Effect of Form of Hay and Carcass Quality on the Economics of Concentrate: Hay Substitution in Cattle Feedlot Diets

Brian S. Freeze and R. Hironaka

A maximum profit linear programming feedlot model is presented to determine if diet combinations of moderate amounts of hay and grain are economically optimal when time (length of the feeding period), form of hay (long versus cubed alfalfa), and carcass quality are considered. While the experiment did not include concentrate-to-hay ratios as high as commercial feedlot use, the results show that highest concentrate diets are economical in all stages of the finishing program, except that when time and carcass grade effects are considered and hay-to-concentrate price ratios are at historical lows, a switch is made for the last 90 kg of gain to diets somewhat above 50% hay.

Key words: beef production, diet formulation, feedlot finishing, isoquant analysis, linear programming, nutritional modeling.

Cattle can be finished on diets varying widely in the proportions of grain and roughage. Until recently, economists theorized a decreasing marginal rate of substitution between concentrate and forage to achieve the same level of gain, as represented by convex isoquants (Brokken et al.; Brokken 1977; Epplin, Bhide, and Heady 1983). This theoretical view contends that the optimum balance of forage and grain is very sensitive to small shifts in the ratios of ingredient prices. Yet, diets in commercial feedlots typically have contained only minimum amounts of roughage to satisfy physiological needs, and very large increases in the grain-to-forage price ratio are required to force movement to higher forage based diets (Brokken 1976).

Recent studies have helped explain this inconsistency between economic theory and feeding practice by showing that the forage-concentrate isoquant may be concave or linear to the origin in the middle range (Brokken et al.; Brokken 1977; Brokken and Bywater; Epplin, Bhide, and Heady 1980, 1983). Sigmoid isoquants obtained by Brokken (1977) for corn-soybean concentrate and alfalfa-orchardgrass forage were concave over the high concentrate diets (5–50% hay region) and convex over the high hay diets. The convexity over the high hay diets was attributed to feed intake depression resulting from hot weather in the latter part of the feeding period for those diets. Isoquants obtained by Bhide et al. (1983) for corn-alfalfa pellet concentrate and corn silage forage were convex over the high concentrate diets and concave over the high silage diets. However, considering the corn silage to be 50% corn grain on a dry matter basis, the concave portion of the isoquant then corresponds to concentrate diets between 15% and 50% forage, which is similar to the result obtained by Brokken (1977). The implication is that diet combinations of forage and grain in the middle range (15–50% forage) are never economically optimal unless differences in output quality,

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Table 1. Composition of the Concentrate Mixture and Diets

Ingredient	NRC Reference Number	Percent as Fed
Steam-rolled wheat	4-05-211	70.0
Steam-rolled barley	4-00-549	22.5
Dried molasses beet pulp	4-00-672	5.0
Limestone ($CaCO_3$)	6-01-069	1.0
Salt (cobalt-iodized)	6-04-152	1.0
Calcium-phosphate	6-01-082	0.5

Note: NRC = National Resource Council.

Table 2. Digestible Energy Concentration of the Diets (Mcal/kg of DM)

Diet Hay % : Conc %	Long Hay DE (Mcal/kg)	Cubed Hay DE (Mcal/kg)
77:23	2.58	2.61
68:32	2.72	2.75
59:41	2.77	2.77
49:51	2.92	2.96
40:60	3.01	3.15

Note: DM = dry matter; DE = digestible energy.

time requirements, or finished cattle price trends in some way offset the additional feed cost (Epplin, Bhide, and Heady 1980; Bhide et al. 1983).

Bhide et al. (1983), for example, demonstrated that diets in the concave portion of the corn silage-concentrate isoquant are not economical. Assuming corn silage to be 50% corn grain on a dry matter basis, the economically optimal diets for all weight gain isoquants switched from 50:50 silage : concentrate diets to 15:85 silage : concentrate diets as the price of corn silage increased relative to concentrate (Bhide et al. 1983).

Since concave or linear regions do exist in beef gain isoquants, it is important to know over what range the isoquants are not convex for a variety of concentrate and forage feeds. In addition, effects of various feeding regimes on the grade of beef produced (carcass quality) may be important in deriving economically optimal feedlot diets.

This article examines the results of a feeding experiment designed to test the effect of form of the roughage (cubed or long hay) and forage-to-concentrate ratio on carcass quality and profitability of beef feedlot finishing.

The Feeding Experiment

Details of the feeding experiment are given in Hironaka, Grigat, and Kozub. Hereford steer calves (n = 160) were purchased at local auctions, average weight 219 kg (range 172–262 kg), and assigned at random to 10 groups of approximately uniform average weight and individually fed diets with hay-to-concentrate ratios on a dry matter weight basis of 77:23, 68:32, 59:41, 49:51, and 40:60. Table 1 displays the composition of the concentrate and table 2 the digestible energy (Mcal/kg) of the diets. At each ratio, one group received long (baled) alfalfa hay and one group received cubed alfalfa hay. The steers were weighed at two-week intervals, and daily feed consumption figures were accumulated to coincide with the weighing intervals. Seven steers were removed or died during the experiment for reasons unrelated to the treatments. Each steer was slaughtered as it reached a market weight of approximately 480 kg. Carcass measurements (Martin, Fredeen, and Weiss) and Canadian beef carcass grades (Anonymous) were obtained.

