Journal of Agricultural and Resource Economics 48(1):1–13 ISSN: 1068-5502 (Print); 2327-8285 (Online)

doi: 10.22004/ag.econ.320680

An Expected Value of Sample Information (EVSI) Approach for Estimating the Payoff from a Variable Rate Technology

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This paper examines the expected payoff to variable rate technology for fertilizer application in terms of a Bayesian expectation of the value of sample information (EVSI). The optimal variable rate for each cell in a field is conditioned on a signal in the form of the electrical conductivity of soil at that cell. Using corn response to nitrogen data from ten on-farm field-level experiments, we calculate the expected payoff from variable rate technology versus a uniform rate applied to all cells to be about $1.81/acre.

Key words: Bayesian decision making, EVSI, precision agriculture, VRT

Introduction

This article addresses the issue of determining in advance whether variable rate technology (VRT) for fertilizer application on a particular maize field is economically justified compared to using standard uniform rate technology (URT). Fertilizer VRT has been commercially available since the early 1990s, and US farmers had adopted VRT over 29% of planted maize area for fertilizer, soil amendments, seed, or plant protection chemicals by 2016 (Lowenberg-DeBoer and Erickson, 2019). Scores of economic evaluations of fertilizer VRT compared with URT in the United States have been published, with widely varying results that tend to suggest that the average benefits are marginal relative to the costs of implementation (for surveys of this literature, see Lambert and Lowenberg-DeBoer, 2000; Bullock and Lowenberg-DeBoer, 2007; Colaço and Bramley, 2018).

Evaluating the economic payoff from VRT presents a difficult conceptual problem. The net benefits of using VRT on a given field in a particular year can be observed, but the net benefits if URT had been used on that same field and year cannot be observed. (And even if they could, the comparison might not be valid for the same field next year or for a different field in the same year.) Therefore, economic evaluation must rely upon some counterfactual outcomes generated by a mathematical model. Published economic evaluations of VRT based on field mappings have relied on such models,[1] ranging from relatively straightforward production economics approaches

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This research is partially supported by the Agriculture and Food Research Initiative (AFRI) Food Security Program Coordinated Agricultural Project, titled "Using Precision Technology in On-Farm Field Trials to Enable Data-Intensive Fertilizer Management," (Accession Number 2016-68004-24769) from the USDA National Institute of Food and Agriculture, by the Nebraska Agricultural Experiment Station with funding from the Hatch Multistate Research capacity funding program (accession numbers NEB 1011054, NEB 227784), and by USDA-NIFA Hatch Project 470-362.

Review coordinated by Dayton M. Lambert.

[1] We should note that VRT in response to real-time signals such as crop color is a somewhat newer technology. Boyer et al. (2011) provide an analysis of sensor-based nitrogen VRT for wheat, while McFadden, Brorsen, and Raun (2018) and Stefanini et al. (2019) both employ Bayesian updating approaches to model sensor-based VRT for nitrogen on wheat and cotton, respectively. All three analyses conclude that uniform rates across the field resulted in higher net returns than the VRTs considered.

(e.g., Boyer et al., 2011) to complex models requiring numerical rather than parametric solutions (e.g., Hurley, Oishi, and Malzer, 2005; Bullock et al., 2009). The complexity of most of these models has prevented them from becoming a standard approach for identifying circumstances under which fertilizer VRT has a sufficient benefit to warrant its cost. This article develops and applies an approach to evaluating the payoff from VRT that is simple enough to promise more widespread use in anticipating payoff on particular fields.

In the first section below, we first develop a general theoretical approach for analyzing the benefit of VRT as the expected value of the sample information (EVSI) contained in the signal from a plot prior to choosing the application rate, based on Bayesian updating of prior expectations about a soil characteristic. We then assume a particular frequency distribution of the soil characteristic and the signal and a particular algebraic function for the yield response that together allow us to obtain a closed-form solution for EVSI as a function of parameters of these distributions and of the response function. As a third step, we specify an econometric model to estimate parameters for the EVSI that now include parameters of the distributions as well as the yield function. The next section describes the data and our estimates of EVSI for variable rate nitrogen application on a sample of corn fields, and the last section discusses the results.

Theoretical Approach

Here we present an approach that leads to identifying the payoff to fertilizer VRT as a function of parameters representing five factors that would intuitively affect that payoff. The value of VRT should increase with

i. the heterogeneity of soil characteristics in cells across the field;
ii. the curvature of the fertilizer response function;
iii. the effect of the soil characteristics on fertilizer response;
iv. the correlation of observable signals and critical soil characteristics; and
v. output price.

We postulate that crop yield (expressed per acre) at each point on a field is a function of the quantity of fertilizer applied and an unobservable soil characteristic at that point. A prior density function describes the decision maker's (DM's) beliefs about the frequency distribution of the unobservable characteristic across the field. There exists an observable signal at each point on the field, the distribution of which is correlated with the unobservable characteristic.

