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March 2022



Working Paper

09.2022

Charging the macroeconomy with an energy sector: an agent-based model

**Emanuele Ciola, Enrico Turco, Andrea Gurgone, Davide
Bazzana, Sergio Vergalli, Francesco Menoncin**

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By Emanuele Ciola, Fondazione Eni Enrico Mattei and Università degli Studi di Brescia
Enrico Turco, Fondazione Eni Enrico Mattei and Università Cattolica del Sacro Cuore
Andrea Gurgone, Fondazione Eni Enrico Mattei and Università Cattolica del Sacro Cuore
Davide Bazzana, Fondazione Eni Enrico Mattei and Università degli Studi di Brescia
Sergio Vergalli, Fondazione Eni Enrico Mattei and Università degli Studi di Brescia
Francesco Menoncin, Fondazione Eni Enrico Mattei and Università degli Studi di Brescia

Summary

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Keywords: Energy Sector; Energy Shocks; Agent-Based Models; Macroeconomic Dynamics

JELClassification: C63, O13, Q41, Q43

Address for correspondence:

Emanuele Ciola
Modelling the Energy Transition
Fondazione Eni Enrico Mattei
Corso Magenta, 63
20123, Milan
Italy
E-mail: emanuele.ciola@feem.it

The opinions expressed in this paper do not necessarily reflect the position of Fondazione Eni Enrico Mattei

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Emanuele Ciola^{1,2,*}, Enrico Turco^{1,3}, Andrea Gurgone^{1,3}, Davide Bazzana^{1,2}, Sergio Vergalli^{1,2}, and Francesco Menoncin^{1,2}

¹Fondazione Eni Enrico Mattei, Italy

²Università degli Studi di Brescia, Dep. of Economics and Management, Italy

³Università Cattolica del Sacro Cuore, Dep. of Economics and Finance, The Complexity Lab in Economics, Italy

*emanuele.ciola@feem.it - Fondazione Eni Enrico Mattei, Corso Magenta, 63, 20123 Milano, Italy

Abstract

The global energy crisis that began in fall 2021 and the following spike in energy price constitute a major challenge for the world economy which risks undermining the post-COVID-19 recovery. In this paper, we develop and validate a new macroeconomic agent-based model with an endogenous energy sector to analyse the role of energy in the functioning of a complex adaptive system and assess the effects of energy shocks on the economic dynamics. The economic system is populated by heterogeneous agents, i.e., households, firms and banks, who take optimal decision rules and interact in decentralized markets characterized by limited information. After calibrating the model on US quarterly macroeconomic data, we investigate the economic and distributional effects of different types of energy shocks, that is an exogenous increase in the price of natural resources such as oil or gas and a decrease in the energy firms' productivity. We find that whereas the two energy shocks entail similar effects at the aggregate level, the distribution of gains and losses across sectors is largely driven by the subsequent impact on the relative energy price, which varies depending on the type of shock. Our results suggest that, in order to design effective measures in response to energy crises, policymakers need to carefully take into account the nature of energy shocks and the resulting distributional effects.

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1. INTRODUCTION

Energy has always been a crucial element in economic development. In recent years, it is assuming an increasingly important role. The fight against climate change (see IPCC 2021 and COP26) has pushed countries to define transition processes towards renewable resources, reducing greenhouse gas emissions and imposing implementation targets, such as net-zero emission (NZE) for 2050 in the European Union (EU). It is widely recognized that the likelihood of a low-carbon transition will crucially depend on the policy makers' ability to spur the economic system toward cost-effective and environmentally sustainable energy sources. However, it is still unclear how long the process will take and what are the major macro-financial risks associated with it. The global decarbonization process has also involved the beginning of the coal phase-out. At the same time, the post-covid recovery has increased energy demand for all countries seeking to restart. Not least, the political crisis between the United States (US) and Russia in Ukraine has further contributed to overheating energy markets and prices. All this has profound repercussions on the level of inflation, which is reaching values not seen for decades. According to the last World Energy outlook (Birol et al., 2021), energy will assume an even-more central role in the lives of households and worldwide economic growth. People's well-being and industrial competitiveness strongly depend on safe, secure and affordable energy. The accessibility and the reliability of energy will become even more critical to all aspects of people's lives (e.g., cooking, heating). For example, the projections on the US energy demand (US Energy Department) forecast a positive trend even if some policies will be adopted to improve energy use efficiency.

In light of this, understanding the major challenges and opportunities resulting from the energy transition requires a modelling framework capable of capturing market imbalances and out-of-equilibrium dynamics that may arise along the transition pathway. According to Hansen et al. (2019), the interactions in the energy sector, influenced by network and governance structures, require a methodological approach that can deal with complexity and non-linear dynamics. As a result, Agent-Based Models (ABM) constitutes a promising approach since they introduce heterogeneous agents whose characteristics and behaviour can enhance the realism of the analysis (Delli Gatti et al., 2011).

An ABM is a computational framework representing the economic system as an evolving environment where autonomous interacting agents behave according to fixed or evolving rules of thumb (Tesfatsion, 2006). Agents may represent individuals, firms, groups of individuals, policymakers or even countries, depending on the problem that is being treated. In the ABM literature, the macroeconomic outcome emerges from the interactions of the heterogeneous agents and does not coincide with the micro-representative agent behaviour (Bonabeau, 2002). Hence, as highlighted by Karimi and Vaez-Zadeh (2021), the ABM framework is an appropriate methodology to investigate the energy transition because it allows studying the (suboptimal) macro-dynamics emerging from locally optimal but uncoordinated micro-interactions. Moreover, agent-based modelling is a useful approach to test policies and better understand the responsiveness of a complex system to small or big changes in its structural parameters or rules of behaviour, allowing for a better description of the system transition (Farmer and Foley, 2009; Helbing, 2012).

Despite this growing interest of the ABM literature in the role of energy in the macroeconomy, little effort has been made to endogenise the energy sector, assuming, on the contrary, an infinitely elastic energy supply and rolling out the possibility of supply shortages and energy market feedback. For this reason, we propose an agent-based model with an endogenous energy sector to improve the analysis of the effects on the real economy of possible energy shortages due to an exogenous increase in the price of natural resources and the forced introduction of less efficient production technology in the energy sector.

The model nests optimal decision rules into an agent-based computational economic framework. Agents are heterogeneous and take decisions derived from profit/utility maximization in a context characterized by limited information and bounded rationality. Further, interactions between agents occur through decentralized matching protocols built to represent real-world markets. The simulated economy is populated by a corporate sector consisting of consumption, capital, and energy firms; a banking sector; a household sector, namely workers and firms' owners (i.e., entrepreneurs and bankers); and a public sector, including a government and a central bank. Moreover, the business sectors have interlocked input demands. Therefore, the energy sector is endogenous to the economic system as it combines labour and capital to generate energy services that are functional to the production of consumption and capital goods. For the sake of realism, we assume that energy firms employ a foreign natural resource (e.g., oil or natural gas) to produce their services and thus control for the presence of exogenous shocks in the global demand for raw materials. Lastly, we calibrate the model on US data to investigate the effects of an increase in the price of natural resources and a reduction in the productivity of the energy sector.

The model presented here includes some novel elements to the energy-related ABM literature. First, since it introduces an endogenous energy sector, it allows to better assess the risks associated with a transition toward less efficient energy production technology and an exogenous increase in the price of natural resources. Second, it introduces a financial accelerator to investigate the direct and indirect effects of corporate defaults (also in the energy sector) on the banking industry. In this way, we can investigate how changes in the net worth of lenders can amplify and propagate fluctuations to the whole system (Van Der Hoog and Dawid, 2019). We expect our framework to be suitable for future policy analysis on energy transition and technological innovation. At the same time, we aim at contributing to the still young and growing ABM literature on energy transition from a macro perspective and provide an additional modelling framework that can be used to address policy-relevant research questions.

The paper is structured as follows: Section 2 reviews the existing literature on energy production in macroeconomic models; Section 3 presents the model; Section 4 illustrates the calibration procedure based on US data showing the capability of the model to reproduce empirical regularities; Section 5 proposes some simulations results exploring the dynamics generated by the model; Section 6 focuses on the energy sector analysing the endogenous response of the aggregate economy to exogenous shocks in energy production. Section 7 concludes and states future lines of research.

2. LITERATURE REVIEW

Given its crucial role in economic development, a large body of literature has addressed issues related to energy economics from different perspectives. For example, Kim and Loungani (1992) and Rotemberg and Woodford (1996) analyse the impact of energy shocks on the business cycle developing a top-down general equilibrium modelling framework. De Miguel and Manzano (2006) and Plante (2014) evaluate the effects of energy policies in a Dynamic Stochastic General Equilibrium (DSGE) model analysing the role of rigidities in the economy. Blazquez et al. (2021) examine the macroeconomic impacts of domestic energy price reforms and renewable energy deployment in the Saudi Arabia economy. In a DSGE model with energy, De Miguel et al. (2009) and Millard (2011) investigate the role of monetary policy on business fluctuations. Lastly, Economides and Xepapadeas (2018) and Niu et al. (2018) analyse fiscal and monetary policies extending the standard DSGE framework with a climate module.

The energy sector has also been incorporated in the standard integrated assessment (IAM) literature since the seminal work of Nordhaus (1994). The author introduces a dynamic integrated model of climate and economy to investigate the impact of policies aimed at controlling climate

change while also accounting for energy services in the production system. Along the same line, Nordhaus and Yang (1996) derive a regional dynamic integrated model of climate and economy to analyse the effects of different national strategies (cooperative and non-cooperative) to address climate change (see Nordhaus and Boyer, 2003, for an extensive overview of the two models).

These models prompted the development of many extensions: for instance, Bosetti et al. (2009) develop the RICE FEEM model to study the effects of endogenous technological change on climate policy. In particular, two different technology inputs (R&D investments and learning by doing) produce two separate environmental outputs (energy-saving and fuel-switching technologies). Other relevant examples in the IAM literature are often identified within extended research initiatives, where the energy sector can assume even very detailed specifications. The World induced Technical Change Hybrid (WITCH) model presents a regionally disaggregated hybrid structure, composed by a top-down neoclassical growth setting and a detailed bottom-up energy sector, framing environmental policies as the outcome of a dynamic game (Bosetti et al., 2006). The International Institute for Applied Systems Analysis (IIASA) developed a modelling framework for the global economy and its main sectors, combining both macroeconomic equilibrium and system engineering modelling tools (Riahi et al., 2007). In particular, the energy supply relies on the Model for Energy Supply Strategy Alternatives and their General Environmental Impact (MESSAGE). This framework features a dynamic linear programming least-cost approach, considering maximum build-up rate, resources production limits and interregional energy trade to develop and assess multi-sector mitigation strategies (Schrattenholzer, 1981). Another example is The Integrated MARKAL-EFOM System (TIMES), which develops a technology-rich model generator for detailed energy systems (Loulou and Labriet, 2008). This model supplies energy services to end-use demands at the minimum cost, based on equipment investment, operations, primary energy supply, and endogenous energy trade.

An additional prominent methodology employed by climate economists are Computable General Equilibrium (CGE) models (Howard and Sterner, 2017).¹ CGE models depict the economy as a set of structural equations governing input-output linkages, with households, firms and a government (and often a “rest of the world” for trade models) as representative agents. As their name suggests, prices and quantities result from general market equilibrium in all sectors. Due to the multi-sectoral nature of this class of models, the energy one is often included among them. A well-known model is the Global Trade Analysis Project (GTAP), that since 1992 spanned a wide community effort and different iterations and extensions (Hertel, 2013; Corong et al., 2017). The GTAP-E is an example of an energy-environmental version of the latter (Burniaux and Truong, 2002). Other relevant examples are the Inter-temporal Computable Equilibrium System (ICES) model (Bosello et al., 2012) and the Environmental Impact and Sustainability Applied General Equilibrium Model (ENVISAGE) (Roson and Van der Mensbrugghe, 2012). The former introduces a power sector divided into renewable and traditional fossil fuel sources. At the same time, the latter includes a fuel bundle comprising electricity, coal, gas, and oil derived from the GTAP database. An example of an energy-related exercise conducted in a CGE framework is the study of Allan et al. (2007), which investigate the effects of a 5% improvement in the energy efficiency of all production sectors in the United Kingdom (UK), identifying rebounds but no backfire.

Nevertheless, all those methodologies show some limitations, often derived from the assumption of full rationality of its agents (Farmer et al., 2015; Fagiolo and Roventini, 2017; Bardazzi and Bosello, 2021). These include underplaying the role of imperfect information, omitting the transition costs, the presence of fixed prices and inputs, the reliance on a large number of calibrated data and parameters, and being usually limited at a country-yearly scale. While several works attempted to solve these limitations within the methodology (e.g., Howard and Sterner, 2017, for CGE models),

¹Other approaches include Structural Change Models (SCM) and Ecological Macroeconomics models in the Keynesian tradition (EMK), (Ciarli and Savona, 2019).

the need to complement the existing literature with new paradigms such as agent-based modelling is still felt (Farmer et al., 2015).

As a result, a growing macroeconomic ABM literature on climate change and energy transition has been developed in recent years (Balint et al., 2017; Castro et al., 2020). For example, Dafermos et al. (2017) develop a stock-flow-fund ecological macroeconomic model to investigate the trajectories of key environmental, macroeconomic and financial variables under different types of green finance policies, whereas Chappin et al. (2017) present an ABM approach to simulate the climate and energy policies in the European Union (EU). Ponta et al. (2018) and Botte et al. (2021) introduce comparable agent-based, stock-flow consistent macroeconomic models addressing the economic, distributional, and financial stability implications associated with a sustainable transition in the energy sector. Lastly, Lamperti et al. (2018) extend the analysis of Dosi et al. (2010, 2013, 2015) integrating the macroeconomic system with energy and climate change to show how the magnitude and the uncertainty of climate damages can increase over time once taking into account agents interactions and market feedbacks.