Gain Isoquants

The growth of beef animals can be viewed in terms of feeding stages. Diet combinations of forage and concentrate that achieve the same weight gain describe a gain isoquant, i.e., a locus of concentrate-forage amounts consumed that results in the same weight gain. The economically optimum combination of concentrate and forage to reach any particular weight gain isoquant depends on the relative costs of concentrate and forage, as well as the time costs and revenue carcass quality differences between diet alternatives. Since time required to achieve a specified gain level may vary with the composition of the feed, the time costs include the cost of labor, medicine, fuel, etc., and the opportunity cost of fixed and variable resources. These can be estimated in terms of a yardage charge per day times the number of days to achieve a particular weight gain and by interest charges on the feed and feeder input costs.

In this article accumulated gain isoquants were estimated directly using the instrumental variables (IV) approach outlined by Sonka, Heady, and Dahm. They used ration protein percentage as the instrument for estimating the quantity of supplement consumed. The isoquant was estimated by regressing recorded values of corn consumption on predicted values of supplement consumption. Burt contends that the isoquant determination is redundant and implied in the estimate of protein consumption as a function of ration protein percentage. Epplin, Bhide, and Heady (1980) note that Burt's contention would hold if a theoretical relationship exists between the random independent variables and the instruments and if the relationship is properly specified by the functional form. In fact, the isoquant shape depends on the functional form chosen, whereas no such constraint holds for the IV procedure.

Since the steers were individually fed in this experiment, individual linear regressions were determined for each steer for accumulated feed intake as a function of accumulated weight and for accumulated days-on-feed as a function of accumulated weight. The equations all had highly significant coefficients and exhibited R-squared values of .99 plus. They were used to estimate accumulated days-on-feed and accumulated feed intake values for each steer from a hypothetical starting weight of 220 kg and a finishing weight of 480 kg. These starting and finishing weights approximated the actual average start and finish weights of steers on the test. The use of the individual regression equations corrected feed intake and days-on-feed values to a uniform weight gain of 260 kg. A time-on-feed relationship (estimated mean days-on-feed as a function of the proportion of forage in the diet and mean accumulated weight gain) was determined using Ordinary Least Squares (OLS) regression. For estimating the relationships, quantities of concentrate and hay were expressed on an as-fed basis. Throughout the trial the moisture content of the feed was approximately constant.

Table 3. Accumulated Cubed Hay-Concentrate Isoquant Functions where $C = \beta_0 + \beta_1 H$

Isoquant Gain Level	Intercept Coefficient β_0	Predicted Hay Coefficient β_1	Correlation Coefficient r	n
50 kg	359.12 (9.03)	−0.5120 (−4.11)	.0141	75
110 kg	730.43 (14.98)	−0.4829 (−6.62)	.4293	75
170 kg	1,102.00 (17.70)	−0.4740 (−7.75)	.5453	75
260 kg	1,659.50 (19.42)	−0.4683 (−8.45)	.6019	75

Concentrate-Hay Isoquants

Four weight gain intervals were defined for estimating the gain isoquants. From the assumed starting weight of 220 kg, accumulated weight gain isoquants were defined for 50, 110, 170, and 260 kg. The durations of the isoquant feeding stages were considered long enough for steers to accommodate changes in diet proportions of concentrate and forage. The last isoquant feeding stage (390 to 480 kg) was assumed long enough for diet to affect carcass quality, i.e., the carcass grades attained were assumed to be unaffected by diets in the previous gain increments. Although this assumption may not be realistic when the early feeding stages involve extremes of very high (near 100%) forage or concentrate diets, it is plausible for the range of forage : concentrate diets considered in this experiment.

Estimating the individual gain isoquants was accomplished using a two-stage least squares (2SLS) implementation of the IV technique (Johnston). For a particular weight gain, G, the isoquant is specified as:

$$C = g(H) + u, \tag{1}$$

where g represents the functional form of the isoquant. Since the animal and not the experimenter controls feed intake in *ad libitum* feeding, the feed variables (on an as-fed basis) of C (kilograms of concentrate consumed) and H (kilograms of hay consumed) have a random component that may not be independent of the error terms (u's) associated with their estimated values. The resultant violation of the OLS regression assumption that the values of the independent variables are fixed or nonstochastic results in estimation of OLS regression coefficients in (1) that are biased and inconsistent.

The IV technique requires creation of new independent variables that are nonstochastic but highly correlated with the stochastic independent variables of the regression model under consideration. Since the researcher may

Table 4. Accumulated Long Hay-Concentrate Isoquant Functions where $C = \beta_0 + \beta_1 H$ or $C = \beta_0 + \beta_1 H + \beta_2 H^2$

Isoquant Gain Level	Intercept Coefficient β_0	Hay Coefficient β_1	Squared Hay Coefficient β_2	Correlation Coefficient r	n
50 kg	316.17 (7.27)	−0.5670 (−3.46)		.1706	78
110 kg	516.43 (2.53)	0.17056 (0.22)	−0.0006300 (−0.95)	.3900	78
170 kg	764.48 (3.14)	0.26224 (0.45)	−0.0004534 (−1.40)	.5732	78
260 kg	1,136.50 (3.57)	0.3198 (0.64)	−0.0003149 (−1.75)	.6638	78

fix the proportion of one of the feeds in the diet, the stochastic component in the measured quantities of hay can be "purged" by regressing H on the proportion of hay in the diet (R). The predicted values of hay (H^p), as well as powers of the predicted values, may then be employed as appropriate instrumental variables in the estimation of the values of the coefficients for equation (1). For each of the cubed hay-concentrate and long hay-concentrate feeding experiments and for each accumulated gain isoquant, various functional forms for (1) were considered including linear, quadratic, and cubic forms. The "best" functional forms for each accumulated weight gain interval, the regression coefficients, and the corrected t-values[1] resulting from the estimation procedure are presented in tables 3 and 4.

