Crop Density and Irrigation with Saline Water

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The economic implications of plant density for irrigation water use under saline conditions are investigated, utilizing the involved physical and biological relationships. The analysis considers a single crop and is applied to cotton data. The results suggest that treating plant density as an endogenous control variable has a substantial impact on profits and the optimal quantities and qualities of the applied irrigation water.

The salinity problem arises from the fact that irrigation water from any source contains a certain amount of soluble salts. During irrigation, as a portion of the water evaporates, these salts accumulate in the soil and adversely affect the growing conditions and crop yields.

Problems of soil salinity and irrigation with saline water have recently become a matter of considerable concern in the arid and semiarid regions of the western United States (van Schilfgaarde). The increasingly intensive use exposes the water resources to a gradual deterioration in quality, and causes increasing salinity in natural aquifers. Good quality water sources are becoming steadily scarce, necessitating increasing use of higher salinity water sources (Bitoun). Sheridan, for example, stated that salinization is expected to be the major threat to the San Joaquin's (the southern half of the great Central Valley of California) productivity within the near future. The expected transition from good quality to saline water should encourage further economic analysis of irrigation with saline water which accounts for the involved physical and biological relationships in the water-soil-plant system.

Economic aspects of irrigation with saline water have been discussed extensively in the literature. In some papers quality (salinity level) of irrigation water was referred to as an exogenous parameter and quantity was assumed as a single control variable (e.g., Yaron and Olian; Matanga and Marino; Feinerman and Yaron, 1983a), while others considered the opposite (e.g., Yaron and Bresler; Feinerman and Yaron, 1983b). Some papers have considered both quality and quantity of irrigation water as endogenous decision variables (e.g., Bresler and Yaron, Moore *et al.*). However, in these and many other studies the dimension of crop density was not considered and a constant number of plants per unit of land area was implicitly assumed.

A comprehensive analysis of plant response to soil salinity was presented in Maas and Hoffman. They stated that "the most common salinity effect is a general stunting of plant growth" and they added, "too often vegetative growth response to salinity is not a reliable guide for predicting fruit or seed production . . . With some crops, e.g., . . . cotton . . . , seed or fiber production are decreased much less than vegetative growth." A recent study by Francois, who conducted a field plot study

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to determine the feasibility of increasing cotton density on highly saline soils, concluded "Although cotton is known to be one of the most salt-tolerant field crops, highly saline soils nevertheless can significantly reduce plant size . . . The smaller plant size leaves a significant space between plant canopies which could support additional plants." Based on these findings, the present study was initiated to explore the implications of plant density for irrigation water use under saline conditions. An economic framework for optimizing irrigation water quantities and qualities and crop densities is presented, utilizing physical and biological relationships involved in irrigation with saline water. The analysis considers a single crop and is applied to cotton (*Gossypium hirsutum*) data. The results demonstrate that the impact of assuming plant density as an endogenous control variable on profits and optimal use of irrigation water is quite substantial.

Objective and Frame of Analysis

The model presented here is aimed at determining the optimal combination of irrigation water quantity and quality and plant density for cotton. The analysis implies knowledge of several functions and parameters which are described below.

The decision maker has at his disposal M sources of water supply, differing in quality and costs, which can be mixed. Let C represent the salt concentration in milliequivalents per liter (meq/l) of one acre-foot (a-f) of irrigation water, mixed from various sources and let P(C) be a cost function (dollars/a-f) which relates the cost of one mixed acre-foot to its associate C.[1] For the sake of simplicity, the following functional form is assumed:

$$P(C) = \alpha - \beta C \qquad \alpha, \beta > 0 \tag{1}$$

In irrigation planning under saline conditions, it is essential to know the dynamics of salts in the soil. Bresler utilized the law of mass conservation to formulate an equation which describes these dynamics. Bresler's equation can be rearranged to yield:

$$S_1 = \frac{WC + S_0(V - \tfrac{1}{2}W + \tfrac{1}{2}ET)}{V + \tfrac{1}{2}W - \tfrac{1}{2}ET} \tag{2}$$

Where

S_0 = soil salinity (meq/l) prior to growing season
S_1 = soil salinity at the end of growing season
W = quantity (a–f/acre) of irrigation water applied
V = known soil moisture content (a–f/acre) at saturated paste
ET = evapotranspiration (a–f/acre)

