Portfolio Analysis Considering Estimation Risk and Imperfect Markets

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Mean-variance efficient portfolio analysis is applied to situations where not all assets are perfectly price elastic in demand nor are asset moments known with certainty. Estimation and solution of such a model are based on an agricultural banking example. The distinction and advantages of a Bayesian formulation over a classical statistical approach are considered. For maximizing expected utility subject to a linear demand curve, a negative exponential utility function gives a mathematical programming problem with a quartic term. Thus, standard quadratic programming solutions are not optimal. Empirical results show important differences between classical and Bayesian approaches for portfolio composition, expected return and measures of risk.

This paper extends the mean-variance model to account explicitly for the possible effects of including an asset traded in an imperfectly competitive market on the composition of an expected utility maximizing portfolio. An imperfect asset is characterized by dependence between the asset's rate of return and its level of holding in a portfolio. Furthermore, an asset's risk is attributed to two sources: the actual random deviation of an asset's return from its mean (market risk) and uncertainty about the true values of the asset's mean and variance (estimation risk). The resulting portfolio problem is illustrated for a small agricultural bank; however, the general modelling approach holds for a wide range of portfolio problems.

Below, we review literature about portfolio analysis considering estimation risk and imperfect markets. An illustrative problem with three assets is specified algebraically, where two of the assets are risky. Optimal portfolios for the banking problem are derived using non-linear programming; then portfolio responses to selected parameter changes are evaluated. The programming results show the effects of estimation risk with imperfectly elastic assets are not trivial and warrant further consideration in more comprehensive empirical models.

Related Studies

Combining the effects of risk and market imperfections in micro models is a demanding task (Baltensperger). Mean-variance (EV) portfolio theory provides one modelling approach, but it was originated by Markowitz under the assumption that assets are traded in perfectly competitive markets. However, studies by Klein and James have considered the theoretical implications for risk efficient sets of including assets traded in imperfect markets.

Klein's approach used a banking situation to derive an equilibrium ratio of loans to total assets under expected utility maximization, where utility was expressed by a quadratic function. The optimal loan-to-asset ratio explicitly accounted for lending risks, differences in loan demand, and differences in demand elasticities, un-

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der the assumption of a linear demand function. That is, the rate of return on loans was a linear function of the amount lent. An important result was the loss in applicability of Tobin's separation theorem; the optimal combination of risky assets, relative to holding a risk-free asset, is no longer independent of the utility function. If one of the risky assets (loans) has less than perfect elasticity, then the expected return on loans depends on the amount of risky assets relative to the risk-free asset, which in turn requires knowledge about the bank utility function (Klein, p. 494).

James extended Klein's analysis to show the relationship between risk and return in a portfolio model with an imperfect, risky asset. James' formulation minimizes the portfolio variance subject to a specified expected income level, where at least one of the assets is traded in an imperfectly competitive market. His analysis shows that introducing market imperfections (specified as a monopoly position), subject to a downward sloping demand curve, does not affect the upward slope of an EV efficient set; the set is still concave, but not necessarily linear as in the purely competitive case. Moreover, the difference between the expected return on the imperfect asset and a risk-free asset is expressed as the risk premium from the capital asset pricing model plus a monopoly premium determined by the demand elasticity. An interaction between the risk and monopoly premiums brings greater risk from expanded holdings of the imperfect asset.

In James' study the mean and variance of the assets' returns are assumed known, as is the case in the Markowitz derivation of the mean-variance frontier. However, a number of studies have suggested approaches to the portfolio problem, for competitive assets when the moments of the distributions are not known with certainty. Fried considers the use of linear regression models to predict the mean return of an asset, given the value of relevant exogenous variables. He observes that the variance associated with an asset's forecasted return has two parts, one representing the uncertainty about the true value of the regression coefficients (estimation risk) and the other due to the variation of the stochastic error term (market risk). Berck employs Fried's methods in a portfolio model for cotton producers.

Other studies have focused on estimating the moments of asset returns from sample observations. Frankfurter, Phillips, and Seagle give Monte Carlo results showing the possible problems of using point estimates in place of population parameters. Barry observes the increase in the variance of predicted returns for optimal portfolios when estimation risk is considered. Jobson and Korkie derive the approximate sampling distribution of the estimators for the return and variance of an optimal portfolio when normally-distributed assets have unknown moments.