Data plots of the estimated accumulated mean hay and concentrate amounts consumed at each isoquant level reveal the shape of the cubed hay-concentrate isoquants to be almost linear, while plots of the long hay-concentrate isoquants appear linear through the high hay region to concave over the high concentrate region (fig. 1). The cubed hay isoquants dominate those of the long hay isoquants at each isoquant level and reflect the higher feed-to-gain ratios of steers on the cubed hay diets versus the long hay diets (Hironaka, Grigat, and Kozub). Steers on the cubed hay diets also had average higher days-on-feed at each isoquant level than steers on the long hay diets.

The linear functional form estimated for the cubed hay-concentrate isoquants provided the best fit on the basis of correlation coefficient values (predicted concentrate correlated with actual concentrate consumed) of all functional forms estimated (linear, quadratic, and cubic). With the exception of the 50-kg gain isoquant (which is linear), the quadratic functional form provided the best fit for the long hay-concentrate isoquants but portrays a concave isoquant over the entire concentrate region as opposed to a concave region only over the high concentrate region as indicated by a plot of the means in figure 1. The correlation of predicted accumulated concentrate consumed at each isoquant to actual values (as illustrated by the correlation coefficient squared) was low (tables 3 and 4) and reflects the large variation in feed intake among individual steers on the same treatment. Figure 2 displays the estimated isoquants.

Estimation of Time-on-Feed Relationship

Accumulated time-on-feed (days) linear relationships were estimated for each of the cubed hay-concentrate and long hay-concentrate experiment groups. Because of the large variation in days-on-feed within diets, mean values of days-on-feed were regressed against the proportion of hay in the diet. Results are reported in tables 5 and 6.

Gain Isoquant, Time, Hay Type, and Carcass Quality Relationships

The estimated hay-concentrate gain isoquants together with the market prices of inputs and

[1] Since the standard errors of the estimates obtained from the second-stage regression of the 2SLS procedure are not "proper" estimates of the "true" standard errors, they were corrected using the procedure outlined in Gujarati, p. 386, and the corresponding t-values were revised.

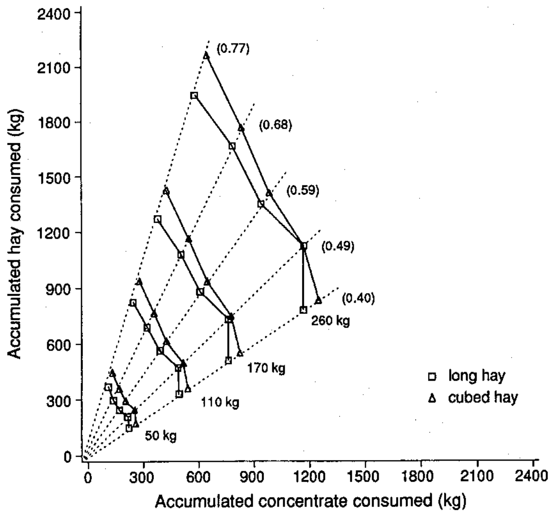

Figure 1. Gain isoquants plotted from accumulated hay and concentrate means

outputs were used to determine economically optimal diet combinations of forage and concentrate. The selection of the optimal diet is dependent upon the feedlot operator's objective. Several economic objectives have been proposed for cattle feeding operations including minimizing time-on-feed, maximizing average daily gain, minimizing costs of feeding, maximizing return per head, and maximizing returns to land (Melton et al.). Where the quality of beef produced is not changed across diets, only the cost aspect of the problem need be considered (Bhide et al. 1980). An appropriate objective in this case would be the minimization of feeding and time costs. However, where quality differences exist across diets a more appropriate objective becomes maximization of graded carcass returns over feeding and time costs. In this study optimal diets were generated for both objective functions of minimizing feeding and time costs and of maximizing returns over feed and time costs. Results were compared to determine the effect of carcass quality differences on the economics of diet selection.

Table 5. Accumulated Days-on-Feed (D) as a Function of Proportion of Cubed Hay in Diet (R) – Linear Regression Coefficients, where $D = \beta_0 + \beta_{1R}$

Isoquant Gain Level	Diet Proportion of Cubed Hay: Intercept β_0	Slope β_1	R^2	No. of Means
50 kg	67.00 (8.74)	27.88 (2.18)	.614	5
110 kg	114.48 (11.34)	60.47 (3.60)	.812	5
170 kg	161.97 (12.41)	93.05 (4.28)	.859	5
260 kg	233.20 (13.02)	141.92 (4.76)	.883	5