At each point in the field, the manager has a choice of observing the signal so as to apply a rate that maximizes expected profit conditional on that signal (a variable application rate) or simply applying the uniform rate that maximizes expected profit conditional on the prior density of the characteristic. We define the expected payoff from VRT as the difference between these two expected payoffs, which in the decision theoretic literature is known as EVSI—the expected value of sample information. To model EVSI, we follow an approach suggested by Kihlstrom (1976) and Lawrence (1999).

In this analysis, VRT is a package consisting of hardware capable of varying the application rate across the field, hardware capable of monitoring the signal, s, and software supporting them. At issue is the expected value of VRT relative to URT for a field, prior to knowing the values of s that would be observed at points within that field. This difference, EVSI, is the *ex ante* expected payoff from adoption, which is the same as the expected economic value of observing s at each cell (equation 4 below). Observing this characteristic itself would lead to the expected value of perfect information. In our study, s is observed electrical conductivity of the soil, though it might be some other characteristic such as soil type or crop (infrared) color if such data had been available.

A Simple Bayesian Decision-Making Framework

At every point on the field, return to fertilizer, or "profit," is determined by a variable input, x, expressed on a per acre basis, and an unknown and unobservable state of nature, γ:

$$\pi(x,\gamma) = pf(x,\gamma) - wx, \tag{1}$$

where $f(x,\gamma)$ is the production function (per acre yield response function) and p and w are the output and input prices. Note that the cost of implementing the technology used (VRT or URT) is not considered in this measure of payoff. In the absence of information about γ at a given point, the expected profit-maximizing choice of x is obtained from

$$\max_x \mathrm{E}_\gamma[\pi(x,\gamma)] = \int_G \pi(x,\gamma)g(\gamma)d\gamma, \tag{2}$$

where $g(\gamma)$ is the density function representing the *prior* probability distribution of γ across the field, and G is its range. We denote as x' the level of input that maximizes expected profits in equation (2). If the signal is not observed at any point, x' is optimal for the entire field and is thus the optimal uniform rate under URT.

Following a general production problem posed by Kihlstrom (1976), we introduce the possibility of obtaining some soil information (a signal), $s \in S$, that is correlated with the true unknown soil characteristic, γ. Having observed s, the expected profit-maximization problem becomes

$$\max_x \mathrm{E}_{\gamma|s}[\pi(x,\gamma)] = \max_x \int_G \pi(x,\gamma)h(\gamma|s)d\gamma, \tag{3}$$

where $h(\gamma|s)$ represents the *posterior* probability distribution of γ given s, obtained by Bayes's rule.[2] We denote $x''(s)$ as the application rate that maximizes expected profits when s is observed, a contingency plan describing the input decision in response to any message s that might be observed.

We assume that prior to adopting VRT on a given field, the DM holds a prior distribution of γ and knows the response function. When the DM decides whether to obtain the signals from individual cells, they do not know what signals will be drawn, but they do know the distribution from which they will be drawn (see below). The adoption decision is thus based on an expected profit maximization in which the profit associated with each possible message provided by the signal is weighted by the probability of receiving that message. EVSI is the extra expected profit from observing s at a given point before choosing the rate to apply at that point, with expectations taken with respect to the density functions of both γ and s:

$$\mathrm{EVSI} = \int_S \int_G \pi\left(\boldsymbol{x}''(s),\gamma\right) h(\gamma|s)d\gamma\phi(s)ds - \int_G \pi\left(\boldsymbol{x}',\gamma\right) g(\gamma)d\gamma, \tag{4}$$

where $\phi(s)$ is the marginal probability density function of the signal.[3] The first expression on the righthand side identifies the expected payoff from first observing the signal s and then applying

[2] Bayes's rule states that the posterior density function describing the probability distribution of γ is

$$h(\gamma|s) = \frac{v(s|\gamma) \cdot g(\gamma)}{\phi(s)}, \text{ where } \phi(\mathrm{s}) = \int_G v(s|\gamma)g(\gamma)d\gamma,$$

where $v(s|\gamma)$ represents the sampling distribution of the signal and $\phi(s)$ is the marginal density function of the signal.

