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We model the dynamic global competition among national fiat currencies, cryptocurrencies, and Central Bank Digital Currencies (CBDCs) in which the strength of a country and of its currency are mutually reinforcing. The rise of cryptocurrencies hurts stronger fiat currencies but can benefit weaker fiat currencies by reducing competition from stronger ones. Countries strategically implement CBDCs in response to competition from emerging cryptocurrencies and other currencies. Our model suggests the following pecking order: Countries with strong but non-dominant currencies (e.g., China) are most incentivized to launch CBDC due to both technological first-mover advantage and potential reduction in dollarization; the strongest currencies (e.g., USD) benefit from developing CBDC early on to nip cryptocurrency growth in the bud and to counteract competitors' CBDCs; nations with the weakest currencies forgo implementing CBDCs and adopt cryptocurrencies instead. Strong fiat competition and the emergence of cryptocurrencies spur financial innovation and digital currency development. Our findings help rationalize recent developments in currency and payment digitization, while providing insights into the global battle of currencies and the future of money.

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Abstract

We model the dynamic global competition among national fiat currencies, cryptocurrencies, and Central Bank Digital Currencies (CBDCs) in which the strength of a country and of its currency are mutually reinforcing. The rise of cryptocurrencies hurts stronger fiat currencies, but can benefit weaker fiat currencies by reducing competition from stronger ones. Countries strategically implement CBDCs in response to competition from emerging cryptocurrencies and other currencies. Our model suggests the following pecking order: Countries with strong but non-dominant currencies (e.g., China) are most incentivized to launch CBDC due to both technological first-mover advantage and potential reduction in dollarization; the strongest currencies (e.g., USD) benefit from developing CBDC early on to nip cryptocurrency growth in the bud and to counteract competitors' CBDCs; nations with the weakest currencies forgo implementing CBDCs and adopt cryptocurrencies instead. Strong fiat competition and the emergence of cryptocurrencies spur financial innovation and digital currency development. Our findings help rationalize recent developments in currency and payment digitization, while providing insights into the global battle of currencies and the future of money.

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In the past decades, privately owned payment systems (e.g., PayPal, M-Pesa, Alipay, and Square) have gained widespread popularity. Recently, various cryptocurrencies further caused a fundamental reorientation of domestic and international monetary and payment technologies, as well as of policies and regulatory frameworks governing payment systems (Brunnermeier, James, and Landau, 2019; Adrian and Mancini-Griffoli, 2019; Cong, Li, and Wang, 2021a). Many countries around the globe react to these trends by actively researching on Central Bank Digital Currencies (CBDCs, see, e.g., Bech and Garratt, 2017; Duffie, 2021; Duffie and Gleeson, 2021), as revealed by the sharp rise in the number of central banks in the process of developing their own digital currencies (Boar, Holden, and Wadsworth, 2020; Boar and Wehrli, 2021).[1] Due to their potential to be safer, cheaper more efficient, interoperable, and versatile, digital currencies have the potential to challenge or even replace traditional fiat currency and other online payment systems.

How does the emergence of cryptocurrencies shape international currency competition? Will digital currencies challenge the dominance of the dollar? Which countries should develop CBDCs and when? How are various currencies differentially affected? To examine these issues, we develop a dynamic model of currency competition among multiple countries (or regions with a shared currency or a "systemic hegemonic currency," as in Carney, 2019) allowing the potential co-existence of fiat money, cryptocurrencies, and CBDCs, a crypto sector with endogenous growth, and governments' endogenous efforts for digitizing money. Our theory helps rationalize international trends in payment and currency digitization, reveals a novel pecking order for CBDC development, and provides insights concerning the implications of the rise of digital currencies for global competition, financial innovation, and the future of money.

Specifically, we consider two countries, A and B, each with its fiat currency, and a digital economy featuring one representative cryptocurrency C as the means of payment. In each period,

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[1] For example, the Bank of Canada (Jasper Project) and the Monetary Authority of Singapore (Ubin Project) have tested the use of token-based CBDCs for cross-border wholesale settlements (Veneris, Park, Long, and Puri, 2021). The Hong Kong Monetary Authority (LionRock Project) and the Bank of Thailand (Inthanon Project) have collaborated in a similar way. European and Japanese central banks have also been actively researching digital currencies through Project Stella. China, in particular, has rolled out its digital currency program in 2020 and conducted more than $5 billion in e-renminbi transactions, reputedly anticipating the world to test drive its technological achievement at the Beijing Winter Olympic Games (Rabouin, 2021). Some suspect China of waging a digital currency insurgency on the global financial system and the primacy of the dollar (Ehrlich, 2020), while others dismiss the impact (Eichengreen, 2021). The key motivations of China for introducing eCNY are cited as limiting the dominance of private payment services. However, both mobile service provision and eCNY, once more international, can challenge U.S. dollars and Euros. After all, eCNY technology likely opens commercial opportunities for China in some emerging markets, amplifying China's influence in emerging economies, something U.S. and EU foreign policy experts may have to consider.

one representative OLG household is endowed with perishable consumption goods, which also serve as the numeraire. Importantly, all three currencies A, B, and C fulfill the standard roles of money as (i) store of value allowing households to store endowments for desired consumption timing, (ii) medium of exchange (generating a convenience yield), and (iii) unit of account (not only domestically but also internationally as a reserve currency). In general, households choose their holdings of currencies A, B, and C to store their consumption goods over time, trading-off the currencies' convenience yield versus inflation and depreciation relative to other currencies which compromise the store of value function.

Importantly, currencies A and B exhibit an endogenous debasement that decreases with the strength of countries' economic fundamentals captured by the countries' expenses, such as the fiscal deficit, international trade costs, or the debt service costs countries A and B incur. For example, a country's high expenses in terms of the consumption good represent weak economic fundamentals, cause a high inflation rate and/or depreciation relative to other currencies, and thus imply a weak national currency. We use A to denote the stronger country and its currency, which is more valuable in terms of the numeraire and can be viewed as the international reserve currency (e.g., the U.S. dollar); then B represents a competitor currency, such as the Yuan. To incorporate that foreign debt is often denominated in the reserve currency, i.e., dollars (Maggiori, Neiman, and Schreger, 2020), or that the U.S. dollar is the global unit of account for invoicing in international trade (Gopinath, Boz, Casas, Díez, Gourinchas, and Plagborg-Møller, 2020), we assume that countries' expenses are partially denominated in currency A.

Households' choice between national currencies induces a feedback and can lead to a vicious circle of inflation and depreciation for weaker currencies. A stronger currency A causes higher inflation and depreciation of currency B. As currency B depreciates, households substitute more towards currency A, aggravating inflation and depreciation of currency B. Country A essentially imposes a pecuniary externality on the relatively weaker country B through a form of dollarization. The mechanism manifests itself clearly in practice in that the strength of the U.S. dollar and the trust the world has in U.S. finance (or submission to its technological and military prowess) are mutually reinforcing.

We then consider the crypto sector in which the growth rates of adoption, usage, and convenience yield of cryptocurrencies endogenously increase with adoption. Cryptocurrencies, including stablecoins pegged to fiat currencies, constitute a viable substitute for fiat currencies as a store of

value and medium of exchange. High inflation rates in fiat currencies spur cryptocurrency usage and growth over time. Intuitively, the absence of strong national fiat currencies implies a vacuum in the currency space, and private cryptocurrencies emerge to fill the demand. Households also hold more cryptocurrencies when the underlying technology is more effective or the crypto sector is more vibrant and creative. Interestingly, the cryptocurrency market acts as a buffer zone amidst the battle between the two fiat currencies and dampens the degree of dollarization and the vicious circle of debasement the weaker currency is exposed to.

As the crypto sector grows and the household substitutes toward crytpocurrency, the strong currency faces more competition from cryptocurrency and depreciates. Because the growth of the cryptocurrency market depends on the strength of currencies A and B, a stronger currency B could benefit A by slowing the growth of the crypto-sector which in turn poses less competition to A. Importantly, the weaker currency B might benefit from the rise of cryptocurrencies, depending on whether the reduction in competition from A outweighs the increase in competition from cryptocurrencies. The model therefore rationalizes why countries with dominant currencies are more eager to ban or regulate cryptocurrencies, whereas countries with the weakest currencies, such as El Salvador and Venezuela, do exactly the opposite and even adopt cryptocurrency as a legal means of payment.

Our framework also applies to the study of fiat-backed cryptocurrencies, especially stablecoins which are typically pegged to the U.S. dollar and (partially) backed by U.S. dollar assets (e.g., USDC). When a cryptocurrency is backed by reserves consisting of currency A, country A can capture part of the seigniorage generated from cryptocurrency usage, which strengthens currency A but weakens other currencies. These findings suggest that the U.S. and the U.S. dollar may benefit from regulation that requires stablecoin issuers to hold U.S. dollar reserves instead of regulation that restricts or bans stablecoin issuance. Furthermore, as an alternative to developing CBDCs to compete with cryptocurrencies, properly regulated stablecoins could potentially allow countries such as the United States to effectively "delegate" the creation of a digital dollar to the private sector, whilst sharing the seigniorage revenues.

We next consider the endogenous development of sovereign digital currencies, notably CBDCs. We model CBDC implementation in a technology-neutral manner that does not rely on any specific design, simply stipulating that it increases the convenience yield of holding the country's currency. Our framework features monetary neutrality and can accommodate possible interest-bearing or

tax-charging digital currencies. We also recognize that launching CBDCs entails tremendous technological, legal, economic, and operational obstacles, therefore modeling it as a Poisson arrival process based on the countries' endogenous (and costly) efforts. Once implemented, CBDCs would immediately alter the endogenous value of other currencies, whether fiat or digital, as well as other countries' incentives to implement their own digital currencies.

Countries' strategic decisions to implement CBDCs reflect competition from both cryptocurrencies and other fiat currencies. The stronger country's incentives to launch CBDC mainly derive from the desire to compete with cryptocurrency. These incentives are high when the cryptocurrency market is in its infancy, because then the launch of CBDC has the largest effect in reducing competition from cryptocurrencies. This effect gives rise to a "cryptocurrency kill zone" that allows for a preemptive "killer adoption" of the technology. If countries with strong currencies adopt the technology underlying cryptocurrencies through launching CBDC early enough, they can nip the future growth and dominance of cryptocurrencies in the bud. Otherwise it is only until the cryptocurrency market has gained widespread adoption that the implementation of CBDC becomes unavoidable to avert a takeover by cryptocurrencies. As a result, the stronger country's strategy for launching CBDC evolves from an offensive, preemptive tactic to a purely defensive measure. Regardless, our model predicts that the digitization of money becomes inevitable in the long run.

We find that CBDCs offer the most advantages for countries with non-dominant currencies (country B), as long as their currencies are not too weak. The non-dominant country B's incentives to launch CBDC is stronger than A's and are primarily shaped by the desire to obtain a technological first-mover advantage to reduce the degree of dollarization country B is exposed to. Our model explains why the first CBDCs have been implemented by countries such as China, rather than the United States. The implementation of CBDCs by such countries poses considerable challenge for both the cryptocurrency market and stronger fiat currencies, such as the U.S. dollar. We also find that the dominance of the U.S. dollar causes "entrenchment" that reduces the incentives of the U.S. to implement CBDC. The recent spike in U.S. inflation, however, potentially undermines the dominance of the U.S. dollar and improves government incentives to venture into digitizing money.

Depending on the circumstances, decisions to launch CBDC can be either strategic substitutes or complements. Our model highlights that through the launch of CBDC, weaker currencies may challenge the dominance of stronger currencies. If it poses a threat on the dominance of the stronger currency, the implementation of CBDC by weaker countries increases the incentives of

the stronger country to launch CBDC too, giving rise to strategic complementarity in CBDC issuance. Consistent with our model, the issuance of CBDC by China is often perceived as such a threat to the dominance of the U.S. dollar and, accordingly, has led calls to action (Ehrlich, 2020, Forbes) for the U.S. to consider the development of CBDC too. In contrast, CBDC issuance by stronger countries eliminates the possibility for weaker countries to attain a technological first-mover advantage, thereby always reducing weaker countries' incentives to develop CBDC and giving rise to strategic substitutability in CBDC issuance.

We further study the implications of CBDC issuance on the currencies of developing countries with particularly weak currencies (bigger gap between A and B). Consistent with Brunnermeier et al. (2019), we find that such countries are particularly prone to digital dollarization: they tend to suffer the most when a country with strong currency implements CBDC. Yet, these developing countries and their currencies do not benefit much from implementing CBDC themselves, because their currency is weak regardless of its underlying technology. Our analysis suggests that developing countries may benefit from adopting cryptocurrency as a legal means of payment within their own territory instead of implementing CBDCs as a way to escape from (digital) dollarization. Overall, the pecking order of CBDC development entails countries with strong but non-dominant currencies (e.g., China) leading the efforts, followed by countries with the strongest currencies (e.g., the U.S.), and then by nations with the weakest or non-existent sovereign currencies (e.g., El Salvador).

Finally, we recognize that fiat currencies and traditional bank-payment-rails are inefficient and fragmented, due to the non-digital nature, the lack of coordination, or the limited competition in the presence of network effects (Rochet and Tirole, 2003). The development of various digital currencies therefore can be viewed as financial innovations that eventually benefit households (and businesses and governments, see, e.g., Duffie, Mathieson, and Pilav, 2021). Our model can be used to understand how currency competition and the strength of national currencies relate to financial innovation. In particular, the weakness of national currencies implies a vacuum in the currency space which favors the emergence of (private) cryptocurrencies and thus financial innovation in the private sector. Moreover, as crytpocurrencies gain widespread adoption, countries' incentives to innovate through the implementation of CBDC increase too, further stimulating financial innovation. Put differently, the dominance of national currencies curbs incentives for innovation both for governments and the private (financial) sector, which is consistent with the view that competition stimulates innovation.

Literature. Our discussion on global digital currency competition is most closely related to on-going policy debates, regulatory hearings, and industry initiatives (Bech and Garratt, 2017; Duffie and Gleeson, 2021; Duffie, 2021; Prasad, 2021; Giancarlo, 2021). Instead of analyzing competitions among platforms (Gandal and Halaburda, 2016; Lyandres, 2020) or reserve assets (He, Krishnamurthy, and Milbradt, 2019), we consider the competition between general payment tokens and digital currencies that aim to compete with fiat as new forms of money. Digitization leads to unbundling and rebundling of the roles of money and fiercer competition of specialized currencies (Brunnermeier et al., 2019), which affects exchange rates and monetary policy (e.g., Benigno, 2019).[2] We analyze both the competition between private and public money and between CBDCs, focusing on the impacts of CBDC introduction on currency competition, price dynamics, and governments' incentives to digitize money.

Our work thus adds to the emerging literature on CBDCs and stablecoins. Bech and Garratt (2017); Auer and Böhme (2020); Auer, Frost, Gambacorta, Monnet, Rice, and Shin (2021); MAS (2021); Mancini-Griffoli, Peria, Agur, Ari, Kiff, Popescu, and Rochon (2018); Duffie et al. (2021) provide overviews and surveys about CBDCs. Many articles analyze the interaction between the banking sector and CBDCs (Fernández-Villaverde, Schilling, and Uhlig, 2020; Bindseil, 2020; Bordo and Levin, 2017; Davoodalhosseini, 2021; Brunnermeier and Niepelt, 2019; Piazzesi and Schneider, 2020; Parlour, Rajan, and Walden, 2020; Fernández-Villaverde, Sanches, Schilling, and Uhlig, 2021). In particular, several studies examine the impact of CBDCs on deposit and lending markets within a country, and its dependence on bank competition, market frictions, and design features (Chiu, Davoodalhosseini, Hua Jiang, and Zhu, 2019; Andolfatto, 2021; Keister and Sanches, 2021; Garratt and Zhu, 2021). Ferrari, Mehl, and Stracca (2020) analyze open-economy implicaitons of CBDCs. Lyons and Viswanath-Natraj (2020), Kozhan and Viswanath-Natraj (2021), Guennewig (2021), Li and Mayer (2021), etc., analyze stablecoins issued by private entities. We are the first to analyze global competition of digital currencies among multiple nations that involves cryptocurrencies, CBDCs, and fiat money simultaneously, and to provide insights on the incentives and tradeoffs for central banks to introduce CBDCs or for governments to adopt cryptocurrencies. We also clarify how stablecoins pegged to fiat currencies alter these tradeoffs.

More broadly, our study contributes to the large literature on blockchain economics and cryp-

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[2] In particular, Benigno, Schilling, and Uhlig (2019) show that a global (crypto)currency, like the global financial cycle (Rey, 2015), transforms the monetary trilemma (Fleming, 1962; Mundell, 1963) into an "irreconcilable duo."

tocurrencies.[3] Biais, Bisiere, Bouvard, Casamatta, and Menkveld (2018), Pagnotta (2021), Cong, Li, and Wang (2021b), etc., provide theoretical foundations for token pricing while Liu and Tsyvinski (2021), Liu, Tsyvinski, and Wu (2019), Makarov and Schoar (2020), Cong, Karolyi, Tang, and Zhao (2021), etc., empirically document cryptocurrency return patterns. While a large part of the literature focuses on consensus generation and the design or functionality of tokens (e.g., Cong and Xiao, 2021; Cong et al., 2021a; Garratt and Van Oordt, 2021; Gryglewicz, Mayer, and Morellec, 2021; Mayer, 2022; Prat and Walter, 2021), they do not examine the competition among various digital currencies, including the ones issued by central banks as well as private parties.

1 An Illustration of Global Currency Competition

We first present a stylized two-period model to introduce the building blocks for the dynamic model, present four fundamental insights, and convey the intuition behind our key findings.

1.1 Fiat Money in the Two-period Economy

One representative household populates the economy and one generic consumption good serves as the numeraire in which prices are quoted. There are two time periods, $t = 0, 1$, without time discounting. Money serves a combination of the standard roles as: (i) a store of value, (ii) a medium of exchange, and (iii) a unit of account. We consider two countries, A and B, with their own native fiat currencies A and B. Country $x \in \{A, B\}$ has one unit of currency outstanding whose time-t price is P_t^x in terms of the numeraire.

At $t = 0$, the representative household is endowed with one unit of perishable consumption good. The household only derives consumption utility at $t = 1$, and thus would like to store the endowment from $t = 0$ to $t = 1$. Because the consumption good cannot be stored directly, money serves as a store of value and, specifically, enables the household to store the consumption good. The household uses its entire endowment to buy money at $t = 0$ and then sells money at $t = 1$ in exchange for consumption goods. We assume that country x buys back its own currency at $t = 1$ using consumption goods at price P_1^x.[4]

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[3] Chiu and Koeppl (2019), Cong and He (2019), and Easley, O'Hara, and Basu (2019) are among the earliest contributions. For a literature review on blockchain economics, see, e.g., Chen, Cong, and Xiao (2021), John, O'Hara, and Saleh (2021), and Irresberger, John, and Saleh (2020).

[4] This is consistent with how a government typically guarantees the value of the currency through its ability to raise real resources via taxation and offer to purchase currency using those resources (Obstfeld and Rogoff, 2017).