The average soil salinity in the root zone during the growing season can be approximated by:

$$S = 0.5(S_0 + S_1) = \frac{WC + 2S_0V}{2V + W - ET} \tag{3}$$

Under given soil properties and climatic conditions as well as irrigation water quantities such that the possibility of soil moisture deficiency is eliminated, the leaf area of a plant is determined by the soil salinity and the plant's age (Hoffman *et al.*). Let L_t be the leaf area of a single plant (dm²/plant) and T_t be the transpiration per unit of leaf-area (ml/dm²) t weeks after planting. Based on data presented in Hoffman *et al.*, the following

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[1] A procedure to derive the P(C) function is:

i) For different levels of C, solve the following linear programming problem:

$$P(C) = \underset{W_m}{\text{Min}} \sum_{m=1}^{M} P_m W_m$$

subject to:

$$\sum_m W_m = 1$$

$$\sum_m W_m C_m = C$$

where W_m and C_m are, respectively, the quantity and the (given) salinity level of the irrigation water from source m.

ii) Regress P(C) on C and identify the functional form which fits the data best. It is easy to verify that if two sources of water are assumed a linear cost function results.

functions for cotton were estimated by ordinary least squares regression models (60 observations):[2]

$$\begin{array}{l} L_{t+1} = -25.5 + 0.83L_t - 0.05S + 0.0002S^2 \\ \text{Standard errors } (0.08) \quad (0.02) \quad (0.00005) \\ \quad + 7.28t - 0.15t^2 - 0.009St \\ \quad\; (1.95) \quad (0.04) \quad (0.004) \\ R^2 = 0.994 \end{array} \tag{4}$$

$$\begin{array}{l} T_t = 159.6 - 1.02L_t + 0.0022L_t^2 - 0.193S \\ \text{Standard errors } \quad (0.082) \quad (0.0003) \quad (0.013) \\ R^2 = 0.853 \end{array} \tag{5}$$

Obviously, relating leaf area at period t to the average soil salinity during the growing season is an approximation. Due to lack of appropriate experimental data, it was not possible to estimate L_t as a function of alternate salinity levels at previous periods. Field experiments aimed at observing the leaf area at different growth stages under conditions of alternate salinity levels will improve the specification and estimation of Equation (4) and will enable the extension of the analysis for several consequent irrigations (such an extension can easily be included in the current analysis but since Equation (4) is based on the average soil salinity, the value of doing so is very limited).

The total weekly transpiration per plant is given by L_tT_t and the total transpiration from a field of one acre is $D \sum_t L_tT_t$ where D represents the plant density (plants/acre). Assuming that soil evaporation is a given fraction (γ) of the total evapotranspiration (Ritchie and Burnett) the total evapotranspiration from the field is given by:

$$ET = \frac{D}{1-\gamma} \sum_t L_tT_t \tag{6}$$

In order to eliminate yield decrease due to competition for radiation between adjacent canopies at high population densities, it is assumed that D can not exceed the number of plants which will yield canopy closure at the end of the growth cycle. More formally, let N represent the number of plants (per acre) which yield canopy closure at the end of the growth cycle under non-saline conditions, and let $\bar{L}$ represent the leaf area per plant. Then, the plant density under saline conditions is constrained by:

$$D \leq N\bar{L}/L_T \tag{7}$$

where T is the end of the growth cycle (since $\partial L_t/\partial S < 0$ for every t, $L_T < \bar{L}$).

To eliminate the possibility of yield reduction due to soil moisture deficiency, the following restriction on W is imposed:

$$W \geq ET \tag{8}$$

It has been well established in the literature that in absence of soil moisture deficiency, crop yield is directly related to the average soil salinity in the root zone during the growing season. A specific formulation for a large number of crops is presented in Maas and Hoffman. They demonstrate that there is some threshold level ($\bar{S}$) beyond which crop yields decline linearly with increasing soil salinity. The basic profit function which follows these relationships can be written:[3]

$$\pi(W,C,D) = \begin{array}{ll} RD[a + bs] - P(C)W & \text{if } S > \bar{S} \\ RDY_{max} - P(C)W & \text{if } S \leq \bar{S} \end{array} \tag{9}$$

where

R = income net of non-water variable costs directly related to yield;

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[2] Necessary conditions for consistent estimates of the model's (first-order autoregressive model) coefficients require that |coefficient of L_t| < 1 and that the error terms are independent (e.g., Theil, p. 412). It was verified respectively by one-sided t-test (2.5% significance level) and Durbin-Watson test (Durbin Watson statistic of 1.879 was calculated) that these two requirements hold.

[3] The formulated profit function assumes that the variable costs are independent of D. Since seed costs are relatively very small compared to the total, the effect of this assumption on the empirical results is negligible.