[3] Equation (4) is conceptually identical to the formulation of the value of VRT identified by Babcock, Carriquiry, and Stern (1996). However, their response specification is linear response and plateau and they specify the prior distribution of the soil characteristic as a three-parameter gamma, so the resulting posterior distribution is a highly complex function (their equation 11) that does not admit to any tractable solutions. Instead, they estimate its parameters using a 20-point Gauss–Legendre quadrature. This complexity makes their approach inconvenient for identifying fields for which VRT might be profitable. Bullock et al. (2009) also use a Bayesian updating approach to identify payoff from VRT (their equations 21 and 17). They specify discrete prior and posterior densities, which require considerable calculations to solve for VRT.

the rate that maximizes expected profit given that signal, evaluated prior to observing the signal (known as preposterior analysis, Raiffa and Schlaifer, 1961). The expectation is taken with respect to the density of s across the field, $\phi(s)$, identified in footnote 2, and since the profit is scaled to the level of 1 acre, this expectation is also the expected per acre profit from using VRT across the field. Similarly, the second expression is the comparable expected per acre profit if the optimal uniform rate is applied across the field. Thus, equation (4) is the expected extra per acre profit from observing and using the signal at given points, compared to profits from an economically optimal uniform rate application. Note that this is the gross value of observing the signal, from which the cost of adopting the VRT package must be deducted to determine the *net* benefit of VRT relative to URT.

A Specification with a Quadratic Response Function and Bivariate Normal Distributions

Lawrence (1999) provides an approach that allows us to algebraically identify the EVSI. He specifies a quadratic response function $f(x,\gamma)$ and a bivariate Normal distribution of γ and s.[4] The quadratic yield function is

$$y = \alpha + \beta_1 x + \beta_2 x^2 + \beta_3 \gamma + \beta_4 \gamma x, \tag{5}$$

where $\beta_2 < 0$. The prior density function, g, and the posterior density function, h, in equation (4) are Normal with means μ_γ and $\mu_{\gamma|s}$, respectively, and correlation ρ; $(\gamma,s) \sim \mathit{Bivariate\ Normal}\ (\mu_\gamma,\sigma_\gamma^2;\mu_s,\sigma_s^2;\rho)$. The solutions to the maximization problems in equations (2) and (3), respectively, are then

$$x'\left(\mu_\gamma\right) = x\left(w,p,\mu_\gamma\right) = \frac{w/p - \beta_1 - \beta_4\mu_\gamma}{2\beta_2} \text{ and} \tag{5a}$$

$$x''\left(\mu_{\gamma|s}\right) = x\left(w,p,\mu_{\gamma|s}\right) = \frac{w/p - \beta_1 - \beta_4\mu_{\gamma|s}}{2\beta_2}. \tag{5b}$$

The two optimal rates in equations (5a) and (5b) yield two expected maximum profit functions, $\mathrm{V}(\mu_\gamma)$ and $\mathrm{V}(\mu_{\gamma|s})$. The theoretical expected value of sample information (EVSI) in equation (4) can be expressed in terms of these expectations rather than the integrals themselves:

$$\mathrm{EVSI} = \mathrm{E_s}\left[\mathrm{V}\left(\mu_{\gamma|s}\right)\right] - \mathrm{V}\left(\mu_\gamma\right), \tag{6}$$

where $\mathrm{E}_s[\cdot]$ indicates the expectation over the distribution of the signal. For the quadratic yield function, these maximum expected profit functions can be expressed as

$$\begin{aligned} \mathrm{V}\left(\mu_\gamma\right) &= c_1\mu_\gamma^2 + c_2\mu_\gamma + c_3; \\ \mathrm{V}\left(\mu_{\gamma|s}\right) &= c_1\mu_{\gamma|s}^2 + c_2\mu_{\gamma|s} + c_3; \end{aligned} \tag{7}$$

where $c_1 = -\frac{p\beta_4^2}{4\beta_2}$, $c_2 = p\beta_3 + \frac{p\beta_4}{2\beta_2}\left(\frac{w}{p} - \beta_1\right)$, and $c_3 = p\alpha + \frac{1}{2\beta_2}\left(w\beta_1 - \frac{w^2}{2p} - \frac{p\beta_1^2}{2}\right)$ are combinations of the known prices and the parameters of the quadratic yield function. Plugging equation (7) into equation (6) and using the law of iterated expectations that implies that $\mathrm{E}_s(\mu_{\gamma|s}) = \mu_\gamma$ yields

$$\mathrm{EVSI} = c_1\left[\mathrm{E}_s\left(\mu_{\gamma|s}^2\right) - \mu_\gamma^2\right]. \tag{8}$$

[4] The quadratic yield specification is required for Lawrence's closed-form analytical results. While some studies of alternative specifications have shown other specifications to be preferable for nitrogen response (i.e., Bullock and Bullock, 1994), other studies have not (Perrin, 1976; Liu, Swinton, and Miller, 2006). In any case, the quadratic is probably the most commonly used in economic studies (see Anselin, Bongiovanni, and Lowenberg-DeBoer, 2004; Liu, Swinton, and Miller, 2006; Bullock et al., 2009).