The household can use either currency A or B as a store of value and takes prices as given. Let $m^A \geq 0$ and $m^B \geq 0$ denote the units of consumption good the household stores at time $t = 0$ in currencies A and B respectively. At $t = 0$, the household invests the whole unit of consumption good in money, i.e., $m^A + m^B = 1$. Denote the time-0 price of currency A by $P^A = P_0^A$. Because each currency is in unit supply, the initial market capitalization of currency x in terms of the consumption good is also P^x. Because the household is the only holder of money, the market clearing conditions require $P^x = m^x$. As a result,

$$P^A + P^B = 1. \tag{1}$$

At $t = 1$, the household sells its holdings of currency x at price P_1^x and consumes the proceeds, so the household's consumption at $t = 1$ reads: $c = P_1^A + P_1^B$. We consider without loss of generality country A as "strong" and country B as "weak," in that $P_0^A \geq P_0^B$ and currency A serves as the reserve currency at $t = 0$ in a way made precise later.

Household's utility. Naturally, money also serves as a medium of exchange (i.e., transaction medium). We account for this function of money in reduced form by stipulating that the household derives a convenience yield from holding money (e.g., [Krishnamurthy and Vissing-Jorgensen, 2012](#)). As such, the household's lifetime utility reads

$$U = c + Z_o(m^A + m^B) + Z^A v(m^A) + Z^B v(m^B), \tag{2}$$

where c is the household's consumption at $t = 1$ and $Z_o(m^A + m^B) + Z^A v(m^A) + Z^B v(m^B)$ is the convenience yield of holding currency from $t = 0$ to $t = 1$. Crucially, currencies A and B offer different convenience yields $Z_o m^A + Z^A v(m^A)$ and $Z_o m^B + Z^B v(m^B)$, with the difference in convenience captured by the coefficients $Z^A \geq 0$ and $Z^B \geq 0$. For illustration, we take the commonly used CRRA specification with $\eta = 2$, which we generalize later in the dynamic model:

$$v(m^x) = \frac{(m^x)^{1-\eta} - 1}{1 - \eta} = \frac{m^x - 1}{m^x}. \tag{3}$$

The household derives a constant (marginal) base convenience yield $Z_o > 0$ regardless of whether she holds A or B. The constant Z_o is chosen large enough to ensure that the convenience yield

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The dynamic model gets rid of this assumption.

$Z_o m^x + Z^x v(m^x)$ to holding currency x is non-negative in equilibrium and is otherwise immaterial. The functional form (3) has several appealing features. First, as m^x approaches zero, the marginal convenience to holding x becomes arbitrarily large, capturing broadly that x cannot be substituted for certain activities and transactions. As a consequence, $m^x > 0$. Second, as m^x becomes large, the convenience yield to holding currency x diminishes.

Global currency, reserve currency status, and inflation. Both countries must cover expenses, such as the cost of servicing of their outstanding debt or their fiscal deficit. We assume that currency A as the reserve currency is the "global" unit of account in debt contracts and trade invoicing, among other "exorbitant privileges."[5] To capture that international trade invoicing and borrowing are often denominated in dollars in practice, we assume that country x's expenses are denominated in currency A.[6] Now, country x covers expenses of π^x units of currency A by inflating its currency and reducing the currency value at time $t = 1$, i.e., $P_1^x - P_0^x = \pi^x P^A$. In essence, any currency holder incurs taxes of $\pi^x(P^A/P^x)$ units of the consumption good, where π^x inversely proxies for the strength of a country's economic fundamentals or a country x's fiscal strength.[7]

We can easily infer this tax as inflation, creating a link between a country's fiscal strength and the strength of its currency: Country x's fiscal strength (i.e., π^x) affects inflation and so the benefits of holding currency x, which in turn determines the strength and value of currency x. The sole purpose of introducing the parameter π^x is to capture this empirically relevant link between a country's fiscal strength or economic fundamentals and the strength of its currency (Jiang, Lustig, Van Nieuwerburgh, and Xiaolan, 2020).[8]

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[5]Du, Pflueger, and Schreger (2020) show that countries which are able to issue more domestic currency debt are also the ones that issue more debt denominated in foreign currency; Maggiori et al. (2020); Maggiori, Neiman, and Schreger (2019) document that U.S. dollar is the primary currency of denomination (over 60%) since the 2008 crisis in cross-border investors portfolio holdings, even when neither the investor nor the issuer are based in the United States; a dollar dominance similarly manifests in invoicing traded goods (e.g., Goldberg and Tille, 2008; Gopinath and Stein, 2021), consistent with the international use of the dollar as a unit of account (e.g., Matsuyama, Kiyotaki, and Matsui, 1993; Doepke and Schneider, 2017); Gourinchas (2019) and Jiang, Krishnamurthy, and Lustig (2020), among others, further elaborate on the dollar dominance.

[6]The dynamic model relaxes this assumption, and considers that country x's expenses are only partially denominated in currency A.

[7]We require π^x to be sufficiently small to ensure positive P_1^x, a restriction relaxed in the dynamic model.

[8]Following **?**, one could also model this link between a country's fiscal strength and the strength of its currency by stipulating that the convenience yield of currency x directly depends on the economic fundamentals of country x. Our results pertain as long as there a country's fiscal strength improves the benefits of holding currency.

1.2 Equilibrium for Traditional Currency Competition

On the margin, the household must be indifferent between allocating funds to currency A and to currency B, subject to $m^A + m^B = 1$. Taking prices P^x as given and considering market clearing for currency x, we then have:

$$\frac{Z^A}{(P^A)^2} - \pi^A = \frac{Z^B}{(P^B)^2} - \frac{\pi^B P^A}{P^B}, \tag{4}$$

which together with (1) pins down the currency values P^A and P^B. Condition (4) states that in equilibrium, the sum of the marginal convenience yield, $\frac{Z^x}{(m^x)^2} = \frac{Z^x}{(P^x)^2}$, and inflation, π^A and $\pi^B P^A / P^B$ respectively, must be equal across currencies x. Under mild regularity conditions, a unique equilibrium ensues.

Proposition 1. *Suppose $\pi^B \geq 4(Z^A + Z^B)$, $\pi^A \in [0, \pi^B)$ is sufficiently small (e.g., $\pi^A = 0$), and $Z^A \geq Z^B$. Then, there exists a unique equilibrium. In equilibrium, (4) holds and the currency value P^A satisfies $P^A > 1/2 > P^B$. Currency A carries less inflation than currency B, in that $\pi^A < \pi^B(P^A/P^B)$.*

Note that the parameter restrictions are simply sufficient conditions to ensure that country A is strong relative to country B. Proposition 1 formalizes these conditions. Our results also hold under symmetry in terms of convenience yields, i.e., $Z^A = Z^B$.

Insight 1: Currency competition features feedback effects. A decrease in demand and value P^B of currency B increases the inflation $\pi^B P^A / P^B$ currency B carries, which in turn reduces the value of currency B further. Notably, this effect is amplified because due to $P^A + P^B = 1$, a decrease in currency value P^B also implies an increase in currency value P^A which further exacerbates inflation of currency B and reduces P^B. Consequently, currency usage and dominance exhibit strong network effects.[9] Due to currency A's dominance, we say that currency A maintains the reserve currency status.[10] Interestingly, the causality runs both ways: Currency A has less inflation than B because it is the more valuable currency (i.e., $P^A > P^B$); A is the stronger currency also because it has less inflation.

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[9]In fact, a similar force could be obtained by modelling network effects in reduced form (e.g., Cong, Li, and Wang, 2021c; Cong et al., 2021a).

[10]That is, we take the currency with higher price at $t = 0$ as the reserve currency.

Currency dominance is therefore reinforcing in our model, which fits real life observations. For example, once the U.S. dollar became stronger after the demise of Bretton Woods, the U.S. government and firms could issue debts in dollars and borrow more easily worldwide, which helped its economic, financial, and military developments. The strong military presence, as well as the trust the world has in U.S. finance, through rule of law, stable and independent central banks, well-functioning open capital markets, on top of the depth and liquidity of US fixed income markets, especially money markets and Treasuries, in turn reinforces and solidifies the dollars' strength and global currency status.

1.3 The Rise of Cryptocurrencies

We now add a representative cryptocurrency C to our two-period model. Specifically, cryptocurrency C has fixed unit supply and is traded in a frictionless secondary market against the consumption good at price P^C which we set to be the same at $t = 0$ and $t = 1$, for simple illustration.[11] The household can now store its wealth from $t = 0$ to $t = 1$ by buying cryptocurrency at $t = 0$ with its endowment and selling at $t = 1$ for consumption goods. Besides, the household derives a convenience yield from holding cryptocurrency, which we assume, for simplicy, to be $v^C(m^C) = Z_o m^C + Y m^C$, where m^C denotes the household's cryptocurrency holdings in terms of consumption goods. We consider that the cryptocurrency convenience yield has the same constant (marginal) base Z_o as the convenience yield of currencies $x = A, B$; the exact value of Z_o is irrelevant in equilibrium.

The dependence on Y captures the technology underlying cryptocurrencies and the prosperity of the cyber economy. Another key distinction from fiat money is that cryptocurrencies are less subject to inflation—a salient feature inherent to many cryptocurrencies, especially those competing with money as a store of value (e.g., Bitcoin), that is cited as a driver for their emergence.[12]

Notice that we implicitly assume that there are no other ways for country x to cover expenses π^x than imposing a tax on currency holdings. Also, unlike government-issued money, cryptocurrency systems do not impose explicit tax and are algorithmically committed to moderate inflation. We incorporate this reality by stipulating that cryptocurrency holdings are not directly taxed.[13]

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[11]We later endogenize the price dynamics and also discuss how the representative cryptocurrency may represent stablecoins potentially pegged to a fiat currency as well as non-pegged decentralized cryptocurrencies such as Ether. Note that at lower frequencies, major cryptocurrency prices relative to USD are not necessarily more volatile than other commonly used fiat currencies such as Euros or Yens (Kikuchi, Onishi, and Ueda, 2021).

[12]Intuitively, blockchain technology and smart contracts allow commitments to a predetermined supply schedule (Cong et al., 2021a), which can be designed to mitigate inflationary pressure from expanding supply.

[13]Admittedly, it is true that governments around the globe are working hard to collect tax from cryptocurrency

The crypto equilibrium. Currency competition occurs now within the triangular relationship between countries A and B as well as the cyber economy induced by the cryptocurrency, leading to both country-to-country and country-to-cryptocurrency competitions. To characterize the effects of cryptocurrencies, we look now for a "crypto equilibrium," with $m^C > 0$ and $P^C > 0$. Also in the presence of cryptocurrencies, the household stores its entire endowment at $t = 0$ in money, so that $m^A + m^B + m^C = 1$. Market clearing for currency x implies $m^x = P^x$, so that:

$$P^A + P^B + P^C = 1. \tag{5}$$

In the equilibrium with cryptocurrencies and $P^C > 0$, the household is indifferent between exchanging a marginal unit of cryptocurrency for one unit of currency A and B. The indifference conditions and market clearing lead to closed-form solutions of currency values:

$$
P^A = \sqrt{\frac{Z^A}{Y + \pi^A}}, \quad \text{and} \quad P^B = \frac{2\sqrt{Z^B(Y + \pi^A)}}{\sqrt{4\,Y^2 + 4\,\pi^A\,Y + (\pi^B)^2(Z^A/Z^B)} + \pi^B\,\sqrt{Z^A/Z^B}},
$$
$$
P^C = 1 - \sqrt{\frac{Z^A}{Y + \pi^A}} - \frac{2\sqrt{Z^B(Y + \pi^A)}}{\sqrt{4\,Y^2 + 4\,\pi^A\,Y + (\pi^B)^2(Z^A/Z^B)} + \pi^B\,\sqrt{Z^A/Z^B}}. \tag{6}
$$

Note that for the crypto equilibrium to exist, it must be that P^C in (6) is positive.

Interestingly, (6) illustrates that the cryptocurrency market acts as a type of buffer zone in the competition between currency A and B. For instance, a decrease in π^B which leads currency B to appreciate causes the cryptocurrency price P^C to fall, but does not affect the price of currency A. In contrast, a decrease in π^A and an appreciation of currency A cause both currency B and cryptocurrencies to depreciate. The underlying reason is that country B's expenses are denominated in terms of currency A. However, the consequences of the appreciation of currency A are partially absorbed by cryptocurrencies. We summarize these findings in the following Proposition.

Proposition 2. *The crypto equilibrium, if it exists, is unique. It features $m^x = P^x$, where currency values P^x for $x \in \{A, B, C\}$ are characterized in (6). There exists a crypto equilibrium if and only if P^C in (6) exceeds zero, which is satisfied if Y is sufficiently large. In the crypto equilibrium, the*

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transactions, but the regulatory actions are very costly and peer-to-peer transactions cannot be taxed as it currently stands. Any tax collected on crypotcurrencies may be expended away on dealing with the monetary and financial instability the lack of investor protection cryptocurrencies cause (Prasad, 2021).
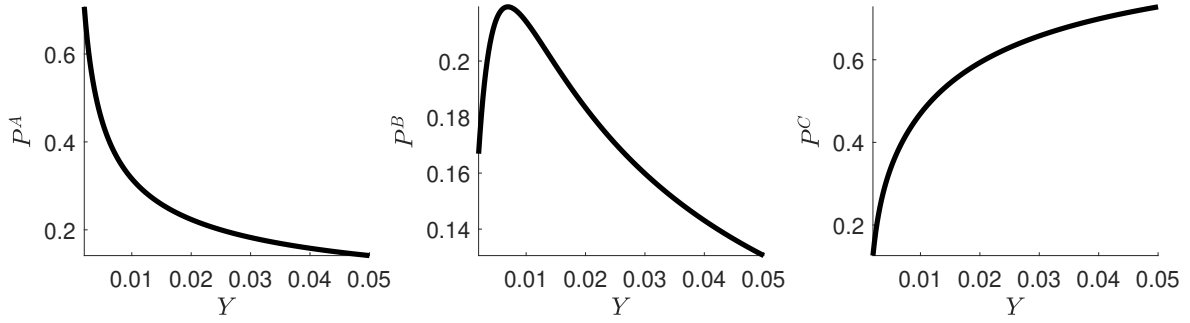
Figure 1: Comparative Statics in a crypto equilibrium. The baseline parameters are $Z^A = Z^B = 0.001$, $\pi^A = 0$, and $\pi^B = 0.008$, so that $\pi^B \geq 4(Z^A + Z^B)$ holds.

value of currency A increases with Z^A, decreases with π^B, and does not depend on Z^B and π^B. The value of currency B decreases with Z^A and π^B, but increases with Z^B and π^A.

Insight 2: Cryptocurrencies harm strong currency A but may benefit weak currency B.
We now study the effects of the rise of cryptocurrencies on currency competition. Importantly, the rise of cryptocurrencies unambiguously harms the strong country A and thus the reserve currency A, in that P^A decreases with Y. The right panel of Figure 1 graphically illustrates this effect by showing that the value of currency A decreases with Y. Also note that not surprisingly, the cryptocurrency value P^C increases with Y, implying that Y quantifies cryptocurrency adoption and the size and value of the cryptocurrency market/sector (or cyber economy).

The rise of cryptocurrencies may benefit the weak country and currency, in that P^B follows an inverted U-shaped pattern in Y in the middle Panel of Figure 1. Intuitively, the rise of cryptocurrencies mitigates the adverse effects of "dollarization" the weak country is exposed to, weakening the feedback between currency usage and inflation/depreciation. The cryptocurrency growth (i.e., an increase in Y) reduces the demand for both currency A and B, thereby decreasing P^A and P^B. However, as currency A depreciates, country B's expenses denominated in currency A fall too, which reduces inflation and benefits currency B. The crypto sector has two opposing effects for the weak country: (i) cryptocurrencies as direct competition for currency B weaken currency B and (ii) cryptocurrencies reduce the degree of competition currency B faces from currency A. Notably, when the strong currency is dominant and π^B is sufficiently large compared with π^A, this second effect may dominate for low values of Y, as the following corollary formalizes.

Corollary 1. *The rise of cryptocurrencies harms the strong currency A, i.e., P^A decreases with Y.*

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But, the rise may benefit the weak currency B: If and only if $\pi^B \geq \sqrt{2}\pi^A$, there exists an interval $[0, \underline{Y})$ with $\underline{Y} > 0$ on which P^B increases with Y. For sufficiently large Y, P^B decreases with Y.

Historically, one salient reason governments have sought to regulate private money has been to curb financial instability. An unbacked, privately issued currency typically faces the dynamic instability problem because its transaction value vaporizes if people suddenly deem it useless in the future. While this fundamental instability leads to hyperinflations and currency unraveling, cryptocurrencies circumvent the issue by committing to little inflation through algorithms. In our setting, therefore, the regulation is really motivated by currency competition, which is especially salient for the stronger country that strives to preserve its global currency dominance.

In our framework, banning or regulating cryptocurrencies by any country (or both) can be interpreted as reducing usability and thus the convenience yield Y to holding cryptocurrencies. As the currency value of the strong country P^A decreases with Y, countries with a strong currency benefit the most from banning and regulating the cryptocurrency market.[14]

In contrast, because the currency value of the weak country may increase with Y, countries with a weak currency benefit less from such regulation or are reluctant to ban and regulate cryptocurrencies at all. Even more, such countries may even want to stimulate cryptocurrency usage within their country, which could be interpreted as an increase of usability and convenience yield Y. Note that according to Corollary 1, the weak country's currency value increases in Y for sufficiently small values of $Y \geq 0$ if and only if the inflation of currency B is sufficiently high ($\pi^B \geq \sqrt{2}\pi^A$). Countries with very weak currencies (e.g., developing countries) therefore benefit from cryptocurrencies and, possibly, from adopting them as means of payment within their country. The model rationalizes why countries with relatively strong currencies, such as China or the US, undertake significant efforts to ban or regulate cryptocurrencies, while countries with a very weak currency, such as El Salvador or Venezuela, do exactly the opposite and actively stimulate the usage of cryptocurrencies (e.g., Bitcoin) within their country so as to mitigate the "dollarization" they are exposed to.

1.4 Central Bank Digital Currencies (CBDCs)

Bans and regulations of cryptocurrencies might not always be feasible and/or are not successful to sufficiently curb cryptocurrencies. Even after bans and regulations, cryptocurrencies may have

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[14]Because the strong country always would like to regulate cryptocurrencies to reduce the convenience yield of cryptocurrencies, one can think of Y also as the convenience net of the effect of regulation or a ban.

value and offer some convenience yield Y, and therefore compete with fiat money. In light of this competition by cryptocurrencies which are at the forefront of technology and payment innovations, a country may respond by adopting technologies and launching a digital currency itself.