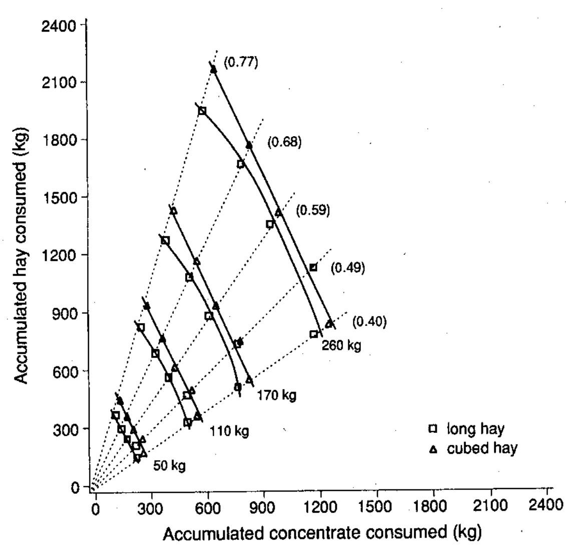

Figure 2. Gain isoquants plotted from regression equations

Input and Output Prices and Coefficients

Input prices considered in each of the two decision models related to the 1988–89 feeding year, with the feeding of steer calves assumed to begin in November of 1988. Concentrate ingredient prices (Canadian dollars) were fixed at \$158 per tonne for wheat, \$132 per tonne for barley, \$145 per tonne for beet pulp, \$120 per tonne for limestone, \$188 per tonne for salt, and \$540 per tonne for calcium. Prices for cubed alfalfa hay and baled alfalfa hay were varied over a range inclusive of their respective November 1988 prices. The November 1988 hay prices were \$127 per tonne for cubed alfalfa hay and \$79.20 per tonne for baled alfalfa hay. Time costs were calculated in terms of a yardage charge of \$.18 per day and an interest charge of 13% per annum on the feeder costs

Table 6. Accumulated Days-on-Feed (D) as a Function of Proportion of Long Hay in Diet (R) – Linear Regression Coefficients, where $D = \beta_0 + \beta_{1R}$

Isoquant Gain Level	Diet Proportion of Baled Hay: Intercept β_0	Slope β_1	R^2	No. of Means
50 kg	51.16 (5.46)	39.64 (2.54)	.683	5
110 kg	93.99 (7.09)	76.22 (3.45)	.799	5
170 kg	136.83 (7.12)	112.80 (3.52)	.806	5
260 kg	201.10 (6.86)	167.63 (3.44)	.797	5

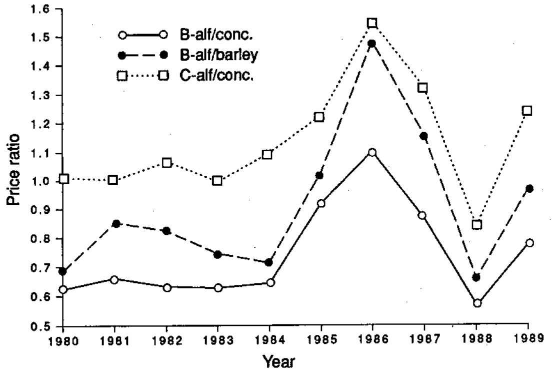

Figure 3. Historical price ratios of forage and concentrate

and one-half of the feed costs. Feeder steer cost was set at \$237.60 per 100 kg. To show the variation in forage-to-concentrate price ratios over time, historical (1980–89) cubed alfalfa-to-concentrate, baled alfalfa-to-concentrate, and baled alfalfa-to-barley price ratios were calculated and are plotted in figure 3.

Tables 7 and 8 display the carcass quality grade distributions and dressing percentages over the various treatment diets for cubed hay and long hay diet combinations. Carcass grade prices for each of the isoquant scenarios and diet combinations were calculated based on the level of average November (approximate time most calves are put on feed in Alberta) 1988 carcass grade prices adjusted by a seasonal index to the 1989 finishing date of the cattle on the different diet treatments. November 1988 carcass grade prices per 100 kg were \$311.40 for A1 steers, \$304.14 for A2 steers, \$274.14 for A3 steers, and \$263.14 for A4 steers.

The significant price differentials among grades and the differences in grade distribution and dressing percentages among treatment diets (tables 7 and 8) indicate that carcass quality might have an effect on diet selection. This is in contrast to some U.S. studies that indicate that carcass quality is not significantly affected by the proportion of forage in the diet (Pope and Heady). The difference may be attributed to the emphasis placed in the Canadian grading system on lean carcasses, which would suggest some premium for higher forage feeding. The top Canadian grade of A1 compares to the U.S. Good and Standard grades, while Canadian A2 compares to U.S. Choice and Prime grades (Considine et al.). Thus, Canadian grades should be more price responsive to higher forage feeding.

Linear Programming Models

Two linear programming formulations (corresponding to the two alternative objective functions of minimizing costs and time-on-feed and of maximizing returns over feed and time costs) were employed to investigate the economics of substituting forage for concentrate. The first model assumes that carcass quality is homogeneous among the diet alternatives within each of the long hay and cubed hay experiments. Given the estimated accumulated hay-concentrate and days-on-feed equations, accumulated levels of hay and concentrate and days-on-feed were respectively generated for each of the eight isoquant gain levels and possible values of proportion of hay in the diet (R) within the range of .40 to .78 in increments of .02.