Equation (8) can also be expressed in terms of the variances of the prior and posterior distributions of the unobservable soil characteristic, as

$$\text{EVSI} = c_1 \left[\sigma_\gamma^2 - \text{E}_\text{s}\left(\sigma_{\gamma|s}^2\right)\right], \tag{9}$$

where σ_γ^2 and $\sigma_{\gamma|s}^2$ are the prior and the posterior variances of γ, respectively. Note that EVSI is a function of output price, p (in coefficient c_1), but not of input price, w, which disappears with the coefficient c_2. The expected value of obtaining information s about the unknown γ is proportional to the reduction in uncertainty about γ (ignoring the extra cost of the VRT technology package).

When γ and s are bivariate Normally distributed with correlation ρ, the posterior variance of the distribution of γ is $\sigma_{\gamma|s}^2 = \sigma_\gamma^2(1-\rho^2)$. This allows us to express equation (9) as a closed-form solution with intuitively appealing parameters:

$$\text{EVSI} = c_1\left[\rho^2\sigma_\gamma^2\right] = -\frac{p\beta_4^2}{4\beta_2}\left[\rho^2\sigma_\gamma^2\right]. \tag{10}$$

Equation (10) is a fundamental contribution of this analytical approach. It expresses the value of VRT as an explicit function of parameters representing the underlying factors we have intuitively identified as affecting the value of VRT: the variance of the state of nature across the field, σ_γ^2; the correlation between the signal and the state of nature, ρ; the curvature of the response function, β_2; the effect of the state of nature on the response, β_4; and the price of output, p. So far as we have been able to determine, no other study of VRT has identified the expected payoff from VRT in terms of fundamental intuitive parameters such as these. This result provides some possibility of identifying, *ex ante*, fields for which VRT has a good possibility of success.

Estimation of Response Function and EVSI

While we cannot observe underlying parameters ρ and σ_γ, the assumed bivariate Normal distribution of γ and s allows us to derive an approximation of EVSI in equation (10). We proceed as follows: We first take expectations of the yield function (5) with respect to the prior distribution of γ and with respect to the posterior distribution of γ, given s, and we obtain estimable reduced-form equations. The equation to be estimated, equation (14), is a reduced form that includes the parameters of the yield function (5) as well as of the bivariate distribution of γ and s. Once the parameters for this reduced form are estimated we can solve for the structural parameters of the yield function (5) and for the key parameters of the bivariate distributions including their correlation all of which are used in estimating EVSI using equation (17). This is the link between the theoretical and the empirical models.

Taking the expectation of the yield function (5) with respect to the prior distribution of γ, we obtain the following expression for given values of x:

$$E_\gamma(y) = \alpha + \beta_1 x + \beta_2 x^2 + \beta_3\mu_\gamma + \beta_4\mu_\gamma x. \tag{11}$$

The posterior mean of the distribution of γ, given s, is given by

$$E(\gamma|s) = \mu_{\gamma|s} = \mu_\gamma + \rho\frac{\sigma_\gamma}{\sigma_s}(s-\mu_s). \tag{12}$$

Using this result, we take the expectation of the yield function (5) with respect to the posterior distribution of γ to obtain

$$E_{\gamma|s}(y) = \alpha + \beta_1 x + \beta_2 x^2 + \beta_3\left(\mu_\gamma + \rho\frac{\sigma_\gamma}{\sigma_s}(s-\mu_s)\right) + \beta_4\left(\mu_\gamma + \rho\frac{\sigma_\gamma}{\sigma_s}(s-\mu_s)\right)x. \tag{13}$$

Substituting $\tilde{s} = \frac{s-\mu_s}{\sigma_s}$ so that $\tilde{s} \sim Normal(0,1)$, rearranging terms, and adding a random error term, we obtain the following reduced-form estimating equation:

$$y = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 \tilde{s} + \theta_4 \tilde{s} x + \epsilon, \tag{14}$$

where $\theta_0 = \alpha + \beta_3 \mu_\gamma$, $\theta_1 = \beta_1 + \beta_4 \mu_\gamma$, $\theta_2 = \beta_2$, $\theta_3 = \beta_3 \rho \sigma_\gamma$, and $\theta_4 = \beta_4 \rho \sigma_\gamma$. Note that the parameters θ of equation (14) to be estimated are combinations of the parameters β of the quadratic yield function (5), and of the mean and variance of the distributions of γ as well as its correlation with the distribution of s, while $\tilde{s}$ appears as a righthand variable. We then use these to estimate EVSI using equation (17).[5]

Equation (14) estimates coefficients that describe the outcome of a Bayesian decision framework but does not necessarily imply Bayesian estimation procedures. Given that the expected value of $\tilde{s}$ is 0, the first-order condition for the expected profit-maximizing uniform application rate without observing s implies that the optimal uniform rate applied to all cells is

$$x'\left(\mu_\gamma\right) = \frac{w}{2\theta_2 p} - \frac{\theta_1}{2\theta_2}, \tag{15}$$

and the first-order condition for the expected profit-maximizing application rate conditional on having observed s (the variable rate for a given cell) yields

$$x''(\tilde{s}) = \frac{w}{2\theta_2 p} - \frac{\theta_1}{2\theta_2} - \frac{\theta_4 \tilde{s}}{2\theta_2}. \tag{16}$$