Y_{max} = maximum yield with no salinity losses (kg/plant);

$a > 0, b < 0$ = known parameters.

There are two counteracting effects of soil salinity on the total yield per acre. Salinity decreases the yield per plant (assuming $S > \bar{S}$) but it also decreases the plant size which is measured by the leaf area. The smaller leaf area leaves a space between adjacent canopies which, via restriction (7), could support additional plants (i.e., higher D). This, coupled with the fact that higher soil salinity is associated with irrigation water of higher salt concentration and hence, of lower cost, suggest that the impact of salinity on profits is not *a priori* clear.

Summarizing the above discussion, the optimization problem may be stated as:

$$J(W,C,D) = \underset{W,C,D}{MAX}\ \pi(W,C,D) \qquad (10)$$

Subject to: (1) and (3) through (8)

This is a non-linear maximization problem which should be solved by numerical method.

Empirical Results

Two sources of irrigation water are assumed with salinity levels of 20 and 100 meq/l and costs of \$25 and \$20 per a-f, respectively. As a result, the parameters of the cost function are $\alpha = 26.25$ dollars and $\beta = 0.0625$ dollars/meq/l. In order to emphasize the salinity effects (the commercial yield of cotton is relatively not sensitive to soil salinity) a high initial soil salinity level of $S_0 = 70$ meq/l is assumed. Data on other parameters were collected from various sources. The assumed values and the specific sources are summarized in Table 1.

The maximization problem (10) was solved by a penalty function computer program for solution of non-linear problems, written by Piacco and Ghaemi.

In order to evaluate the importance of considering the plant density as an endogenous control variable, the problem was resolved, assuming constant density at the conventional level of 26,000 plants/acre (Francois).

In the following, the cases of variable density and predetermined density will be referred to as Case 1 and Case 2 respectively. The results are presented in Table 2.

Several observations and conclusions can now be made:

(I) The impact on profits of considering plant density as a control variable is substantial (the profits under Case 1 are 1.57 times as large as the profits under Case 2). This finding suggests that where soil salinity is a problem, for some situations controlling population density might be at least as important as controlling irrigation water quantities and qualities;

(II) Under Case 2, most of the applied water is from the less saline (and the more costly) source and the average soil salinity is determined in a level equal to the threshold salinity level for cotton (92). As a result, the optimum yield is identical to the maximum yield. Under Case 1, the relative amount of water from the more saline source is much higher than under Case 2, the average soil salinity (153.2) is much higher than the threshold and the optimum yield per plant (0.0392) is smaller than the maximum. However, the total yield per acre under Case 1 (2050.9 kg) is 1.56 times as large as the total yield under Case 2 (1310.4). Obviously, population density is, in effect, substituted for yield per plant.

(III) Under both cases, the constraint aimed at eliminating soil moisture deficiency ($W \geq ET$) is effective. The difference between the values of ET, however, is quite substantial. It

TABLE 1. Assumed Parameter Values and Their Sources.

Parameter	Value	Source
V	1.25 a-f/acre	H. J. Vaux, Jr. (personal communication)
γ	0.34	Ritchie and Burnett
N	26,000 plants/acre	François
$\bar{L}$	292 dm²/plant	Hoffman et al.
R	0.364 dollars/kg	Cost analysis worksheet, Tulare County, CA, 1979
Y_{max}	0.0504 kg/plant	H. J. Vaux, Jr. (personal communication)
a	0.0675 kg/plant	Maas and Hoffman; U.S. Salinity Laboratory
b	−0.000185 (kg/plant)/(meq/liter)	Maas and Hoffman; U.S. Salinity Laboratory
$\bar{S}$	92 meq/liter	Maas and Hoffman; U.S. Salinity Laboratory

suggests that in future studies, ET should not be assumed as an exogenous parameter (an assumption which is frequently made in the economic literature dealing with salinity problems) but as an endogenous state variable. It can be readily verified that constraint (7) (which is relevant only for Case 1) is also effective.

(IV) The empirical results are obviously conditional on the quality of the physical data and incorporated assumptions and should be considered with some caution. Nevertheless, they enable us to learn the order of magnitude of the differences between Cases 1 and 2 and to draw operative conclusions about the desired combination of W, C and D.

Limitations of the Analysis and Recommendations for Further Research

Two major difficulties exist with the problem as formulated in the previous sections. First is that a single irrigation season was assumed when salinity problems may involve long-run salt accumulation and leaching processes. The short-run objective function is based solely on immediate profits and ignores the effect of the terminal soil salinity level on the succeeding seasons.