3. MODEL

3.1. Overview

The economy is populated by households $h = 1, \dots, n^H$, firms $f = 1, \dots, n^F$, banks $b = 1, \dots, n^B$, the government and a central bank. Figure 1 provides a visual representation of the model setup.

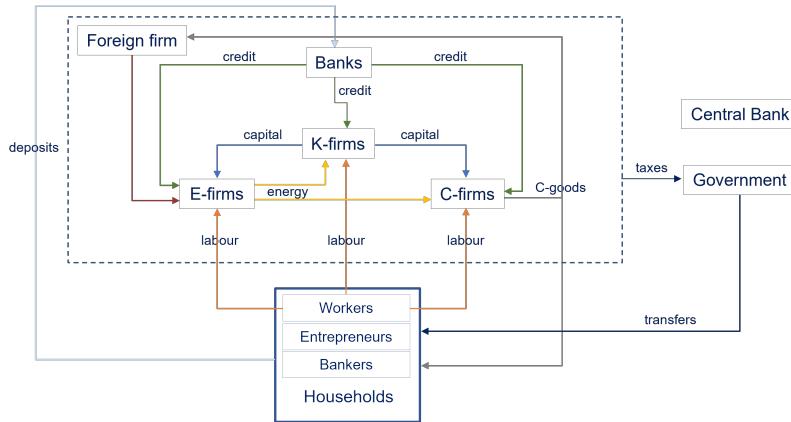


Figure 1: The model setup

- *Household sector*: households are divided between workers, entrepreneurs and bankers. The former supply labour in exchange for a wage, while entrepreneurs and bankers own firms and banks and get dividends. They consume final goods from C-firms and accumulate wealth in the form of deposits at banks.
- *Corporate sector*: firms are distributed across private sectors representing energy (E), consumption (C) and capital (K) goods. The production is carried out by means of a constant elasticity of substitution (CES) technology with constant returns to scale and diminishing marginal returns. Since firms must pay for production inputs (capital, energy and labour) upfront, if internal resources (deposits) are not sufficient to cover production costs, they borrow from banks.

- *Banking sector*: banks receive deposits from households and provide credit to firms at a rate depending on borrowers' financial fragility (i.e., the leverage ratio). The maximum amount of outstanding credit depends on the net worth of banks and financial regulation.
- *Public sector*: the government collects taxes from agents' income and distributes transfers to low-income households. If public spending is greater than tax revenues, the government runs a budget deficit by issuing additional bonds. The central bank adjusts the policy rate following an inertial Taylor rule.
- *Foreign sector*: energy firms purchase an energy input from abroad, say a fossil fuel such as oil or natural gas, which is then converted into energy services by means of labour and capital. To preserve the stock-flow consistency of the model, proceeds deriving from sales of the foreign energy input are redistributed to domestic households in the form of dividends according to the share of individual deposits.

3.2. Sequence of events

In line with the agent-based approach, the economic system in our model evolves according to a recursive and iterative process by which agents interact with each other following a given sequence of steps in each time period:

1. At the beginning of each time period, firms are endowed with a predetermined level of capital, debt and deposits. Moreover, firms have already set their desired level of production, selling prices and inputs demand.²
2. Workers inelastically supply up to one unit of labour in exchange for a salary, while firms hire and fire workers to meet their labour demands. The wage bill is paid upfront and is uniform across firms. Workers pay income taxes on wages and update their consumption budget.
3. Firms produce output (energy, capital, consumption goods) and sell it in the related markets. Market transaction are carried out sequentially:
 - i. E-firms purchase the energy input from the foreign sector;
 - ii. E-firms produce and sell energy services to C- and K-firms by using the foreign energy input, labour and capital inherited from the past;
 - iii. C-firms combine labour, energy and capital stock to produce and sell the final goods to households.
 - iv. K-firms supply capital goods commissioned from C- and E-firms for the subsequent period by producing them through labour and energy.
4. Firms compute profits and pay taxes to the government, instalment on outstanding loans, and dividends to the owners.
5. If the net worth of a firm turns negative or it becomes illiquid (and the owner has not sufficient resources to cover the liquidity shortage), the firm declares bankruptcy and banks account for the non-performing loans. The owner initializes a new enterprise by purchasing the assets of the defaulted firm using her private wealth and, if not sufficient, a new bank credit.

²At the time $t = 0$, the system is initialized at the perfect competition steady state solution as shown in the Appendix A.3.

6. After computing profits, banks pay taxes to the government and dividends to the owners. If the non-performing loans result in a bank default, the bank compensates the losses by recurring to the banker's private wealth. If this is not sufficient, the Central Bank bails failed institutions through a variation in the monetary base.
7. Firms update their desired price-quantity strategy for the subsequent period and calculate conditional input demands accordingly.
8. If the expected cost of inputs exceeds available liquid resources (i.e., the deposits), firms resort to the credit market to cover the financing gap.
9. At the end of the period, the government updates its fiscal position and adjusts the level of tax rate and transfers to the households to comply with a target debt-to-GDP ratio. The central bank sets the policy rate following an inertial Taylor rule.

3.3. Matching protocol

One of the key elements of ABMs is the presence of decentralized markets (Chen, 2016). By decentralization, we mean that agents undertake individual transactions without a central exchange guaranteeing market clearance at a single price. Accordingly, demand (i.e., buyers) meets supply (i.e., sellers) through a predefined matching protocol in our model. In what follows, we describe the mechanism by which agents interact in decentralized credit, goods, and labour markets.

At every time-step of the simulation,³ when a market opens, all buyers enter the market in a random order. Since agents have limited information on the market, each buyer is associated with a list of $[\mathcal{Z}]$ potential sellers chosen at random (plus the selling partner from the previous period) and can observe each new seller with a probability $p = \mathcal{Z} / [\mathcal{Z}]$. If a new seller is observed, the buyer compares the available quantity she can purchase from her current partner with the new one, given her consumption budget and the sellers' supply. Should the latter be greater, the buyer switches to the new seller, otherwise she stays with her current partner. Once the partner is chosen, the bought quantity is given by the minimum between the buyer's budget and the seller's supply. A pseudo-code representation is provided in Algorithm 1.

The matching protocol above is a general description of the interactions within each market. However, there are some market-specific peculiarities that have to be taken into account:

- In the credit market, buyers are firms, and sellers are banks. A firm switches to a new bank if it offers a lower interest rate and fully satisfies its credit demand.
- In the goods market, buyers are households, and sellers are C-firms. In the capital and the energy markets, buyers are firms demanding capital or energy, and sellers are producers. Buyers switch to a new partner if it supplies a higher quantity given the buyers' budget constraints.
- In the labour market, buyers are workers, and sellers are firms. Unemployed and part-time workers switches to a new firm if it offers a higher number of working hours.

³When the system is initialized at the time $t = 0$, each buyer is randomly matched to one seller.

Algorithm 1: Matching algorithm

Data: buyers' j nominal demand $P_j Q_j^d$ and existing partner \bar{z}_j ; sellers' z supply price P_z and quantity Q_z

for a randomly chosen buyer j **do**

make a list with $[\mathcal{Z}]$ new sellers

foreach $z = 1, \dots, [\mathcal{Z}]$ **do**

if random number $< \mathcal{Z}/[\mathcal{Z}]$ **then**

if $\min(P_b Q_b^d / P_z, Q_z) > \min(P_b Q_b^d / P_{\bar{z}_j}, Q_{\bar{z}_j})$ **then**

switch to the new partner z and set $\bar{z}_j = z$

j buys from \bar{z}_j

3.4. Households

3.4.1 Consumption

There are three types of households ($h = 1, \dots, n^H$): entrepreneurs (n^F), bankers (n^B), and workers (n^W). Households buy consumption goods and save in the form of bank deposits. At each time t , they set a consumption budget $C_{h,t}$, visit a subset $\mathcal{Z}_{h,t}^C$ of C-firms chosen at random, and purchase goods from the firm offering the largest amount, given the buyer's budget and the seller's price and quantity. If the amount of goods sold by firms is lower than the consumption budget, consumers accumulate involuntarily savings.

The nominal income $Y_{h,t}$ of consumer h at the time t depends on her type:

$$Y_{h,t} = \begin{cases} W_t N_{w,t} & \text{for workers} \\ DIV_{f,t} - REC_{f,t} & \text{for entrepreneurs} \\ DIV_{b,t} - REC_{b,t} & \text{for bankers} \end{cases} \quad (1)$$

where W_t is the wage rate, $N_{w,t}$ is labour supplied by worker w , $DIV_{f,t}$ and $DIV_{b,t}$ are dividends distributed by firms and banks', and $REC_{f,t}$ and $REC_{b,t}$ are the costs born by entrepreneurs f and bankers b for recapitalization.

As we describe in section 3.8, the government collects taxes on personal income and distributes social transfers to poor individuals. As such, the net income $Y_{h,t}^{net}$ is given by the minimum between after-tax income and the social transfer received by the households:

$$Y_{h,t}^{net} = \max [(1 - \tau_t^{tax}) Y_{h,t}, TRA_{h,t}] . \quad (2)$$

Following Assenza et al. (2015), to determine their consumption demand, households seek to estimate a measure of permanent income $\bar{Y}_{h,t}$, i.e., a proxy for future expected nominal income, by means of a learning adaptive rule, that is:

$$\bar{Y}_{h,t} = (1 - \xi) Y_{h,t}^{net} + \xi (1 + \mathbb{E}_{h,t}[g_{C,t}]) \bar{Y}_{h,t-1} \quad (3)$$

where $\xi \in (0, 1)$ is a memory parameter, and $\mathbb{E}_{h,t}[g_{C,t}]$ is the expected inflation rate on consumption goods by the household h ($\mathbb{E}_{h,t}[g_{C,t}] = \mathbb{E}_{h,t}[P_{C,t}/P_{C,t-1}] - 1$).⁴ From equation (3), the level of

⁴The price $P_{C,t}$ is expected at this stage since consumers do not know the actual price they will find in the market. Here we assume that households take the past growth rate of wages as a proxy for predicting current inflation (see Section 3.5.6).

permanent income is given by a weighted-average of current and past income levels, updated by observed inflation, with geometrically decaying weights.

The consumption budget expressed in nominal terms is defined as a linear combination between permanent income and a fraction of accumulated financial wealth (i.e., deposits $D_{h,t}$):

$$C_{h,t} = \mathbb{E}_{h,t}[P_{C,t}]C_{h,t}^d = \bar{Y}_{h,t} + \chi D_{h,t} \quad (4)$$

where $\chi \in (0, 1)$ is the marginal propensity to consume out of financial wealth.⁵ The difference between income and consumption represents household's savings which pile up to existing deposits:

$$D_{h,t} = D_{h,t-1} + Y_{h,t}^{net} - C_{h,t}. \quad (5)$$

3.5. Firms

3.5.1 Price and Quantity Adjustment

Because agents have limited information and can only visit a subset of the market, firms do not know their demand function in advance. Under these conditions, there is no a unique price-quantity $\{P, Q\}$ combination that guarantees the market equilibrium. Instead, each firm has certain degree of monopolistic power on its own local market and seeks to improve its current position by adopting a $\{P, Q\}$ strategy that allows to achieve higher profits.

To determine the desired combination $\{P, Q\}$, we assume that firms explore a space of $\{P, Q\}$ strategies by imitating other competitors with a similar size and adopt the most successful strategy, that is the $\{P, Q\}$ combination delivering the highest pay-offs. This mechanism can be defined as an *evolutionary algorithm* of price/quantity setting with *strategic complementarities*.⁶

First of all, the distance $d_{s,f,t}$ between firm f from all other firms $s \neq f \in \mathcal{N}^F$ belonging to the same sector $F = \{E, C, K\}$ is computed. The distance is measured as the difference between firms' relative positions in the firm size distribution in terms of nominal production, namely:

$$d_{f,s,t} = |\hat{y}_{f,t} - \hat{y}_{s,t}| \quad (6)$$

where $\hat{y}_{f,t} \equiv \frac{P_{f,t}Q_{f,t} - \min(P_{f,t}Q_{f,t})}{\max(P_{f,t}Q_{f,t}) - \min(P_{f,t}Q_{f,t})}$.

The best competitor to target is obtained by weighted sampling from the list of all firms but f , with weights corresponding to the probabilities in (7):

$$Pr_{f,s,t} = \frac{\exp(-\omega d_{f,s,t})}{\sum_{s \neq f \in \mathcal{N}^F} \exp(-\omega d_{f,s,t})}. \quad (7)$$

In words, the probability for firm f of observing the target competitor s is a decreasing function of their distance $d_{f,s,t}$. The parameter $\omega > 0$ represents the intensity of choice, i.e., how fast firms target the closest competitor in the distribution.

Finally, once the target competitor has been identified, the firm f updates its price and quantity for the subsequent period $\{Q_{f,t+1}, P_{f,t+1}\}$ as described below:

$$Q_{f,t+1} = \begin{cases} \zeta^Q Q_{f,t} + (1 - \zeta^Q) Q_{s,t} & \text{if } \Pi_{s,t} \geq \Pi_{f,t} \\ \zeta^Q Q_{f,t} + (1 - \zeta^Q) \hat{Q}_{f,t} & \text{otherwise} \end{cases} \quad (8)$$

⁵The formulation of consumption budget in equation (4) captures in a reduced form the optimal solution of an utility maximization problem with log-utility function, as discussed in Appendix A.1.