CBDCs are digital currencies backed by governments or central banks. They are generally believed to have advantages over fiat in, e.g., improving cross-border payments, lowering the cost of providing physical money, promoting financial inclusion, enabling smart contracting and programmable money, reducing depository counterparty risk, and help monetary policy implementation such as the dissemination of government relief payments during the pandemic (e.g., Foundation and Accenture, 2020; Duffie et al., 2021, Page 7).[15] Moreover, CBDCs are a source of profit and seigniorage revenue, but with reduced cost to taxpayers for production and for Anti-Money Laundering (AML) and tax collection; interest-paying CBDCs may also reduce intermediary rent to the banks. A retail CBDC would also preserve the relevance of generally-accessible central bank money in a digital economy, safeguarding consumer and merchant interests as commerce moves further online (MAS, 2021), as well as increasing interoperability across platforms to keep public money relevant Brunnermeier et al. (2019). Note that our specification of the convenience yield is consistent with how central banks will unlikely allow significant CBDC circulate outside their own territory, for fear of losing control of its currency.[16]

More broadly, launching a CBDC here can be interpreted as fully adopting and regulating large stablecoins or digital payment systems so as to derive comparable benefits of an actual CBDC.[17] Carney (2019), for example, has spoken about a globally coordinated "systemic hegemonic currency," perhaps a stablecoin backed by a basket of deposits at different central banks, that could dampen the spillover shocks associated with the dominance of the US dollar as a reserve currency.

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[15]See also the witnessing for Economic Affairs Committee, House of Lords, UK Parliament (Duffie and Gleeson, 2021), October 12, 2021. We recognize that CBDC designs are work in progress and some of the advantages are a promise but not a guarantee. Depending on the design, CBDCs have downsides such as breaking the complementarity of deposit and credit lines, exacerbating lending inequalities, or reducing deposits and investments (Piazzesi and Schneider, 2020; Parlour et al., 2020; Keister and Sanches, 2021), and the alteration of the informational environment through smart contracting and tokenization (Cong and He, 2019; Lee, Martin, and Townsend, 2021). Our specification captures the net benefits of the digitization of payment systems and currencies, which are well-recognized (e.g., Prasad, 2021). Notably, in a New York Times interview on February 22, 2021, Treasury Secretary Yellen remarked: "Too many Americans don't have access to easy payments systems and banking accounts, and I think this is something that a digital dollar, a central bank digital currency, could help with."

[16]Although people are concerned that China may be internationalizing its currency, the data to date has shown no evidence that the Chinese government desires to do anything in that direction.

[17]In fact, in many developed and emerging economies, the ratio of monetary base (MB) to M2 has been declining since 2003, due to the advancement of electronic payment systems such as PayPal and Alipay (Yao, 2020). Privately produced monies such as stablecoins resemble privately produced monies during the pre-Civil War Free Banking Era in 1834-1863, and may not be effective media of exchange until properly regulated (Gorton and Zhang, 2021).

Implementing CBDC in our model could also be interpreted as existing banking/payment agencies all coordinate to develop a private but interoperable digital currency or payment rail system under compliance with central bank regulation. As discussed earlier, such a private federated digital currency or payment initiative is hard to incentivize and coordinate, as seen in the case of JPMorgan coin or Facebook's initial Libra project.[18] Without a credible threat of CBDC, incumbent banks are reluctant to improve the current system.

CBDCs could be directly managed by the central banks or indirectly managed through banks. Direct CBDCs are also divided into (deposit) account-based or token-based, with the former most closely resembling electronic payment systems such as PayPal or Alipay while the latter potentially involving both digital tokens issues by central banks and technology firms or conventional banks providing customers with synthetic CBDCs fully backed by segregated central bank deposits. Our modelling of CBDCs is technology-neutral and agnostic of the (technical) details on the design and implementation. We merely assume that their implementation improves upon fiat currency in terms of technology, thereby increasing Z^x of country x and the convenience yield.

The effects and incentives behind CBDC issuance. Implementing CBDC constitutes a way to compete in technology with other (digital) currencies. Depending on the parameter values, in particular that of Y, the implementation can have a differential impact on fiat-to-fiat and fiat-to-cryptocurrency competitions. To start, note that (6) reveals that CBDC issuance by either country weakens the cryptocurrency value and adoption P^C, in that $\frac{\partial P^C}{\partial Z^x} < 0$. Importantly, sufficiently large values of Z^A and Z^B due to CBDC issuance spell the demise of the crypto sector.

To gain more intuition on the benefits and incentives behind countries' CBDC issuance, consider $Y = 0$ and parameters ensuring a crypto equilibrium with $P^C > 0$ (e.g., $\sqrt{\frac{Z^A}{\pi^A}} + \frac{Z^B}{\pi^B \sqrt{Z^A/\pi^A}} < 1$). Suppose a country cares about its seigniorage revenue through maximizing its currency value. We then have:

$$P^A = \sqrt{\frac{Z^A}{\pi^A}} \quad \text{and} \quad P^B = \frac{Z^B}{\pi^B P^A} \tag{7}$$

---

[18]The renamed Diem project innovates by committing to phase out its own tokens, reducing fragmentation in payments, and potentially supporting CBDC (Catalini, 2021).

as well as $P^C = 1 - P^A - P^B$. As we show in Appendix D, it holds that

$$\frac{\partial P^A}{\partial Z^A} = \frac{1}{2\sqrt{Z^A \pi^A}} \quad \text{and} \quad \frac{\partial P^B}{\partial Z^B} = \frac{1}{\pi^B P^A}. \tag{8}$$

First, $\frac{\partial P^A}{\partial Z^A}$ and $\frac{\partial P^B}{\partial Z^B}$ are decreasing in π^A and π^B respectively. CBDC issuance therefore benefits country x more if the inflation π^x is lower and currency x is more valuable (higher P^x).

Note that $\frac{\partial P^x}{\partial Z^x}$ captures to a first order the potential benefits accruing to country x upon the implementation of CBDC. We show in Appendix D:

$$sign \left(\frac{\partial P^A}{\partial Z^A} - \frac{\partial P^B}{\partial Z^B} \right) = sign(\pi^B - 2\pi^A). \tag{9}$$

Thus, when $\pi^B \in (\pi^A, 2\pi^A)$, the weak country benefits more from the issuance of CBDC than the strong country does. As a result, CBDC issuance offers the largest advantages for countries with non-dominant but relatively strong currencies, such as China or strong emerging economies like India. These countries should also have the strongest incentives to launch CBDC, which is consistent with the first large scale CBDC launch by China and not the United States.

Insight 3: Country B's CBDC poses a greater threat to cryptocurrencies. Given $\frac{\partial P^C}{\partial Z^B} = -\frac{\partial P^B}{\partial Z^B} < -\frac{\partial P^A}{\partial Z^B} \leq \frac{\partial P^C}{\partial Z^A}$ for $\pi^B < 2\pi^A$, our earlier findings also suggest that CBDC issuance by countries with strong but non-dominant currencies like China or India pose a bigger threat to cryptocurrencies than CBDC issuance by the United States does. The intuition is that cryptocurrencies mainly compete with weaker currencies rather than the reserve currency, so that any appreciation by weaker currencies harms the cryptocurrency market value more.

Insight 4: Pecking order of CBDC issuance. Overall, we observe a pecking order of CBDC issuance. Non-dominant but vibrant emerging economies such as China or India, benefit the most from implementing CBDC, followed by the strong countries such as the United States that are already dominant in the global currency competition. Countries with very weak currencies (e.g., $\pi^B > 2\pi^A$), such as El Salvador, benefit the least from CBDC issuance, because $\frac{\partial P^B}{\partial Z^B}$ decreases with π^B. Intuitively, the currency of these countries is weak regardless of the implementation of CBDC, and CBDC issuance by such countries has negligible impact on the strong country's currency or the cryptocurrency market. As mentioned earlier, these countries may find it advantageous to adopt

cryptocurrency as legal means of payment within their territory.

1.5 Stablecoins and Fiat-backed Cryptocurrency

In the past years, stablecoins, such as Tether or USDC, have gained tremendous popularity. Stablecoins are cryptocurrencies pegged to a reference unit, typically the U.S. Dollar. Stablecoins are often (or at least claim to be) backed by fiat-denominated reserve assets, such as T-bills, commercial papers, or fiat currency itself. The tremendous growth of stablecoins, with their market value more than tripling from January to November 2021, attracted enormous attention from policy makers and regulators.[19]

Our model can accommodate that some cryptocurrencies, especially stablecoins, being partially backed by the dominant national currency A (i.e., U.S. dollars). For this sake, consider that fraction $\theta \in [0, 1)$ of aggregate cryptocurrency value P^C is backed by currency A, i.e., empirically, θ can be seen as the fraction of aggregate cryptocurrency market capitalization that stems from U.S. dollar backed stablecoins. In that case, $\theta P^C / P^A$ units of currency A are kept as reserves backing cryptocurrency and thus are locked up, which leaves $1 - \theta P^C / P^A$ units of currency A as the circulating supply held by the household. That is, $m^A = P^A(1 - \theta P^C / P^A) = P^A - \theta P^C$, while $m^B = P^B$ and $m^C = P^C$, which implies the market clearing condition

$$P^A(1 - \theta P^C / P^A) + P^B + P^C = 1 \quad \Longleftrightarrow \quad P^A + P^B + P^C(1 - \theta) = 1. \tag{10}$$

For simplicity, we do not consider that the degree of reserves backing cryptocurrency affects the convenience yield to holding cryptocurrency.[20] Appendix E presents the solution to this model extension with fiat-backed cryptocurrencies, and solves for currency values P^A, P^B, and P^C in closed-form. Figure 2 plots the equilibrium currency values P^A (left panel), P^B (middle panel), and P^C and $P^C(1 - \theta)$ (right panel) against θ. Interestingly, both the value of currency A and

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[19]On November 1, 2021, U.S. President's Working Group on Financial Markets, joined by the Federal Deposit Insurance Corporation (FDIC) and the Office of the Comptroller of the Currency (OCC), released a report on the recent developments of stablecoins. U.S. Secretary of the Treasury Janet Yellen emphasized the potential of stablecoins as beneficial payments options and risks due to the lack of legal oversight. In response, U.S. Senate held a hearing on the risks of stablecoins on December 14, 2021.

[20]Admittedly, in practice, reserves backing cryptocurrency could have ambiguous effects. For instance, a higher level of reserves backing a stablecoin improves its stability, which is beneficial to users, but may come at the expense of higher fees and a reduced degree of decentralization. Moreover, the level of reserves also affects the profitability of stablecoin issuers, which endogenously should affect their incentives to develop and to issue stablecoins in the first place.
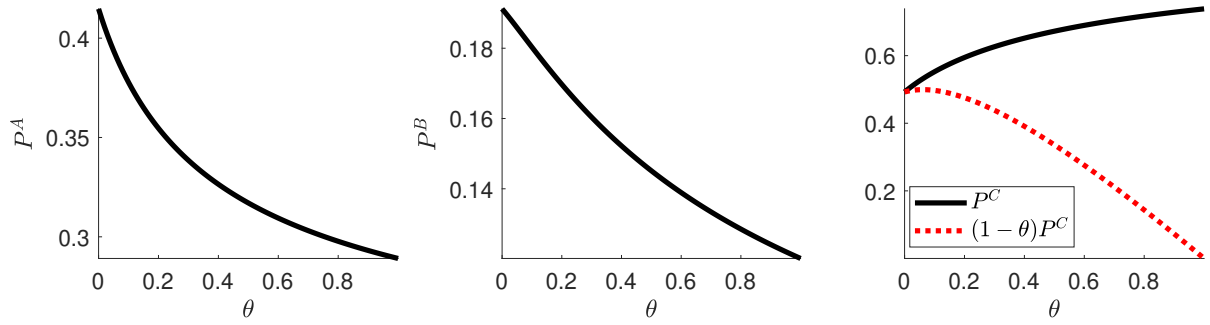
Figure 2: Comparative Statics in a crypto equilibrium with respect to θ. The baseline parameters are $Z^A = Z^B = 0.001$, $\pi^A = 0$, $\pi^B = 0.008$, so that $\pi^B \geq 4(Z^A + Z^B)$ holds. In addition, we set $Y = 0.01$.

cryptocurrency C increase with θ, while P^B decreases with θ. In addition, the market value of cryptocurrency in excess of its reserves, $(1 - \theta)P^C$, decreases with θ.

To gain some intuition about the mechanisms at work, note that if cryptocurrencies are (partially) backed by reserves consisting of currency A (or assets denominated in currency A), demand for cryptocurrencies also stimulates demand for currency A. Put differently, the seigniorage from cryptocurrency usage partially accrues to country A which in turn harnesses part of the cryptocurrency convenience yield. This effect implies that a higher collateralization ratio θ raises demand for currency A and therefore currency value P^A, i.e., P^A increases with θ (left panel). At the same time, a stronger currency A exacerbates competition for currency B, so that the value of currency B falls with θ (middle panel).

Interestingly, the cryptocurrency market value also benefits from being backed by reserves of currency A, in that P^C increases with θ. The underlying reason is that an increase in θ strengthens currency A and, because some of country B's expense are denominated in currency A, raises the inflation of currency B. The increase in inflation, in turn, makes households substitute their holdings of currency B toward currency A and cryptocurrency. However, the actual seigniorage revenue accruing to the issuer of cryptocurrency is only $(1 - \theta)P^C$ units of the consumption goods, because θP^C units of the consumption are used to build reserves (i.e., as collateral). As Panel C illustrates, the seigniorage captured by the cryptocurrency sector decreases with θ, as A now seizes part of the seigniorage generated by cryptocurrencies. Our findings generate insights regarding the benefits, risk, and regulation of (U.S. dollar) stablecoins.

Insight 5: Regulated stablecoins as digital dollar. Our analysis reveals that requiring stablecoins pegged to the U.S. dollar to be backed by U.S. dollar assets can strengthen the dominance of the U.S. dollar, while weakening other national currencies. When stablecoins are backed by U.S. dollar assets, the U.S. can capture part of the seigniorage created by the cryptocurrency sector. U.S. dollar stablecoins effectively export a digital version of the U.S. dollar to other countries or the digital economy in which cryptocurrency is adopted, possibly increasing the "reach" and global influence of the U.S. dollar.

As a result, regulation that restricts or bans stablecoin issuance may not be optimal for the United States. Instead, the U.S and government could benefit from regulation that requires stablecoin issuers to hold U.S. dollar reserves, so as to reclaim seigniorage from the cryptocurrency sector and to benefit from the adoption of these stablecoins. Moreover, appropriately regulated stablecoin issuance through private entities effectively creates a digital dollar which could complement a digital dollar CBDC issued by the central bank. Facilitating regulated issuance of U.S. dollar stablecoins, the U.S. could "delegate" the creation of a digital dollar to the private sector, whilst capturing part of the generated seigniorage revenues.

More broadly, requiring cryptocurrencies and digital payment systems to use a fiat currency or CBDC as collateral or reserve would have a similar effect as the stablecoin here. Given that digital payment systems such as Alipay enjoys a liquidity premium as money or treasury debt do Chen and Jiang (2022), our analysis provides insights on how they affect currency competition.

2 Dynamic Model of Currency Digitization and Competition

The battle of currencies and the decision to launch CBDC are inherently dynamic. To analyze countries' incentives to implement CBDC against the backdrop of an evolving crypto sector, we develop a generalized dynamic model. The insights from the illustrative model are further enriched with dynamic patterns. Moreover, we obtain several novel predictions about the long-term impact of strong initial competition, the interaction between currency competition and financial innovation, and strategic responses to the launching of CBDCs in other countries.

2.1 Model Setup

Time t is infinite without any discounting. To introduce households and money, we set up the model "as if" time runs discretely with time increments $dt > 0$, so that $t = 0, dt, 2dt, 3dt, ...$ Once the model description is complete, one can take the continuous limit $dt \to 0$.

Households and consumption. The economy is populated by one representative OLG household which takes prices as given. Cohort t is born at t and lives until $t + dt$ when a new cohort is born. At birth, cohort t is endowed with one unit of the perishable generic consumption good which serves as the numeraire that all prices are quoted in. Cohort t only derives utility from consumption at time $t + dt$ and thus would like to store their endowment (consumption good) from t to $t + dt$, yet the consumption good cannot be stored.

Global currency supplies. Currency or money comes in different forms. As before, countries A and B have their fiat currencies, and there is one representative decentralized cryptocurrency (currency C). Each currency $x \in \{A, B, C\}$ is in fixed unit supply and has equilibrium price P_t^x in terms of the consumption good. No matter which form a currency takes, it serves a combination of the three functions of money. Later, we allow the two countries to exert effort to introduce CBDCs to replace their fiat currencies. As in our static model, we refer to the country with the stronger currency (higher value in consumption goods) at time $t = 0$ as the "strong" country, and the other country the "weak" one. Without loss of generality, we set country (currency) A to be strong and currency A can be viewed as reserve currency, in that $P_0^A > P_0^B$.

For simplicity, cryptocurrencies are not backed by reserves. Appendix F solves the more general model in which some cryptocurrencies are (partially) backed by reserves consisting of currency A, thereby modeling reserve-backed stablecoins like Tether or USDC.[21]

Money as a store of value and market clearing. Money serves as a store of value, and allows OLG households to delay their consumption. To obtain consumption at time $t + dt$, cohort t uses their consumption good endowment to buy money from the previous cohort (i.e., cohort $t - dt$) at time t. At time $t + dt$, cohort t exchanges money for the consumption good with cohort $t + dt$ and

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[21]That is, we set $\theta = 0$ in terms of the notation of Section 1.5. Appendix F presents the model in which fraction θ of the cryptocurrency market value is backed by reserves consisting of currency A, thereby modeling U.S. dollar-backed stablecoins.

so on. To initialize the model, we assume that the first cohort born at time $t = 0$ buys currency $x = A, B$ from the central bank (government) of country x at prices P_0^A and P_0^B respectively as well as the cryptocurrency C from its developers at price P_0^C.

We denote by m_t^x cohort t's holdings of currency x in terms of the consumption good over their lifetime $[t, t + dt]$. As cohort t does not derive any utility from consuming early at time t, cohort t invests their entire endowment of one consumption good into money, which implies

$$m_t^A + m_t^B + m_t^C = 1. \tag{11}$$

Because cohort t is the only holder of money, it follows that $m_t^x = P_t^x$ which is the market clearing condition for currency x. As a result,

$$P_t^A + P_t^B + P_t^C = 1. \tag{12}$$

Note that according to (11) and (12), at each point in time, the aggregate value of money in terms of the consumption good equals the endowment. That is, the real value of the economy is fixed, and currency competition is a zero sum game in terms of the consumption good. If one currency appreciates in terms of the consumption good, another currency must depreciate.