One possibility for overcoming this shortcoming is to introduce a penalty term in the objective function (9) which includes a terminal value for salinity. This obviously requires a long-run analysis. But as a first approximation and for demonstration purposes, penalties of 0.5DRb ($=\delta\pi/\delta S_1$) and 0.5 × 26000Rb dollars per meq/l ("basic" penalties) were imposed on the terminal soil salinity level (S_1) for Cases 1 and 2 respectively. Then, the penalties were doubled and the problem was resolved. The results are reported in Table 3.

As expected, average soil salinity and profits are decreasing with the penalty level (see also the results of Table 2 when no penalty was assumed). Water is applied only from the less saline source in excess of ET and (with the exception of 0.5DRb penalty level under Case 1) the

TABLE 2. Empirical Results at the Optimal Solutions of Cases 1 and 2.

	W (a-f/Acre)	C (meq/liter)	D (Plants/Acre)	ET (a-f/Acre)	$L_T(T = 20)$ (dm²/Plant)	S (meq/liter)	Y (kg/Plant)	J ($/Acre)
Case 1	3.69[a]	56.21	52,319	3.69	145.11	153.2	0.0392	661.9
Case 2	2.31[b]	23.8	26,000[c]	2.31	190.5	92.0	0.0504	420.5

[a] Consists of 2.02 a-f from the less saline source and 1.67 a-f from the more saline source.
[b] Consists of 2.2 a-f from the less saline source and 0.11 a-f from the more saline source.
[c] The "potential" D based on (7) is 39,853.

TABLE 3. Empirical Results for Cases 1 and 2 With a Penalty on Terminal Soil Salinity.

	Penalty Level ($/meq/liter)	W (a-f/ Acre)	C (meq/ liter)	D (Plants/ Acre)	ET (a-f/ Acre)	$L_T(T = 20)$ (dm^2/Plant)	S (meq/liter)	Y (kg/Plant)	J ($/Acre)
Case 1	0.5DRb	3.76	20.0	39,851	3.54	190.51	92.0	0.0504	485.3
	DRb	5.55	20.0	35,459	3.57	214.11	63.84	0.0504	374.2
Case 2	13,000Rb	3.07	20.0	26,000[a]	2.47	203.48	76.3	0.0504	327.9
	26,000Rb	4.82	20.0	26,000[b]	2.68	218.79	58.5	0.0504	274.1

[a] The "potential" D based on (7) is 37,310.

[b] The "potential" D based on (7) is 34,700.

optimal level of S is lower than the threshold level $\bar{S} = 92$ (obviously, an unoptimal solution when only immediate profits are considered). Although the ratio between profits under Cases 1 and 2 decreases with the penalty level (661.9/420.5 = 1.57 under no penalty, 485.3/327.9 = 1.48 under the basic penalties and 374.2/274.1 = 1.37 under the doubled penalties), it is still quite substantial. This suggests that the previous conclusion about the importance of treating crop density as a control variable remains valid under long-run analysis.

The second major difficulty is due to the fact that yield per plant (as well as the leaf area) was assumed to be a function of soil salinity only. Allowing yield per plant to be a function of the soil moisture content and the density as well and omitting the restrictions imposed on W and D by (8) and (7) respectively, is a direction in which the analysis might profitably be extended. Unfortunately, the data base required for estimating a multiple input production function which relates crop yield to soil salinity, soil moisture content and crop density is not available in the literature. Economic theory has the capability of allowing any number of inputs to vary, although the complexity increases. The thrust of research in this area should be an increasing scientific knowledge about the relationships involved. The empirical work of Francois, which investigated the options of increasing cotton density on highly saline soils, is a step in the right direction.

Other extensions of this analysis include incorporation of irrigation timing within the growing season and extension of the economic framework to a multi-output farm. The methodology described in this paper can serve as a building block in such extended analyses.

Summary

The dimension of crop density has been ignored in previous economic analyses dealing with salinity problems. The results reported here suggest that this should not be the case. The impact of plant density on profits and irrigation water use (quantity and quality) is quite substantial. Under certain conditions, controlling population density might be at least as important as controlling irrigation water quantities and qualities.

The difficulties exist with the problem here formulated and the directions into which the analysis might profitably be extended were discussed in the previous section. The methodology discussed in this paper can serve as a building block in such extended analyses. Its main advantage seems to be in providing conceptual and methodological framework to investigate the issue of crop density under saline conditions as well as guidance in the design of experiment to generate the data needed to get a better handle on the relationships involved.

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