Klein and Bawa follow a Bayesian approach to maximizing expected utility when the population parameters are unknown. In this case, the predictive distribution of an asset's returns combines any priors the decision maker may have about the population parameters with the sample data via Bayes formula. Using two normally-distributed assets and a quadratic utility function, Klein and Bawa show that estimation risk changes the optimal portfolio substantially for small samples.

Summarizing, past research has treated the following: a) theoretical problems of banks facing a downward sloping demand for loans; b) the general problem of deriving a mean-variance efficient portfolio when at least one of the assets is traded in an imperfectly competitive market; and c) the problems of deriving efficient portfolios for competitive assets when the moments of the distributions of asset returns are unknown. In the next section we introduce the portfolio problem when one of the assets in the choice set is traded in an imperfectly competitive market and is subject to estimation risk.

Theoretical Framework

We illustrate the effects of an imperfect asset on an optimal portfolio for a risk averse banker under the assumptions that the returns are normally distributed and the utility function is expressed by the negative exponential $U(\Pi) = 1 - e^{-\rho\Pi}$ where Π is the return from investment and ρ is the degree of risk aversion. The expected value of a negative exponential function integrated over a normal density function for Π is

$$E[U(\Pi)] = E(\Pi) - \rho\sigma_{\Pi}^2 \qquad (1)$$

where $E(\Pi)$ and σ_{Π}^2 are a portfolio's expected return and variance, respectively (Freund). Thus, maximizing $E[U(\Pi)]$ is equivalent to maximizing $E(\Pi) - \rho\sigma_{\Pi}^2$. We select the negative exponential because of its plausible use in empirical studies and well behaved algebraic properties. It has the property of constant absolute risk aversion. As shown below, this portfolio problem requires iterative solution techniques, even with a simple algebraic form. Also, iterative solution of the problem over a grid of values for ρ yields the EV (mean-variance) frontier.

The bank may allocate a fixed amount of funds (Y) among three assets. Asset X_1 is a risk-free asset with return r_1. Asset X_2 is a risky asset traded in a competitive market with return $r_2 = R_2 + e_2$, where R_2 is the mean of r_2 and e_2 is a random variable with mean zero and variance σ_2^2. Asset X_3 is a risky asset traded in an imperfect market, subject to a linear demand function, so that its return is $r_3 = A + BX_3 + e_3$.[1] The parameters A and B are assumed to be unknown population constants and e_3 is a random variable with mean zero and variance σ_3^2. A linear equation for r_3 is used for simplicity and to permit linear regression techniques in the empirical analysis.[2]

The traditional approach for selecting EV efficient, or expected utility maximizing, portfolios is to replace the parameters in (1) with their point estimates and then maximize (1) with respect to the asset levels. However, this approach tends to underestimate portfolio risk by ignoring the error in estimating the unknown parameters. This estimation risk is in addition to the market risk generated by the variability of e_2 and e_3.

A Bayesian approach, employed by Klein and Bawa, is used here to maximize the expected value of (1). Maximization occurs in two steps. First, the predictive distribution of the returns is obtained by integrating the distribution of the returns, given the parameters, over the posterior density of the parameters. That is, the predictive distribution of r_2, $g(r_2)$, is:

$$g(r_2) = \int f(r_2 | R_2) p(R_2) dR_2$$

where $p(R_2)$ is the posterior density of R_2 and $f(r_2|R_2)$ is the density of r_2 given R_2. Second, expected utility is maximized by using $g(r_2)$ as the distribution of r_2. The optimal portfolio is thus derived in accordance with Von-Neumann-Morgenstern axioms (Klein and Bawa).[3]

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[1] A negative slope coefficient (B) is anticipated for loan demand; however, requiring $B < 0$ is not necessary to satisfy the second order maximization conditions.

[2] The models for r_2 and r_3 need not be as simple as they appear. R_2 and A can be both linear and nonlinear functions of exogenous variables, but not a function of the X_i. Forecasting models based on exogenous variables are discussed by Fried.

[3] The difference between classical and Bayesian methods can be illustrated for a risky, competitive asset. Under traditional methods X_2 would have a population mean equal to the sample mean, $\bar{r}_2$, and variance equal to the unbiased estimate of σ_2^2, s_2^2. Using Bayesian methods, and assuming normality and a large sample, r_2 has approximately a normal distribution with mean $\bar{r}_2$ and variance $s_2^2(1 + 1/n)$ where n is the sample size. Thus, including estimation risk increases the variance of r_2 which is what one intuitively expects.