Convenience and money as a medium of exchange. Next to its function as a store of value, money also serves as a medium of exchange, both across and within cohorts, and/or provides liquidity services to its holders. Again, we account for these functions in reduced form by assuming that households derive a covenience yield from holding money, reminiscent of the money-in-the-utility-function approach frequently adopted in the classical monetary economics literature (e.g., Feenstra, 1986). Formally, the lifetime utility of cohort t reads

$$dU_t = dc_t + Z_o(m_t^A + m_t^B + m_t^C)dt + Z_t^A v(m_t^A)dt + Z_t^B v(m_t^B)dt + Y_t v(m_t^C)dt, \tag{13}$$

where dc_t denotes cohort t's consumption at time $t + dt$ and the remainder terms capture the convenience yield of holding money over $(t, t + dt)$. As in the baseline, cohort t derives a constant (marginal) base convenience yield Z_o regardless of which currency she holds. The constant $Z_o \geq 0$ is chosen large enough to ensure that the convenience yield to holding currency x, that is, $Z_o m_t^x + Z_t^x v(m_t^x)$ for $x = A, B$ and $Z_o m_t^C + Y_t v(m_t^C)$ for $x = C$, is positive and is otherwise

immaterial. The convenience yield cohort t derives from holding m_t^x numeraire units in currency x grows with Z_t^x for $x = A, B$ and Y_t for $x = C$ respectively, and, as in Krishnamurthy and Vissing-Jorgensen (2012), is further characterized by a concave function $v(m_t^x)$, satisfying $v'(m_t^x) > 0$, and $v''(m_t^x) < 0$. The parameters Z_t^x and Y_t relate to the (payment) technology underlying currency $x = A, B$, and C respectively, and capture all differences in currencies' convenience yields. As we will see later, better payment technology (i.e., higher Z_t^x or Y_t) stimulates usage and holdings of currency x. To ensure that equilibrium currency holdings are strictly positive $m_t^x > 0$ and each currency x has positive value, we assume $\lim_{m_t^x \to 0} v'(m_t^x)/m_t^x = \infty$. In particular, the marginal utility of holding currency x goes to infinity. This reflects that for certain activities and transactions, currency x cannot be substituted with another currency.[22]

Inflation and money as a unit of account. We maintain that country x levies "inflation taxes" $\tau_t^x dt$ in terms of the consumption good from its currency holders so as to cover its expenses, such as the costs of servicing debt, international trade expenses, or its fiscal deficit. Note that as in the static model, the assumption that country x covers per-period expenses $\tau_t^x dt$ by levying inflation taxes on currency holders is a tractable way to model the empirically relevant link between a country's fiscal strength and the strength of its currency (Jiang et al., 2020).[23]

To capture that countries with weaker currencies oftentimes borrow debt denominated in terms of stronger currencies and that international trade is mostly invoiced in terms of the reserve currency (e.g., the US dollar), we assume that part of countries' expenses are denominated in terms of the reserve currency A and the remainder is denominated in terms of the consumption good. Recall that $P_0^A > P_0^B$ at time $t = 0$, supporting the notion that currency A is the reserve currency. It also holds that in the long-run, currency A remains stronger than currency B, in that $\lim_{t \to \infty} Prob(P_t^A > P_t^B) = 1$, while it might be the case that temporarily, $P_t^A < P_t^B$. We assume that even if the value of currency B temporarily exceeds the value of currency A, currency A continues to serve as the global unit of account. This assumption reflects that the reserve currency/unit of account status is typically sticky and does not change with transitory value fluctuations (Gopinath et al., 2020).

Formally, over $[t, t + dt)$, country x raises $\pi^x dt$ units of currency A plus $\kappa^x dt$ units of the consumption good as taxes, where $\kappa^x \geq 0$ and $\pi^x \geq 0$ are exogenous. Expressed in terms of the

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[22]For instance, transaction within a certain country most of the time have to settled with the local currency.

[23]Our results are likely to hold as long as there is a positive link between a country's fiscal strength (economic fundamentals) and the benefits of holding its currency.

consumption good, total taxes of country x are $\tau_t^x dt := (\kappa^x + \pi^x P_t^A) dt$. As a result, holding one unit of currency A over $(t, t+dt)$ incurs a tax of $\tau_t^x dt$ units of the consumption good. Thus, holding one unit of the consumption good in currency x or, equivalently, $1/P_t^x$ units of currency x, one incurs taxes $(\tau_t^x/P_t^x) dt$ in terms of the consumption good over $(t, t+dt)$. The tax τ_t^x country x levies can be thought of as inflation, which is formally equivalent. Crucially, the strong country imposes a pecuniary externality on the weak country: If the strong currency P_t^A appreciates, the inflation rate τ_t^B of currency B increases, so that currency B depreciates in terms of the its consumption value. As in the static model, cryptocurrency holdings are not taxed, but, due to endogenous price dynamics, might be subject to inflation and depreciation when P_t^C decreases over time.

Technology, cryptocurrency, and CBDC. Government-issued currencies and cryptocurrencies differ in their convenience yield and, in particular, in the technology parameters Z_t^x and Y_t. The cryptocurrency market and its underlying technology grow endogenously with adoption and usage of cryptocurrencies in that

$$\frac{dY_t}{Y_t} = \mu m_t^C dt, \tag{14}$$

where $\mu \geq 0$ is a constant. Note that under the specification in (14), cryptocurrency usage, as captured by m_t^C, stimulates the growth of the technology underlying cryptocurrencies and so financial innovation. The idea behind (14) is that the profits that cryptocurrency developers earn increase with cryptocurrency usage and adoption m_t^C. As such, a higher level of m_t^C motivates developers to improve the technology underlying cryptocurrencies and expand use cases. We assume that the potential convenience yield of cryptocurrencies is bounded, in that $Y_t \leq \overline{Y}$ for some exogenous constant $\overline{Y}$. Formally, the drift of dY_t vanishes as it reaches $\overline{Y}$ (i.e., $dY_t = 0$ if $Y_t = \overline{Y}$) while (14) holds for $Y_t < \overline{Y}$. Intuitively, the cryptocurrency market has reached widespread adoption and its capacity, once Y_t has reached $\overline{Y}$.

Finally, we discuss the role for CBDC in our model. As in the static model, we introduce CBDC in a technology-neutral fashion that does not rely on any specific design. We merely interpret CBDC as a technological innovation which improves upon traditional fiat money. Formally, when a country $x = A, B$ launches CBDC at some time T^x, CBDC fully replaces traditional fiat money and therefore represents currency x as did fiat money before time T^x. To capture that CBDC

constitutes a technological improvement, we assume that for $x = A, B$:

$$Z_t^x = \begin{cases} Z_L & \text{for} \quad t < T^x \\ Z_H + \alpha Y_t & \text{for} \quad t \geq T^x \end{cases},$$

where $\alpha \geq 0$ and $Z_L \leq Z_H$ are positive constants. In other words, at time T^x, Z_t^x jumps up. Note that the gains of CBDC implementation partially depend on the the state of the cryptocurrency market and its underlying technology, which reflects the notion that the technology underlying CBDC to some extent derives from the technology underlying cryptocurrencies (e.g., blockchain technology and smart contracts).[24]

The implementation of CBDC constitutes a major disruption to incumbents and requires the support from multiple parties and regulatory approval that all take great time and effort.[25] To capture this friction, we assume that the time T^x arrives according to a jump process $dJ_t^x \in \{0, 1\}$ with intensity λe_t^x, where $\lambda > 0$ and $e_t^x \geq 0$ is the endogenous effort of country x to implement CBDC. Effort e_t^x is costly and entails private flow costs or disutility $\frac{(e_t^x)^2}{2}$. The government or central bank of country x would like to maximize the value of its currency, P_t^x. Therefore, country x chooses e_t^x to maximize the expected change in currency price P_t^x net the disutility of effort:

$$e_t^x = \arg \max_{e \geq 0} \left(\mathbb{E}[dP_t^x] - \frac{e^2}{2} dt \right). \tag{15}$$

For simplicity, the costs $\frac{(e_t^x)^2}{2}$ take the form of disutility and do not affect taxes τ_t^x. More importantly, the costs of implementing CBDC are denominated in consumption goods.

Discussion: Monetary neutrality, interest rates, and stablecoins.   Our assumptions that money holdings do not bear interest and that the supply of currency x is fixed to one are without

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[24]Arguably, development in cryptocurrencies would spur traditional banking and payment systems to innovate to improve services and increase users' convenience. In fact, if banks were incentivized to and well-coordinated, they can achieve many benefits associated with a digital currency system (without introducing digital currencies). However, a digital currency could allow additional benefits by enabling users, for example, to tap into the cyber economy and metaverse across jurisdiction boundaries and digital networks. CBDC taps into digital networks (relating to Y), effectively internationalizing the currency through the digital network, in part complementing the role of trade on making a currency achieve international status (Gopinath and Stein, 2021). What matters is the relative gain in convenience once CBDC is introduced.

[25]For example, many see support from the banking sector as vital to the success of a digital U.S. dollar, however commercial banks in the U.S. have taken a largely adversarial stance. According to Duffie (2021), "the development of an effective and secure digital dollar will require significant resources and time, perhaps more than five years."

loss of generality. The reason is that our framework features monetary neutrality: If the supply of currency x changes by a factor ω, the price of currency x in terms of the consumption good changes by a factor $1/\omega$, while the total value of all currency x outstanding remains unchanged at P_t^x. In particular, if the supply of currency x changes by a factor ω and the proceeds from this supply change are distributed pro-rata among the holders of currency x via interest payments (if the proceeds are positive) or taxes (if the proceeds are negative), then the real value of any household's currency x holdings and thus the household's utility remain intact.

In addition, note that it is always possible to transform changes in currency value, dP_t^x, into a tax or interest payment or vice versa.[26] In other words, changes in currency price can be arbitrarily transformed into changes in currency supply and interest payments or taxes for currency holders and vice versa in a way that leaves real quantities and real returns to holding currency x unchanged. As a result, the taxes country x levies can be interpreted as depreciation or inflation of currency x. Under any of these transformations, P_t^x denotes the (total) value of currency x (i.e., the market capitalization of currency x) in terms of the consumption good. In particular, it is possible to peg the price of currency x to one unit of the consumption good. In our setting, even if we allow CBDCs to be interest-bearing, remuneration would not mitigate the currency devaluation against the consumption goods or inflation.

The above logic also extends to cryptocurrencies. An appropriate fee (i.e., tax) and interest payment schedule could implement the price of cryptocurrency being pegged to the price of currency A (e.g., the US dollar) in a way that leaves the real returns to holding cryptocurrency unchanged. In practice, such a peg would pertain for stablecoins (e.g., Tether or DAI). In fact, stablecoins such as Diem are very much designed as a complement to CBDCs (Catalini, 2021). For simplicity, we abstract from any payoff-neutral supply and price changes and, without loss of generality, fix the supply of currency x to one. We thus use the terms currency value (market capitalization) and currency price interchangeably.

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[26]For example, when currency x, which is in unit supply, appreciates (i.e., $dP_t^x > 0$) so that at $t + dt$, total value of currency x reads $P_t^x + dP_t^x$. This price change induces returns dP_t^x to holding one unit of currency x. Notably, country x could issue additional dP_t^x / P_t^x units of currency x to drive down currency-x price P_{t+dt}^x to P_t^x, while leaving the total value of currency x at time $t + dt$, i.e., $P_t^x(1 + dP_t^x / P_t^x) = P_t^x + dP_t^x$, unchanged. The proceeds from this supply change is dP_t^x units of the consumption good. The country pays these proceeds to currency holders as interest payments on currency x on a pro-rata basis to its currency holders, yielding interest payments of dP_t^x units of the consumption good per unit of currency x.

2.2 Solving for the Markov Equilibrium

We now characterize the equilibrium in our dynamic model. Let the state variable $z \in \{0, A, B, AB\}$ denote which countries have launched CBDC up to date. Specifically, $z = 0$ means that no country has launched CBDC yet, $z = A$ means that only country A has launched CBDC, $z = B$ means that only country B has launched CBDC, and $z = AB$ means that both countries A and B have launched CBDC. We focus on a Markov equilibrium with state variables (Y, z), so that all equilibrium quantities can be expressed as functions of (Y, z). In equilibrium, at any time $t \geq 0$, cohort t chooses the holdings of currencies $\{x\}$ to maximize the expected utility $\mathbb{E}[dU_t]$, given prices $\{P_t^x\}$. The markets for all currencies clear, i.e., $m_t^x = P_t^x \ \forall x \in \{A, B, C\}$.

To solve for the equilibrium, we define the expected returns of currency x in terms of the consumption good as

$$r_t^x = \frac{\mathbb{E}[dP_t^x]}{P_t^x dt}. \tag{16}$$

Notice that r_t^x is the expected rate of appreciation of currency x in terms of consumption good. That is, if $r_t^x > 0$, currency x is expected to appreciate, causing deflationary pressure in terms of the consumption, and, if $r_t^x < 0$, currency x is expected to depreciate, causing inflationary pressure.

Next, we can write cohort t's consumption dc_t at $t + dt$ as:

$$dc_t = \sum_{x \in \{A,B,C\}} \left(\frac{m_t^x P_{t+dt}^x}{P_t^x} - \sum_{x \in \{A,B\}} \left(\frac{\tau_t^x m_t^x}{P_t^x} dt. \right. \right. \tag{17}$$

Basically, cohort t's consumption consists of the proceeds from selling their holdings of currency x in terms of the consumption good, m_t^x / P_t^x, at price P_{t+dt}^x to cohort $t + dt$ minus the taxes cohort t pays to countries A and B. As argued above, these taxes are equivalent to inflation. The interpretation is that country x could collect taxes by printing/selling more money and keeping the proceeds from doing so, while the households bear the costs of this inflation.

Heuristically, we can write $P_{t+dt}^x = P_t^x + dP_t^x$ and, inserting this relation into (55), we obtain:

$$dc_t = \sum_{x \in \{A,B,C\}} m_t^x + \sum_{x \in \{A,B,C\}} \left(\frac{m_t^x dP_t^x}{P_t^x} - \sum_{x \in \{A,B\}} \left(\frac{\tau_t^x m_t^x}{P_t^x} dt. \right. \right. \tag{18}$$

Because cohort t only derives utility from consuming at time $t + dt$, it is optimal to use the entire endowment one to purchase money at time t, so that $\sum_{x \in \{A,B,C\}} m_t^x = 1$ must hold in optimum

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given prices $\{P_t^x\}$. As a result, cohort t maximizes

$$\max_{m_t^x} \mathbb{E}[dU_t] \quad \text{s.t.} \quad \sum_{x \in \{A,B,C\}} \left(m_t^x = 1, \right. \tag{19}$$

taking prices $\{P_t^x\}$ as given. Using (13), (18), and $\sum_{x \in \{A,B,C\}} m_t^x = 1$, we can rewrite the objective function in (19) as:

$$\mathbb{E}[dU_t] = 1 + Z_o dt + \sum_{x \in \{A,B,C\}} m_t^x r_t^x dt - \sum_{x \in \{A,B\}} \left(\frac{\tau_t^x m_t^x}{P_t^x} dt + Z_t^A v(m_t^A) dt + Z_t^B v(m_t^B) dt + Y_t v(m_t^C) dt. \right.$$

The first three terms represent the expected consumption of cohort t at time $t+dt$, which is the unit endowment plus the expected returns to investing in currencies A, B, and C, less the taxes levied by countries A and B. The last three terms represent the convenience yield to holding currencies.

In light of $\sum_{x \in \{A,B,C\}} m_t^x = 1$, it must be in optimum that

$$\frac{\partial \mathbb{E}[dU_t]}{\partial m_t^A} = \frac{\partial \mathbb{E}[dU_t]}{\partial m_t^B} = \frac{\partial \mathbb{E}[dU_t]}{\partial m_t^C},$$

provided $m_t^x \in (0,1)$. That is, in equilibrium, the household is on the margin indifferent between substituting a marginal unit of currency x towards another currency. As stated in Proposition 3 below, this relationship implies the following equilibrium pricing equations:

$$Y_t v'(P_t^C) + r_t^C = Z_t^A v'(P_t^A) + r_t^A - \frac{\tau_t^A}{P_t^A},$$
$$Y_t v'(P_t^C) + r_t^C = Z_t^B v'(P_t^B) + r_t^B - \frac{\tau_t^B}{P_t^B}. \tag{20}$$

In equilibrium, the sum of the marginal convenience yield to holding cryptocurrencies and expected cryptocurrency returns equals the sum of the marginal convenience yield to holding national currency x and its returns net the inflation currency x carries due to taxation.

To complete the description of the Markov equilibrium, the following Proposition shows that prices as well as returns can be expressed as functions of (Y, z) only and do not explicitly depend on calendar time.

Proposition 3. *In a Markov equilibrium with state variables (Y, z), the equilibrium pricing equation (20) holds, the price of currency $x = A, B, C$ satisfies $P_t^x = P^x(Y, z)$, and expected returns satisfy*

$r_t^x = r^x(Y, z)$. *The launch of CBDC is characterized by effort* $e_t^x = e^x(Y, z)$ *for* $x = A, B$.

In the Appendix F, we provide the detailed solution to a more general version of the model which also allows that cryptocurrency is partially backed by reserves consisting of currency A. We then characterize the model solution in terms of a system of coupled ODEs that describe the dynamics of the currency prices P_t^x. We also discuss how to numerically solve the model to obtain currency values $P_t^x = P^x(Y, z)$ and countries' effort to launch CBDC $e_t^x = e^x(Y, z)$ as well as existence and uniqueness of the equilibrium. Finally, note that because costs of developing CBDC are quadratic, country x always exerts non-zero effort at any point in time t it has not launched CBDC yet (i.e., $t < T^x$). In equilibrium, as time t approaches infinity, any country x has launched CBDC eventually so that $T^x < \infty$. And, once all countries have launched CBDC, the currency A dominates currency B, so that $\lim_{t \to \infty} P_t^A > \lim_{t \to \infty} P_t^B$. Thus, even if currency B is temporarily stronger than currency A, the "initial order of dominance" will be restored eventually, suggesting that currency A can be viewed as reserve currency irrespective of temporary fluctuations in currency values.

3   Model Implications and Discussion

Once we solve the system of coupled ODEs, we derive predictions on how the rise of cryptocurrencies shapes currency competition and shape incentives of various governments to launch their own digital currencies. For the numerical solution, we follow Li (2021), and assume that convenience yield takes the CRRA functional form

$$v(m_t^x) = \left(\frac{(m_t^x)^{1-\eta} - 1}{1 - \eta} \right) ($$

We make the following parameter choices, $Z_L = 0.5$, $Z_H = 2$, $\alpha = 0.15$, $\pi^A = 1$, $\pi^B = 4$, $\eta = 2$, and $\overline{Y} = 75$. We normalize $\kappa^A = \kappa^B = 0$, so that the differences between countries A and B are captured entirely by the differences in π^A and π^B. We initialize the model at $Y_0 = 0.01$, and over time the growth of Y_t is endogenously determined according to (14).

The strong country A can be interpreted as the United States, and the U.S. dollar is then the international reserve currency and unit of account. Our baseline specification considers π^A and π^B that are not too divergent, which describes the competition between fiats of major nations or regions, such as the US dollar and the Euro or the US dollar and the Chinese Yuan. Alternatively, country B could also be interpreted as a relatively strong emerging economy like India.
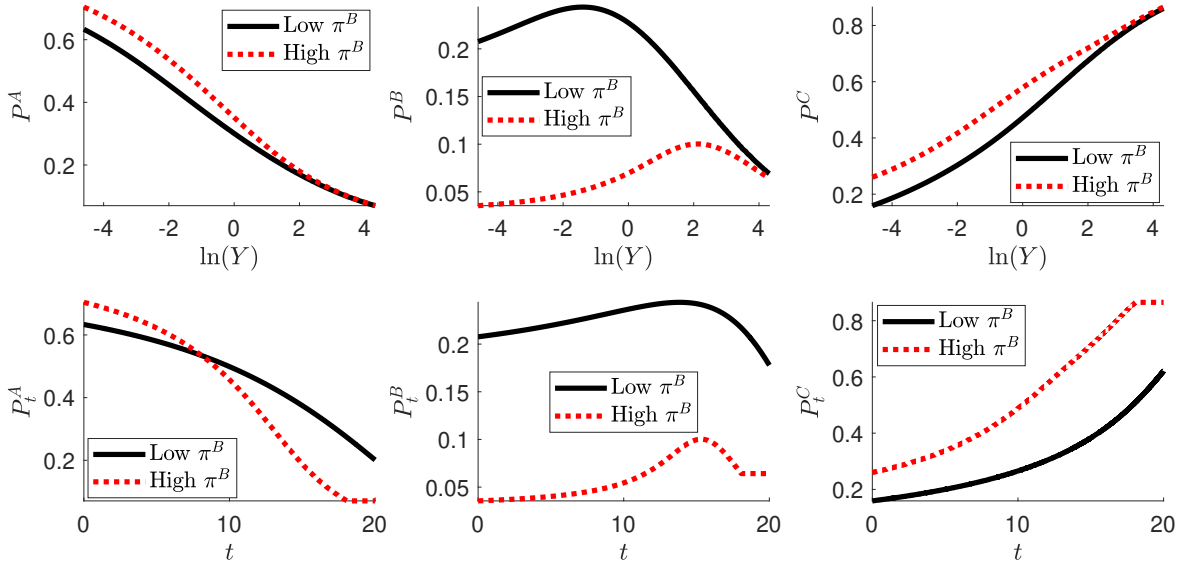
Figure 3: Currency value as a function of Y (upper panels) and a function of time t (lower panels).