Notably, the EVSI measure of the value of VRT in equation (10) becomes

$$\text{EVSI} = -\frac{p\beta_4^2}{4\beta_2}\left[\rho^2 \sigma_\gamma^2\right] = -\frac{p\theta_4^2}{4\theta_2}, \tag{17}$$

where the θ are coefficients of the estimating equation (14). As a reviewer has pointed out, this solution for EVSI is a nonlinear function of parameters, and plugging estimated parameters into equation (17) will, by Slutsky's theorem (Wooldridge, 2010, section 3.2), provide a consistent estimator of EVSI with a large number of observations (which in our data is 7,294).

Sensitivity of EVSI

Equation (17) repeats some of the determinants of the payoff from VRT. In brackets, we see that EVSI increases with the correlation between the signal and the state of nature, and with the variability of the state of nature across the field. There is zero expected benefit to variable rate application for a uniform field with $\sigma_\gamma^2 = 0$ or, if there is no correlation between the signal and the state of nature, $\rho = 0$. From equation (17), the elasticity of EVSI with respect to the correlation ρ is

$$\frac{\partial \ln \text{EVSI}}{\partial \ln \rho} = 2. \tag{18}$$

Similarly, elasticity of EVSI with respect to variance of the state of nature, σ_γ^2, is 1.0, and the elasticity with respect to curvature, β_2, is −1.0, while the elasticity with respect to the interaction coefficient, β_4, is 2.0. We now turn to an empirical application of the approach developed above.

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[5] Given that we cannot directly observe γ, a reviewer has questioned whether the estimation of parameters in equation (14) suffers from attenuation due to errors in variables. However, we assume x and $\tilde{s}$ to be measured without error, so we expect no such attenuation.

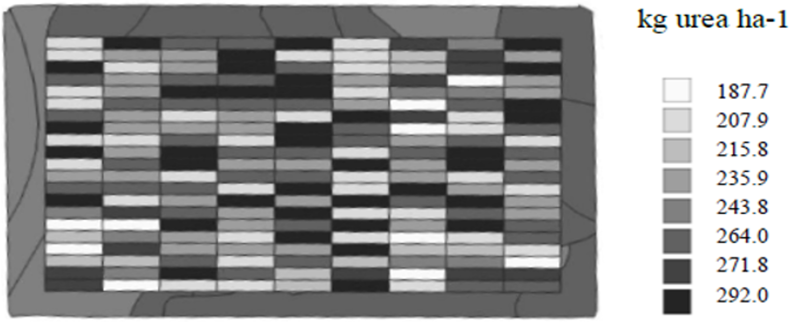

Figure 1. Layout of a 2018 On-Farm N Fertilizer Trial Conducted on a 32-ha Field in Central Illinois
Source: Data Intensive Farm Management (2018).

Table 1. Descriptive Statistics (N = 7,294)

Variable	Units	Mean	Std. Dev.	Min.	Max.
Corn dry yield	bushels/acre	232.65	31.45	0	315.67
Applied nitrogen (N)	pounds/acre	174.15	35.89	29.88	315.94
Soil EC	Veris EC scale	37.32	9.36	7.60	80.22
Standardized EC ($\widetilde{EC}$)	Veris EC scale	0	1	-3.17	4.58

Notes: Statistics describe cells within ten fields in Illinois (4 fields in 2016, 5 in 2017) and Ohio (1 in 2017).
Source: Data Intensive Farm Management (2018).

Data

We use a rich set of experimental data from the Data-Intensive Farm Management (Data Intensive Farm Management, 2018) project at University of Illinois. For the ten fields available for this paper, farmers' fields were divided into equal-sized cells, ranging from 127 to 2,347 cells per field, with cell dimensions varying across fields due to field size, shape, and the size of farmers' equipment. We refer to subunits within fields as "cells," but in the precision agriculture literature they are also referred to variously as "management zones," "sites," "plots," or "grids," depending somewhat on how the subunits are identified. Figure 1 depicts the whole-field experimental layout of a field typical of the DIFM project. To each cell a fertilizer and seeding rate treatment was randomly assigned, with the fertilizer levels for each field bracketing the farmer's choice for the field. For a more complete description of the experimental approach, see Bullock et al. (2019).