Given the expository purposes of this study, the predictive distributions of r_2 and r_3 are assumed to be normal. Thus, the expectation of the negative exponential can be evaluated in terms of mean and variance. The assumption of normality is not necessarily unrealistic. Considerable evidence indicates that distributions of returns on financial assets are not normal. However, if these returns are adjusted for predictable effects of exogenous forces, then the normality assumption becomes more tenable, as discussed in Fried (p. 553). The decision maker is assumed to have diffuse priors on the unknown parameters.

To complete the analysis, it is also assumed e_2 and e_3 are independently distributed.[4] Thus the predictive distribution for r_2 in large samples is approximately normal with mean $\bar{r}_2$, the sample mean, and variance $s_2^2(1 + 1/n)$, where n is the sample size and s_2^2 is the unbiased estimate of σ_2^2. For r_3 the predictive distribution of r_3 in large samples is approximately normal with mean $a + bX_3$. If no exogenous variables other than X_3 influence r_3, then a and b are derived by regressing r_3 on X_3 and an intercept term.[5] The variance of the predictive distribution of r_3 is $[1\ X_3]S_{AB}[1\ X_3] + s_3^2$ where s_3^2 is the unbiased estimate of σ_3^2 and S_{AB} is the covariance matrix of A and B. Clearly, the means and variances above are identical to those given by classical least squares for a forecast of the dependent variable and the variance of the forecast. Technically the posterior distributions are of the student "t" form; however, for large samples the t is closely approximated by the normal. Given X_3, the forecasted mean return for X_3r_3 is $aX_3 + bX_3^2$, and the forecast variance, X_3^2 times the variance of r_3, which, given the properties of matrix multiplication, is

$$X_3([1\ X_3]S_{AB}[1\ X_3]' + s_3^2)X_3 = [X_3\ X_3^2]S_{AB}[X_3\ X_3^2]' + X_3^2s_3^2$$

Or, in scalar algebra,

$$X_3^2s_A^2 + 2X_3^3s_{AB} + X_3^4s_B^2 + X_3^2s_3^2$$

where s_A^2, s_B^2, and s_{AB} are the posterior variances of A and B and the covariance of A and B, respectively. Under these specifications, maximizing the negative exponential for the three asset case requires maximizing J where

$$\begin{aligned} J = {} & r_1X_1 + \bar{r}_2X_2 + aX_3 + bX_3^2 \\ & - \rho\big[s_2^2X_2^2 + s_3^2X_3^2 + s_{2/n}^2X_2^2 \\ & + s_A^2X_3^2 + 2s_{AB}X_3^3 + s_B^2X_3^4\big] \end{aligned} \qquad (2)$$

subject to

$$X_1 + X_2 + X_3 \leq Y \qquad X_1, X_2, X_3 \geq 0.$$

The variance of the expected return is the sum of the bracketed expression in (2). The first two terms are the traditional variances in EV analysis. The next two terms account for error in estimating R_2 and A. The last two terms are attributed to the imperfect asset. The cubic term reflects correlation between the slope coefficient and the intercept of the return equation for the imperfect asset. The variance of B is multiplied by a quartic term. Thus, the imperfect asset with a linear return and uncertain parameters results in a portfolio model that is solved by quartic programming. If, however, the slope of the return function is known with certainty, the model is solved by quadratic programming. Thus, the requirement for quartic programming is based on uncertainty about the elasticity of the imperfect asset.

Empirical Relevance of Estimation Risk

A fair question in empirical studies is whether estimation risk is relevant compared with market risk, particularly if sample sizes are large, say in excess of

[4] If e_2 and e_3 are not distributed independently then one is faced with deriving the posterior distribution for a set of seemingly unrelated regressions. The independence assumption seems reasonable here because the market for a bank's imperfect asset is likely local, while the markets for competitive assets are likely national or international in scope.

[5] If other independent variables were used in the regression for r_3, then the estimated coefficients would be multiplied by the projected levels of their independent variables for the future period and summed to give A. Corresponding adjustments would have to be made to get S_{AB}.

thirty observations. The answer hinges on the structure of the regression model. If r_2's value is not conditioned by any exogenous variables, then the estimation error of R_2 is roughly of order $1/n$ compared with market risk. It can be ignored for large samples. The traditional and Bayesian approaches will give essentially the same answers. If, however, r_2 is conditioned by exogenous variables, then the comparative magnitude of the estimation error may not dissipate as quickly as when r_2 is explained only by a constant population mean. This is particularly true if the values of the exogenous variables for which r_2 is being forecasted differ substantially from their sample means. The reduction in estimation risk from larger samples may be more than counterbalanced if the levels of the exogenous variables for the forecast period are far from their sample means.

