arbitrarily set at X^0 = (.5, .5, .5). In order to account for a stochastic environment for each of these combinations, a set of ten years (randomly picked from the available set of 30 years) of environmental data was selected randomly for a total of 150 evaluations per iteration. These iterations are composed of ten years of weather conditions and fifteen different soil moisture triggers as given by the experimental design matrix (appendix).

Results

Three case studies are presented to illustrate the flexibility of the optimization via experimentation methodology. In Case I a random set of ten years' weather patterns is selected. These same ten years are then used during each iteration of the experimentation process. For Case II a different set of ten years is randomly selected from the thirty years of data for each experimentation iteration. These years are drawn with replacement occurring. Case II provides for a more stochastic setting. Finally, the optimal soil moisture levels are determined using all the thirty years of weather data. In all cases the convergence, or stopping, criterion used is

$$[|\Sigma F(X^{k+1}) - \Sigma F(X^k)|/\Sigma F(X^k)] \leq 0.015.$$

Case I

Table 1 shows the optimal decision sets, the mean, and standard deviation of the net income per acre for each iteration. It is interesting to note that not only the mean net income increases toward achieving optimal solution, but the solution is becoming more robust. Apparently, the improved policies are working primarily at increasing the net returns in poor years and much less so at increasing the net returns from good years.

The contribution of the optimal policy over the arbitrarily picked initial decision levels is depicted in figure 2, with the curves representing C^0 and C^4 from the above table 1. It is clear that, while both curves oscillate at basically similar frequencies, the optimal policy leads to a much more stable net income. It appears that the optimal set of the decision

[2] This level of α (parameter of the design matrix), which is needed for greater stability and concavity of the estimated response function, corresponds to six central points. Since randomness is introduced here by drawing particular years, each such central point would generate an identical response. Hence, only one central point is used.

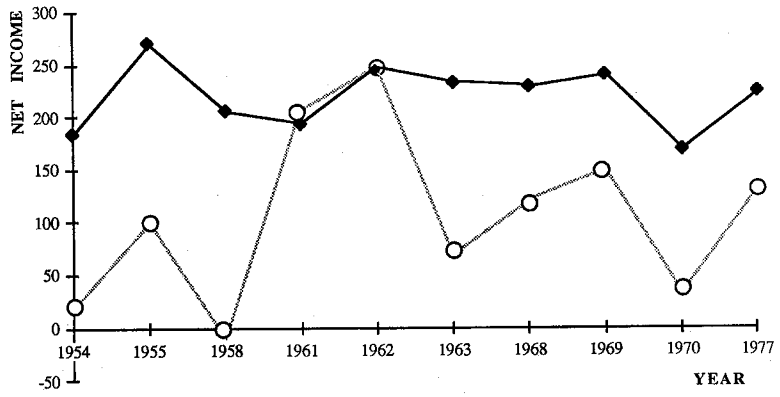

Figure 2. Optimal (solid) vs. initial guess (gray) lines of net income (in $/acre)

variables cannot completely override the influence of weather; but, by following the optimal decision criteria, a producer can substantially moderate fluctuations caused by weather.

Cases II and III

In these cases a new set of randomly picked years was used in evaluating each iteration. By picking the years randomly, a stochastic situation is maintained (not quite in the event of a small population, as in our case where random drawings for weather conditions must proceed with replacement to avoid exhausting all choices). Comparing the results for each iteration, however, is meaningful only for the moments of the samples. Table 2 shows the results of this procedure.

It is instructive to note the similarity between the final optimal decision sets (the C^4 column) in both cases despite the relative dissimilarity in years selected between them. As a final test for the above apparent robustness, this procedure was performed on the entire thirty years of data. The final optimal solution set was X_{opt} = (.203, .318, .310), with a mean net income of 226.97 and a standard deviation of 22.61. These results compare better with Case II than Case I, but do not differ drastically. The practical meaning of all three cases is to initiate the first irrigation when soil moisture reaches about 0.2 and to irrigate again once it falls below 0.3.

The strategy as reflected from the optimal solutions is very simple: hold on "longer" or drier early in the growing season, and be more "protective" of the plants as the season progresses. Such a strategy is intuitively appealing when one considers that in the later part of the season reproductive stages occur and grain mass is being generated. Agronomic research has shown that the corn plant is more sensitive to soil moisture stress later in the growing season (Rhodes, Shaw and Newman).

Remarks

The methodology described and demonstrated here provides an approach to develop *ex ante*

Table 2. Mean, Standard Deviation, and Optimal Decision Set of Net Income ($/acre) for Different Sets of Ten Randomly Selected Years for the Simulated Corn Crop at Each Iteration C^k

Iteration	C^0	C^1	C^2	C^3	C^4
Mean	106.780	30.440	222.460	226.420	229.810
Standard deviation	82.430	51.070	32.920	10.480	25.940
x_1	0.500	0.015	0.142	0.201	0.215
x_2	0.500	0.260	0.371	0.273	0.297
x_3	0.500	0.107	0.333	0.309	0.307

decision rules. Development of *ex ante* rules allows for the implementation of the rules in the stochastic decision making environment. Most previous studies utilizing crop growth models and optimization procedures have developed *ex post* decision rules that are by definition not implementable. As such, optimization via experimentation has a fundamentally important advantage over previous methodologies. Furthermore, it is potentially applicable to any simulation model (e.g., crop-growth, farm-level, macropolicy model) in which a decision variable(s) is under the control or discretion of a decision maker.