To estimate the response function (14), we pooled data from ten farm fields, four in Illinois in 2016, five in Illinois in 2017, and one in Ohio in 2017, with a total of 7,294 cells. The data consists of corn yields, applied nitrogen (N) and soil electrical conductivity (EC), which is the observed signal for each cell i in field j. EC has been associated with the availability of nitrate in the soil in which high levels are expected to increase yields (Johnson et al., 2003; Liu, Swinton, and Miller, 2006). It is a soil signal that may be correlated with some soil properties such as texture, drainage, cation-exchange capacity, and subsoil characteristics (Grisso et al., 2005). Data on EC can be obtained in a shorter period and is cheaper than traditional grid-based soil testing. We do not employ EC as the signal because we believe it to be the best proxy for nitrogen response but simply because it is available on all fields and is plausibly correlated with nitrogen response. Figure 2 illustrates a scatter plot of pooled yields versus N rates. Figure 3 illustrates the locations of the 2017 trials. Table 1 reports descriptive statistics for the ten fields used in this study.

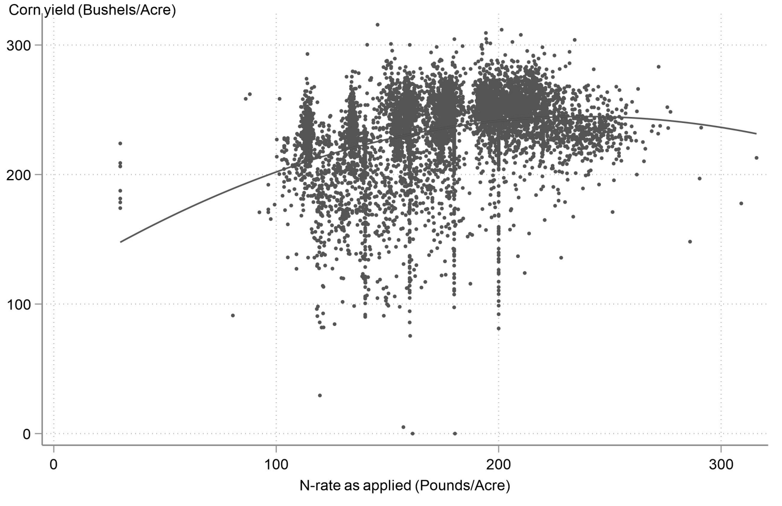

Figure 2. Scatter Plot of (N, Corn Yield) Data Used in this Analysis

Source: Data Intensive Farm Management (2018).

Figure 3. Location of Six 2017 Trials Used in the Study

Source: Data Intensive Farm Management (2018).

Results

Estimated Response Function

To estimate equation (14), the independent variable is observed yield; x is pounds of nitrogen applied per acre; the signal, $\tilde{s}$, is represented as standardized observed ECs, referred to as $\widetilde{EC}$; and we add a random disturbance, u_i, to capture the differences in field location. The estimation procedure was maximum likelihood with random effects for fields and error terms clustered at the field level. Table 2 presents the estimate of the yield response function for all ten fields, 7,294 observations. Coefficients for N are statistically significant at 5% and at 10% for N^2, the rest are statistically significant at 1%. From the variable N response equation (14), we conclude that, because estimates of θ_2 and θ_4 are both negative, there is an inverse relationship between $\widetilde{EC}$ and optimal N rate. In other words, $\widetilde{EC}$ is a substitute for N.

Table 2. Estimated Nitrogen Response Equation (equation 14)

Variable	Coefficient	Estimate
N_{ij}	θ_1	0.455**
		(0.2247)
N_{ij}^2	θ_2	−0.00101*
		(0.000546)
$\widetilde{EC}_{ij}$	θ_3	9.542***
		(2.181)
$\widetilde{EC}_{ij} \times N_{ij}$	θ_4	−0.0494***
		(0.01126)
Constant		169.33***
		(28.642)

Notes: Pooled data from ten fields in Illinois (four fields in 2016, five in 2017), and Ohio (one in 2017), 7,294 cells (independent variable is observed yield at each cell). Estimation procedure is maximum likelihood with random effects for fields and error terms clustered at the level of the field. Standard errors in parentheses. Single, double, and triple asterisks (*,**,***) indicate significance at the 10%, 5%, and 1% level, respectively.

Table 3. Soil Electrical Conductivity (EC) and Estimated Optimal VRT Nitrogen Application per Field, Illinois (four fields in 2016, five in 2017) and Ohio (one in 2017)

	Obs.	EC				Optimal VRT N Application (estimated)[a]			
Field	(cells)	Mean	Std. Dev.	Min.	Max.	Mean	Std. Dev.	Min.	Max.
1	127	42.25	6.66	32.68	66.97	143.64	17.48	78.72	168.76
2	160	38.42	3.24	31.40	44.26	153.69	8.51	138.35	172.12
3	160	35.58	4.05	24.73	42.48	161.15	10.63	143.03	189.65
4	256	47.15	6.31	34.58	60.41	130.78	16.58	95.99	163.78
5	581	35.47	8.86	20.12	79.57	161.45	23.26	45.63	201.76
6	1,548	40.44	6.22	22.81	59.22	148.39	16.34	99.08	194.68
7	682	28.08	11.48	7.60	78.85	180.86	30.14	47.52	234.63
8	819	33.06	9.07	14.57	59.45	167.77	23.82	98.46	216.33
9	2,347	41.28	6.98	22.68	80.22	146.19	18.32	43.94	195.04
10	614	27.15	6.17	15.92	48.32	183.29	16.21	127.69	212.78
Pooled cells	7,294	37.32	9.36	7.60	80.22			43.94	234.63