⁶Evidence about the role of strategic complementarities in price setting is reported by Pitschner (2020) for the US, Koga et al. (2020) for Japan, Amiti et al. (2019) for Belgium, and Keeney et al. (2010) for Ireland.

$$P_{f,t+1} = \begin{cases} \zeta^P P_{f,t} + (1 - \zeta^P) P_{s,t} & \text{if } \Pi_{s,t} \geq \Pi_{f,t} \\ \zeta^P P_{f,t} + (1 - \zeta^P) \hat{P}_{f,t} & \text{otherwise} \end{cases} \quad (9)$$

Equation (8) and (9) state that if profits realized by the targeted firm s are higher than f 's, price and quantity are smoothly adjusted towards the target's values, with ζ^Q and ζ^P being the speeds of adjustment. Otherwise, the firm f updates its strategy, whereby \hat{Q} is the observed demand in the previous period, and $\hat{P} = P \cdot [1 + U(-\gamma^P, \gamma^P)]$ is a neighbourhood of its current price, with $U(-\gamma^P, \gamma^P)$ being a random drawn from a uniform distribution on $[-\gamma^P, \gamma^P]$. In other words, firms seek to maximize profits by locally searching for the most successful competitors and mimicking their $\{P, Q\}$ strategy if more profitable, or by exploring a neighbourhood of their current strategy. Finally, all the prices are revised to account for the expected increase in marginal costs, which is approximated by wage inflation (see later), so that firms strive to maintain the mark-up unchanged.

3.5.2 Production technology

We assume that firms use a Constant Elasticity of Substitution (CES) technology (Arrow et al., 1961) with constant returns to scale and diminishing marginal returns. While the Leontief and the Cobb-Douglas production functions are a common standard in the ABM literature (see, among others, Cincotti et al., 2010; Dosi et al., 2010; Assenza et al., 2015; Caiani et al., 2016; Dawid et al., 2019), we deviate from these hypotheses for two main reasons. First, the implicit elasticity of substitution in the Leontief and the Cobb-Douglas cases is at odds with empirical evidence. In particular, empirical studies find neither perfect complementary (i.e., the Leontief) nor unitary elasticity of substitution (the Cobb-Douglas) between labour and capital (Duffy and Papageorgiou, 2000; Gechert et al., 2021). Second, the CES production function reduces to the Leontief and the Cobb-Douglas cases under specific assumptions.⁷ Accordingly, the adoption of a CES technology allows us to increase the flexibility of the model and improve its empirical validation.

Overall, the production function of a firm f at the time t is defined as follows:

$$Q_{f,t} = Z_{f,t} \left[\sum_{j=1}^n A_{j,f} (X_{j,f,t})^{\rho_f} \right]^{\frac{1}{\rho_f}} \quad (10)$$

where $X_{j,f,t}$ is the quantity of input $j = 1, \dots, n$ in the firm f at the time t , $\sum_{j=1}^n A_{j,f} = 1$ are the factor shares, $\rho_f = \frac{\sigma_f - 1}{\sigma_f}$ is the substitution parameter, σ_f is the Hicks elasticity of substitution, and $Z_{f,t} = 1$ is total factor productivity.⁸

As stated earlier, we assume three different sectors in the economy, namely energy services, consumption and capital goods. All firms in each sector have the same production technology, require the same inputs, and have the same values for the factor shares and the elasticity of substitution. In particular, E-firms employ capital, labour, and a foreign energy input (e.g., oil or natural gas, see section 3.5.3) to produce, while K-firms require energy services and workers to supply capital. Lastly, C-firms combine energy services, labour, and physical capital to produce final consumption goods. Table 1 summarizes the calibrated values of these parameters.

⁷It is worth recalling that the CES production function reduces to a Cobb-Douglas for $\sigma_f = 1$, a Leontief function when $\sigma_f = 0$, or a linear function for $\sigma_f = +\infty$.

⁸We normalize total factor productivity equal to one for the sake of simplicity. We test the effects of a temporary reduction in its value for the energy sector in Section 6.

3.5.3 Foreign energy input

As stated before, a stylized foreign sector acts as provider of an exogenous natural resource (i.e., a fossil fuel like oil or natural gas) for the energy sector, which combines it with labour and capital to generate the energy services sold to consumption and capital good firms. Following Ponta et al. (2018), we assume there is a representative foreign firm that offers the energy input at a fixed price⁹ and infinite supply. Therefore, this firm can always satisfy the demand for energy input from E-firms. In order to close to model, all proceeds coming from the sales of foreign input are channelled back into the economy in the form of dividend payment to domestic households according to their share of personal wealth.

3.5.4 Conditional input demand

Given the desired quantity for the subsequent period $Q_{f,t+1}$ and the set of expected input prices $\{\mathbb{E}_{f,t}[P_{j,t+1}]\}_{j=1}^n$, each firm f sets the conditional input demands that minimize the expected direct costs¹⁰ subject to production technology and irreversible investments. The cost minimization problem for the firm thus reads:

$$\min_{\{X_{f,j,t+1}; \Delta X_{f,j,t+1}\}_{j=1}^n} \mathbb{E}_{f,t} [DC_{f,t+1}] = \sum_{j=1}^n \mathbb{E}_{f,t} [P_{j,t+1}] \Delta X_{j,f,t+1} \quad (11)$$

$$\text{s.t. } Q_{f,t+1} = \left[\sum_{j=1}^n A_{j,f,t+1} (X_{j,f,t+1})^{\rho_f} \right]^{\frac{1}{\rho_f}} \quad (12)$$

$$X_{j,f,t+1} = \Delta X_{j,f,t+1} + (1 - \delta_j) X_{j,f,t} \quad (13)$$

$$\Delta X_{j,f,t+1} \geq 0 \text{ when } j \text{ indicates physical capital} \quad (14)$$

where $\Delta X_{j,f,t+1}$ and δ_j are the additional demand and the depreciation rate¹¹ of input j , respectively. The demand for additional input obtained by solving the problem above (see Appendix A.2) are determined by:

$$\Delta X_{j,f,t+1}^d = \left(\frac{A_{j,f,t+1} \psi_{f,t+1}}{\mathbb{E}_{f,t}[P_{j,t+1}]} \right)^{\sigma_f} Q_{f,t+1} - (1 - \delta_j) X_{j,f,t} \quad \forall j = 1, \dots, n \quad (15)$$

where

$$\psi_{f,t+1} = \left[\sum_{j=1}^n (\mathbb{E}_{f,t}[P_{j,t+1}])^{1-\sigma_f} (A_{j,f,t+1})^{\sigma_f} \right]^{\frac{1}{1-\sigma_f}} \quad (16)$$

are the expected marginal costs.

⁹We set this value at the beginning of the simulation such that the aggregate expenditure for foreign input over GDP is equal to the average oil expenditure over GDP in the US between 1973 and 2019, namely 3%. Source: U.S. Energy Information Administration.

¹⁰The costs are expected at this stage as firms do not know the actual quantity they will find in the markets. Here we assume that the current prices of inputs, updated for inflation, are taken as a reference for the expected values.

¹¹We assume that all inputs fully depreciates from the previous period, except for physical capital. In other words, firms can use energy inputs and hours worked only in the current period and must repurchase them to produce new goods in the future.

3.5.5 Credit demand

Since firms pay for production factors in advance, if the expected direct costs of production exceed the total amount of available liquid resources (i.e., the deposits $D_{f,t}$), firms ask for a bank loan to cover the financing gap. Hence, the additional credit demand is given by:

$$\Delta L_{f,t+1}^d = \mathbb{E}_{f,t} [DC_{f,t+1}] - D_{f,t}. \quad (17)$$

Firms try to secure bank credit in a decentralized market (see Section 3.3) and repay principal at a fixed instalment rate θ^B . Accordingly, firms do not necessarily have to pay off outstanding debt before they can access to new credit but, if need be, can refinance loans at the current interest rate at every time unit. As a result, firms' debt and deposit update following the rule:

$$L_{f,t+1} = \Delta L_{f,t+1} + (1 - \theta^B)L_{f,t} \quad (18)$$

$$D_{f,t} = \Delta L_{f,t+1} + D_{f,t} \quad (19)$$

where $L_{f,t}$ is total debt, and $0 \leq \Delta L_{f,t+1} \leq \Delta L_{f,t+1}^d$ is the new credit line. If liquid resources are still insufficient to purchase the production factors, (i.e., $D_{f,t} < \mathbb{E}_{f,t} [DC_{f,t+1}]$), firms reduce proportionally their input demands:

$$\Delta \bar{X}_{j,f,t+1}^d = v_{f,t+1} \Delta X_{j,f,t+1}^d - (1 - v_{f,t+1})(1 - \delta_j) X_{j,f,t} \quad \forall j = 1, \dots, n \quad (20)$$

where

$$v_{f,t+1} = \frac{D_{f,t} + \sum_{j=1}^n \mathbb{E}_{f,t} [P_{j,t+1}] (1 - \delta_j) X_{j,f,t}}{\psi_{f,t+1} Q_{f,t+1}} \quad (21)$$

is the ratio between the available liquid resources and the expected total costs.

3.5.6 Labour market and wage setting

In the labour market, workers inelastically supply up to one unit of labour at the prevailing wage rate, while firms can hire and fire workers at no costs based on their labour requirements resulting from the cost minimization problem. Since workers can only visit a subset of firms, search frictions may prevent the adjustment of labour to market equilibrium resulting in unfulfilled demand.¹² As a result, there is no a market-clearing wage rate. Instead, at the beginning of each period, when the labour market closes, the wage rate is adjusted through an insider-outsider bargaining process.

In particular, given the labour supply, N^s , i.e., the total number of workers, n^W , the aggregate labour demand as obtained by the sum of individual firms' optimal labour demand, $N_t^d = \sum N_{f,t}^d$, the current amount of hours worked, N_t^{emp} and the current wage W_t , the wage rate for the subsequent period is determined by:

$$W_{t+1} = (1 + g_{W,t+1})W_t = \left[\theta^W \left(\frac{N_t^d}{N_t^{emp}} \right) + (1 - \theta^W) \left(\frac{N_t^{emp}}{N^s} \right) \right] W_t \quad (22)$$

where θ^W represents the insider-outsider bargaining power. Equation (22) states that the higher the labour demand with respect to current employment, the faster the wage growth, in line with the insiders' interests; on the other hand, the higher the labour supply with respect to current employment (i.e., the higher the unemployment rate), the lower the wage growth, following the outsiders' objective of full occupation.

¹²For details about the interaction of firms and workers on the labour market the reader is addressed to the description of the matching protocol in Section 3.3.

3.5.7 Profits, net worth and bankruptcy

Once the respective market closes, the selling firms compute profits as revenues net of direct costs for employed inputs and debt service. The equation of profits and net worth accumulation are defined as:

$$\Pi_{f,t}^{net} = (1 - \tau_t^{tax}) \left(P_{f,t} Q_{f,t} - DC_{f,t} - \sum_{b=1}^{n^B} i_{f,b,t} L_{f,b,t} \right) \quad (23)$$

$$NW_{f,t} = NW_{f,t-1} + \Pi_{f,t}^{net} \quad (24)$$

with τ_t^{tax} being the tax rate on profits. If solvent, firms pay out a fraction of equity to the owner in the form of dividends, namely:

$$DIV_{f,t} = \max \left[0, \min \left(\mu^F NW_{f,t}, D_{f,t} \right) \right] \quad (25)$$

where $\mu^F = 1 - \beta^F$ is the payout ratio of firms.

A firm might default when it becomes either insolvent (i.e., the net worth turns negative, $NW_{f,t} < 0$) or illiquid (i.e., the deposits are negative, $D_{f,t} < 0$). If the firm is illiquid but the owner has sufficient resources to cover the liquidity shortage (i.e., $D_{h,t} > |D_{f,t}|$), the firm is rescued by the entrepreneur without entering the default procedure. In the other cases, the firm is declared bankrupt and, to keep the number of agents constant, is replaced by a new entrant, whose initial conditions are set in the following way. First, the residual deposits of the bankrupt firm are used to partly repay the outstanding debt. Second, the entrepreneur seeks to buy out the assets of the old firm at their historical cost by using her personal wealth and, if not sufficient, by asking for a new bank loan in the credit market. Third, the lending bank records a loss equal to the difference between the funds raised by the entrepreneur and the remaining value of the debt. Overall, the balance sheet of the new firm will have, on the asset side, the purchase value of the assets that the entrepreneur managed to buy out from the old firm, and, on the liability side, the value of the new loan if present.

3.6. Banks

In real economies, the banking system exploits its unique informative advantage to collect information about borrowers and solve an asymmetric information problem when issuing new credit. For the sake of simplicity, we assume a stylized pricing mechanism for the banking sector in our model such that banks supply credit to the corporate sector and make profits by charging differentiated interest rates on loans. Moreover, a fraction θ^B on the principal plus interests on outstanding loans are repaid in each period. Lastly, banks hold firms and households' deposits and buy the T-bills issued by the government in proportion to their net worth.

3.6.1 Interest rate

The interest rate charged by bank b to the borrowing firm f is given by:

$$i_{b,f,t} = i_t^{CB} + \varrho^B \left(1 - \frac{NW_{b,t}}{\max_{s=1,\dots,n^B} NW_{s,t}} \right) + \rho^B \frac{L_{f,t}}{NW_{f,t}} + \iota^B \frac{NPL_{t-1}}{L_{t-1}} \quad (26)$$

where $\varrho^B, \rho^B, \iota^B > 0$ are interest rate-related parameters. Equation (26) states that the interest rate on loans consists of four components: (i) the policy rate set by the central bank; (ii) a bank-specific

financial soundness component, which is inversely proportional to the bank's net worth, reflecting the role of economies of scale; (iii) a firm-specific risk component, which is a function of the borrower's leverage; (iv) a systemic risk component, which is a function of the ratio of aggregate non-performing loans on the total amount of credit extended by the banking sector in the previous period.