On the assumption that weight gain in a particular interval is independent of the feeding history of the steer (Bhide et al. 1980), estimates of concentrate and hay eaten within each gain interval were determined by subtracting the appropriate cumulative estimate on one isoquant gain level from that above it. With four weight gain isoquants and 20 possible levels of hay in the diet, the results represented $(4 \cdot 20) = 80$ possible diet activities in the model, 20 in each weight gain interval. This allowed the determination of the optimal diet R value for each weight gain interval. Mathe-

Table 7. Canadian Carcass Grade Distribution by Cubed Hay-Concentrate Diets

Carcass Grade and Dressing Percent (DRS)	Proportion of Hay in the Diet									
	0.77	(n)	0.68	(n)	0.59	(n)	0.49	(n)	0.40	(n)
A1	0.6875	(11)	0.5000	(7)	0.6667	(10)	0.3125	(5)	0.2143	(3)
A2	0.1875	(3)	0.3571	(5)	0.2667	(4)	0.4375	(7)	0.5714	(8)
A3	0.1250	(2)	0.1429	(2)	0.0	(0)	0.1250	(2)	0.2143	(3)
A4	0.0	(0)	0.0	(0)	0.0666	(1)	0.1250	(2)	0.0	(0)
	1.0	(16)	1.0	(14)	1.0	(15)	1.0	(16)	1.0	(14)
DRS	0.5607		0.5685		0.5690		0.5794		0.5621	

matically, the model specification was similar to that given in Melton as:

$$\text{Min } Z = \sum_{f=1}^{6}\sum_{i=1}^{4}\sum_{r=1}^{20} C_f\, FEED_{fir}$$

$$+ \sum_{i=1}^{4}\sum_{r=1}^{20} pHAY\; HAY_i r + YARD\ TIME$$

$$- \sum_{r=1}^{20} (pFDR\; BWT\; FDR_r)$$

$$- \sum_{i=1}^{4}\sum_{r=1}^{20} FEEDINT_{ir} - \sum_{i=1}^{4} FDRINT_i$$

subject to:

$$-100\, FEED_{fir} + (PCONC_f\, 100)\, CONC_{ir} = 0 \quad \text{for } f = 1 \text{ to } 6,\ i = 1 \text{ to } 4,\ r = 1 \text{ to } 20$$

$$-100\, CONC_{ir} + IC_{ir}\, FA_{ir} \le 0 \quad \text{for } i = 1 \text{ to } 4,\ r = 1 \text{ to } 20$$

$$-100\, HAY_{ir} + IH_{ir}\, FA_{ir} \le 0 \quad \text{for } i = 1 \text{ to } 4,\ r = 1 \text{ to } 20$$

$$-\, TIME + \sum_{i=1}^{4}\sum_{r=1}^{20} DYS_{ir}\, FA_{ir} \le 0$$

$$\left\{\left[\left(\sum_{f=1}^{6} C_f\, FEED_{fir} + pHAY \times HAY_{ir}\right)\Big/2\right] RINT_{ir}\right\} - FEEDINT_{ir} \le 0 \quad \text{for } i = 1 \text{ to } 4,\ r = 1 \text{ to } 20$$

$$\sum_{r=1}^{20} [pFDR\; BWT\; INT\,(DYS_{ir}/365) FA_{ir}] - FDRINT_i \le 0 \quad \text{for } i = 1 \text{ to } 4$$

$$-FDR_r + FA_{ir} \le 0 \quad \text{for } i = 1$$

$$-\sum_{r=1}^{20} FA_{ir} + \sum_{r=1}^{20} FA_{(i+1)r} \le 0 \quad \text{for } i = 1 \text{ to } 7$$

$$\sum_{r=1}^{20} FA_{ir} = 1 \quad \text{for } i = 1 \text{ to } 4$$

Table 8. Canadian Carcass Grade Distribution by Long Hay-Concentrate Diets

Carcass Grade and Dressing Percent (DRS)	Proportion of Hay in the Diet									
	0.77	(n)	0.68	(n)	0.59	(n)	0.49	(n)	0.40	(n)
A1	0.6875	(11)	0.4375	(7)	0.2857	(4)	0.4375	(7)	0.0625	(1)
A2	0.2500	(4)	0.3125	(5)	0.4287	(6)	0.3125	(5)	0.6250	(10)
A3	0.0	(0)	0.1875	(3)	0.1428	(2)	0.1875	(3)	0.3125	(5)
A4	0.0625	(1)	0.0625	(1)	0.1428	(2)	0.0625	(1)	0.0	(0)
	1.0	(16)	1.0	(16)	1.0	(14)	1.0	(16)	1.0	(16)
DRS	0.5542		0.5773		0.5894		0.5861		0.5830	

Concentrate Feedstuffs (FEED f1r's: where f=WHT,BLY; 1='50','260'; r=1,2)