This argument is stronger for an imperfectly elastic asset. While the variance of the intercept, which may include any number of shifters, is multiplied by the squared level of the asset, the variance of the slope coefficient is multiplied by the fourth power of the asset level. Thus, even though a larger sample size may increase parameter precision, the overall risk effect may be substantial, particularly if the optimum level of the imperfectly elastic asset is substantially different from its sample mean.

In empirical analysis the relevance of estimation risk compared with market risk will depend on the sample data and characteristics of the problem. In this paper we examine the relevance issue in detail in order to gain further insight about the importance of estimation risk.

Programming Analysis

The effects of risk and market imperfections are evaluated in a nonlinear programming analysis of the three asset case with solutions for five levels of risk aversion under various specifications of the parameters in equation (2).[6] The setting is a small agricultural bank with \$6 million of funds (Y) available to invest in risk-free treasury bills (X_1) having a 5 percent annual return, corporate securities (X_2) having an estimated expected annual return of 5.714 percent and an estimated population variance of 0.3322,[7] and farm loans (X_3) having an expected return of

$$\bar{r}_3 = \underset{(.230)}{8.024} - \underset{(.0274)}{.07546}X_3 \qquad (3)$$

with standard errors in parentheses. Using these returns, optimal portfolios are derived for a static problem. A dynamic model, while more realistic, could tend to obscure the effects of the two sources of uncertainty.

The parameters of the loan demand function were estimated from a sample of agricultural banks, which included annual data on amounts lent and interest rates on farm loans over the 1972 to 1979 period (Barnard). The constant term in (3) is the sum of an intercept term plus six independent variables evaluated at their sample means multiplied by the respective estimates of their coefficients.[8] The results

[6] The solutions were obtained using a non-linear optimization package called Generalized Reduced Gradients by Lasdon et al. Convergence was obtained when the Kuhn-Tucker conditions were satisfied to within .001.

[7] The rate of return of 5.714 was computed as the sample mean of 52 observations on the one-year U.S. Treasury Bill rate for 1977. Thus, the variance due to estimation is $(1/52)(.3322) = .0064$, an almost negligible proportion of the asset's total risk.

[8] The regression equation from which (3) is derived regressed observed r_3 on six independent variables, an intercept term, and X_3. The posterior covariance matrix for these parameters is assumed equal to the generalized least squares estimate of the covariance matrix of the generalized least squares estimator of the unknown coefficients. To get the vector [A B], we simply multiplied the estimated coefficient vector by the matrix

$$\begin{bmatrix} 1, z_1, z_2, \ldots, z_6, 0 \\ 0, 0, \ldots, 0, 1 \end{bmatrix}$$

where the z_i are the forecasted values of the six independent variables. To get S_{AB}, the estimated covariance of the coefficients was premultiplied by the above matrix and postmultiplied by its transpose.

TABLE 1. Optimal Portfolios for the Base Problem.

Risk Coefficient	Activity Levels X_1	X_2	X_3	Mean Net Return[a]	Variance[b] Market Risk	Estimation Risk	Total Risk
A. Estimation Risk Included							
(ρ)							
0.0	0.0	0.0	6.0	45.4	18.5	2.07	20.6
50	0.0	1.91	4.09	42.5	9.83	.863	10.7
100	2.58	1.05	2.37	37.5	3.25	.278	3.53
150	3.63	.692	1.68	35.4	1.61	.140	1.75
200	4.22	.527	1.26	34.1	.907	.080	.987
B. Estimation Risk Excluded							
(ρ)							
0.0	0.0	0.0	6.0	45.4	18.5	2.07	20.6
50	0.0	1.69	4.31	42.8	10.5	.960	11.5
100	2.37	1.07	2.56	38.0	3.76	.324	4.08
150	3.50	.717	1.78	35.7	1.81	.158	1.96
200	4.10	.538	1.37	34.4	1.06	.094	1.15

[a] Mean net returns are computed as gross return less the initial six million of investable funds, measured in units of \$10,000.
[b] Measured in \$10,000^2.
Source: computed.

show a highly elastic demand for farm loans.