The interest rate mechanism is in line with the external finance premium hypothesized by Bernanke et al. (1999) but adds some important features. First, the firm-specific interest rate on loans is a mark-up on the risk free interest rate and depends on the firms' financial fragility, as in De Grauwe and Macchiarelli (2015), Assenza et al. (2015) and Delli Gatti and Desiderio (2015).¹³ Second, it introduces the financial soundness of banks in their own credit policy definition: banks with higher financial strength (i.e., higher equity ratio) extend credit at more favourable terms (Delli Gatti et al., 2010). Moreover, the cost of borrowing depends on the financial soundness of the banking sector as a whole. As the share of non-performing loans goes up, credit becomes more costly for all borrowers because banks become more reluctant to lend. Finally, equation (26) introduces a feedback between macro and micro dynamics because the monetary policy spreads on the individual cost of credit through the prevailing risk-free policy rate, i_t^{CB} .

3.6.2 Prudential regulation

Following the Basel III framework, the credit supply is subjected to the compliance with macro-prudential requirements set by the regulatory authority. The maximum level of credit that banks are allowed to supply (L^{max}) is constrained by a minimum capital requirement such that the amount of risk-weighted assets cannot exceed a fraction of equity capital:

$$L_{b,t}^{max} = \frac{1}{\omega^B \gamma^B} NW_{b,t} \quad (27)$$

where $E_{b,t}$ represents the bank's net worth, $\gamma^B \in (0, 1)$ is the minimum capital adequacy ratio, and ω^B is the risk-weight on lending to firms.¹⁴

Additionally, the maximum exposure to a single counterpart cannot exceed a given fraction of the bank's equity, i.e.:

$$L_{b,f,t}^{max} = \kappa^B NW_{b,t} \quad (28)$$

with $\kappa^B > 0$.

3.6.3 Profits

The b -th bank's net worth evolves as:

$$NW_{b,t} = NW_{b,t-1} + (1 - \tau_t^{tax}) \Pi_{b,t} - DIV_{b,t} \quad (29)$$

where τ_t^{tax} is the tax rate, $\Pi_{b,t}$ are bank profits, and $DIV_{b,t}$ are dividends (see (32)). Profits are defined as:

$$\Pi_{b,t} = \sum_{f=1}^{n^f} i_{b,f,t} L_{b,f,t} - \sum_{f \in \mathcal{D}^F} NPL_{b,f,t} \quad (30)$$

where the first term on the right hand side is the sum of returns on solvent firms' loans, while the second represents the total value of non-performing loans, with \mathcal{D}^F being the set of bankrupted firms.

¹³We show this result in the Appendix A.4.

¹⁴The underlying assumption is that the risk-weight on cash is zero.

3.6.4 Bankruptcies, recapitalization, and deposit guarantee scheme

Each bank b must respect the following constraint so that equity ($NW_{b,t}$) covers at least a fraction $1/\nu^B$ of its deposits:

$$\nu^B NW_{b,t} \geq D_{b,t}. \quad (31)$$

If condition (31) is satisfied, the equity in excess is paid to banks' owners as dividends, that is:

$$DIV_{b,t} = \max(0, \nu^B NW_{b,t} - D_{b,t}). \quad (32)$$

When a bank's net worth becomes negative, the bank is counted as bankrupt. This model assumes that the banking sector participates in a Deposit Guarantee Scheme (DGS) by which it commits to protect households and firms from losses on deposits. Moreover, the DGS works as a safety net to avoid bank runs following the illiquidity of financial institutions.¹⁵

The recapitalization and default process follow these steps. First, the bankers try to employ their wealth to recapitalize the defaulted banks. Each bank b evaluates the needed additional equity:

$$\Delta_{b,t} = \frac{L_{b,t}}{\nu^B} - NW_{b,t} \quad (33)$$

with ν^B being the capital requirements. If the required capital $\Delta_{b,t}$ is less than the banker's own deposits $D_{b,t}^B$ the net worth is updated as follows:

$$NW_{b,t} = NW_{b,t} + \Delta NW_{b,t} \quad (34)$$

with $\Delta NW_{b,t}$ selected from:

$$\Delta NW_{b,t} = \min \left[D_{b,t}^B, \max (\Delta NW D_{b,t}, \Delta_{b,t}) \right] \quad (35)$$

where $\Delta NW D_{b,t}$, the delta between the bank's net worth and own deposits, is defined as:

$$\Delta NW D_{b,t} = \frac{D_{b,t}^B - \nu^B NW_{b,t}}{1 + \nu^B}. \quad (36)$$

These resources are deducted from the banker's own deposits and income accordingly:

$$D_{b,t}^B = D_{b,t}^B - \Delta NW_{b,t} \quad (37)$$

$$Y_{b,t} = Y_{b,t} - \Delta NW_{b,t} \quad (38)$$

If instead the bank's own deposits are not sufficient, such that $\Delta_{b,t} \geq D_{b,t}^B$, the banker cannot recapitalize. In this case the bank losses are bailed by the Central Bank, through a variation in the monetary base that covers the gap:

$$M_t = M_t + \Delta NW_{b,t} - D_{b,t}^B. \quad (39)$$

Notice that deposits in the bankrupted bank held by workers, firms and entrepreneurs ($D_{b,t}^B$) are set to zero. The needed net worth variation $\Delta NW_{b,t}$ is selected from:

$$\Delta NW_{b,t} = \max (\Delta_{b,t}, \Delta NW D_{b,t}) \quad (40)$$

¹⁵Deposits insurance is in force in Europe since the Directive 2014/49/EU. In the US, almost all banks adhere to the FDIC insurance; the few banks that do not are still covered by state-level guarantees. <https://www.sapling.com/4600324/banks-that-not-fdic-insured>

with

$$\Delta NWD_{b,t} = \frac{D_{b,t}^B - D_{b,t}}{\nu^B} - NW_{b,t}. \quad (41)$$

The banker's net worth and income are then updated as follows:

$$NW_{b,t} = NW_{b,t} + \Delta NWD_{b,t} \quad (42)$$

$$Y_{b,t} = Y_{b,t} - D_{b,t}^B. \quad (43)$$

3.7. Central Bank

The central bank sets the policy rate and re-finances banks when bankers lack the necessary resources. The policy rate i_t^{CB} in equation (44) is set according to an inertial Taylor rule depending on the deviation of inflation and unemployment rates from policy target values, respectively p^* and u^* , given the steady state interest r^* :¹⁶

$$i_t^{CB} = \rho^{CB} i_{t-1}^{CB} + (1 - \rho^{CB}) \max [0, r^* + p^* + \lambda^y(u^* - u_t) + \lambda^p(p_t - p^*)]. \quad (44)$$

To avoid abrupt changes in firms' financing conditions, the central bank follows an interest rate smoothing behaviour, with ρ^{CB} being the speed of adjustment to the original Taylor rule rate (Castelnuovo, 2003).

Furthermore, the central bank intervenes to refinance the banking sector whenever the condition in equation (31) is violated and bankers cannot provide enough capital to comply with regulatory requirements.

3.8. Government

The government collects taxes from firms and households, and distribute transfers to low-income individuals. If public spending is greater than tax revenues, the government runs a budget deficit and issues additional bonds which are bought by the banking sector.¹⁷ In other words, government bonds are issued to fund transfers TRA_t to households, interest payment on the outstanding debt $i_{t-1}^{CB} B_{t-1}$, net of the tax revenue TAX_t . The stock of bonds evolves to accommodate the change in the net position of government

$$B_t = (1 + i_{t-1}^{CB}) B_{t-1} + TRA_t - TAX_t. \quad (45)$$

Tax revenues are the sum of taxes on workers' wages, firms' and banks' profits. At the macroeconomic level, taxes and social transfers are defined as a fraction of current GDP: $TAX_t = \tau_t^{tax} GDP_t$, and $TRA_t = \tau_t^{tra} GDP_t$. Equations (50) and (49) specify the way in which the government dynamically adjusts the level of tax rate and transfer, respectively τ_t^{tax} and τ_t^{tra} . Given the total amount of allocated funds, government distributes transfers to all the individuals earning less than a marginal household \hat{h}_t in the income distribution. The latter is determined such that the amount of social transfer, TRA_t , is completely depleted and everyone earning less than \hat{h}_t reaches her income through a transfer payment.

¹⁶The inflation rate p is defined as the change in consumer price index. The last is built as total revenues by C-firms weighted by sold quantities. The unemployment rate u is the share of unemployed workers to total, where a worker is defined unemployed if he is not hired by any firms.

¹⁷We assume that the interest rate paid on public debt is equal to the risk-free policy rate i_t^{CB} .

Fiscal Sustainability The government is subject to a budget constraint to keep public debt from moving on an explosive path; therefore, debt cannot expand indefinitely to finance interests on outstanding debt. The issue of public finance sustainability is addressed through an inter-temporal budget constraint. Equation (46) describes the dynamics of the debt-to-GDP ratio:

$$b_{t+1} = \frac{1 + i_t^{CB}}{1 + g_t} b_t - f_{t+1} \quad (46)$$

where b_t is debt-to-GDP ratio, $f_t \equiv (TAX_t - TRA_t)/GDP_t$ is the primary budget-to-GDP, i_t^{CB} is the nominal interest rate on public bonds (equal to the monetary policy rate), and g_t is the expected nominal growth rate of GDP.

To achieve fiscal sustainability, the government smoothly adjusts the current debt-to-GDP ratio to a target value b^* at a rate ρ^G , that is

$$b_{t+1} = b_t + \rho^G (b^* - b_t) \quad (47)$$

which, by substituting (47) into (46), implies a primary balance equal to

$$f_t = \frac{1 + i_t^{CB}}{1 + g_t} b_t - [b_t + \rho^G (b^* - b_t)]. \quad (48)$$

The primary balance is revised according to equation (48), where f_{t+1} represents the planned budget for the subsequent period. To reach that value, the government revises the level of tax rate, keeping the share of social transfer over GDP essentially unchanged.¹⁸ In particular, the share of social expenditures, τ_t^{tra} , is anchored to a reference value, ψ^G .¹⁹ This value can be revised only if the government has some fiscal space allotted through its expected primary balance f_{t+1} , such that:

$$\tau_t^{tra} = \max(\psi^G, -f_{t+1}). \quad (49)$$

For a given τ_t^{tra} , the tax rate τ_t^{tax} for the current period is then updated accordingly in order to match the public budget:

$$\tau_t^{tax} = \max(0, f_{t+1} + \tau_t^{tra}). \quad (50)$$

If negative, the tax rate is set to zero, as subsidies are not addressed in this version of the model.

4. VALIDATION

In this section, we illustrate the methodology we follow to calibrate the model on US data and analyze the goodness of fit of simulated output to the empirical data. The model contains 56 parameters, most of which (48) can be obtained from existing data or can be derived from economic theory as a combination of other coefficients. The remaining parameters (8) are not directly observable since they refer to the microeconomic behaviour of agents, which is private information and is not freely available. Accordingly, we set the value of the parameters following two different approaches. On the one hand, we calibrate the coefficients that we can derive from micro and macro data or that have been estimated in other empirical studies. On the other hand, we assign the remaining values to fit the simulated time series to US data.²⁰

¹⁸Notice that to not over-complicate the convergence towards a stable path of the economy, the fiscal rule sets off after some periods (500) in the simulation.

¹⁹This reflects the stable ratio of social expenditures net of pension spending to GDP observed in countries such as the US.

²⁰At the moment, we set those values to have a qualitative replica of the US time series. As a future line of research, we plan to estimate the parameters through indirect inference (see Smith Jr, 1993; Gourieroux et al., 1993).

4.1. Parameters calibration

4.1.1 Workers and discount factors

We assume a reasonably high number of workers ($n^W = 1000$) to ensure sufficient interactions between agents and allow the system to generate endogenous dynamics. We set the discount factor of households ($\beta^C = 0.993$) and firms ($\beta^F = 0.985$) to replicate the average quarterly real interest rates observed on the risk-free T-bill ($E[r_t^{RF}] = 0.7\%$) and the stock market ($E[r_t^E] = 1.5\%$). Indeed, we assume as in standard macroeconomic models the long-term relationships (see Cochrane, 2009):

$$\beta^C(1 + E[r_t^{RF}]) = 1 \quad \text{and} \quad \beta^F(1 + E[r_t^E]) = 1, \quad (51)$$

where the difference $E[r_t^E] - E[r_t^{RF}] = \frac{1}{\beta^F} - \frac{1}{\beta^C}$ captures the equity risk premium. Accordingly, we set the marginal propensity to consume out of wealth ($\chi = 1 - \beta^C$), the memory parameter of the households ($\varepsilon = \beta^C$), and the dividend payout ratios of firms ($\mu^F = 1 - \beta^F$) using the estimated values of those coefficients. Lastly, we assume that employed workers have a greater bargaining power than the unemployed and set the value of this parameter ($\theta^W = 0.75$) to reproduce the dynamics observed in the US economy (see the next section).

4.1.2 Firms

Focusing on the business environment, we set the same elasticity of substitution in consumption, capital, and energy sectors following the empirical analysis of Van der Werf (2008), who finds a value of that parameter close to 0.50 in developed countries ($\sigma_C = \sigma_E = \sigma_K = 0.5$). At the same, we use the ratio between the current-cost depreciation of fixed assets and the current-cost net stock of fixed assets calculated by the US Bureau of Economic Analysis to estimate the annual depreciation rate of capital in the US. Our analysis indicates an average annual depreciation rate of 5% between 1925 and 2020, implying a quarterly rate of 1.25% ($\delta_K = 0.0125$). Lastly, we assume that the remaining production factors (i.e., energy services, hours worked, and foreign natural resource) and consumption goods can be used only in the current period, and firms and households must repurchase them at every time step ($\delta_E = \delta_N = \delta_O = \delta_C = 1$).²¹

We employ the Eurostat annual use tables at basic prices to determine the factor shares of sectoral production functions in our model. The database records the annual nominal value of intermediate goods transactions, labour cost, final consumption expenditure, and gross capital formation in the EU and the US from 1990 to 2019. Since the dataset breaks down the aggregate economy by 65 main activities (according to NACE Rev. 2, Statistical classification of economic activities in the European Community), we group them into consumption or capital sectors depending on whether the total demand of the economic activity is mainly directed toward final consumption expenditure or the purchase of capital and intermediate goods. At the same time, we identify the energy sector with the “Electricity, gas, steam and air conditioning” activity already defined in the original tables. Lastly, we calculate a proxy of the sectoral factor shares by dividing the nominal expenditure on intermediate inputs (i.e., the capital in our framework), labour, and energy by the total sectoral costs (see Table 1). The results of our analysis indicate that both the consumption and the energy sectors are capital intensive, while most of the costs in capital firms depend on labour (see Table 1). As expected, energy represents only a marginal share of total inputs (Kilian, 2008).