Developing countries or relatively weak emerging economies are heavily dependent on dollar financing (Du et al., 2020) and so are characterized by a sufficiently large value of π^B (i.e., $\pi^B \to \infty$). When π^B is sufficiently big, the presence of the weak country would not have a significant impact on currencies A or C. We study the incentives and effects of CBDC issuance by such countries in Section 3.6 where we formally consider the case of large π^B.

3.1 The Rise of Cryptocurrencies and Currency Competition, and Pricing

We start by discussing the currency value dynamics under currency competition. Figure 3 displays currency values P^x both as a function of Y and calendar time t before any CBDC is launched ($z = 0$). Note that Y_t increases over time, and the rate of increase endogenously depends on cryptocurrency adoption and thus the cryptocurrency price. The solid black line depicts the baseline scenario $\pi^B = 4$, and the dotted red line depicts a scenario with a higher value of $\pi^B = 20$.

The upper and lower left panels display the price of currency A for different values of Y and t. The rise of cryptocurrencies unambiguously hurts the strong currency A, in that the value of currency A decreases with Y and over time t. At the same time, the cryptocurrency price in the upper and lower right panels increases with Y and over time t. Notice that before reaching the upper bound $\overline{Y}$, the growth of the cryptocurrency market is effectively exponential and P_t^C is increasing and convex in time t, which reflects dynamic network and feedback effects: Higher cryptocurrency

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usage and adoption at time t contributes to the growth in the underlying technology Y_t and boosts cryptocurrency adoption in the future.

The upper and lower middle panels display the price of currency B for different values of Y and t. Notably, under either value of π^B, the price of currency B is hump-shaped in Y and over time. Put differently, the price of currency B first increases and then decreases with Y and over time. As such, the weaker country initially benefits from the rise of cryptocurrency. However, as the cryptocurrency market grows sufficiently large, it eventually limits usage of currency B, thereby damaging its value. The underlying reason is that a stronger cryptocurrency market (i.e., an increase in Y) has two opposing effects on currency B. First, an increase in Y exacerbates direct competition currency B faces from cryptocurrencies, which makes households partially substitute their holdings of currency B for cryptocurrencies. Second, an increase in Y weakens currency A and therefore alleviates competition currency B faces from currency A. Weaker currency A reduces the inflation rate τ_t^B of currency B, which encourages households to hold more currency B. The first effect dominates for large values of Y while the second dominates for small values, leading to the aforementioned hump-shaped pattern of country B's currency value in $\ln(Y)$ and t. We further elaborate on the implications of this result in Section 3.6, where we show that developing countries are more likely to benefit from the rise of cryptocurrencies than advanced economies.

Crucially, the endogenous growth of cryptocurrencies depends on the strength of the national currencies. For instance, when both currency A and B are relatively strong and so have low inflation rates, the household's incentives to hold cryptocurrency are low too. By (14), low cryptocurrency usage and adoption today stifles the growth of the crypto economy and the underlying technology, therefore implying low cryptocurrency usage and adoption in the future. In other words, the presence of strong national currencies hampers the emergence of privately-issued (crypto-) currencies. In contrast, a vacuum generated by weak national currencies, which prevails, e.g., when π^B is large, favors the emergence of cyptocurrencies, thereby spurring the growth of the cryptocurrency market and boosting the competition national currencies face from cryptocurrencies in the longer run.

As a result, the strong country may actually benefit in the longer run from a stronger competitor B which is characterized by a lower value of π^B, in that the price P_t^A may increase in π^B at later times (see lower left panel). The reason is that when π^B is low, country A faces fierce competition from country B ex-ante, but a strong currency B limits the growth of the cryptocurrency market and so limits competition from cryptocurrencies in the longer run. Conversely, when π^B is high,
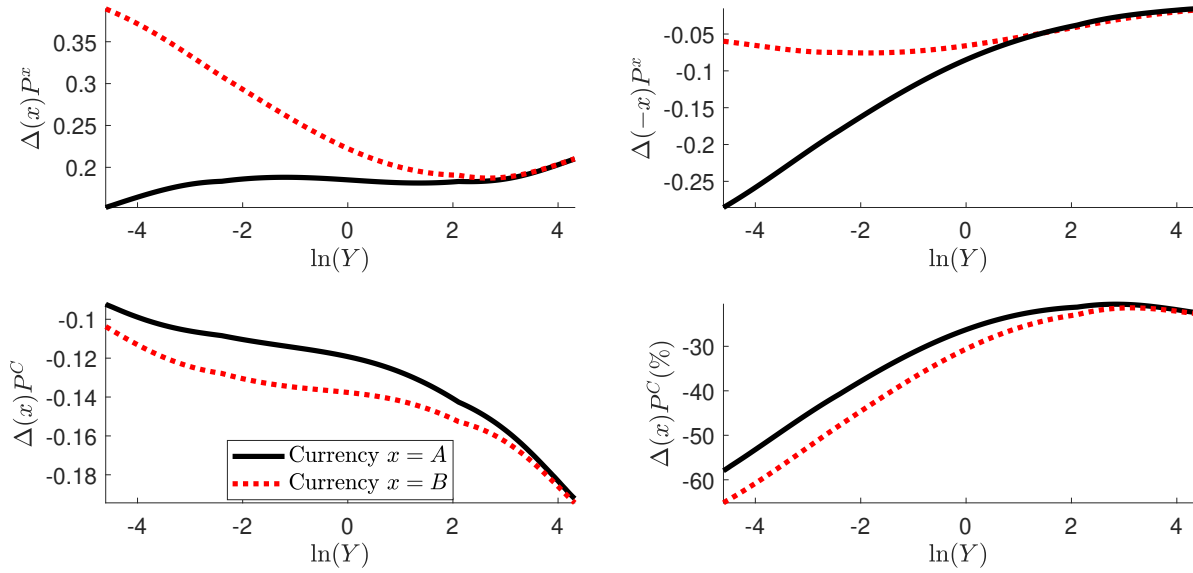
Figure 4: The effects of CBDC issuance on currency competition.

there is relatively low competition for currency A from currency B. However, the weakness of currency B facilitates the rise of cryptocurrencies as competitor to currency A in the longer run.

3.2 The Effects of CBDC issuance

We now study how the launch of CBDC by either country affects currency competition and pricing. Figure 4 plots the change in currency x's value if country x launches CBDC in state $z = 0$ (upper left panel), the change in currency x's value if the other country (i.e., country $-x$) launches CBDC (upper right panel), and the change in cryptocurrency value if country x launches CBDC both in absolute (lower left panel) and percentage terms (lower right panel). The solid black line refers to currency A, and the dotted red line refers to currency B. The upper left panel shows that upon the implementation of its own CBDC, the weak country's currency appreciates more than the strong country's currency. This suggests that the implementation of CBDC offers greater advantages for weaker countries than for stronger ones.

The upper right panel of Figure 4 depicts the effects of CBDC issuance by one country on the other's currency. Notice that currency A is harmed more by CBDC issuance of country B than currency B is harmed by CBDC issuance by country A. In other words, the strong currency suffers more from the CBDC implementation by its competitor than the weak currency.

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The lower two panels of Figure 4 plot the change in cryptocurrency value when country A launches CBDC (solid black line) and country B launches CBDC (dotted red line) both in absolute terms (lower left panel) and percentage terms (lower right panels). Provided currency B is sufficiently valuable (i.e., π^B is not too large), CBDC issuance by the weak country has a more negative impact on the cryptocurrency value than CBDC issuance by the strong country. Intuitively, by implementing CBDC, a country can regain the fraction of currency usage it initially loses to the cryptocurrency, and this fraction tends to be larger for weaker currencies to begin with.

We conclude that (i) weaker currencies benefit more from CBDC issuance, (ii) stronger currency values tend to suffer more from competitor CBDCs, and (iii) the cryptocurrency suffers the most when countries (currency unions) with relatively strong but not the dominant nations/regions, (e.g., China, India, or the Euro zone) implement CBDCs. According to our model, the implementations of CBDC by these countries pose more danger to the cryptocurrency market than the launch of CBDC by the dominant currency country, i.e., the United States. The intuition underlying this result is that cryptocurrencies mainly compete with digital currencies of relatively weaker countries rather than from the digital reserve currency.

Finally, note that for low values of $\ln(Y)$, i.e., when the cryptocurrency is still in its infancy, the implementation of CBDC by the strong or weak country has the largest negative impact on cryptocurrency value and adoption in relative terms. As contemporaneous cryptocurrency adoption determines the growth of Y and thus future cryptocurrency adoption, this effect gives rise to a cryptocurrency "kill zone" which allows for a "killer adoption" of cryptocurrency technology: If countries implement CBDC before cryptocurrency adoption has grown large, they can substantially limit cryptocurrency adoption and preclude future cryptocurrency growth and dominance. Essentially, early implementations of CBDC nips the emergence of crytpocurrencies in the bud.

3.3   The Incentives to Implement CBDC

Having studied the ex-post effects of CBDC issuance in state $z = 0$, we now characterize country x's incentives to launch CBDC captured in e_t^x. Crucially, these incentives depend on the size of the cryptocurrency market (Y) as well as on whether the other country has already launched CBDC (z). Importantly, according to (15), country x has high-powered incentives to launch CBDC if the contemporaneous currency price is low and/or the future (expected) currency price after launching CBDC is high. Both the currency value prior and after the launch of CBDC reflect the prevailing

levels of currency competition and so do the countries' incentives to implement CBDC.

In state $z = 0$, no country has launched CBDC yet. For $x = A, B$, at the time country x implements CBDC, the value of currency x jumps up by $P^x(Y, x) - P^x(Y, 0)$ units of the consumption good. According to (15), Country A chooses now effort $e^A(Y, 0)$ to maximize:

$$\lambda e^A(Y, 0)\big(P^A(Y, A) - P^A(Y, 0)\big)\left(-\frac{(e^A(Y, 0))^2}{2},\right.$$

leading to

$$e^A(Y, 0) = \lambda\big(P^A(Y, A) - P^A(Y, 0)\big).\Big(\tag{21}$$

Analogously, we obtain

$$e^B(Y, 0) = \lambda\big(P^B(Y, B) - P^B(Y, 0)\big).\Big(\tag{22}$$

Note that in state $z = 0$, the incentives to implement CBDC reflect both a rising competition from cryptocurrencies and the prospect of attaining a technological edge over other national currencies. That is, the implementation of CBDC not only allows a country to compete more effectively with cryptocurrency but also gives the country an edge over the country which has not launched CBDC yet. This is a first-mover advantage which persists for a while after the successful launch of CBDC because CBDC implementation takes time and thus the other country cannot react immediately.[27]

In states $z = A$ and $z = B$, one country has attained such a first-mover advantage in terms of technology and CBDC. As a result, the other country launches CBDC both to compete with the cryptocurrency market and to catch up to the other country in terms of technology. Formally, the incentives to launch CBDC are captured by the realized currency value increase upon launching CBDC. Performing similar calculations as before, we obtain:

$$e^A(Y, B) = \lambda\big(P^A(Y, AB) - P^A(Y, B)\big), \Big(\tag{23}$$

$$e^B(Y, A) = \lambda\big(P^B(Y, AB) - P^B(Y, A)\big).\big\{ \tag{24}$$

Note that once a country implements CBDC, it exerts no more effort.

Figure 5 displays the efforts (incentives) of both countries (upper two panels) as well as their
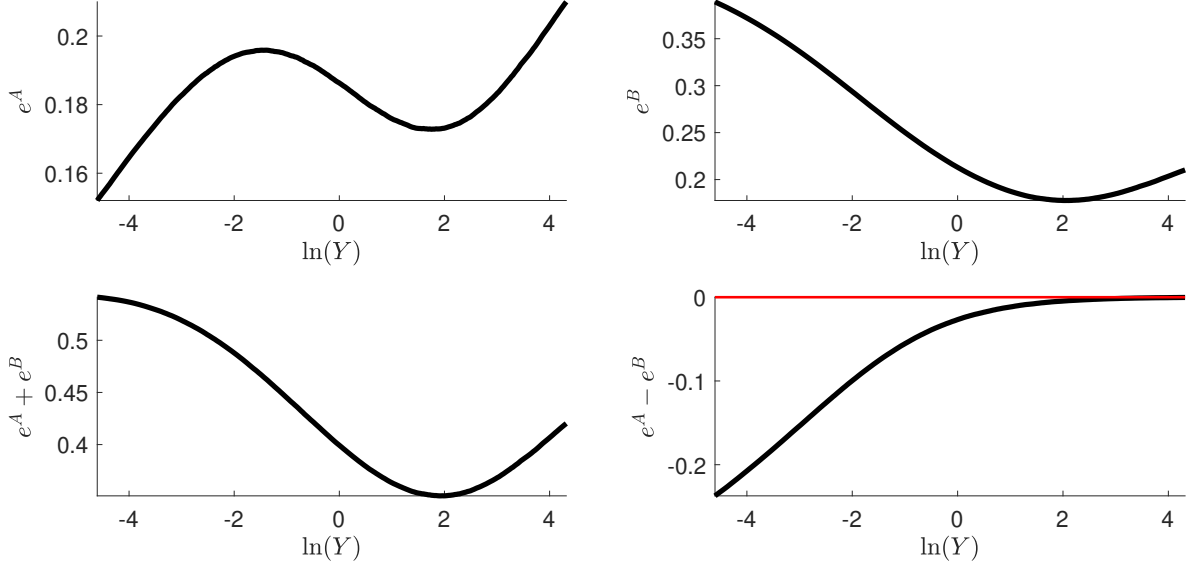

Figure 5: The countries' incentives to launch CBDC.

differences and sums (lower two panels) for different values of Y in state $z = 0$ when no country has implemented CBDC yet. We start by discussing the strong country's incentives to launch CBDC in the upper right panel of Figure 5. Note that the strong country's effort is initially low when there is little competition from cryptocurrencies, in which case $P^A(Y, 0)$ is large and so the incentives to launch CBDC, $P^A(Y, A) - P^A(Y, 0)$, are limited. In other words, the initial dominance of currency A reduces country A's incentives to innovate by developing CBDC. Over time, the cryptocurrency market rises as a competitor, thereby weakening currency A. As Y and cryptocurrency adoption increase, $P^A(Y, 0)$ decreases and, in turn, the incentives to launch CBDC ramp up. The competition from cryptocurrencies essentially incentivizes country A to adopt CBDC.

Because the cryptocurrency market's growth rate depends on level of adoption m_t^C (see (14)), any reduction in m_t^C has persistent negative impact on future cryptocurrency adoption and value. Note that if country A launches CBDC relatively early (i.e., for low values of Y), the implementation of CBDC causes a significant reduction in future cryptocurrency adoption and value m_t^C. As a result, the launch of CBDC in the early stages of cryptocurrency adoption effectively "kills" the cryptocurrency market, hampering cryptocurrency adoption in the longer run. The possibility to cut down the cryptocurrency market in its early stages incentivizes country A to launch CBDC early on. In turn, the strong country's incentives to launch CBDC reach their peak in the so-called kill zone characterized by low values of Y where CBDC implementation by the strong country cuts
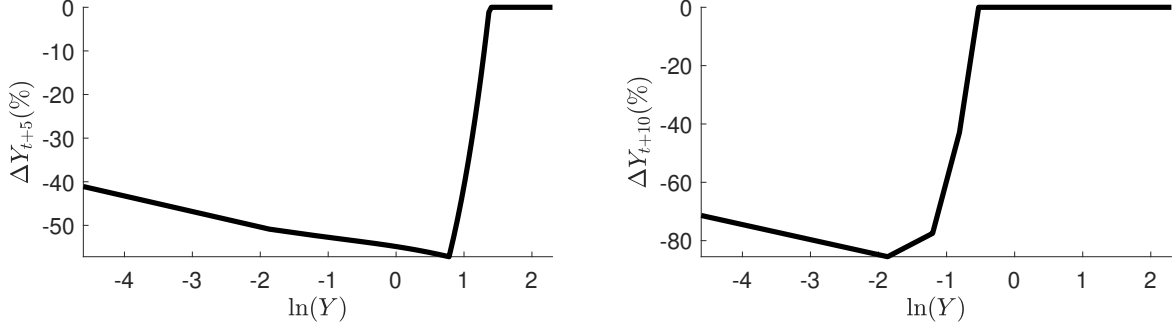
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Figure 6: The persistent effects of CBDC issuance by the strong country A.

down the cryptocurrecy market and, again, nips its growth in the bud.

Figure 6 provides a an illustration. To understand this figure, consider two scenarios at time t when $Y_t = Y$: (i) country A launches CBDC and (ii) country A does not launch CBDC. Figure 6 plots the percentage change in Y at time $t + 5$ (left panel) and time $t + 10$ (right panel) when country A launches CBDC as opposed to the scenario that no country launches CBDC and state $z = 0$ prevails until time $t + 5$ and $t + 10$ respectively. According to Figure 6, if country A launches CBDC early enough, it can achieve a significant (percentage) reduction in future cryptocurrency covenience or technology Y_{t+5} and Y_{t+10}. In contrast, if Y_t exceeds a critical threshold, CBDC issuance at time t no longer reduces the value of Y_t at future times $t + 5$ and $t + 10$. In other words, the earlier country A launches CBDC, the more persistent the effects of CBDC issuance on cryptocurrency adoption and Y.[28]

Loosely speaking, when the cryptocurrency market has grown sufficiently large and has reached a sufficient level of adoption, it is no longer possible to stifle its growth through the launch of CBDC, which reduces the benefits of launching CBDC. Thus, after the initial peak, country A's incentives to launch CBDC decrease again. Eventually, for sufficiently large values of Y, it becomes unavoidable to launch CBDC as a defensive measure to avoid full dominance of cryptocurrency. This leads to a double-peaked incentives to launch CBDC by the strong country over time and across $\ln(Y)$. Notice that the strong country's strategy for launching CBDC evolves from an offensive, preemptive tactic to a purely defensive measure.