Notes: [a] In lb/acre, using a corn price of \$3/bu and nitrogen price of \$0.42/lb, equation (16) and estimated coefficients from (14).

Table 3 presents for each field the mean, standard deviation, minimum and maximum of EC, and optimal VRT rates across the cells within each field, calculated using equation (16) with estimated parameters from equation (14), evaluated at the values of x and s observed at each cell in each field. Ordering from lowest to highest average EC, we observe the clear inverse relationship between optimal VRT rate and EC, as implied by the negative estimate of the interaction coefficient for N and EC, θ_4. For example, fields 7 and 10, which have the lowest average EC, have the largest average optimal VRT applications of 180.86 lb/acre and 183.29 lb/acre, respectively. Fields 1 and 4, with the highest average EC, have the lowest average optimal VRT application of 143.64 lb/acre and 130.78 lb/acre, respectively. The relationship between the standard deviation of EC and the standard deviation of optimal VR application is similarly monotonic.

Using equation (17) with the parameter estimates for the pooled sample of ten fields, we estimate EVSI to be 1.81/acre:

$$\widehat{\text{EVSI}} = -\frac{p\hat{\theta}_4^2}{4\hat{\theta}_2} = -\frac{3 \times (-0.0494)^2}{4(-0.00101)} = \$1.81/\text{acre}. \tag{19}$$

Table 4. Estimated EVSI by Cell, Average and Range per Field in Illinois and Ohio, 2016–2017 ($US/a)

Field ID	Obs. (cells)	Mean (Std. Dev.)	Min.	Max.
1	127	1.42 (3.29)	0	18.27
2	160	0.24 (0.23)	0	0.99
3	160	0.40 (0.73)	0	3.29
4	256	2.83 (2.60)	0	11.08
5	581	1.70 (3.50)	0	37.11
6	1,548	1.00 (1.17)	0	9.97
7	682	4.51 (3.98)	0	35.86
8	819	2.09 (2.13)	0	10.77
9	2,347	1.34 (2.31)	0	38.25
10	614	2.94 (2.29)	0	9.53
Pooled cells	7,294	1.82 (2.64)	0	38.25

Here we use the Slutsky theorem, as indicated earlier, to assert that inserting estimated values into equation (17) provides a consistent estimator of EVSI, given the large number of observations we have. This is our best *ex ante* out-of-sample estimate of the expected gross return to VRT for fields drawn from a distribution of fields similar to those in our sample.[6]

We also calculate the expected payoff from using the variable rate x'' compared to the uniform rate x' at each individual cell, using equation (14) to estimate yields. This provides a sample-based estimate of EVSI, as opposed to the parameter-based estimate in equations (17) and (19). Table 4 presents the average and range of cell-level EVSI estimates by field. These estimates range from \$0/acre to \$38.25/acre but average \$1.82/acre, essentially the same as the parameter-based estimate in equation (19).

The \$1.81/acre is the expected return from using the signal from each cell to generate an optimal rate, ignoring the costs of the VRT technology package. While it is below estimates in the literature of those costs, this result is similar to estimates in other empirical studies. Babcock, Carriquiry, and Stern (1996) estimate higher comparable payoffs between \$2.93/acre and \$10.03/acre. Bullock et al. (2009) find comparable payoffs to be \$1/acre or less and concluded that prospects for VRT "are generally dim."

Sensitivity

The estimated EVSI of \$1.81/acre is too low to warrant adoption, at least on the "flat and black" US Corn Belt fields studied here. In this section we examine changes in various parameters that would be sufficient to achieve an EVSI of (arbitrarily) \$10/acre, which is about 5.52 times higher than the estimated level. We solve equation (17) for the various parameters to derive estimates of the changes in individual parameters that would be sufficient to increase the EVSI by about 5.52 times, to \$10/acre. Solving equation (17) for $d\ln\rho$, for example,

$$d\ln\rho = \frac{5.52}{2} = 2.76. \tag{20}$$

We estimate that a ρ 2.76 times larger than the ρ of these fields would be sufficient to raise EVSI to \$10/acre. But of course, we do not have an estimate of ρ. Judging from the generally low VRT payoff measured here, this correlation must be low, perhaps $\rho = 0.1$ or as little as $\rho = 0.01$. If

[6] As a reviewer has noted, we can use our results to approximate the results from an alternative scenario: Consider fields to be cells, calculate an optimal uniform rate within each field, then apply uniform rates within fields, as compared to a uniform rate for the entire set of fields. If the variance of gamma were the same as the variance of EC, we would calculate from equation (10) and data in Table 3 that the EVSI for this scenario would be 45%$[=(6.31/9.36)^2]$ of the EVSI for the scenario of our paper, or \$0.82/acre ($= 0.45 \times \$1.81$).