	WHT(50,1) 100kg	WHT(50,2) 100kg	BLY(50,1) 100kg	BLY(50,2) 100kg	WHT(260,1) 100kg	WHT(260,2) 100kg	BLY(260,1) 100kg	BLY(260,2) 100kg	CONC(50,1) 100kg	CONC(50,2) 100kg	CONC(260,1) 100kg	CONC(260,2) 100kg	HAY(50,1) 100kg	HAY(50,2) 100kg	HAY(260,1) 100kg	HAY(260,2) 100kg	FEEDINT(50,1) dollars
OBJ-Minimize Costs	$18.00	$18.00	$15.80	$15.80	$18.00	$18.00	$15.80	$15.80					$12.70	$12.70	$12.70	$12.70	1
WHT(50,1)	-100								70								
WHT(50,2)		-100								70							
BLY(50,1)			-100						30								
BLY(50,2)				-100						30							
WHT(260,1)					-100						70						
WHT(260,2)						-100						70					
BLY(260,1)							-100				30						
BLY(260,2)								-100				30					
CONC(50,1)									-100								
CONC(50,2)										-100							
CONC(260,1)											-100						
CONC(260,2)												-100					
HAY(50,1)													-100				
HAY(50,2)														-100			
HAY(260,1)															-100		
HAY(260,2)																-100	
FEEDINT(50,1)	1.31		1.15										0.93				-1
FEEDINT(50,2)		1.16		1.02										0.82			
FEEDINT(260,1)					1.31		1.15								0.93		
FEEDINT(260,2)						1.16		1.02								0.82	
FDRINT(50)																	
FDRINT(260)																	
FDRSTR(1) BALANCE																	
FDRSTR(2) BALANCE																	
DAYS ON FEED BALANCE																	
STEER TRANSFER ROW																	
ACTIVITY CONSTRAINT 1																	
ACTIVITY CONSTRAINT 2																	

	FEEDINT(50,2) dollars	FEEDINT(260,1) dollars	FEEDINT(260,2) dollars	FDRINT(50) dollars	FDRINT(260) dollars	FDRSTR(1) head	FDRSTR(2) head	TIME days	FA(50,1) head	FA(50,2) head	FA(260,1) head	FA(260,2) head		RHS
OBJ-Minimize Costs	1	1	1	1	1	$541.00	$541.00	$0.18						
WHT(50,1)													=E=	0
WHT(50,2)													=E=	0
BLY(50,1)													=E=	0
BLY(50,2)													=E=	0
WHT(260,1)													=E=	0
WHT(260,2)													=E=	0
BLY(260,1)													=E=	0
BLY(260,2)													=E=	0
CONC(50,1)									107				=L=	0
CONC(50,2)										148			=L=	0
CONC(260,1)											470		=L=	0
CONC(260,2)												636	=L=	0
HAY(50,1)									359				=L=	0
HAY(50,2)										314			=L=	0
HAY(260,1)											1578		=L=	0
HAY(260,2)												1345	=L=	0
FEEDINT(50,1)													=L=	0
FEEDINT(50,2)	-1												=L=	0
FEEDINT(260,1)		-1											=L=	0
FEEDINT(260,2)			-1										=L=	0
FDRINT(50)				-1					16.96	15.03			=L=	0
FDRINT(260)					-1						43.16	43.55	=L=	0
FDRSTR(1) BALANCE						-1			1				=L=	0
FDRSTR(2) BALANCE							-1			1			=L=	0
DAYS ON FEED BALANCE								-1	88	78	224	226	=L=	0
STEER TRANSFER ROW									-1	-1	1	1	=L=	0
ACTIVITY CONSTRAINT 1									1	1			=E=	1
ACTIVITY CONSTRAINT 2											1	1	=E=	1

Figure 4. A reduced example of the minimize costs linear programming model tableau

where, C_f is the cost per 100 kg of the fth concentrate ingredient, $FEED_{fir}$ represents the amount of concentrate feedstuff f consumed per steer on the rth diet to the ith isoquant gain level (100 kg units), $pHAY$ is the price of hay ($ per 100 kg), HAY_{ir} is the amount of HAY consumed per steer on the rth diet to the ith isoquant gain level (100 kg units), $pFDR$ is the price of feeder steers ($ per 100 kg), BWT is the beginning weight of feeder steers (100 kg units), FDR_r represents the feeder steers purchased (no. head) for the rth diet, $YARD$ is the yardage charge per day to cover fixed costs and labor ($ per day), $TIME$ is the days-on-feed for the feeding regime selected, $FEEDINT_{ir}$ is the interest charge ($) on the value of one-half of the feed for the rth diet between the $i - 1$th and ith isoquant gain levels, $FDRINT_i$ is the interest charge ($) on the value feeder steer for the ith isoquant gain level, $PCONC_f$ is the proportion of feedstuff f in the concentrate (decimal), $CONC_{ir}$ is the amount of concentrate consumed per steer (100 kg units) to reach the ith isoquant gain level on the rth diet, IC_{ir} is the mean concentrate consumed (kg) to reach the ith gain isoquant from the previous $(i - 1)$th gain isoquant using the rth diet, FA_{ir} is the proportion of feeder steers using the rth diet to reach the ith gain isoquant, IH_{ir} is the mean hay consumed (kg) to reach the ith gain isoquant from the previous $(i - 1)$th gain isoquant using the rth diet, INT is the current interest rate (prime plus 1%) as a decimal, DYS_{ir} is the mean number of days-on-feed to reach the ith gain isoquant from the previous $(i - 1)$th gain isoquant using the rth diet, and $RINT_{ir}$ is the prorated interest rate for the rth diet and for $i = 4$ isoquant, i.e., the total number of days-on-feed between the ith and $i - 1$th isoquants for the rth diet times the interest rate (INT) divided by the number of days in a year (365). A reduced example of the initial tableau using this notation is presented in figure 4.