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[28]Admittedly, without further assumptions, the probability that Y_t reaches $\overline{Y}$ in the long run (i.e., as $t \to \infty$) is one. However, one could introduce a negative component to the drift of dY_t, say $\frac{dY_t}{Y_t} = \mu m_t^C dt - \delta dt$, in which case a reduction in m_t^C could imply for the long-run $Y_t \to 0$ instead of $Y_t \to \overline{Y}$. For simplicity, we do not formally introduce this effect.

Interestingly, according to the upper right panel in Figure 5, the weak country has high-powered incentives to launch CBDC early on, so as to attain a first-mover advantage in terms of technology and to reduce the degree of dollarization and competition from currency A. Note that competition from currency A is particularly strong for low values of Y, when the cryptocurrency market is in its infancy and currency A is strong. As the cryptocurrency market grows, currency A depreciates and so do the degree of dollarization and competition currency B faces from currency A. As a result, country B's incentives to launch CBDC, which stem mainly from the desire to obtain a competitive advantage over currency A, taper off over time with the rise of cryptocurrencies. Importantly, we also find that the weak country's incentives to launch CBDC exceed the ones of the strong country (see the lower right panel of Figure 5). Again, these high-powered incentives to implement CBDC that country B is provided with reflect the competitive pressure currency B faces from currency A.

Our findings suggests the following pecking order for implementing CBDCs. First, relatively strong countries that do not have a dominant currency, such as China, the Eurozone, or India, have the highest incentives to launch CBDC, and so are likely to launch CBDC first. Second, the United States (country A in our model), follows the weaker countries in developing CBDC. The model therefore rationalizes why the first CBDC issuances have not been carried out by the US, but rather by countries with weaker currencies, such as China. Third, as we show shortly, very weak countries, characterized by a very large value of π^B, have negligible advantages from launching CBDC. Intuitively, such countries possess weak currencies regardless of whether they launch CBDC or not, which mechanically limits the benefits of launching CBDC.

Finally, the lower left panel of Figure 5 illustrates that countries' joint incentives to launch CBDC, $e^A + e^B$, tend to be highest for low values of Y. As such, our results suggest that the recent hype about CBDC issuances might be transitory and may taper off over time, as the cryptocurrency market expands further. However, eventually the (national) digitization of money is inevitable, in that joint effort to launch CBDC increases again for larger values of $\ln(Y)$.

3.4 Strategic Effects of CBDC issuance

The decision on whether to implement CBDC is strategic and crucially depends on whether other countries have launched CBDC. As discussed above, in state $z = 0$, countries' incentives to implement CBDC reflect the hope to attain a technological first-mover advantage over the other country and, in states $z = A, B$, they reflect the need to catch up with the other country.
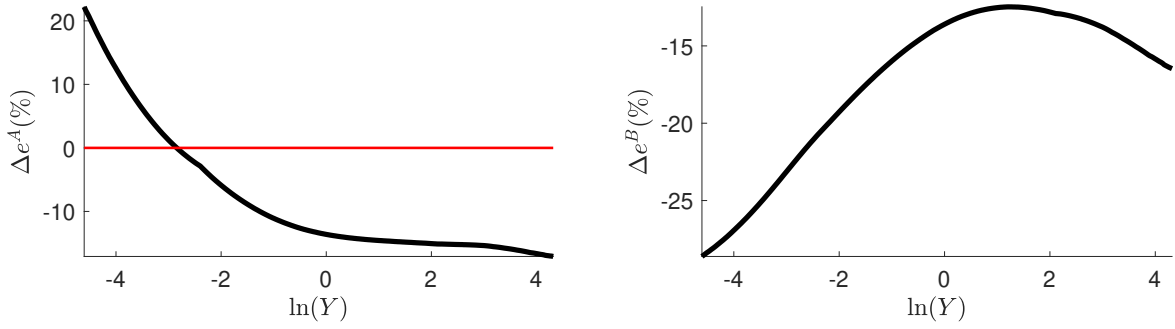
Figure 7: Country x's effort change when the other country launches CBDC.

We now study how country x's effort changes when the other country launches CBDC. Figure 7 shows the percentage changes in country A's effort when country B launches CBDC (left panel) and the percentage changes in country B's effort when country A launches CBDC (right panel) for different values of $\ln(Y)$.

CBDC implementation by the strong country always reduces the weak country's incentives to implement CBDC, in that Δe^B is negative. The intuition is that for the weak country the main motive to launch CBDC is to gain a first-mover advantage over currency B in terms of technology. However, once the strong country has already launched CBDC, it is no more possible to gain this first-mover advantage, curbing the incentives to launch CBDC.

Next, note that CBDC issuance by weak countries may increase or decrease the strong country's CBDC implementation effort. The intuition is that when Y is low and the value of currency A is large (Figure 3), CBDC issuance by the weak country causes drastic reduction in the value and dominance of currency A (Figure 4). In turn, the strong country would like to launch CBDC as well to defend or restore the dominance of its currency, which leads to this strategic complementarity.[29] Consistent with our model results, to the extent that the issuance of CBDC by China can be seen as such a threat to the dominance or reserve currency status of the U.S. dollar, it has led calls to action (Ehrlich, 2020, in Forbes) for America to consider the development of CBDC more seriously too. The recent hearing on stablecoins constitutes a salient example (United States Senate Committee on Banking and Affairs, 2021). That said, the incentive is still smaller than country B's, as Duffie (2021) aptly puts, "Much has been written about the potential impact of eCNY,

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[29]Outside the scope of our paper, it may advantage the United States to develop CBDC technology to offer the technology to countries that wish to lower the costs or advance the development time for introducing their own CBDCs (see, e.g., Duffie, 2021).
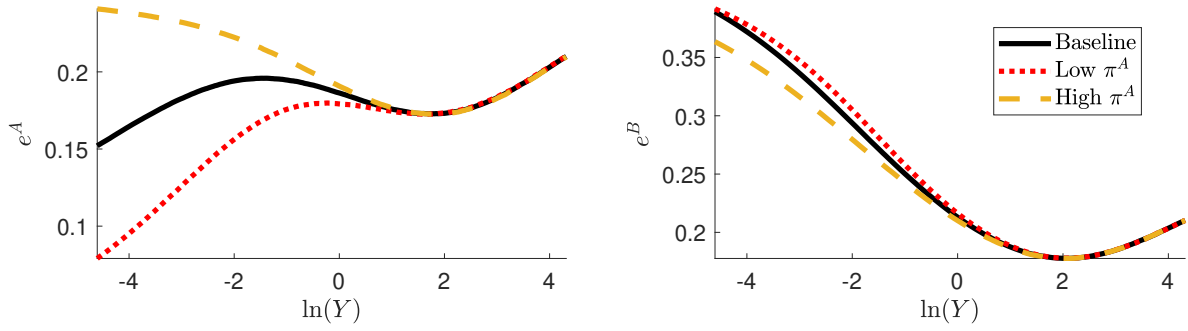
Figure 8: Inflation and CBDC. We use our baseline parameters (i.e., $\pi^A = 1$). For the low value of π^A, we pick $\pi^A = 0$, and for the high value of π^A, we pick π^A.

China's new CBDC, on the international dominance of the U.S. dollar. Concerns that the renminbi will rival the dollar in international markets are not warranted at this time, and these concerns are not a good reason to rush out a digital dollar before it is carefully designed."

3.5 Currency Dominance, CBDC Issuance, and Financial Innovation

We interpret currency A as the reserve currency which in practice maps to the U.S. dollar. Thus, country A can be interpreted as the United States with its economic fundamentals captured by π^A. An increase in π^A means that the economic fundamentals of country A worsen, which feeds back into inflation and the currency value A, undermining the dollar dominance. Similarly, a decrease in π^A can be interpreted as a positive shock to economic fundamentals or as a negative shock to core inflation, reinforcing the dominance of the US dollar.

How does a more dominant US dollar affect countries' incentives to launch CBDC? How do changes and, in particlar, an increase in US dollar inflation affect the cryptocurrency market and the development of CBDC? To address these questions, Figure 8 plots the incentives of country A and B to launch CBDC against $\ln Y$ both under our baseline parameters (solid black line; $\pi^A = 1$) and for a lower value of π^A (dotted red line; $\pi^A = 0$).

As show in the left panel of Figure 8 for any value of $\ln(Y)$, a stronger currency A (due to lower π^A) weakens country A's incentives to innovate by launching CBDC for the strong country, but has only small effects on the incentives of the weak country. This effect is exacerbated through the endogenous channel of the cryptocurrency market growth: Stronger currency A reduces cryptocurrency adoption and growth, which implies less competition for national currencies in the longer

run and so undermines incentives to launch CBDC further. In contrast, the right panel Figure 8 suggests that the dominance of the US Dollar has no significant effects on the weaker country's efforts to implement CBDC.

Our analysis implies that a more dominant dollar makes the U.S. government less likely to implement CBDC. Conversely, weaker fundamentals, higher inflation, and thus more fierce competition among national currencies increases the incentives to implement CBDC, ceteris paribus. The recent rise of emerging economies as well as the high core inflation in the United States (Santilli and Guilford, 2021) therefore challenge the predominance of the dollar, and can increase the government's incentives to venture into dollar digitization.

Both the rise of cryptocurrencies and the prospective implementation of CBDC can be considered financial innovations that disrupt banks and eventually benefit consumers (Duffie, 2021). For example, a viable CBDC may spur firms that currently provide (costly yet inefficient) bank-railed payment services to compete more aggressively, in terms of both pricing and technology innovation. We also study how the strength of national currencies and specifically the dominance of currency A affect financial innovation. Recall that the endowment in our economy is fixed to one unit of the consumption "per period dt." Financial innovation only matters for the convenience yield households derive from holding currency. We consider two different measures of financial innovation: (i) Y_t which can be viewed as the technology underlying cryptocurrencies as a payment system; (ii) countries' propensity to innovate their currency by implementing CBDC, as quantified by the probability $Prob_t$ that at least one country has launched CBDC up to time t. In essense, Y_t measures financial innovation originating in the private (financial) sector, and $Prob_t$ measures government-induced financial innovation.

To examine how the strength of currency and country A, quantified by π^A, relates to financial innovation through the emergence of cryptocurrencies and the implementation of CBDC, Figure 9 plots the cumulative probability $Prob_t$ that at least one country has launched CBDC up to time t and the level of Y_t against time t both under our baseline parameters (i.e., $\pi^A = 1$) and a lower value of π^A (i.e., $\pi^A = 0$). Note that for any t, both measures of financial innovation, the probability that any CBDC is launched and, increase with π^A, meaning that weaker national currencies and in particular a weaker currency A stimulate financial innovation.

The intuition behind this finding is as follows. Relatively weak national currencies (i.e., a weaker country A) imply a vacuum in the currency space that is filled by crypocurrencies. Put differently,
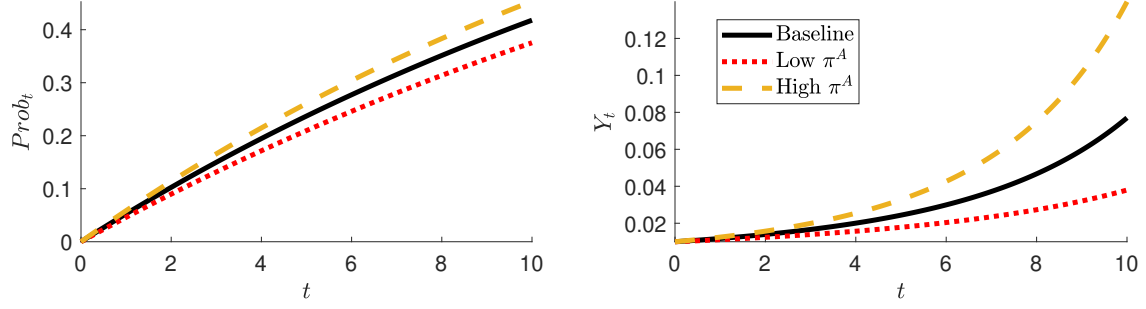
Figure 9: Financial Innovation. We use our baseline parameters (i.e., $\pi^A = 1$). For the low value of π^A, we pick $\pi^A = 0$, and for the high value of π^A, we pick π^A.

an increase in π^A weakens national currencies, and boosts value and adoption of cryptocurrencies. High cryptocurrency adoption stimulates the growth of their underlying technology Y, triggering financial innovation. And, the growth of cryptocurrencies feeds back into countries' decisions to innovate and eventually provides countries with high-powered incentives to launch CBDC, further increasing the degree of financial innovation.

These results also suggest that the recent rise in core inflation in the US, which corresponds to an increase in π^A, as well as the high inflation rates in other developed economies contributes to the growth of the cryptocurrency market. Facilitating the growth of the cryptocurrency market, these high inflation rates stimulate financial innovation in the (private) cryptocurrency sector and provide countries' with higher incentives to implement CBDC, which also contributes to financial innovation and development.

3.6 Developing Countries and Digital Currencies

In this Section, we study the setting in which the weak country is characterized by a sufficiently large value of π^B, as is the case for El Salvador or Venezuela. In our model, a high value of π^B corresponds to high inflation, weak economic fundamentals, and a weak currency of country B.

A first observation is that $\lim_{\pi^B \to \infty} P_t^B = 0 \ \forall \ t \geq 0$. As such, $\lim_{\pi^B \to \infty} \mathbb{E}[dP_t^B]/dt = 0$, which — by (15) — implies that countries with sufficiently high inflation rates and weak currencies do not benefit from implementing CBDC. Intuitively, the currency of a developing country is weak regardless of its underlying technology, which mechanically limits the gains from launching CBDC.

Instead, our analysis suggests that these countries tend to benefit the most from adopting cryptocurrency as a legal means of domestic payment. As shown in Figure 3, weak countries may
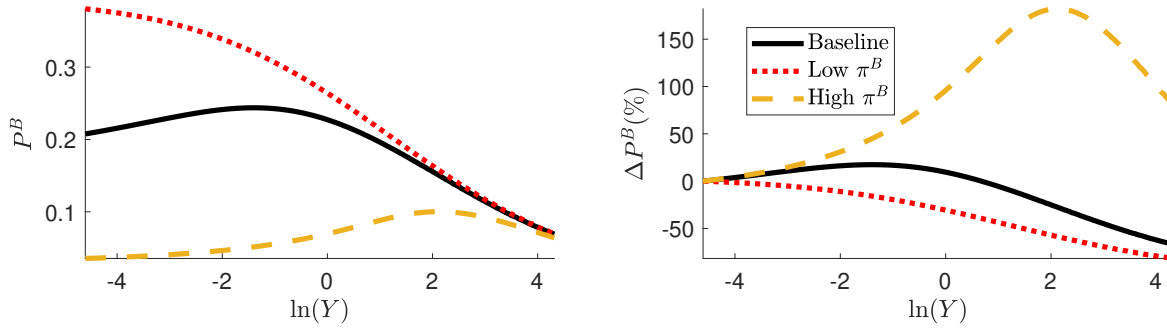
Figure 10: Which countries benefit from and adopt cryptocurrencies? We use our baseline parameters (i.e., $\pi^B = 4$). For the low value of π^B, we pick $\pi^B = 2$, and for the high value of π^B, we pick $\pi^B = 20$.

benefit from the rise of cryptocurrencies, in that P^C increases with Y for low values of Y. However, the extent of the benefit crucially depends on its fundamentals. We argue that developing countries characterized by large values of π^B are more likely to benefit from the rise of cryptocurrencies.

To formalize this argument, the left panel in Figure 10 plots the price of currency B against $\ln(Y)$, which quantifies technology, size, and adoption of cryptocurrencies for different values of π^B. Notice that for a low level of π^B, the rise of cryptocurrencies unambiguously harms currency B, but benefits currency B for larger levels of π^B. Interestingly, the higher π^B, the more currency B benefits from the rise of cryptocurrencies, in that P^C reaches its peak for a larger value of $\ln(Y)$ if π^B is larger. More formally, the value of $\ln(Y)$ maximizing the value of currency B, which is the peak of P^B in Figure 10, is larger for higher values of π^B.

The right panel plots $\Delta P^B = (P^B / P_0^B - 1) \cdot 100$ which measures the percentage value gain currency B experiences in response to the growth of the cryptocurrency market relative to its initial value P_0^B. This relative value gain is negative for low values of π^B, positive for larger values of π^B, and, notably, highest for high values of π^B. Loosely speaking, the larger π^B, the more country B benefits from the rise of cryptocurrencies.

Consistent with Duffie (2021), our findings suggest that small open economies can mitigate the threat of an invasive digital currency through the early adoption of an effective domestic digital currency. In fact, many developing countries may find it optimal to adopt cryptocurrency as a legal means of payment within their country, especially when they do not have high incentives to issue CBDC. A unilateral adoption of cryptocurrency as a legal means of payment in country B increases the usage of cryptocurrencies and thus could be interpreted in our model as an exogenous, positive
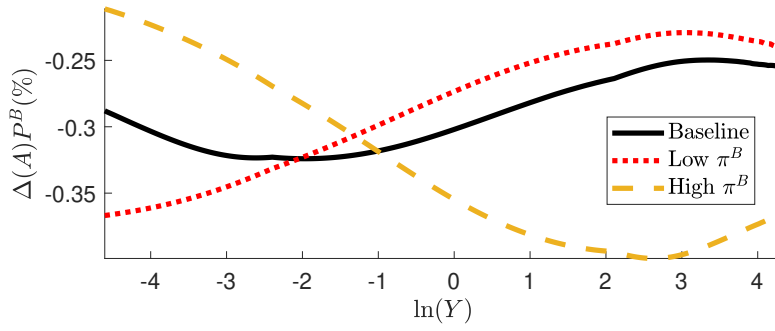
Figure 11: Which countries benefit from and adopt cryptocurrencies? We use our baseline parameters (i.e., $\pi^B = 4$). For the low value of π^B, we pick $\pi^B = 2$, and for the high value of π^B, we pick $\pi^B = 20$.

shock to the convenience yield parameter Y_t. Again, developing countries (i.e., characterized by low values of π^B) are more likely to benefit from an increase in Y and so have more incentives to adopt cryptocurrency. These findings rationalize that while countries with stronger currencies, such as the United States and China, try to ban and regulate cryptocurrency, developing countries with very weak currencies and high inflation rates do the opposite and adopt cryptocurrency as a means of payment in addition to its fiat currency.

Finally, we examine whether developing countries and particularly small economies are more prone to digital dollarization than more developed ones. Figure 11 plots the percentage change in currency B's value P^B given Y when the strong country launches CBDC. Notably, it depends on the value of Y (or $\ln(Y)$) and thus on the state of the cryptocurrency market whether developing countries are more prone to digital dollarization. For low values of $\ln(Y)$, the cryptocurrency market is in its infancy, and the degree of dollarization a developing country is exposed to is massive regardless of whether country A has launched CBDC or not. Under these circumstances, CBDC issuance by the strong country hurts weaker but not the weakest currencies (characterized by a low value of π^B) more than it hurts the weakest currencies of some developing countries (characterized by a high value of π^B). As discussed previously, the rise of cryptocurrencies benefits developing countries and their currencies the most, while it challenges strong currencies. Once the cryptocurrency market has gained sizeable adoption and $\ln(Y)$ is big, developing countries benefit particularly from less competition from stronger currency (i.e., less dollarization). Intuitively, the implementation of CBDC by the strong country then restores the old currency's dominance with digital dollarization. As such, for larger values of Y, developing countries suffer the most from the

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implementation of CBDC by the strong country. As Y_t grows over time, we conclude that in the longer run, developing countries are most prone to digital dollarization, which is consistent with predictions in Brunnermeier et al. (2019).