Decision rules pertaining to corn production in the High Plains area of Texas derived using the optimization via experimentation methodology indicate that the producer should be more protective of the corn plant during the second and third irrigations than during the first. For a producer who is going to irrigate three times, the soil moisture levels which trigger the three irrigations are approximately .2, .3, and .3, respectively. It is somewhat lower than the general practice in that area, where irrigation usually takes place with soil moisture at or below the .5 level.

The optimization via experimentation has certain advantages over previous methodologies, yet some improvements are necessary and disadvantages are evident. Being a procedure relatively new to the agricultural literature, a summary of these points follows. This is a somewhat incomplete list, and more research and experience is required to enhance it.

(*a*) Optimization via experimentation is a distinctly separated two-stage iterative procedure of experimentation and optimization. The probing phase is performed by conducting a full-scale experiment with "treatments" determined by the described designed matrix. The experiment evaluation is performed completely in a black box fashion; and, furthermore, the quality of the response is a function of the simulation model's representation of reality. Only trivial communication links are required between the simulator and the optimizer subsystems. This is an important advantage because the current simulation model could be exchanged for any new, state-of-the-art competing model with a minimal effort.

(*b*) The response information generated for the optimizer can be controlled by the user, who determines the resolution of the perturbation matrix. Prior knowledge of the system's behavior can be constructive here. Note that the role of the Taylor's series approximation is to estimate the response surface.

(*c*) Adaptation to new regions can be performed provided information exists on the environment's behavior. Past scenarios or Monte-Carlo-simulated environments can be adapted at ease.

(*d*) Convergence rates depend on the shape of the response envelope. Well-behaved estimated envelopes (a strictly concave polyhedron, for example) should lead to very fast convergence via the QP algorithm. Nonconcave response functions may require the first-order gradient process, which is much less efficient and may lead to divergence or non-damped oscillating solutions.

(*e*) A major improvement for irrigation scheduling can be expected if good workable rainfall forecasting can be installed as a part of the simulation model. Such a forecast may delay the need for, or even save, irrigation(s) if the decision criteria is modified to be conditional on the probability to rain in the next decision period. Note that substantial rain occurring shortly after irrigation may be damaging to plants and, if forecast, could have saved the cost of irrigation. This aspect is strongly recommended for future research.

(*f*) Improvement in defining the decision set is also desired. The soil moisture measure used above was applied primarily for demonstration purposes, although it may be used in practice as well. It measures the soil moisture conditions but completely ignores the state of the plant. It is quite conceivable that adding such plant indicators, or combinations of plant/soil indicators, would provide a better decision base.

(*g*) The computer resources required for implementing this methodology are actually the same as those used to run the simulation program. That is, if such a program is run on a personal computer using Fortran, the above procedure can be easily implemented on it. A more powerful computer may of course speed up the optimization process. The number of iterations depends on the behavior of the response surface. A smooth and consistently concave surface (under stochastic conditions) may require only two iterations in terms of figure 1, while in some cases, having a non-concave surface, convergence may not be achieved at all. In the cases studied above no more than five iterations were needed.

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Appendix

Experimental Design

The experimental design matrix gives the soil moisture trigger levels for given starting values on X (the first row of the table A1) and changes in X. Trigger levels give the soil moisture level at which the grower is assumed to irrigate once this level is realized. The design matrix gives fifteen different trigger level sets around the initial set. The number of simulations per iteration is therefore 150 (=15 levels * 10 years). Box and Hunter discuss the efficiency related to the characteristics of such a design matrix. The values of X in table A1 are calculated by

$$X = \delta \Delta X + (X_1 © W),$$

where X_1 is the three-element row vector of starting values on X; W is a fifteen-element column vector of ones; © is the Kroneker multiplication operator.

Table A1. The Experimental Design Matrix with X^0 = [.5, .5, .5], and δ = 0.15

	ΔX			X	
0	0	0	0.50	0.50	0.50
1	1	1	0.65	0.65	0.65
1	1	−1	0.65	0.65	0.45
1	−1	1	0.65	0.45	0.65
1	−1	−1	0.65	0.45	0.45
−1	1	1	0.45	0.65	0.65
−1	1	−1	0.45	0.65	0.45
−1	−1	1	0.45	0.45	0.65
−1	−1	−1	0.45	0.45	0.45
1.684	0	0	0.7526	0.50	0.50
−1.684	0	0	0.2474	0.50	0.50
0	1.684	0	0.50	0.7526	0.50
0	−1.684	0	0.50	0.2474	0.50
0	0	1.684	0.50	0.50	0.7526
0	0	−1.684	0.50	0.50	0.2474