The regression results reported in (3) are for a sample of 67 banks, with data collected over a seven-year period, certainly not a small sample by most standards. Nonetheless, the ratio of estimation risk for A to market risk is about 10 percent. This is probably the minimum ratio for these data since the demand shifters were set at their sample means. Indeed, if the problem were being solved for ex ante actions, the values of the exogenous variables that are incorporated into A would be set at their projected levels. These levels could differ substantially from their sample means, thus increasing the importance of estimation risk.

Based on these data, the optimal portfolio of bank assets results from the maximization of

$$\begin{aligned}1.05X_1 &+ 1.05714X_2 + 1.08024X_3\\ &- 0.0007546X_3^2 - \rho(10^{-4})\\ &\cdot[.3322X_2^2 + .5151X_3^2 + .006388X_2^2\\ &+ .05313X_3^2 - 2(.001909)X_3^3 + .0007539X_3^4]\end{aligned}$$

subject to

$$X_1 + X_2 + X_3 \leq 6.0 \text{ and } X_1, X_2, X_3 \geq 0.$$

Optimal asset levels are shown in Table 1 for five levels of risk aversion. The results show an emphasis on the risk-free asset for higher levels of risk aversion, a more balanced portfolio for intermediate levels of risk aversion, and specialization in the higher yielding farm loans for smaller ρ. The risk neutral solution (ρ = 0.0) shows complete specialization in farm lending, despite the less than perfectly elastic return function. In this case the complete specialization results from the highly elastic demand and from the specific numerical values on returns and fund availability.

For each level of ρ the corresponding quadratic programming (QP) solution that recognizes only market risk is given in part B of Table 1. As expected, disregarding estimation risk yields solutions with higher mean returns and total risk for each non-zero level of risk aversion. Compared with the quartic solutions, the QP solutions show greater investment in the imperfect asset and lower investment in the risk-free asset for non-zero risk aversion. When estimation risk is added to the market risk for the QP results, the underestimation of to-

TABLE 2. Optimal Portfolios with Decreased Elasticity for the Imperfect Asset.

Risk Coefficient	Activity Levels X_1	X_2	X_3	Mean Net Return[a]	Variance[b] Market Risk	Estimation Risk	Total Risk
A. Estimation Risk Included							
(ρ)							
0.0	0.0	4.47	1.53	36.1	7.84	1.02	8.86
50	2.67	2.12	1.21	34.1	2.25	.407	2.65
100	3.99	1.06	.957	33.0	.843	.172	1.01
150	4.49	.700	.807	32.4	.499	.096	.595
200	4.77	.527	.699	32.1	.344	.061	.405
B. Estimation Risk Excluded							
(ρ)							
0.0	0.0	4.47	1.53	36.1	7.84	1.02	8.86
50	2.36	2.15	1.49	34.4	2.68	.844	3.53
100	3.74	1.07	1.19	33.3	1.11	.362	1.47
150	4.29	.716	.990	32.8	.675	.188	.863
200	4.62	.537	.847	32.4	.465	.111	.576

[a] Mean net returns are computed as gross return less the initial six million of investable funds, measured in units of $10,000.
[b] Measured in $\$10,000^2$.
Source: Computed

tal variance can be computed for the various solutions. For example, when $\rho = 50$, the total variance of the QP solution (11.5) is underestimated by about 8.3 percent (10.5 versus 11.5). The impact of estimation risk on the optimal portfolio composition varies with ρ. For high values of ρ, little difference occurs in the optimal portfolios. However, for intermediate levels of ρ, the optimal portfolios respond more strongly to whether estimation risk is acknowledged. Nonetheless, the mean returns vary by no more than two percent for any level of ρ.

Results in Table 2 indicate the effects of a less competitive, more volatile market for farm lending. The slope coefficient for the loan return function is multiplied by 10, giving a more steeply sloped demand, and the variance of the slope is increased so that the estimated coefficient is double its standard error. The change in slope is maintained in the remaining models. Compared with the results in Table 1, the optimal portfolios respond to those changes by reduced holdings of the imperfect risky asset and greater holdings of the risk-free and risky competitive assets (for $\rho = 50$). Thus, the more steeply sloped loan demand and greater risk combine to reduce the attractiveness of the imperfect asset. Moreover, diversity between the two risky assets occurs in the risk neutral solution. When estimation risk is deleted, the QP solutions underestimate total variance of their portfolios by about 20 percent, and indicate greater holdings of the imperfect asset for $\rho \geq 50$. The mean returns drop considerably for the less elastic demand as shown by a comparison of mean returns between the corresponding portfolios in Tables 1 and 2. Estimation risk again causes a substantial difference in the composition of the optimal portfolio, with about a 1 percent decrease in mean returns.