4 Conclusions

We develop a dynamic general equilibrium model of global competition among national fiat currencies, cryptocurrencies (stablecoins included), and Central Bank Digital Currencies (CBDCs). The strength of a country and of its currency are mutually reinforcing, leading to global currency dominance by the strongest countries. The endogenous rise of cryptocurrencies hurts the stronger country, but may benefit weaker currencies by reducing fiat competition and dollarization. Reserve requirements on stablecoins mitigate the impact of cryptocurrencies on the fiat currencies they are pegged to. Because countries' strategic decisions to implement CBDCs reflect the competition from both emergent cryptocurrencies and other fiat currencies, a pecking order for digital currency development emerges: Countries with non-dominant currencies tend to have the highest-powered incentives to launch CBDC first so as to attain a technological and cumulative first-mover advantage; countries with dominant currencies are motivated to launch CBDC early on both to nip cryptocurrency growth in the bud and to counteract a competitor's CBDC; nations with the weakest or without a sovereign currency may opt for cryptocurrencies or stablecoins pegged to a basket of currencies to avoid (digital) dollarization. In general, weaker national currencies favor the emergence of cryptocurrencies as competitor and boost countries' incentives to implement CBDC, both spurring valuable financial innovations. Our findings help rationalize recent developments in the digitization of money and provide insights into the future of money and the global battle of currencies.

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Appendix

A Proof of Proposition 1

Part I discusses the household optimization, and derives the (necessary) equilibrium condition (4). Part II establishes existence and uniqueness of the equilibrium under the following conditions: i) $Z^A \geq Z^B$, ii) $\pi^B \geq 4(Z^A + Z^B)$, and iii) $\pi^A \in [0, \pi^B)$ is sufficiently small (e.g., $\pi^B = 0$).

For the proof, we define $v^x(m^x) \equiv Z_o m^x + Z^x v(m^x)$ for $x = A, B$, with the function $v(m^x)$ defined in (3).

A.1 Part I — Household Optimization

We start by discussing the representative household's optimization. First, note that at time $t = 0$, the household acquires m^x/P_0^x units of currency x which equals m^x units of currency x in terms of the consumption good. At time $t = 1$, the household sells m^x/P_0^x units of currency x at price P_1^x and consumes the proceeds. Thus, consumption at time $t = 1$ reads

$$c = \frac{P_1^A m^A}{P_0^A} + \frac{P_1^B m^B}{P_0^B} \tag{25}$$

As the household does not derive any utility from consuming at time $t = 0$, it invests its entire endowment in money at time $t = 0$, so $m^A + m^B = 1$.

Recall the household optimizes lifetime utility in (2), that is, the representative household solves

$$\max_{m^A, m^B \geq 0} \left(\frac{P_1^A m^A}{P_0^A} + \frac{P_1^B m^B}{P_0^B} + v^A(m^A) + v^B(m^B) \right) \quad \text{s.t.} \quad m^A + m^B = 1, \tag{26}$$

taking prices P_t^A and P_t^B as given, where we used (25). We now can insert $m^A + m^B = 1 \iff m^B = 1 - m^A$ into the objective in (27) and rewrite (27) as

$$\max_{m^A \in [0,1]} \left(\frac{P_1^A m^A}{P_0^A} + \frac{P_1^B (1 - m^A)}{P_0^B} + v^A(m^A) + v^B(1 - m^A) \right) \tag{27}$$

Provided $m^A \in (0, 1)$ is interior, the following first order condition must hold:

$$\frac{P_1^A}{P_0^A} + \frac{\partial}{\partial m^A} v^A(m^A) = \frac{P_1^B}{P_0^B} - \frac{\partial}{\partial m^A} v^B(1 - m^A). \tag{28}$$

The second order condition to (27)

$$\frac{\partial^2}{\partial(m^A)^2} \left(v^A(m^A) + v^B(1 - m^A) \right) < 0 \tag{29}$$

must hold for an interior maximum m^A.

Next, notice that
$$P_1^x = P_0^x - \pi^x P_0^x,$$
so that
$$\frac{P_1^A}{P_0^A} = 1 - \pi^A \quad \text{and} \quad \frac{P_1^A}{P_0^A} = 1 - \frac{\pi^B P_0^A}{P_0^B}.$$

Using these relations, we can rewrite equilibrium first order condition (28) as

$$\frac{\partial}{\partial m^A} v^A(m^A) - \pi^A = -\frac{\partial}{\partial m^A} v^B(1 - m^A) - \frac{\pi^B P_0^A}{P_0^B}, \tag{30}$$

This condition (30) simplifies after calculating $\frac{\partial}{\partial m^A} v^A(m^A)$ and $-\frac{\partial}{\partial m^A} v^B(1 - m^A)$ and using $m^x = P^x$ and $P^A + P^B = 1$ to

$$Z^A(P^A)^{-2} - \pi^A = Z^B(P^B)^{-2} - \frac{\pi^B P^A}{P^B}$$

or, equivalently,

$$(P^B)^2[Z^A - \pi^A(P^A)^2] = (P^A)^2[Z^B - \pi^B P^A P^B].$$

Inserting $P^B = 1 - P^A$, we obtain

$$Z^A(P^A)^{-2} - \pi^A = Z^B(1 - P^A)^{-\eta} - \frac{\pi^B P^A}{1 - P^A}, \tag{31}$$

which is the equilibrium condition (4) in terms of only P^A. To characterize an interior equilibrium, it therefore suffices to solve (31) for $P^A \in (0, 1)$.

Finally, note that because $\eta > 1$, it follows that the left-hand-side of (31) tends to $+\infty$ as P^A tends to zero, while the right-hand-side remains finite. Likewise, the right-hand-side of (31) tends to $+\infty$ as P^A tends to one, while the left-hand-side remains finite. As such, there cannot exist an equilibrium with $P^A = 0$ or $P^A = 1$.

A.2 Part II — Existence and Uniqueness

In what follows, to establish uniqueness and existence of the equilibrium under the following conditions: i) $Z^A = Z^B = Z$, ii) $\pi^B \geq \eta 2^\eta$, and iii) $\pi^A \in [0, \pi^B)$ is sufficiently small (e.g., $\pi^B = 0$).

Define
$$f(P^A) = Z^A(P^A)^{-2} - \pi^A - Z^B(1 - P^A)^{-2} + \frac{\pi^B P^A}{1 - P^A}.$$

By (31), $f(P^A) = 0$ in equilibrium. To establish uniqueness and existence, we will show that $f(P^A)$ has a unique root on $(0, 1)$.

We can calculate

$$f'(P^A) = -2Z^A(P^A)^{-3} - 2Z^B(1 - P^A)^{-3} + \frac{\pi^B}{1 - P^A} + \frac{\pi^B P^A}{(1 - P^A)^2}$$

We can multiply $f'(P^A)$ by $(1 - P^A)^2$ to obtain

$$(1 - P^A)^2 f'(P^A) = -2Z^A(P^A)^{-3}(1 - P^A)^2 - 2Z^B(1 - P^A)^{-1} + \pi^B$$

For $P^A \in [1/2, 1]$, we obtain

$$(1 - P^A)^2 f'(P^A) \geq \pi^B - 2Z(1/2)^{-3}(1/2)^2 - 2Z^B(1/2)^{1-\eta}$$
$$\pi^B - 4Z^A - 4Z^B = \pi^B - 4(Z^A + Z^B).$$

Thus, if

$$\pi^B \geq 4(Z^A + Z^B), \tag{32}$$

then $f(P^A)$ increases in P^A on $[1/2, 1]$.

Note that

$$f(1/2) = \pi^B - \pi^A + 4(Z^A - Z^B) > 0$$

and

$$\lim_{P^A \to 1} f(P^A) = \lim_{P^A \to 1} \left(\pi^A - Z^B(1 - P^A)^{-\eta} + \frac{\pi^B P^A}{1 - P^A} \right) = -\infty.$$

As such, when (32) holds, the function $f(P^A)$ has a unique root P_0^A on the interval $(1/2, 1)$.

Finally, suppose that $\pi^A = 0$. In equilibrium,

$$Z^A(P^A)^{-2} = Z^B(1 - P^A)^{-2} - \frac{\pi^B P^A}{1 - P^A}$$

must hold. As $Z^A \geq Z^B$ and $P^A > 0$, it follows that $P^A > 1/2$. Thus, by continuity, if π^A is sufficiently small, it holds that $P^A > 1/2 > P^B$. If π^A is sufficiently small and (32) holds, then there exists a unique equilibrium with $P^A > 1/2 > P^B$.

B Proof of Proposition 2

Part I discusses the household optimization, and derives the (necessary) equilibrium condition (38). Part II discusses existence and uniqueness of the equilibrium, when $\eta = 2$, and also characterizes currency values in closed-form. For the proof, we define $v^x(m^x) \equiv Z_o m^x + Z^x v(m^x)$ for $x = A, B$, with the function $v(m^x)$ defined in (3). We also set $v^C(m^C) \equiv (Z_o + Y)m^C$.

B.1 Part I — Household Optimization

We start by discussing the representative household's optimization. First, note that at time $t = 0$, the household acquires m^x/P_0^x units of currency x which equals m^x units of currency x in terms of the consumption good. At time $t = 1$, the household sells m^x/P_0^x units of currency x at price P_1^x

and consumes the proceeds. Thus, consumption at time $t = 1$ reads

$$c = \frac{P_1^A m^A}{P_0^A} + \frac{P_1^B m^B}{P_0^B} + m^C, \tag{33}$$

where we used that $P_0^C = P_1^C$. The lifetime utility of the representative household reads

$$c + v^A(m^A) + v^B(m^B) + v^C(m^C)$$

As the household does not derive any utility from consuming at time $t = 0$, it invests its entire endowment in money at time $t = 0$, so $m^A + m^B + m^C = 1$.

The household maximizes lifetime utility in (2), that is, the household solves

$$\max_{m^A, m^B, m^C \geq 0} \left(\frac{P_1^A m^A}{P_0^A} + \frac{P_1^B m^B}{P_0^B} + m^C + v^A(m^A) + v^B(m^B) + v^C(m^C) \right) \left(\text{ s.t. } m^A + m^B + m^C = 1, \right. \tag{34}$$

taking prices P_t^A, P_t^B, and P_t^C as given. We can substitute $m^C = 1 - m^A - m^B$ and rewrite (34) as

$$\max_{m^A, m^B \geq 0} \left(\frac{P_1^A m^A}{P_0^A} + \frac{P_1^B m^B}{P_0^B} + 1 - m^A - m^B + v^A(m^A) + v^B(m^B) + v^C(1 - m^A - m^B) \right) \left(\tag{35} \right.$$

subject to $m^C \geq 0 \iff m^A + m^B \leq 1$.

In optimum when $m^A + m^B \in (0, 1)$ and $m^x \in (0, 1)$, the following two first order conditions (with respect to m^A and m^B) must hold:

$$\frac{P_1^A}{P_0^A} - 1 + \frac{\partial}{\partial m^A} \left[v^A(m^A) + v^C(1 - m^A - m^B) \right] = 0 \tag{36}$$

$$\frac{P_1^B}{P_0^B} - 1 + \frac{\partial}{\partial m^B} \left[v^B(m^B) + v^C(1 - m^A - m^B) \right] = 0. \tag{37}$$

We know that $P_1^A / P_0^A = 1 - \pi^A$ and $P_1^B / P_0^B = 1 - \pi^B P_0^A / P_0^B$. Inserting these relations and $m^x = P^x$ into (36), we obtain

$$\frac{\partial}{\partial m^A} \left[v^A(m^A) - \pi^A + v^C(1 - m^A - m^B) \right] = 0$$

$$\frac{P_1^B}{P_0^B} - \pi^B P_0^A / P_0^B + \frac{\partial}{\partial m^B} \left[v^B(m^B) + v^C(1 - m^A - m^B) \right] = 0.$$

Using the explicit expressions for $v^x(m^x)$ and doing some algebra, we then obtain

$$Z^A(m^A)^{-2} - \pi^A = Z^B(m^B)^{-2} - \frac{\pi^B P^A}{P^B} = Y, \tag{38}$$

which becomes after inserting $m^x = P^x$:

$$Z^A(P^A)^{-2} - \pi^A = Z^B(P^B)^{-2} - \frac{\pi^B P^A}{P^B} = Y. \tag{39}$$

In the following part (i.e., Part II), we combine (38) and (5) to solve for currency values in closed-form.

B.2 Part II — Existence and Uniqueness

In this part of the proof, we consider only the case $\eta = 2$. Suppose there exists a cryptocurrency equilibrium, which is characterized by (38). With $\eta = 2$, the equilibrium condition (38) becomes

$$Z^A(P^A)^{-2} - \pi^A = Z^B(P^B)^{-2} - \frac{\pi^B P^A}{P^B} = Y,$$

which has to be solve for P^A and P^B. First, we can solve $Z^A(P^A)^{-2} - \pi^A = Y$ for

$$P^A = \sqrt{\left(\frac{Z^A}{Y + \pi^A}\right)}.$$

Inserting this expression for P^A into (38), we obtain

$$Z^B(P^B)^{-2} - \left(\frac{\pi^B}{P^B}\right)\sqrt{\left(\frac{Z^A}{Y + \pi^A}\right)} = Y \iff Z^B - \pi^B P^B\left(\sqrt{\frac{Z^A}{Y + \pi^A}}\right) - Y(P^B)^2 = 0.$$

Thus, we have to solve a quadratic equation in P^B, which admits two solutions

$$P^B = \frac{1}{2Y}\left[-\pi^B\left(\sqrt{\left(\frac{Z^A}{Y + \pi^A}\right)}\right) \pm \sqrt{\frac{Z^A(\pi^B)^2}{Y + \pi^A} + 4YZ^B}\right].$$

One solution is clearly negative and thus constitutes no equilibrium. The positive solution can be rewritten as

$$P^B = \frac{1}{2Y}\left(\sqrt{\frac{Z^A(\pi^B)^2}{Y + \pi^A} + 4YZ^B} - \pi^B\left(\sqrt{\frac{Z^A}{Y + \pi^A}}\right)\right). \tag{40}$$

Expression (40) readily implies that P^B increases with π^A, but decreases with Z^A.

Multiplying and dividing both sides of (40) by $\sqrt{\frac{Z^A(\pi^B)^2}{Y+\pi^A} + 4YZ^B} + \pi^B\left(\sqrt{\frac{Z^A}{Y+\pi^A}}\right)$ and simplifying, one can rewrite (40) as

$$P^B = \frac{2Z^B}{\sqrt{\frac{4Z^B Y^2 + 4Z^B \pi^A Y + Z^A \pi^{B2}}{Y + \pi^A}} + \pi^B\sqrt{\frac{Z^A}{Y+\pi^A}}},$$

which, in turn, can be written as

$$P^B = \frac{2\sqrt{Z^B(Y + \pi^A)}}{\sqrt{4\,Y^2 + 4\,\pi^A\,Y + (\pi^B)^2(Z^A/Z^B)} + \pi^B\,\sqrt{Z^A/Z^B}}. \tag{41}$$

Finally, we can use $P^A + P^B + P^C = 1$ to calculate

$$P^C = 1 - \sqrt{\frac{Z^A}{Y + \pi^A}} - \frac{2\sqrt{Z^B(Y + \pi^A)}}{\sqrt{4\,Y^2 + 4\,\pi^A\,Y + (\pi^B)^2(Z^A/Z^B)} + \pi^B\,\sqrt{Z^A/Z^B}}. \tag{42}$$

The crypto equilibrium exists as long as $P^C \geq 0$, that is, when

$$\sqrt{\frac{Z^A}{Y + \pi^A}} + \frac{2\sqrt{Z^B(Y + \pi^A)}}{\sqrt{4\,Y^2 + 4\,\pi^A\,Y + (\pi^B)^2(Z^A/Z^B)} + \pi^B\,\sqrt{Z^A/Z^B}} \leq 1$$

holds. Provided its existence, the cryptocurrency equilibrium is unique.

C   Proof of Corollary 1

The Corollary follows by direct calculation. We already impose $Z^A = Z^B = Z$ to ease the calculations and to simplify the expressions. Inserting, $Z^A = Z^B = Z$ into the expression for P^B in (6) yields:

$$P^B = \frac{2\sqrt{Z(Y + \pi^A)}}{\sqrt{4\,Y^2 + 4\,\pi^A\,Y + (\pi^B)^2} + \pi^B}.$$

We can can calculate

$$\frac{dP^B}{dY} = \frac{\left(\sqrt{4\,Y^2 + 4\,\pi^A\,Y + (\pi^B)^2} + \pi^B\right)\sqrt{\frac{Z}{Y+\pi^A}} - \left(\sqrt{Z(Y + \pi^A)}\right)\left(\frac{8Y+4\pi^A}{\sqrt{4\,Y^2+4\,\pi^A\,Y+(\pi^B)^2}}\right)}{\left(\sqrt{4\,Y^2 + 4\,\pi^A\,Y + (\pi^B)^2} + \pi^B\right)^2}$$

Note that the denominator of above expression is unambiguously positive. Thus, the sign of the derivative is obtained by inspecting the numerator. The numerator has the same sign as

$$\left(\sqrt{4\,Y^2 + 4\,\pi^A\,Y + (\pi^B)^2} + \pi^B\right)\sqrt{\frac{1}{Y + \pi^A}} - \sqrt{Y + \pi^A}\ \frac{8Y + 4\pi^A}{\sqrt{4\,Y^2 + 4\,\pi^A\,Y + (\pi^B)^2}}$$

which has the same sign as

$$\left(\sqrt{4\,Y^2 + 4\,\pi^A\,Y + (\pi^B)^2} + \pi^B\right)\left(-\frac{(8Y + 4\pi^A)(Y + \pi^A)}{\sqrt{4\,Y^2 + 4\,\pi^A\,Y + (\pi^B)^2}}\right)$$

For $Y = 0$, above expression simplifies to

$$2\pi^B - \frac{4(\pi^A)^2}{\pi^B},$$

which is positive if and only if

$$\pi^B \geq \sqrt{2}\pi^A.$$

Provided $\pi^B \geq \sqrt{2}\pi^A$, by continuity in Y, there exists an interval $[0, \underline{Y})$ with $\underline{Y} > 0$, such that P^B increases with Y on $[0, \underline{Y})$.

Finally, observe that

$$\lim_{Y \to \infty} \left(\sqrt{4Y^2 + 4\pi^A Y + (\pi^B)^2} + \pi^B \right) \left(-\frac{(8Y + 4\pi^A)(Y + \pi^A)}{\sqrt{4Y^2 + 4\pi^A Y + (\pi^B)^2}} = \lim_{Y \to \infty} (2Y - 4Y) < 0.$$

Thus, by continuity, $\frac{dP^B}{dY} < 0$ and P^B decreases with Y for Y sufficiently large.