To accommodate the objective function of maximizing returns above feed, feeder, and time costs, the above model was augmented with carcass grade meat selling activities. Since carcass quality data were only available for the experimental diet treatments, only five diet treatments were considered, i.e., $r = 1$ to 5. Accumulated hay and concentrate consumed and days-on-feed were generated using the developed equations and, for comparison, by using the actual means of concentrate and hay consumed and days-on-feed (by diet treatment and gain isoquant). The objective function was:

$$\begin{aligned}\text{Max } Z = & \sum_{f=1}^{6}\sum_{i=1}^{4}\sum_{r=1}^{5} C_f\, FEED_{fir} \\ & + \sum_{i=1}^{4}\sum_{r=1}^{5} pHAY\, HAY_{ir} + YARD\ TIME \\ & - \sum_{r=1}^{5} (pFDR\ BWT\ FDR_r) \\ & - \sum_{i=1}^{4}\sum_{r=1}^{5} FEEDINT_{ir} - \sum_{i=1}^{4} FDRINT_i \\ & + \sum_{g=1}^{4}\sum_{r=1}^{5} FPR_{gr}\, MEAT_{gr},\end{aligned}$$

where FPR_{gr} represents forecasted 1989 carcass grade prices ($ per 100 kg) for the gth grade of carcass produced off the rth diet fed in the last isoquant gain level feeding stage, and $MEAT_{gr}$ is the finished carcass beef produced (100 kg units) by gth grade and rth diet fed in the last isoquant gain level feeding stage. An additional meat balance constraint was added as:

$$-(DRS_r\ SWT\ GRADE_{gr}\ FA_{ir}) + (100\ MEAT_{gr}) = 0 \qquad \text{for } i = 4,$$

where DRS_r is the dressing percentage in decimal form obtained for beef steers finished on the rth diet in the last isoquant gain level feeding stage, SWT is the slaughter live weight of finished steers (480 kg), and $GRADE_{gr}$ is the proportion (as a decimal) of finished carcass produced on the rth diet that falls into the gth grade on average.

Each of the two model formulations was used to generate results for both the cubed hay-concentrate and long hay-concentrate scenarios. The only adjustment necessary was the use of the appropriate accumulated hay-concentrate and days-on-feed equations or means for each scenario.

The effect of substituting concentrate for forage was investigated at various price ratios of hay-to-concentrate for each scenario. For the cubed hay-concentrate analysis the price of hay was varied in increments of $5 per tonne from $45 to $150 per tonne. For the long hay-concentrate analysis the price of hay was varied in increments of $5 per tonne from $70 to $120 per tonne. Price of the concentrate was fixed. For each scenario and objective function formulation the results generated indicated the economically optimal proportion of hay in the diet to achieve each isoquant gain level given the stated hay-to-concentrate price ratio.

Table 9. Economically Optimal Diets for the Four Cubed Hay-Concentrate Gain Isoquants for the Minimum and Maximum Models at Selected Price Ratios

	Price Per Tonne of Cubed Alfalfa[a]						
	$45	$55	$65	$45	$55	$65	$150
	Ratio of Hay Price to Concentrate Price						
	0.302	0.369	0.436	0.302	0.369	0.436	0.671
	Minimum Cost Model			Maximum Returns Model			
Isoquant 1 (220–270 kg)							
Diet	0.78	0.78	0.40	0.77	0.77	0.40	0.40
Isoquant 2 (270–330 kg)							
Diet	0.78	0.40	0.40	0.77	0.40	0.40	0.40
Isoquant 3 (330–390 kg)							
Diet	0.78	0.78	0.40	0.77	0.40	0.40	0.40
Isoquant 4 (390–480 kg)							
Diet	0.78	0.40	0.40	0.59	0.59	0.59	0.40
Total Feed and Days							
Cubed Hay (kg as fed)	2,151.6	1,067.9	797.4	1,896.0	1,293.0	1,029.6	792.4
Concentrate (kg as fed)	652.0	1,147.6	1,286.1	769.3	1,045.0	1,179.9	1,286.1
Days	344	300	290	334	310	299	290
Net Revenue Difference ($/head) from 40:60 diet	13.66	1.76	0	34.39	24.09	9.92	0

[a] Canadian dollars.

Optimal Diets

Table 9 displays the optimal diet results for the cubed hay-concentrate scenario at alternative price ratios. That the estimated isoquants are linear with slopes varying from −.4683 to −.512 implies that, unless the price ratios of hay-to-concentrate are equal to the corresponding isoquant slopes, it would be less profitable to employ any combinations of hay and concentrate other than those specified at the end points of the isoquant. The data range in this study spanned diet hay-to-concentrate ratios from a 78:22 hay-to-concentrate diet to a 40:60 hay-to-concentrate diet. The minimum cost diet in each isoquant feeding stage is either of the endpoint diets. From 1980 to 1989 the price ratio of cubed alfalfa-to-concentrate ranged from .8 to 1.5. Within this range the high concentrate diet is least cost. Not until the price ratio declines to .369 does the diet with the highest proportion of cubed hay become least cost and then only for the first 50 kg of gain. This price ratio is 46% as high as the lowest price ratio in the decade of the 1980s. The switching pattern exhibited at a price ratio of .369 results because the first (50 kg) gain isoquant is relatively flatter than the isoquants for the remaining gain increments. This agrees with Brokken and Bywater's results which also show isoquants for the initial increments of gain to be relatively flatter than those for subsequent increments of gain. The implication of this pattern is that high roughage diets are relatively more efficient in the early stages of the finishing program.