Similarly, we use the sectoral classification defined above to group the latest business demography statistics produced by the US Economic Census (arranged in 65 main activities according to NACE

²¹Note that this hypothesis does not alter the existing relationships between firms and employees but implies that the former cannot use the hours worked in the past to produce current goods, that is, time flies away.

Rev. 2) in consumption, capital, and energy enterprises. We do so to assume a realistic proportion of firms in different sectors in our model. As with the workers, we set the number of consumption enterprises ($n^C = 100$) to ensure sufficient interactions between agents and define the rest of the firms' population by multiplying this value by their actual shares in the US economy (see Table 1). As expected, the consumption sector represents the majority of firms, followed by capital ($n^K = 50$) and energy ($n^E = 15$) enterprises, and banks ($n^B = 10$).

Table 1: CES production function parameters

		Consumption (C)	Capital (K)	Energy (E)
Number of firms	n^f	100	50	15
Capital share	$A_{K,f}$	0.60		0.60
Labour share	$A_{N,f}$	0.38	0.98	0.38
Energy share	$A_{E,f}$	0.02	0.02	0.02*
Elasticity of substitution	σ_f	0.50	0.50	0.50

*: foreign input share in energy production.

Lastly, we set the parameters regulating the adjustment process of prices and desired quantities, on the one hand, to avoid excessive volatility in the system ($\zeta^Q = \zeta^P = 0.75$) and, on the other hand, to ensure a sufficient exploration of the current strategy by leading firms ($\gamma^P = 0.15$). At the same time, we assume a relatively high intensity of choice of the enterprises ($\omega = 15$) to model the large differences in firms' production observed in real-world data (Okuyama et al., 1999; Ramsden and Kiss-Haypál, 2000; Axtell, 2001; Gaffeo et al., 2003). Moreover, we select those values as they provide a good fit of our simulated time series to the US data (see the next section).

4.1.3 Credit market

The Basel III international regulatory framework for banks defines a minimum ratio of capital to risk-weighted assets equal to 8% and sets a maximum single exposure limit of 25% of Tier 1 capital. Accordingly, we assign the value of those parameters following the above-mentioned regulation ($\gamma^B = 0.08$; $\kappa^B = 0.25$) and assume a risk weighting equal to one ($\omega^B = 1$). Moreover, we assume an average loan maturity of 20 years, which implies a quarterly repayment rate of 1.25% of the principal ($\theta^B = 0.0125$), in line with the average depreciation rate of physical capital. The remaining parameters of the interest rate rule ($\varrho^B = 0.01/4$; $\rho^B = 0.015/4$; $\iota^B = 0.005/4$) are set in such a way to reproduce the dynamics observed in the US economy (see the next section).

4.1.4 Matching protocols

Consumer markets are traditionally characterized by a low switching probability, with an average annual rate of change between 10% and 20% (Harold et al., 2020). Accordingly, we assume that households can observe a restricted number of new firms ($\mathcal{Z}^C = 0.5$), namely they can search for a new supplier with a 50% probability. Similar reasoning applies to the credit market, which is also characterized by information frictions (Diamond, 1984; Ramakrishnan and Thakor, 1984), relationship lending (Berger and Udell, 1995; Elyasiani and Goldberg, 2004), and credit rationing (Jaffee and Russell, 1976; Stiglitz and Weiss, 1981). As a result, it is reasonable to assume a low switching probability ($\mathcal{Z}^B = 0.5$), in line with the findings of Kim et al. (2003). At the same time, the empirical literature on job search indicates that it is a pervasive phenomenon, with the US employed and unemployed workers submitting an average of 1.06 and 8.50 applications per month, respectively (Faberman et al., 2017). Accordingly, we set a relatively high value for this parameter to capture this feature of the US economy ($\mathcal{Z}^W = 10$). Lastly, the international empirical evidence

shows that firms rely on 3 to 5 suppliers on average (Bernard et al., 2018, 2019). In light of this, we set the number of observable enterprises in the energy and the capital markets equal to $\mathcal{Z}^E = 4$ and $\mathcal{Z}^K = 4$, respectively.

4.1.5 Public sector

As stated in the description of the model, we assume that the central bank adjusts the risk-free interest rate using an inertial Taylor rule (44). We set the quarterly inflation rate target ($p^* = 0.02/4$) following the existing literature (Taylor, 1993; Taylor and Williams, 2010) and define the target unemployment rate equal to the average value observed in the US between 1950 and 2019 ($u^* = 0.06$). Lastly, we rely on the empirical analysis of Rudebusch (2006) to determine the remaining parameters of the rule ($\rho^{CB} = 0.80$; $\lambda^u = 1.3$; $\lambda^p = 1.3$).

We estimate the target debt to GDP ratio and the adjustment coefficient of public finances on a panel of 20 Western countries²² between 1995 and 2019. Given the theoretical assumptions of equation (47), the empirical model is defined as:

$$\Delta b_{c,t} = \beta_0 + \beta_1 b_{c,t-1} + \beta_2 \Delta b_{c,t-1} + e_{c,t} \quad (52)$$

where $\Delta b_{c,t}$ is the variation of the annual debt to GDP ratio in country c at the time t . From this specification we can then calculate the unconditional estimators of the long-term parameters:

$$\rho^g = -\frac{\beta_1}{1 - \beta_2} \quad \text{and} \quad b^* = -\frac{\beta_0}{\beta_1}. \quad (53)$$

Table 2 illustrates the results of our estimation, which indicate approximatively a 80% target debt to GDP ratio ($b^* = 0.80$) and an annual adjustment rate of 4%, namely a one percent quarterly value ($\rho^g = 0.01$). Lastly, we set share of social expenditures equal to the average value observed in the US between 1947 and 2020 ($\psi^G = 0.066$).

Table 2: *Debt to GDP ratio estimation*

Pooled OLS, using 447 observations
 Included 20 cross-sectional units
 Time-series length: minimum 17, maximum 23
 Dependent variable: $\Delta b_{c,t}$
 Robust (HAC) standard errors

	Coefficient	Std. Error	t-ratio	p-value
β_0	0.014	0.005	2.860	0.010
β_1	-0.017	0.009	-1.819	0.085
β_2	0.565	0.085	6.624	0.000
Mean dependent var	0.0083	S.D. dependent var	0.0550	
Sum squared resid	0.9327	S.E. of regression	0.0458	
R^2	0.3097	Adjusted R^2	0.3065	
$F(2, 19)$	61.243	P-value(F)	5.20e-09	
$\hat{\rho}$	-0.0598	Durbin-Watson	2.0734	

²²The countries included are: Belgium, Czech Republic, Denmark, Germany, Ireland, Greece, Spain, France, Italy, Cyprus, Luxembourg, Malta, Netherlands, Austria, Portugal, Slovenia, Slovakia, Finland, United Kingdom, and United States.

4.2. Goodness of fit of the model

We conclude this section by analysing the goodness of fit of the model to US data. The time series that we include in the analysis are the logarithm of real gross domestic product (GDP), aggregate investments, consumer (CPI) and energy (EPI) price indexes, the unemployment rate, and the policy interest rate. We collect the US data from the FRED database on a quarterly frequency between 1957-Q1 to 2019-Q4. At the same time, we simulate 1,000 independent replicas of the model with different random seeds to calculate the empirical moments of the distributions. Lastly, since our theoretical framework does not include any long-term technological advancement, we focus on the cyclical behaviour of the time series by filtering them through the Hodrick-Prescott filter.

The standard deviations of the simulated time series are in line with the observed data (table 3). In particular, real investments show higher volatility than the aggregate production, while the magnitude of fluctuations in the remaining time series is less pronounced. However, the model appears to overstate the standard deviation of investments and underestimate the volatility of energy prices. In this sense, further investigations are required, in particular for the price of energy, which depends not only on the endogenous fluctuations of local demand but also on global factors (Baumeister and Kilian, 2016).

Table 3: Standard deviation of filtered time series

	US data	Simulations	
		Mean	90% C.I.
Real GDP	0.015	0.021	0.016 0.025
Real investment	0.065	0.141	0.117 0.165
Unemployment rate	0.008	0.018	0.015 0.021
CPI	0.009	0.015	0.01 0.018
EPI	0.057	0.017	0.012 0.021
Policy rate	0.003	0.008	0.006 0.009

Note: mean and 90% confidence intervals of simulated data calculated on 1,000 independent replicas.

Following Sichel (1993), we analyse the *depth* and the *slope* of business cycles, namely the skewness of the time series in levels and differences (table 4). Our results are in line with the empirical evidence, which indicates that troughs are deeper than peaks (depth) and contractions are steeper than expansions (slope). In other words, the simulated time series display asymmetric cycles, characterized by sharp recessions and slow but steady recoveries. This result is also confirmed by a visual inspection of the unconditional distribution of the real GDP growth rate, in which shocks appear to take larger values during negative episodes than in positive periods (figure 2, left pane). Moreover, the simulated data is not only left-skewed but also follows a Laplace distribution with tails much fatter than a Gaussian (Fagiolo et al., 2008).

Table 4: Depth and slope of business cycles

		US data	Simulations		
			Mean	90% C.I.	
Depth	Real GDP	-0.343	-0.776	-1.017	-0.550
	Unemployment rate	0.577	0.696	0.276	1.046
Slope	Real GDP	-0.476	-0.215	-0.400	-0.027
	Unemployment rate	1.136	0.164	-0.050	0.377

Note: mean and 90% confidence intervals of simulated data calculated on 1,000 independent replicas.

Since the model introduces an endogenous energy sector, we test the observed long-term positive relationship between energy demand and aggregate production (see Ozturk, 2010, for a review). The central pane of figure 2 depicts the logarithm of real GDP and energy demand for 1,000 independent replicas of the model. The simulations confirm the empirical evidence of a strong, positive relationship between energy demand and aggregate production with an estimated elasticity equal to 35%, in line with the findings of Burke and Csereklyei (2016). Moreover, the model can reproduce the average expenditure on oil over GDP observed in the US between 1973 and 2019, that is around 3%, as shown in the right pane of figure 2.

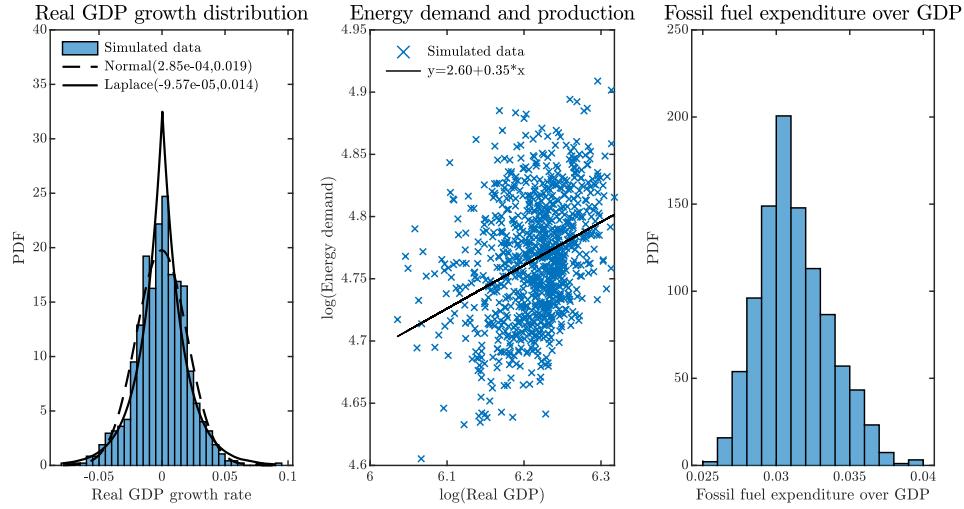


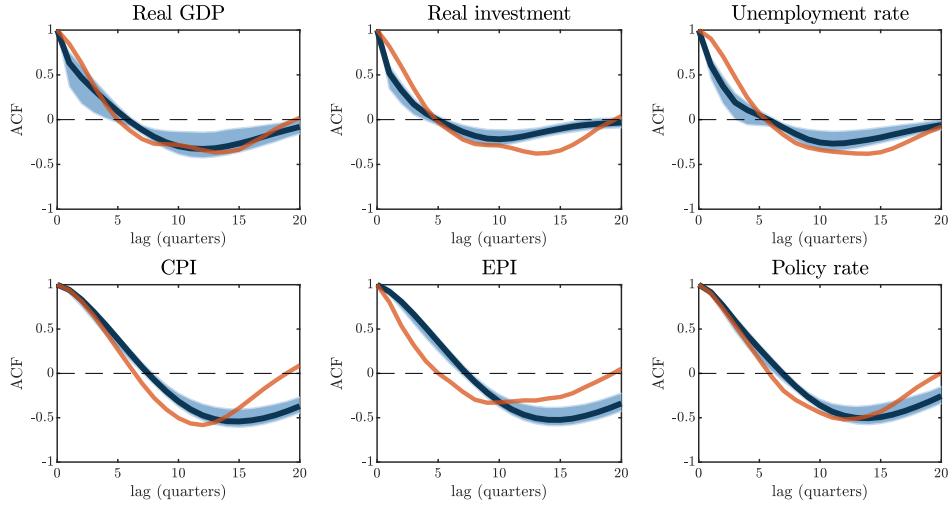
Figure 2: Aggregate production and energy demand

Note: simulated distribution of the unconditional GDP growth rate compared to the Normal and the Laplace distributions (left pane). Unconditional (long-term) relationship between energy demand and aggregate production (central pane). Simulated distribution of fossil fuel expenditure over GDP (right pane). Results calculated on the last observations of 1,000 independent replicas.