Solutions in Table 3 show the effects on the optimal portfolios of less certainty about the value of the slope coefficient, B, for the imperfectly elastic asset. The loan demand characteristics (2) for X_3 were revised so that the ratio of the estimate of B to its standard error equals minus one, indicating statistical insignificance by conventional econometric practices. A comparison of the holdings of X_3 between

TABLE 3. Optimal Portfolios with Decreased Elasticity and Greater Uncertainty for the Imperfect Asset.

Risk Coefficient	Activity Levels			Mean Net Return[a]	Variance[b]		
	X_1	X_2	X_3		Market Risk	Estimation Risk	Total Risk
A. Estimation Risk Included							
(ρ)							
0.0	0.0	4.47	1.53	36.1	7.84	3.36	11.2
50	2.93	2.11	.965	33.7	1.96	.568	2.52
100	4.15	1.04	.814	32.7	.701	.291	.992
150	4.65	.700	.654	32.2	.383	.129	.511
200	4.90	.526	.575	31.9	.262	.081	.343
B. Estimation Risk Excluded							
(ρ)							
0.0	0.0	4.47	1.53	36.1	7.84	3.37	11.2
50	2.36	2.15	1.49	34.4	2.68	2.97	5.66
100	3.74	1.07	1.19	33.3	1.11	1.22	2.33
150	4.29	.716	.990	32.8	.675	.598	1.27
200	4.62	.537	.847	32.4	.465	.331	.796

[a] Mean net returns are computed as gross return less the initial six million of investable funds, measured in units of \$10,000.
[b] Measured in \$10,000^2.
Source: computed.

Tables 2 and 3 for the solutions which include estimation risk shows that greater uncertainty about the true value of B leads the risk averse investor to hold less of X_3. Additionally, the difference in mean returns for the intermediate levels of ρ between the quartic and quadratic solutions in Table 3 are slightly more than 2 percent. Also, the total risk of a portfolio is more grossly underestimated if estimation risk is ignored when uncertainty about B is high. This is true for both quartic and quadratic solutions compared with their counterparts in Table 2.

While the mean returns for risk averse portfolios vary by no more than about 2 percent between recognizing and ignoring estimation risk, the change in portfolio composition is more pronounced. Note in Table 3 for $\rho = 50$, disregarding estimation risk leads to a 54 percent increase in the amount of funds allocated to the loan alternative. Such shifts by a bank would have a substantial impact on local markets. Thus, the incorporation of estimation risk into portfolio analysis may have a larger impact on asset markets than on mean returns for the investor.

Implications for Modeling and Analysis

The numerical results of the programming analysis indicate an optimal portfolio may respond significantly to the risk and market characteristics of its assets and to changes in risk aversion. Comparing solutions for the various programming models shows that an asset's degree of market imperfection has a marked effect on the portfolio's composition and mean returns. Including estimation risk in the programming analysis also influences the optimal portfolios, with the effects on portfolio risk being greater than the effects on expected returns. These effects of estimation risk would be magnified considerably if the costs of funds acquisition were included in the analysis. For the banking example, the high financial leverage of commercial banks means that relatively small changes in expected returns to assets yield proportionately large swings in expected returns to equity, after the costs of acquiring debt capital are paid for. The effects of estimation risk would also be greater if smaller sample sizes were used to estimate the model's parameters.

These results support the need to account jointly for the effects of risk and market imperfections in studies of banking and other empirical situations where these phenomena are important. The model illustrated here has a simplified specification of assets and constraints in order to focus on the portfolio effects of the assets' risk and market characteristics. More complete models that include activities for acquiring resources, their risk and market characteristics, and other constraints would add realism and likely reduce the sensitivity of the portfolio responses to the variations induced in this study. But these added details would obscure the fundamental effects of the risk and market characteristics as well. Thus, including the effects of the risk and market imperfections provides a richer, although more complex analytical framework for evaluating risk efficient portfolios.

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