D Details on the Derivations of Section 1.4

Consider $Y = 0$. Inserting $Y = 0$ into the price expression P^A in (6), we readily obtain $P^A = \sqrt{\frac{Z^A}{\pi^A}}$. Solving (38) for $Y = 0$ and $\eta = 2$ for P^B is equivalent to solving

$$Z^B(P^B)^{-2} - \left(\frac{\pi^B}{P^B}\right)P^A = 0 \quad \Longleftrightarrow \quad Z^B - \pi^B P^B P^A = 0.$$

for P^B. Thus,

$$P^B = \frac{Z^B}{\pi^B P^A} = \frac{Z^B}{\pi^B \sqrt{Z^A/\pi^A}},$$

where we used $P^A = \sqrt{\frac{Z^A}{\pi^A}}$.

Next, taking the derivatives with respect to Z^A and Z^B, we calculate

$$\frac{\partial P^A}{\partial Z^A} = \frac{1}{2\sqrt{Z^A \pi^A}} \quad \text{and} \quad \frac{\partial P^B}{\partial Z^B} = \frac{1}{\pi^B P^A},$$

which is (8). Now, observe that

$$\frac{\partial P^A}{\partial Z^A} - \frac{\partial P^B}{\partial Z^B} = \frac{1}{2\sqrt{Z^A \pi^A}} - \frac{1}{\pi^B P^A} = \frac{1}{2\sqrt{Z^A \pi^A}} - \frac{1}{\pi^B}\sqrt{\frac{\pi^A}{Z^A}},$$

where the second equality uses $P^A = \sqrt{\frac{Z^A}{\pi^A}}$. Multiplying both sides by $2\sqrt{Z^A \pi^A}\pi^B > 0$, we note that $\frac{\partial P^A}{\partial Z^A} - \frac{\partial P^B}{\partial Z^B}$ has the same sign as

$$\pi^B - 2\pi^A,$$

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which was to show.

E   Model Extension with Fiat-Backed Cryptocurrency

Suppose that fraction $\theta \in [0, 1]$ of cryptocurrency is backed by currency A. That is, total reserves backing cryptocurrency are θP^C units of the consumption good. Thus, the reserves backing cryptocurrency consist of $\theta P^C / P^A$ units of currency A, which implies a circulating supply of currency A at $(1 - \theta P^C / P^A)$ units. For the market for currency A to clear, the household holds the remainder, i.e., the circulating supply

$$m^A = (1 - \theta P^C / P^A) \tag{43}$$

of units of currency A. As a consequence, the household's holdings of currency A in terms of the consumption good read

$$m^A = P^A - \theta P^C.$$

The market clearing condition $m^A + m^B + m^C = 1$ therefore becomes

$$P^A + P^B + P^C(1 - \theta) = 1.$$

Thus, we can solve for

$$P^C = \frac{1 - P^A - P^B}{1 - \theta} \tag{44}$$

and, inserting P^C into (43), we solve for

$$m^A = P^A - \theta P^C = P^A - \frac{\theta(1 - P^A - P^B)}{1 - \theta} = \frac{P^A - \theta(1 - P^B)}{1 - \theta}. \tag{45}$$

As before, in a cryptocurrency equilibrium with positive price $P^C > 0$, the indifference conditions (38) must hold, that is,

$$Z^A(m^A)^{-2} - \pi^A = Z^B(m^B)^{-2} - \frac{\pi^B P^A}{P^B} = Y.$$

Intuitively, the household must be indifferent between substituting one marginal unit of currency x with a marginal unit of another currency.

After inserting $m^A = \frac{P^A - \theta(1 - P^B)}{1 - \theta}$ from (45) and $m^B = P^B$, we obtain

$$Z^A \left(\frac{P^A - \theta(1 - P^B)}{1 - \theta} \right)^{-2} - \pi^A = Y \tag{46}$$

and

$$Z^B(P^B)^{-2} - \frac{\pi^B P^A}{P^B} = Y. \tag{47}$$

The equilibrium is obtained by solving (46), (47), and (44) for P^A, P^B, and P^C.

To solve this system, note that one can solve for (46) and (47), which do not depend on P^C, for P^A and P^B and then plug the solution into (44) to obtain P^C. To begin with, use (47) to solve for

$$P^A = \frac{Z^B/P^B - YP^B}{\pi^B} \tag{48}$$

and insert this expression into (46) to obtain after rearranging

$$\left(\frac{Z^B/P^B - YP^B}{\pi^B} - \theta(1 - P^B) \right)^{-2} = \frac{Y + \pi^A}{Z^A(1 - \theta)^2}$$

Thus,

$$Z^B - Y(P^B)^2 - \theta\pi^B(1 - P^B)P^B = P^B(1 - \theta) \cdot \pi^B \sqrt{\left(\frac{Z^A}{Y + \pi^A} \right)} \tag{49}$$

Define

$$K := \pi^B \sqrt{\left(\left(\frac{Z^A}{Y + \pi^A} \right) \cdot \right(}$$

Provided their existence, equation (49) admits two solutions

$$\frac{K(1 - \theta) + \pi^B \theta \pm \sqrt{K^2(1 - \theta)^2 - 2\,K\,\pi^B\,\theta^2 + 2\,K\,\pi^B\,\theta + (\pi^B)^2\,\theta^2 - 4\,Z^B\,\pi^B\,\theta + 4\,Y\,Z^B}}{-2\,(Y - \pi^B\,\theta)}$$

For now, we consider that $Y > \pi^B\theta$, and discard the clearly negative solution to (49) to arrive at

$$P^B = \frac{-K(1 - \theta) - \pi^B\,\theta + \sqrt{K^2(1 - \theta)^2 + 2\,K\,\pi^B\,\theta(1 - \theta) + (\pi^B)^2\,\theta^2 + 4\,Y\,Z^B(1 - \pi^B\theta)}}{2\,(Y - \pi^B\,\theta)}. \tag{50}$$

Inserting P^B into (48), we can solve P^A in closed-form, and, inserting P^A and P^B into (44), we can solve P^C in closed-form.

Finally, note that at time $t = 0$, the cryptocurrency sector collects P^C units of the consumption good from households. Out of these revenues, θP^C units of the consumption good are used to buy currency A which is the reserve backing cryptocurrency. As such, the actual seigniorage revenue of the cryptocurrency sector is $(1 - \theta)P^C$.

F   Solution to the Dynamic Model and Proof of Proposition 3

F.1   Part I — Market Clearing Conditions

We consider that fraction θ of cryptocurrency value P_t^C is backed by currency A reserves, where $\theta \in [0, 1)$ is an exogenous constant. This way, we model dollar-backed stablecoins, such as USDC, since we associate currency A with the U.S. dollar.

As a result, total reserves backing cryptocurrency are worth θP_t^C units of the consumption good. Thus, the reserves backing cryptocurrency consist of $\theta P_t^C/P_t^A$ units of currency A, leaving

the circulating supply of currency A at $(1 - \theta P_t^C / P_t^A)$ units. For the market for currency A to clear, the household holds this circulating supply, i.e.,

$$m_t^A = (1 - \theta P_t^C / P_t^A) \tag{51}$$

units of currency A. Therefore, that the household's holdings of currency A in terms of the consumption good read

$$m_t^A = P_t^A - \theta P_t^C.$$

The market clearing condition $m_t^A + m_t^B + m_t^C = 1$ therefore becomes

$$P_t^A + P_t^B + P_t^C (1 - \theta) = 1. \tag{52}$$

Thus, we can solve for

$$P_t^C = \frac{1 - P_t^A - P_t^B}{1 - \theta} \tag{53}$$

and, inserting P_t^C into (51), we obtain

$$m_t^A = P_t^A - \theta P_t^C = P_t^A - \frac{\theta(1 - P_t^A - P_t^B)}{1 - \theta} = \frac{P_t^A - \theta(1 - P_t^B)}{1 - \theta}. \tag{54}$$

F.2   Part II — Household Optimization

To start with, recall $r_t^x = \frac{\mathbb{E}[dP_t^x]}{P_t^x dt}$. Next, we can write cohort t's consumption dc_t at $t + dt$ as

$$dc_t = \sum_{x \in \{A,B,C\}} \left(\frac{m_t^x P_{t+dt}^x}{P_t^x} - \sum_{x \in \{A,B\}} \left(\frac{\tau_t^x m_t^x}{P_t^x} dt. \right. \right. \tag{55}$$

Observe that $P_{t+dt}^x = P_t^x + dP_t^x$. Recall that the representative household uses its entire endowment one to buy currencies at time t, so $\sum_{x \in \{A,B,C\}} m_t^x = 1$. We can use this relation to rewrite (55) as

$$dc_t = 1 + \sum_{x \in \{A,B,C\}} \left(\frac{m_t^x dP_t^x}{P_t^x} - \sum_{x \in \{A,B\}} \left(\frac{\tau_t^x m_t^x}{P_t^x} dt. \right. \right. \tag{56}$$

Now, note that the representative household maximizes her utility

$$\max_{m_t^x \geq 0} \mathbb{E}[dU_t] \quad \text{s.t.} \quad \sum_{x \in \{A,B,C\}} \left(m_t^x = 1, \right.$$

taking prices P_t^x as given. The utility dU_t is defined in (13).

In light of the optimization constraint $\sum_{x \in \{A,B,C\}} m_t^x = 1$, it must be in optimum that

$$\frac{\partial \mathbb{E}[dU_t]}{\partial m_t^A} = \frac{\partial \mathbb{E}[dU_t]}{\partial m_t^B} = \frac{\partial \mathbb{E}[dU_t]}{\partial m_t^C},$$

provided $m_t^x \in (0, 1)$.

Inserting the market clearing condition $m_t^A = \frac{P_t^A - \theta(1 - P_t^B)}{1 - \theta}$ from (54), $m_t^B = P_t^B$, and $m_t^C = P_t^C$ and doing some simplifying algebra, we can rewrite the above to

$$Y_t v'(P_t^C) + r_t^C = Z_t^A v' \left(\frac{P_t^A - \theta(1 - P_t^B)}{1 - \theta} \right) \left(+ r_t^A - \frac{\tau_t^A}{P_t^A} \right.$$

$$Y_t v'(P_t^C) + r_t^C = Z_t^B v'(P_t^B) + r_t^B - \frac{\tau_t^B}{P_t^B}, \tag{57}$$

which is (20) as desired. Due to $\eta > 1$, it follows that any solution to (20) or (57) must satisfy $P_t^x \in (0, 1)$. Also note that the constant base (marginal) convenience Z_o, which is the same across all currencies, does not enter the equilibrium pricing condition (20) or (57).

F.3   Part III — Markovian Representation

We now express the currency values as well as returns as a functions of Y and state $z \in \{0, A, B, AB\}$, and we omit time subscripts unless necessary. We call z also the CBDC state. In equilibrium, $P_t^x = P^x(Y, z) = m_t^x = m^x(Y, z)$ for all $z \in \{B, C\}$ and $m^A(Y, z) = \frac{P^A(Y,z) - \theta(1 - P^B(Y,z))}{1 - \theta}$, Also recall (52), that is

$$P^A(Y, z) + P^B(Y, z) + P^C(Y, z)(1 - \theta) = 1.$$

Before proceeding, we postulate

$$\frac{dP_t^x}{P_t^x} = \mu^x(Y, z)dt + \Delta^x(Y, z; z')dJ_t^{z,z'}, \tag{58}$$

where $\mu^x(Y, z)$ is the endogenous price drift in state $(Y_t, z_t) = (Y, z)$. In (58), $\Delta^x(Y, z; z')$ is the relative value change of currency x if the CBDC changes from z to z'. The jump process $dJ_t^{z,z'} \in \{0, 1\}$ equals one if and only if the CBDC state changes from z to z'; otherwise, $dJ_t^{z,z'} = 0$. Notice that

$$\Delta^x(Y, z; z') = \frac{P^x(Y, z')}{P^x(Y, z)} - 1,$$

and $\Delta^x(Y, z; z')P^x(Y, z) = P^x(Y, z') - P^x(Y, z)$.

Next, we characterize the equilibrium efforts $e_t^x = e^x(Y, z)$ for $z = 0, A, B$ and $x = A, B$, determined according to the optimization in (15). Clearly, $e^A(Y, A) = e^B(Y, B) = e^x(Y, AB) = 0$. The equilibrium effort levels in the remaining cases read:

$$e^x(Y, 0) = \lambda(P^x(Y, x) - P^x(Y, 0)) \tag{59}$$

$$e^A(Y, B) = \lambda(P^x(Y, AB) - P^x(Y, B)) \tag{60}$$

$$e^B(Y, A) = \lambda(P^B(Y, AB) - P^x(Y, A)). \tag{61}$$

To get some intuition behind (59), note that in state $z = 0$, country A maximizes $\lambda e^A(P^A(Y, A) - P^A(Y, 0) - \frac{(e^A)^2}{2}$ over e^A, yielding optimal interior effort $e^A(Y, 0) = \lambda(P^A(Y, A) - P^A(Y, 0))$. The

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other efforts derive similarly.

Let $(P^x)'(Y, z) = \frac{\partial}{\partial Y} P^x(Y, z)$. For $Y = \overline{Y}$, the drift $\mu_t^x = \mu^x(Y, z)$ equals zero. Otherwise, for $Y < \overline{Y}$, the price drifts satisfy $\mu_t^x = \mu^x(Y, z)$ and by Ito's Lemma read

$$\mu^x(Y, z) = \left(\frac{(P^x)'(Y, z)}{P^x(Y, z)} \right) \mu Y P^C(Y, z), \tag{62}$$

where $\mu Y m^C(Y, z) = \mu Y P^C(Y, z)$ is the drift of dY_t in (14).

Also note that because $P_t^A + P_t^B + P_t^C(1 - \theta) = 1$, we have $dP_t^A + dP_t^B + dP_t^C(1 - \theta) = 0$, so that

$$\mu^A(Y, z) P^A(Y, z) + \mu^B(Y, z) P^B(Y, z) + \mu^C(Y, z) P^C(Y, z)(1 - \theta) = 0 \tag{63}$$

as well as

$$\Delta^A(Y, z, z') P^A(Y, z) + \Delta^B(Y, z, z') P^B(Y, z) + \Delta^B(Y, z, z') P^C(Y, z)(1 - \theta) = 0. \tag{64}$$

In light of (63), (64), or $P_t^A + P_t^B + P_t^C(1 - \theta) = 1$, it suffices to characterize the currency values and dynamics for currencies A and B, and the value and dynamics for currency C follows.

Next, we can characterize expected returns r_t^x, and write $r_t^x = r^x(Y, z)$. It holds that

$$\begin{aligned}
r^x(Y, 0) &= \mu^x(Y, 0) + \lambda e^A(Y, 0)\big(P^x(Y, A)/P^x(Y, 0) - 1\big) + \lambda e^B(Y, 0)\big(P^x(Y, B)/P^x(Y, 0) - 1\big) \\
r^x(Y, A) &= \mu^x(Y, A) + \lambda e^B(Y, A)\big(P^x(Y, AB)/P^x(Y, A) - 1\big) \\
r^x(Y, B) &= \mu^x(Y, B) + \lambda e^A(Y, B)\big(P^x(Y, AB)/P^x(Y, B) - 1\big) \\
r^x(Y, AB) &= \mu^x(Y, AB).
\end{aligned} \tag{65}$$

We also know that $\tau^A(Y, z) = \kappa^A + \pi^A P^A(Y, z)$ and $\tau^B(Y, z) = \kappa^B + \pi^B P^A(Y, z)$.

Inserting these relations into the equilibrium condition (57) yields for $x = A, B$:

$$Y v'\big(P^C(Y, z)\big) + r^C(Y, z) = Z^x(Y, z) v'\big(m^x(Y, z)\big) + r^x(Y, z) - \frac{\tau^x(Y, z)}{P^x(Y, z)}, \tag{66}$$

where $Z^A(Y, z) = Z_L$ for $z = 0, B$ and $Z^A(Y, z) = Z_H + \alpha Y$ for $z = A, AB$. Likewise, $Z^B(Y, z) = Z_L$ for $z = 0, A$ and $Z^B(Y, z) = Z_H + \alpha Y$ for $z = B, AB$. Note that by (54), $m^A(Y, z) = m^A(Y, z) = \frac{P^A(Y, z) - \theta(1 - P^B(Y, z))}{1 - \theta}$, and $m^B(Y, z) = P^B(Y, z)$. As a result, we have verified that all model quantities can be expressed in terms of (Y, z).

Inserting (65) and (62) into (66), one obtains a system coupled first order ODEs in Y for the currency prices $P^x(Y, z)$, which can be solved numerically on $[0, \overline{Y}]$ for all states $z \in \{0, A, B, AB\}$ to obtain $P^x(Y, z)$. We assume that such a solution exists and is unique. A formal proof is beyond the scope of this paper.

Because the currency values in states $z = A$ and $z = B$ depend on the currency values in state $z = AB$, one has to solve the model backward in terms of the state variable z, starting with state $z = AB$. Having obtained $P^x(Y, AB)$ for $Y \in [0, \overline{Y}]$, one can solve for currency values $P^x(Y, A)$

and $P^x(Y, B)$. Having obtained $P^x(Y, A)$ and $P^x(Y, B)$, one can solve for currency values $P^x(Y, 0)$.

F.4 Existence and Uniqueness

A formal proof of equilibrium existence and uniqueness is beyond the scope of the paper. We assume that a unique equilibrium exists. In this Section, we provide a sketch of the arguments that could be used to establish equilibrium existence and uniqueness. To be able to characterize existence and uniqueness of a Markov equilibrium with state variable (Y, z), it is necessary to study the boundary behavior of the system (66) at $Y = \overline{Y}$ for all $z \in \{0, A, B, AB\}$.

We start by analyzing state $z = AB$. At $Y = \overline{Y}$, we have $\mu^x(\overline{Y}, z) = 0$. As such, $r^x(\overline{Y}, AB) = 0$. Inserting this relation into (66), we obtain

$$\overline{Y}v'\big(P^C(\overline{Y}, AB)\big) = Z^x(\overline{Y}, AB)v'\big(m^x(\overline{Y}, AB)\big) \left(r^x(\overline{Y}, AB) - \frac{\tau^x(\overline{Y}, AB)}{P^x(\overline{Y}, AB)}. \right. \tag{67}$$

Note that for $z = AB$, (66) characterizes a system of ODEs with boundary behavior at $Y = \overline{Y}$ characterized in (67), where $P^A + P^B + P^C$. When (67) combined with (52) yield a unique solution $P^x(\overline{Y}, AB)$ for $x = A, B, C$, then the Picard-Lindeloef theorem implies that under mild regularity conditions on the assumed functional forms, there exists a unique solution to the system of ODEs (66). Thus, the Picard Lindeloef theorem applies that existence and uniqueness of an equilibrium follow from existence and uniqueness of a solution to (67).

Likewise, using $\mu^x(\overline{Y}, z) = 0$, we can characterize the boundary behavior of (66) for all z and $x = A, B$. Solving the equilibrium for $Y = \overline{Y}$ does not require an ODE but requires to solve four non-linear equations. Provided (66) for all z and $x = A, B$ admits a unique solution $(P^A(\overline{Y}, z), P^B(\overline{Y}, z), P^C(\overline{Y}, z))$, then the Picard-Lindeloef theorem implies a unique solution to (66) on $(0, \overline{Y}]$. Under these circumstances, there exists a unique equilibrium.