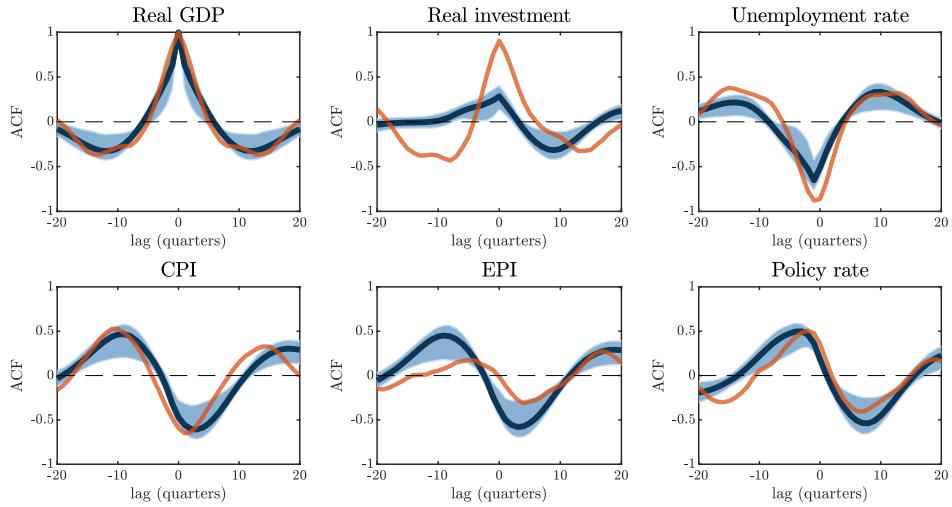
Moving to the dynamical properties of the times series, the autocorrelation functions of simulations are generally correct and follow the patterns observed in US data (figure 3a). In particular, the key macroeconomic variables display a positive auto-correlation in the first four to eight quarters, while negative in the medium term. Similarly, our model correctly predicts the co-evolution of US economic variables with the unemployment rate, the consumer price index, and the policy rate as shown in the cross-correlation functions of real time series with aggregate production (figure 3b).

However, the model underestimates the strength of the relationship between GDP and investments, while it overstates the cyclical behaviour of energy prices. Nevertheless, albeit the size of their comovements is slightly inaccurate, their sign is always correct. At the same time, the model can reproduce the observed inverse relationship between inflation and unemployment rate (i.e., the Phillips curve) without explicitly assuming it (figure 4, left pane). Moreover, an increase in the output gap (measured in log deviations from the mean) reduces unemployment in the simulated economy (figure 4, right pane), although to a lesser extent than in real-world data.

Lastly, a distinctive feature of agent-based models is their ability to generate granular data and fat tail distributions. Accordingly, we take micro-level data per class of agents to investigate the wealth distributions arising from our simulations. From the left pane of figure 5, we observe that the empirical distributions of firms are right-skewed and present long tails (Ijiri and Simon, 1964,



(a) Time series autocorrelation



(b) Time series cross-correlation with real aggregate production

Note: cross-correlation functions of simulated (blue error bars) and US time series (red lines). Median and 90% confidence intervals of simulated data calculated on 1,000 independent replicas.

1974; Okuyama et al., 1999; Ramsden and Kiss-Haypál, 2000; Axtell, 2001; Gaffeo et al., 2003). The wealth ranking is similar for households and firms, where the richest are bankers and banks, while the poorest are workers. In that sense, the model can reproduce the large differences observed in wealth distribution in US (Quadrini and Rios-Rull, 1997).

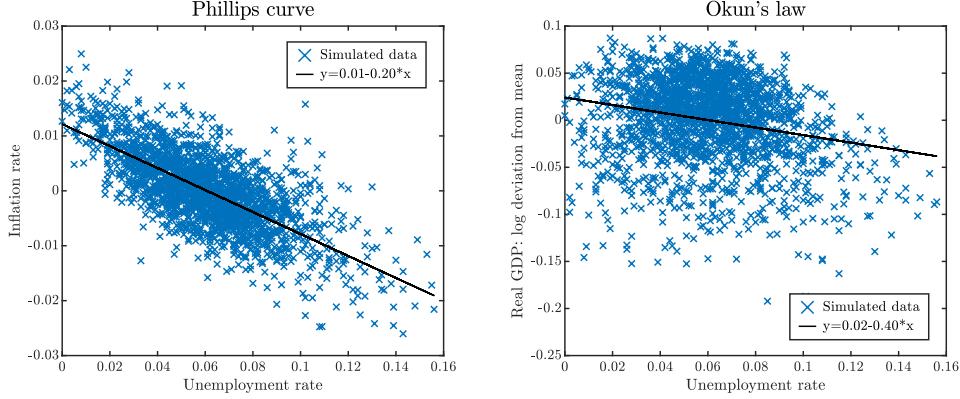


Figure 4: Phillips curves and Okun's law

Note: quarterly inflation rate (left pane) and real GDP log deviation from the mean (right pane) against unemployment rate. Results calculated on the last observations of 1,000 independent replicas.

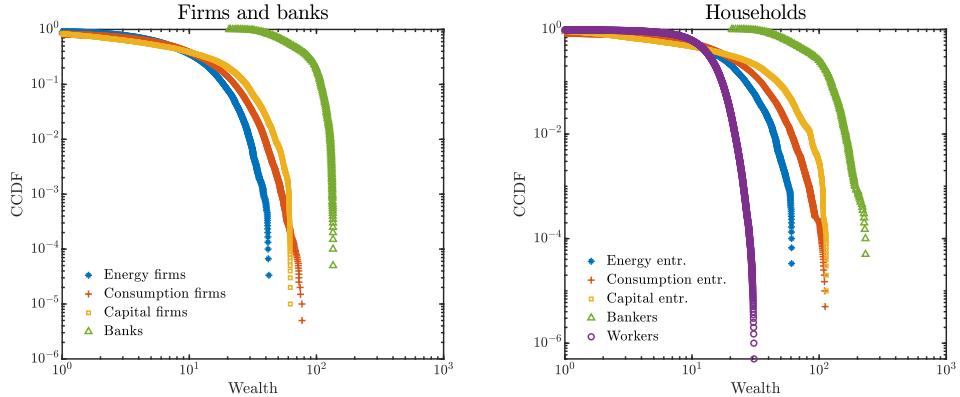


Figure 5: Complementary Cumulative Distribution Functions (CCDF) of wealth by class of agents.

Note: log-log plot of CCDF calculated on the last observation of a single run.

5. SIMULATION RESULTS

This section explores the long-term properties of the simulated time series and describes the mechanisms underlying the endogenous formation of business cycles.

5.1. Long-term convergence

Figure 6 shows the unconditional dynamics of exchanged quantities and prices in all markets, averaged across 1,000 independent replicas of the model. The upper panes focus on energy, capital,

and consumption markets. The dashed lines represent the price and quantity values we set in the initialization phase of the simulations, pointing to the market equilibrium under perfect competition and full employment. The lower figures show the unconditional dynamics of price and quantity in the credit and labour markets.

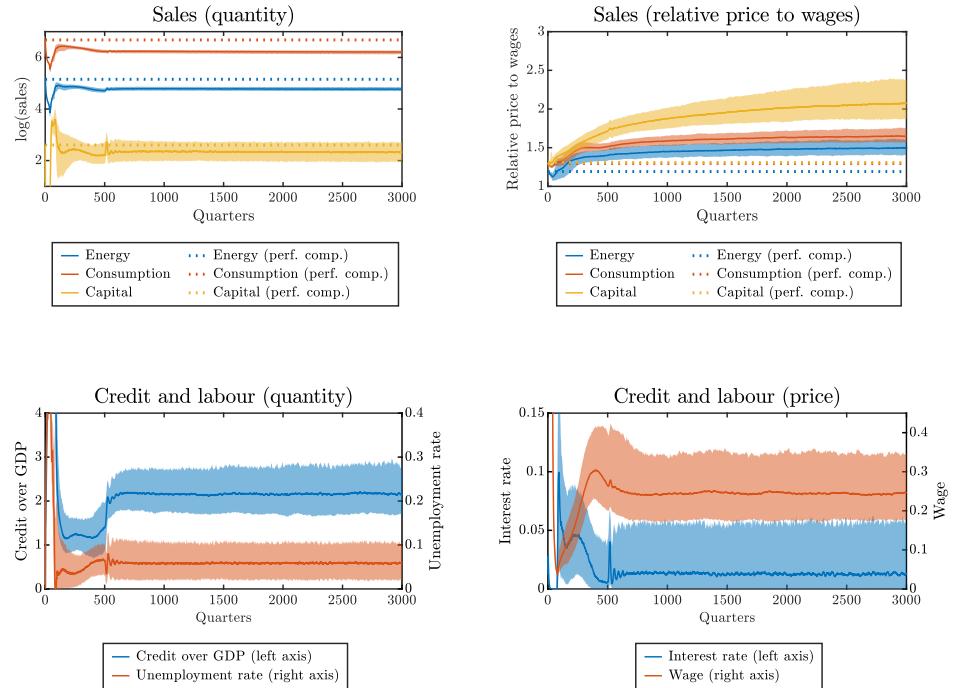


Figure 6: Market equilibrium: convergence of prices and quantities.

Note: median (solid line) and 90% confidence intervals (shaded area) of simulated prices and quantities calculated on 1,000 independent replicas. Dashed lines in the upper panes indicate the market equilibrium under perfect competition and full employment.

The simulated time series converge to a quasi-steady state after an initial burn-in period of 750 quarters, with a sharp change at the time 500 due to the later introduction of public expenditure and taxation. The upper panes of figure 6 show that quantities (relative prices) are below (above) the perfect competition solution, suggesting that firms operate in a context characterized by monopolistic competition. Moreover, the unemployment rate converges to the 6% target value set in the validation phase of the model (figure 6, bottom left pane), while the overall quantity of credit in the system reaches a steady level of 200% of GDP. At the same time, wages – that act as a numerator in our model – converge to a stable value (figure 6, bottom right pane), indicating the absence of long-term inflation in our model. Lastly, the policy rate fluctuates around its steady-state target value (i.e., $i^* = r^* + p^* = 1.1\%$, see equation (44)).

5.2. Cycles

In the remainder of this section we describe the endogenous dynamics of business cycles. Figure 7 shows the evolution of selected variables in relation with real GDP on a sub-period of 25 years. As expected, the unemployment rate, credit over GDP and non-performing loans decrease during periods of high production. Nevertheless, as the system approaches full employment, prices start to increase, leading the central bank to raise the policy rate. In line with the financial instability hypothesis theorized by Minsky (1976), firms default on their debts and, as aggregate production, income, and demand collapse, the leverage of the system spikes, leading to a prolonged wave of bankruptcies. The crisis ends when the central bank, observing an excessively high level of unemployment and declining prices, loses its monetary policy. Accordingly, the availability of new credit at cheaper conditions allows firms to finance their production without defaulting on their debts and hire additional workers. Overall, the new flow of income boosts aggregate demand and production, leading the system to a slow but steady recovery.

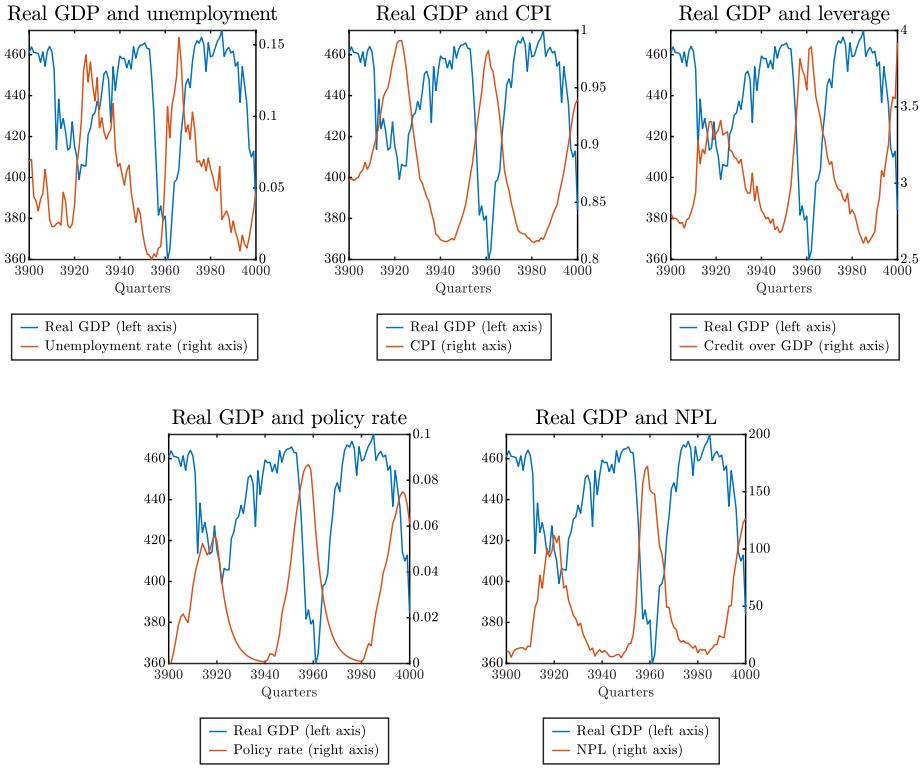


Figure 7: Endogenous business cycle.