The addition of carcass grade and price information produces some change in the results for the maximum returns over feed, feeder, and time costs model. Providing the assumption that the final isoquant period is long enough for diet to affect carcass quality, the results are similar to the minimum costs model except that the diet selected in the last isoquant feeding range is the 59:41 hay-to-concentrate diet. This reflects the shift in the grade distribution of steers fed this diet toward a higher percentage of carcasses grading A1 and A2, relative to the other hay-to-concentrate diet treatments of 77:23, 68:32, 49:51, and 40:60 (table 7). This result only holds for price ratios of cubed alfalfa-to-concentrate below 1.067. Above this level, the 40:60 cubed alfalfa-to-concentrate diet is optimal in all stages. From 1980 to 1989 the only year that the price ratio fell below 1.067 was in 1988 when it was .85. Similar results were obtained using the actual mean days-on-feed and mean hay and concentrate consumed data.

The addition of carcass grade information to the models also alters the results for the long hay-concentrate scenario. Due to the concave

Table 10. Economically Optimal Diets for the Four Long Hay-Concentrate Gain Isoquants for the Minimum and Maximum Models at Selected Price Ratios

	Minimum Cost Model	Maximum Returns Model	
Ratio of Hay : Concentrate Price	0.470	0.470	0.671
Isoquant 1 (220–270 kg)			
Diet	0.40	0.40	0.40
Isoquant 2 (270–330 kg)			
Diet	0.40	0.40	0.40
Isoquant 3 (330–390 kg)			
Diet	0.40	0.40	0.40
Isoquant 4 (390–480 kg)			
Diet	0.40	0.49	0.40
Total Feed and Days			
Baled Hay (kg as fed)	799.3	896.4	799.3
Concentrate (kg as fed)	1,190.9	1,168.2	1,190.9
Days	268	273	268
Net Revenue Difference from 40:60 diet ($/head)	0	3.29	0

nature of the isoquants, only the high concentrate endpoint diet (40:60) is selected in the minimize costs model as economically optimal (table 10). However, in the maximum returns model for the final isoquant feeding stage, the optimum diet selected switches from the high forage diet (40:60) chosen in the minimum cost model to a middle range hay : concentrate diet (49:51). This occurs because of the high proportion of A1 and A2 carcasses (75%) and the relatively high dressing percentage (58.61%) that resulted on this diet (table 8). Since the November 1988 long (baled) alfalfa hay price was approximately $80 per tonne, the high concentrate diet (40:60) would be optimal through all feeding stages except the last, where the intermediate diet (49:51) would be optimal. However, at a baled alfalfa hay price of $100 (price ratio hay-to-concentrate equal to .670), the high concentrate diet (40:60) becomes optimal in the last feeding stage. For the years 1980 to 1984 the baled alfalfa-to-concentrate price ratio ranged from .62 to .67. For all other years except 1988 it was far above the .67 level at which the high concentrate (40:60) diet becomes optimal. When the actual days-on-feed, baled hay, and concentrate means were used in the maximum returns model, the results were similar.

Discussion and Conclusions

This study was undertaken to determine the effect of form of the hay and consideration of carcass quality and time-on-feed information on the economics of substitution of alfalfa hay for concentrate in beef feeding diets. The feeding trial results agreed with other studies by indicating that the forage-concentrate weight gain isoquant is concave or linear to the origin in the middle range (Brokken et al.; Brokken 1977; Brokken and Bywater; Epplin, Bhide, and Heady 1980, 1983). Estimated long hay-concentrate gain isoquants were concave while cubed hay-concentrate gain isoquants were linear. The difference in isoquant shape was likely due to the influence of hay particle size on digestible energy (DE) and feed intake (Hironaka, Grigat, and Kozub). Particle size of the hay cubes was sufficiently fine that its influence on feed intake resembled that of concentrate more than that of long hay. The fact that steers fed the cubed hay diets required more feed on a dry matter basis and more days-on-feed to achieve the slaughter weight of 480 kg than steers on the baled hay diets indicates that cubed hay may never be economic relative to baled hay, unless it has some transportation, storage, and/or feed handling benefits unaccounted for in this study.

The results of the minimize costs model supported the hypothesis that the diets in the concave or linear regions of the isoquant will never be economically optimum. Optimal diets were either high concentrate or high forage diets, with switching to high concentrate diets as the price ratio of hay-to-concentrate increased. At historical (1980 to 1989) price ratios of cubed and baled alfalfa-to-concentrate, the high concentrate diet (40:60) was optimal. However, the addition of carcass quality and price information in the maximize returns model revealed that instances may arise (such as in 1988) where diets in the middle range of the isoquant (somewhat above 50% concentrate) are economically superior, given the range of diets in the experiment.

A limitation of the feeding experiment was absence of a forage : concentrate diet comparable to that typically fed by commercial feedlots in western Canada. A commercial diet is usually 10% to 15% forage on a dry matter basis, with a digestible energy concentration of about 3.3 Mcal/kg, which is higher than the cubed hay 40:60 diet value of 3.15 Mcal/kg. The commercial diet most likely would be selected by the feeding model as optimal in all stages of the feeding program because of its associated fewer total days-on-feed (about 240 days) over the optimal diet regimes in this experiment. Future experiments are planned to include a wider range of diets (100:0 to 0:100 on an as-fed basis) and more typical feedstuffs applicable to western Canada; namely, barley silage forage and barley grain-based concentrate.

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