$\rho = 0.1$, then from the equation above, ρ would need to increase from 0.1 to 0.376. If $\rho = 0.01$, ρ would need to increase from 0.01 to 0.038. However, if $\rho \geq 0.362$, apparently there is no increase that would yield an EVSI of \$10/acre or more. In any case, if a 2.8-fold increase in ρ is required, it seems that electrical conductivity is not sufficiently correlated with N response on these fields to be a profitable signal.

Considering the variance of the state of nature over the field, σ_γ^2, from equation (11) and given that the elasticity of EVSI with respect to σ_γ^2, is 1.0, the necessary percentage increase in variance to achieve an EVSI of \$10/acre is 10/1.81= 5.52. If the distribution of γ is similar to that of EC, this would imply an increase of σ_γ^2, from about 87.6 (the variance of EC across all fields) to 484, which is much higher than the EC variance of 132 in field 7, the most variable of any of the fields. Similarly, the curvature coefficient, β_2, would need to increase from absolute value of 0.001 to 0.0065, which is an indication that profits in our sample are not highly sensitive to the level of N applied. The interaction coefficient, β_4, would need to change from −0.0494 to −0.186, a further indication that N response in this sample is not greatly affected by the level of EC.

Conclusions

In this paper, we have adapted insights from the decision theory literature on the value of information to provide a relatively simple economic model of the value of VRT (variable rate technology) as the expected value of sample information (EVSI). We consider a signal, upon which the variable rate for an individual cell within the field is based, to be an imperfect reflection of soil characteristics that determine fertilizer response. The expected value of the extra payoff from the variable rate at that cell versus a field-uniform rate is the gross payoff of VRT at that cell. This expected value, a version of EVSI, is the expected payoff over all cells in the field.

Using results from literature on the economic value of information, when the signal and the unknown soil characteristic are bivariate Normally distributed and the response function is quadratic, we are able to obtain a closed-form solution that expresses EVSI as a function of parameters representing field and yield response circumstances: heterogeneity across the field, the curvature of the fertilizer response function, the effect of the soil characteristic on fertilizer response, the correlation of observable signals and critical soil characteristic, and the output price. We then show how to econometrically estimate EVSI having information on yields, applications, and the signal. This provides promise for the approach to be a practical tool for identifying, in advance, which fields and circumstances are likely to have a sufficiently high expected return from VRT to warrant the investment. So far as we have been able to determine, results such as this are unique in the economic literature applied to variable rate application or to other precision agriculture technologies.

We have empirically applied this approach to the case of nitrogen application to maize, using data from an elaborate set of whole-field experiments during 2016 and 2017 on ten US Corn Belt farms through the Data Intensive Farm Management project. The sample information at each cell within a field is the observed electrical conductivity (EC), which we take as a signal that is plausibly correlated with whatever unobservable soil characteristic affects nitrogen response.

Our EVSI estimate of the *ex ante* expected payoff of VRT is \$1.81/acre (prior to subtracting VRT implementation costs). This is insufficient to warrant VRT implementation costs, which we believe to be in the range of \$10/acre. Our analysis suggests that for VRT benefits to reach this level, the correlation between state of nature and signal would need to increase by roughly 2.8 times, though we are not able to estimate the level of that correlation. Higher correlations might be achieved with other signals such as sensing data or soil classification, for example. Alternatively, the same improvement in VRT value could be attained by an increase of similar size for either the curvature coefficient or the N times EC interaction coefficient, or a five-fold increase in the variance of the state of nature. Clearly, some of these changes could occur if we had a more robust measure than electrical conductivity (EC) of the state of nature affecting N response, or if fields were more heterogeneous.

Our theoretical approach has provided some evidence for determinants of VRT payoff that had previously been understood intuitively but not analytically or quantitatively. Our claim is that for our results to be a measure of the expected payoff of VRT on a similar field, $1.81/acre is a plausible estimate of the expected gross benefit of VRT across a field with cells drawn from the same distribution as the 7,294 cells in our sample. The lack of profitability of VRT in this analysis may reflect the fact that electrical conductivity is a poor signal for representing the underlying soil characteristic that determines nitrogen response. It is possible that variables such as soil classification and remote sensing data may provide better and/or cheaper signals for calibrating application levels.

[First submitted July 2021; accepted for publication May 2022.]

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