Note: evolution of selected variables (right axis) in relation with real GDP (left axis) on a sub-period of 25 years for a representative simulation.

6. ENERGY SHOCKS

Having explored the properties of the model and its ability to replicate empirical regularities, we carry out a battery of experiments to assess the impact of energy shocks on the economic system in

this section. We simulate two scenarios corresponding to different types of energy shocks:

1. a positive shock to (foreign) energy input price (EIP);
2. a negative shock to energy firm's productivity (EP).

We choose those shocks since the former simulate an exogenous increase in the global price of natural resources, while the latter resembles a mandatory shift towards a sustainable and climate-friendly but economically inefficient energy technology.

For each scenario, we run the model 250 times with different random seeds. For a given run, for each variable expressed *in level* (e.g., GDP, final good price), we compute the *percentage deviation* from the baseline scenario without shock. Then we compute the mean of percentage deviation across 250 runs. For variables expressed *in percent terms* (e.g., public debt to GDP ratio, unemployment rate), we compute the *absolute deviation* from the baseline scenario and then the cross-run average. Figures 8 and 9 display the time series of the mean deviations (along with 50% and 90% confidence interval) for a set of economic variables in response to the energy shock for different scenarios. The economy is hit after 2,000 time steps of the simulation, corresponding to quarter zero in the figures below. The size of the shock varies depending on the type, while its degree of persistence is uniform, with an auto-correlation coefficient equal to 0.80, reflecting a total length of 6.5 years and a halving rate of one year.

Energy input price shock Figure 8 shows the effects of a 300% increase in the price of the foreign energy input.²³ Since such input is used by energy firms, together with capital and labour, to produce and sell energy services to the other sectors, they react to a substantial increase in their costs of production by almost doubling the energy price, leading to a significant increase in the final good price. At the aggregate level, the economy experiences a nearly 5% decline in real GDP followed by an increase in the unemployment rate and a slowdown in nominal wages. This scenario is reminiscent of *stagflation*, i.e., a situation characterized by low output growth and high inflation rate that prevailed in the 1970s following the sudden rise in the price of oil, which is comparable to the foreign energy input in our model. From the bottom panes, we can observe that, after an initial decline, the government debt to GDP ratio sharply increases. The reason is that, despite the fall in real production, the accelerating price inflation following the spike in energy price leads to a higher GDP in nominal terms, hence improving the public debt sustainability which allows for a greater fiscal space. The resulting increase in public transfers eventually leads to a higher budget deficit and a subsequent rise in public debt, before gradually declining towards the baseline value. Interestingly, from the top-left pane it can be noticed that, after the initial shock, the energy price converges towards a level that is systematically higher than the baseline scenario. It is reasonable to expect that this will cause a redistribution effect among sectors – we investigate this question further below.

Energy productivity shock Figure 9 presents the results of an energy productivity shock, consisting of a 25% reduction in the energy firms' total factor productivity. This scenario may refer to a policy-driven transition that involves a systematic and mandatory shift towards a sustainable and climate-friendly but economically inefficient energy technology (e.g., the coal-to-gas shift, Kost et al., 2021). As in standard macroeconomic models, the productivity shock constitutes a supply-side constraint for the entire production system, and its effects have a strong impact on the performance of the simulated economy. Nevertheless, we notice that the system experiences a double-dip recession

²³According to the IMF Primary Commodity Price System, the global price of natural gas increased by 287% in 2021, with a peak of 390% in October.

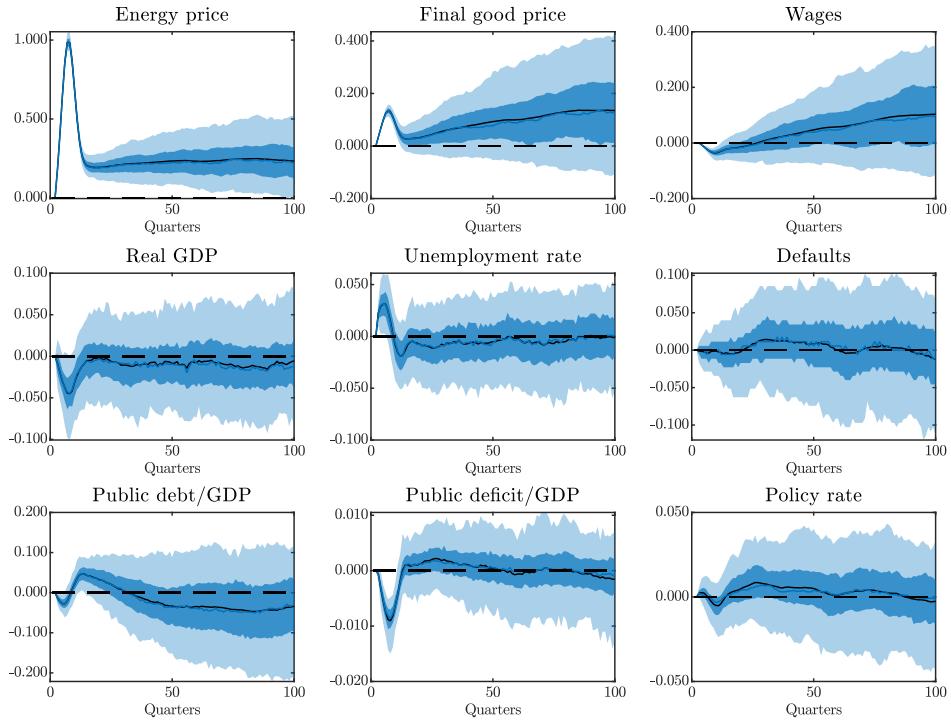


Figure 8: Energy input price shock.

Note: median (solid lines), mean (black lines), and 50-90% bootstrap confidence intervals (shaded areas) calculated on 250 independent runs. Variables: energy price index, consumption price index, nominal wages, real GDP, unemployment rate, aggregate default rate, public debt/GDP ratio, public deficit/GDP ratio, Central Bank's policy rate.

in our model because of the complex interaction between supply and demand. Indeed, the economy falls into recession, begins to recover within a period since the autocorrelation structure of the shock leads to a reduction in its size, and then falls back into recession as a consequence of the resulting increase in unemployment and slowdown in aggregate demand. In other words, the system experiences a supply-induced demand crisis, with falling prices, increasing unemployment and a worsening of public finances.

Effects of energy shocks by sector So far we have explored the effects of different types of energy shocks on the economic system as a whole. It emerged that the effects on price and quantity vary depending on the type of shock: an energy input price shock leads to a higher price inflation and a lower GDP, while a shock to energy firms' productivity have a negative impact on both price and aggregate production. However, previous findings seem to suggest that an energy shock may have different impacts across sectors. To address this question, figure 10a shows for each scenario how the energy shocks affect the net worth of energy, consumption and capital good industries. The two scenarios are displayed on different layers of figure 10a. Starting from the EIP scenario on the top, it can be seen that a positive shock to the energy input price entails a redistribution effect toward the energy sector at the expense of the capital sector, while the consumption sector is mostly unaffected. In the EP scenario the sector hit is the energy one. This effect is redistributed to the

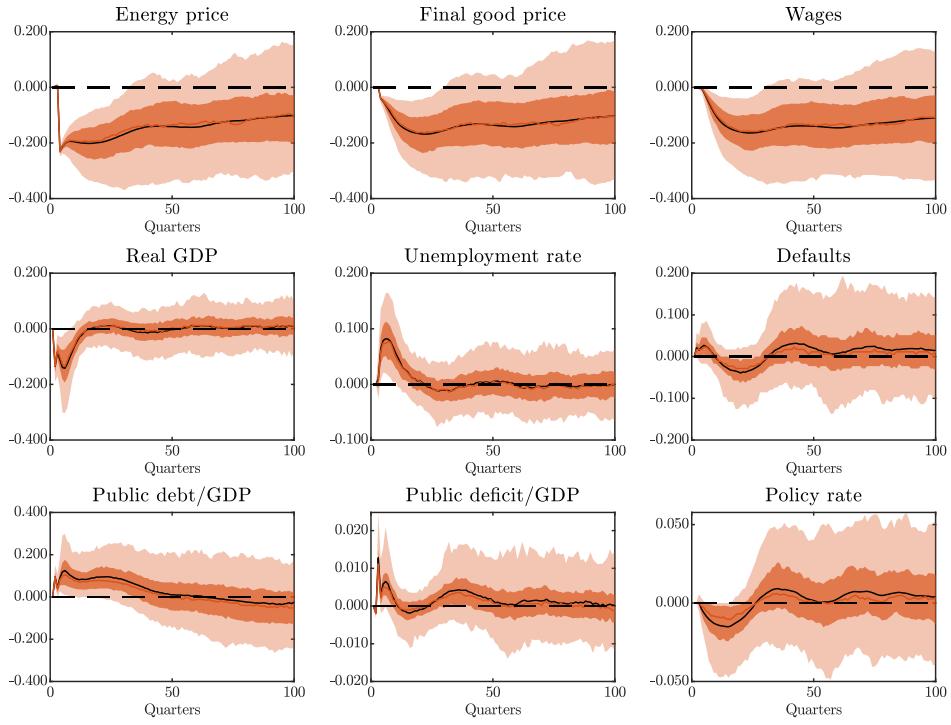
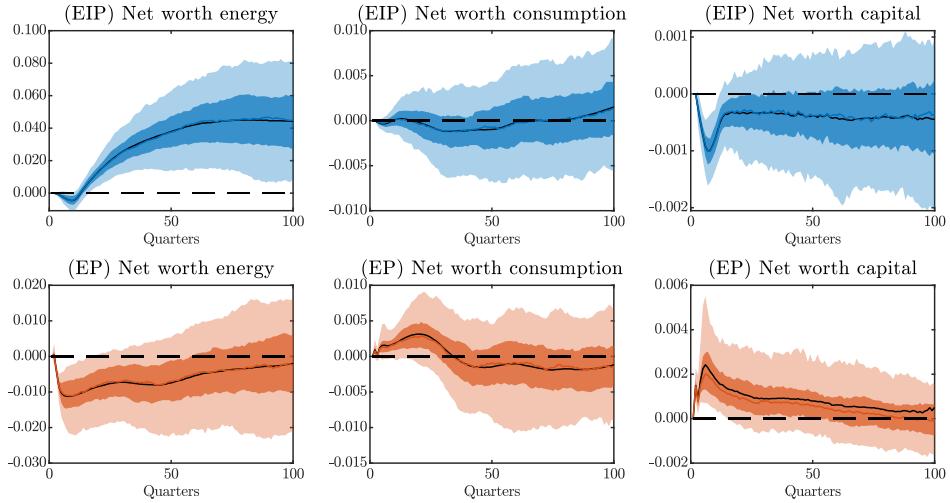


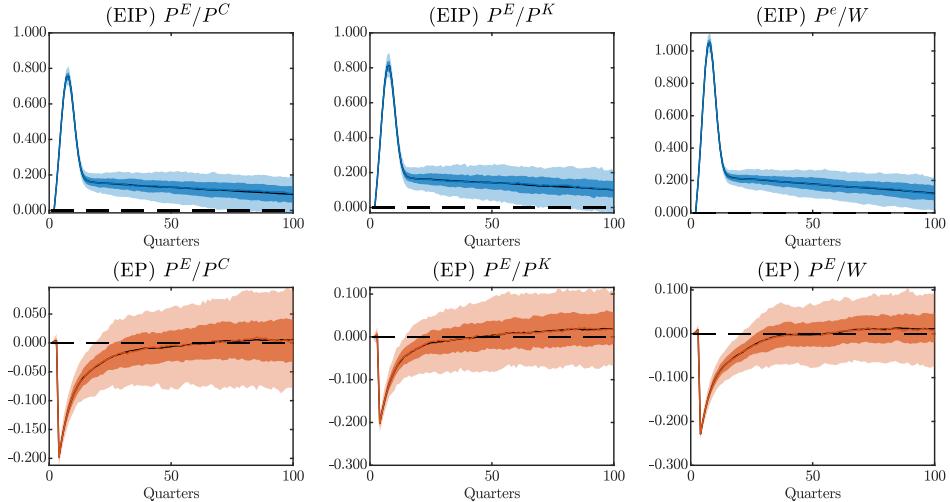
Figure 9: Energy firms' productivity shock.

Note: median (solid lines), mean (black lines), and 50-90% bootstrap confidence intervals (shaded areas) calculated on 250 independent runs. Variables: energy price index, consumption price index, nominal wages, real GDP, unemployment rate, aggregate default rate, public debt/GDP ratio, public deficit/GDP ratio, Central Bank's policy rate.

consumption and capital sectors, although the former see a decrease under zero after approximately thirty quarters while the economy is transitioning back. To understand the economic mechanisms underlying these dynamics, figure 10b reports for each scenario the changes in the relative price of energy with respect to consumption (first column), capital (second column) and nominal wages (third column). Starting from the top panes, we can see that the energy input price shock determines a permanent increase in the relative price of energy with respect to consumption, capital and labour. Because the increase in the price of output is higher than the increase in the cost of inputs, i.e., capital and labour, the EIP shock represents a positive income effect for energy firms – and negative for capital good firms –, resulting in a permanent improvement in the energy sector's net worth (see figure 10a). Moving to the EP scenario, the shock to energy firms' productivity, by reducing the level of output per worker, leads to an increased input demand by energy firms, which strive to keep the production level unchanged. As a result, the price of capital and labour rises relative to energy. The subsequent decline in the relative price of energy represents a negative income effect for energy firms, with a harmful impact on equity. If we treat the energy productivity shock as equivalent to the introduction of a less efficient production technology, our findings enable us to understand the reluctance of energy firms to face the energy transition. In the absence of appropriate choice of policy measures, indeed, the adoption of environmentally friendly but economically inefficient techniques may constitute a serious risk for the financial viability of the energy sector.



(a) Effects of energy shocks on net worth by sectors.



(b) Effects of energy shocks on relative price by sectors.

Note: median (solid lines), mean (black lines), and 50-90% bootstrap confidence intervals (shaded areas) calculated on 250 independent runs. Scenarios: Energy Input Price shock (EIP) and Energy Productivity shock (EP). Variables: average net worth of energy, consumption and capital good sectors (upper figure), and relative energy price with respect to consumption price index, capital price index and nominal wages (lower figure).

7. FINAL REMARKS

Notwithstanding the key importance of energy for economic activity, to date, there are still relatively few macroeconomic ABMs that incorporate this source of production. Additionally, most of the existing models assume an infinitely elastic energy supply, thus ruling out the possibility of endogenous and self-reinforcing supply shortages (see Castro et al., 2020). As a result, our effort in this paper is oriented to bridge this gap by presenting a new full-fledged macroeconomic ABM with an energy sector that endogenously provides energy services to consumption and capital good sectors. Moreover, compared to standard neoclassical models grounded on the representative agent hypothesis and general equilibrium framework, our agent-based setting can generate endogenous fluctuations resulting from decentralized interactions, economic-financial inter-relations and coordination failures, providing a suitable framework for studying the several facets of the energy transition. Overall, the goals of this paper are (i) to develop and validate a novel macroeconomic ABM including consumption, capital and energy sectors with interlocked input demands; (ii) to assess the model's ability to replicate a wide set of empirical regularities; (iii) to analyse the economic effects of energy shocks both at the aggregate and sectoral level.

The model is calibrated on US quarterly macroeconomic data and can replicate selected statistics from real-time series in terms of aggregate volatility and business cycles' properties. In particular, the sample auto- and cross-correlation functions closely match those obtained from US data between 1957 and 2019. At the same time, the model reproduces additional empirical evidence like the positive relationship between aggregate production and energy demand, and the right-skewed distribution of firms' size and households' wealth.

Lastly, we employ our model to investigate the economic impact of different types of energy shocks: an exogenous increase in the global price of natural resources (EIP), and the forced introduction of less efficient production technology in the energy sector (EP). Our analysis indicates that their effects on prices and quantities vary substantially: an energy input price shock leads to higher inflation and lower GDP, reminiscent of the stagflation scenario of the 1970s, while a shock to energy firms' productivity has a negative impact on prices and aggregate production. The reason is that energy shocks eventually trigger a demand crisis due to the presence of feedback loops and coordination failures giving rise to non-linear dynamics. This holds especially true for the EP scenario, whereby the complex interaction between supply and demand leads to a W-shaped recession which amplifies the effects of the original shock (i.e., after the initial supply shock, the system enters a supply-induced demand crisis). Furthermore, we investigate the economic impact of the energy shocks at the sectoral level. We find that an energy input price shock, by fostering a permanent increase in the relative price of energy, leads to a redistributive effect in favour of the energy sector at the expense of workers and other firms. On the contrary, we find that the net worth of the former declines in response to a productivity shock because of the reduction in the relative price of energy. In light of this, policymakers need to carefully account for the distributional effects of a transition to a low carbon economy. In the absence of appropriate choice of policy measures, indeed, the adoption of environmentally friendly but economically inefficient techniques may constitute a serious risk for the financial viability of the energy sector.

The model presented here could be extended to address several questions related to energy transition and climate change. In particular, introducing a climate box would allow exploring the relationship between GHG emissions, global warming and climate damages. At the same time, the transition to a low-carbon economy has always been studied focusing on the production side of the problem. Accordingly, exploring the role of social interactions in shifting consumer preferences towards low-carbon footprint goods or firms would enrich the existing analysis of this issue.

ACKNOWLEDGEMENTS

This work is part of the "Modelling the Energy Transition" (MET) research program, initiated by Fondazione Eni Enrico Mattei (FEEM) with the aim of contributing to the analysis of energy transition by providing new perspectives and insights to economic research. The authors thank, without implication, the participants in the workshop "Research dialogues on the complexity of the energy transition" for their helpful comments.

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A. APPENDIX

A.1. Consumption from utility maximization

In this section, we show that the consumption budget rule defined in equation (4) can be thought of as a reduced form of an optimization problem with a log-utility function.

Given the expected sequences of consumption good prices $\{\mathbb{E}_{h,t}[P_{C,t+s}]\}_{s=0}^{\infty}$, risk-free policy rates $\{\mathbb{E}_{h,t}[i_{t+s}^{CB}]\}_{s=0}^{\infty}$, and nominal income $\{\mathbb{E}_{h,t}[Y_{h,t+s}]\}_{s=0}^{\infty}$, the objective of the consumer h is to maximize the inter-temporal utility function subject to the budget constraint:

$$\max_{\{C_{h,t+s}; D_{h,t+s}\}_{s=0}^{\infty}} \mathbb{E}_{h,t} \left[\sum_{s=0}^{\infty} \beta^s \log(C_{h,t+s}) \right] \quad (54)$$

$$\text{s.t. } D_{h,t+s} + P_{C,t+s} C_{h,t+s} = (1 + i_{t+s-1}^{CB}) D_{h,t+s-1} + Y_{h,t+s} \quad (55)$$

where $C_{h,t+s}$ and $D_{h,t+s}$ are real consumption and nominal savings of the h -th consumer at the time $t+s$.

Since the consumer reoptimizes the inter-temporal problem every period $t+s \forall s = 0, \dots, \infty$, the first-order conditions imply the standard Euler equation:

$$\frac{1}{P_{C,t+s} C_{h,t+s}^d} = \mathbb{E}_{h,t+s} \left[\beta \frac{1 + i_{t+s}^{CB}}{P_{C,t+s+1} C_{h,t+s+1}^d} \right] \quad \forall s = 0, \dots, \infty \quad (56)$$

and the transversality condition:

$$\lim_{s \rightarrow \infty} \mathbb{E}_{h,t} \left[\beta^s \frac{1 + i_{t+s-1}^{CB}}{P_{C,t+s} C_{h,t+s}^d} D_{h,t+s-1} \right] = 0. \quad (57)$$

We derive the solution from equations (55), (56), and (57):

$$(1 + i_{t-1}^{CB}) D_{h,t-1} - \frac{P_{C,t} C_{h,t}^d}{1 - \beta} + \lim_{s \rightarrow \infty} \sum_{j=0}^{s-1} \mathbb{E}_{h,t} \left[\beta^j \frac{P_{C,t} C_{h,t}^d}{P_{C,t+j} C_{h,t+j}^d} Y_{h,t+j} \right] = 0. \quad (58)$$

Under the assumption that the nominal income $Y_{h,t+j}$ is independent from the stochastic discount factor $m_{h,t+z}$, namely:

$$\mathbb{E}_{h,t+z} [m_{h,t+z} Y_{h,t+j}] = \mathbb{E}_{h,t+z} [m_{h,t+z}] \mathbb{E}_{h,t+z} [Y_{h,t+j}] \quad \forall z = 0, \dots, j \quad (59)$$

where $m_{h,t+z} = \beta (1 + i_{t+z}^{CB}) \frac{P_{C,t+z} C_{h,t+z}^d}{P_{C,t+z+1} C_{h,t+z+1}^d}$, we can approximate the discounted flow of exogenous income as follows:

$$\mathbb{E}_{h,t} \left[\beta^j \frac{P_{C,t} C_{h,t}^d}{P_{C,t+j} C_{h,t+j}^d} Y_{h,t+j} \right] \approx \mathbb{E}_{h,t} \left[\prod_{z=1}^j \frac{Y_{h,t+z}}{1 + i_{t+z-1}^{CB}} \right]. \quad (60)$$

We assume that the real risk-free interest rate $\frac{1}{1+r_{t+z}^{CB}} = \frac{1}{1+i_{t+z}^{CB}} \frac{P_{C,t+z+1}}{P_{C,t+z}}$ is independent from the future real income $Y_{h,t+j}^R = \frac{Y_{h,t+j}}{P_{C,t+j}}$ $\forall z = 0, \dots, j-1$ and is approximatively equal to the steady-state value $\mathbb{E}_{h,t} \left[\frac{1}{1+r_{t+z}^{CB}} \right] \approx \beta \forall z = 1, \dots, \infty$. Accordingly, we obtain:

$$\mathbb{E}_{h,t} \left[\beta^j \frac{P_{C,t} C_{h,t}^d}{P_{C,t+j} C_{h,t+j}^d} Y_{h,t+j} \right] \approx \mathbb{E}_{h,t} \left[\prod_{z=1}^j \frac{Y_{h,t+z}}{1 + i_{t+z-1}^{CB}} \right] \approx \beta^j P_{C,t} \mathbb{E}_{h,t} [Y_{h,t+j}^R] \quad (61)$$

which simplifies (58) to:

$$\mathbb{E}_{h,t}[P_{C,t}]C_{h,t}^d = (1 - \beta)(1 + i_{t-1}^{CB})D_{h,t-1} + (1 - \beta)Y_{h,t} + \beta\mathbb{E}_{h,t}[P_{C,t}]Y_{h,t}^R \quad (62)$$

where we assume that the expected real future income is constant and equal to $\mathbb{E}_{h,t}[Y_{h,t+j}^R] = Y_{h,t}^R \forall j = 1, \dots, \infty$. In other words, the consumption budget $\mathbb{E}_{h,t}[P_{C,t}]C_{h,t}^d$ of the h -th consumer at the time t depends on the interest received on deposits and a linear combination between the nominal current and permanent income $Y_{h,t}$ and $\mathbb{E}_{h,t}[P_{C,t}]Y_{h,t}^R$. The latter can also be estimated through the adaptive rule:

$$(1 - \beta)Y_{h,t} + \beta\mathbb{E}_{h,t}[P_{C,t}]Y_{h,t}^R \approx \bar{Y}_{h,t} = (1 - \beta)Y_{h,t} + \beta\mathbb{E}_{h,t}\left[\frac{P_{C,t}}{P_{C,t-1}}\right]\bar{Y}_{h,t-1}. \quad (63)$$

Lastly, if we consider that consumers cannot borrow from banks (i.e., $D_{h,t} \geq 0$), the consumption budget boils down to:

$$\mathbb{E}_{h,t}[P_{C,t}]C_{h,t}^d = \min\left\{(1 - \beta)\left(1 + i_{t-1}^{CB}\right)D_{h,t-1} + \bar{Y}_{h,t}; \left(1 + i_{t-1}^{CB}\right)D_{h,t-1} + Y_{h,t}\right\}. \quad (64)$$

A.2. Conditional input demand

In this section, we detail the derivation of conditional input demands for a generic firm f at the time t .

A.2.1 Minimization problem

For a representative firm f at the time t , the conditional input demand for the subsequent period minimizes the expected direct costs ($\mathbb{E}_{f,t}[DC_{f,t+1}]$) given a production target $Q_{f,t+1}$, a set of expected input prices $\{\mathbb{E}_{f,t}[P_{j,t+1}]\}_{j=1}^n$, and non-negative demand, namely:

$$\min_{\{X_{f,j,t+1}; \Delta X_{f,j,t+1}\}_{j=1}^n} \mathbb{E}_{f,t}[DC_{f,t+1}] = \sum_{j=1}^n \mathbb{E}_{f,t}[P_{j,t+1}] \Delta X_{j,f,t+1} \quad (65)$$

$$\text{s.t. } Q_{f,t+1} = \left[\sum_{j=1}^n A_{j,f} (X_{j,f,t+1})^{\rho_f} \right]^{\frac{1}{\rho_f}} \quad (66)$$

$$X_{j,f,t+1} = \Delta X_{j,f,t+1} + (1 - \delta_j)X_{j,f,t} \quad (67)$$

$$\Delta X_{j,f,t+1} \geq 0 \quad (68)$$

where $X_{j,f,t}$ is the quantity of input $j = 1, \dots, n$ with depreciation rate δ_j in the firm f at the time t , $\Delta X_{j,f,t+1}$ is the additional demand for the production factor j , $\sum_{j=1}^n A_{j,f} = 1$ are the factor shares, $\rho_f = \frac{\sigma_f - 1}{\sigma_f}$ is the substitution parameter, and σ_f is the Hicks elasticity of substitution.

The optimal solution of the minimization problem implies the following additional demand for input:

$$\Delta X_{j,f,t+1}^d = \left(\frac{A_{j,f} \psi_{f,t+1}}{\mathbb{E}_{f,t}[P_{j,t+1}] - \lambda_{j,f,t+1}} \right)^{\sigma_f} Q_{f,t+1} - (1 - \delta_j)X_{j,f,t} \quad \forall j = 1, \dots, n \quad (69)$$

where

$$\psi_{f,t+1} = \left[\sum_{j=1}^n (\mathbb{E}_{f,t}[P_{j,t+1}] - \lambda_{j,f,t+1})^{1-\sigma_f} (A_{j,f})^{\sigma_f} \right]^{\frac{1}{1-\sigma_f}} \quad (70)$$

are the expected marginal costs, and

$$\lambda_{j,f,t+1} = \mathbb{E}_{f,t} [P_{j,t+1}] - \psi_{f,t+1} A_{j,f} \left[\frac{Q_{f,t+1}}{(1-\delta_j) X_{j,f,t}} \right]^{\frac{1}{\sigma_f}} \geq 0 \quad \forall j = 1, \dots, n \quad (71)$$

is the Lagrange multiplier associated to the non-negativity constraint.

A.2.2 Insufficient funds

If the available liquid resources ($D_{f,t}$) are not sufficient to purchase the production factors, the firm reduces proportionally input demands by maximizing the attainable production given the budget constraint:

$$\max_{\{\bar{X}_{f,j,t+1}; \Delta \bar{X}_{f,j,t+1}\}_{j=1}^n} \bar{Q}_{f,t+1} = \left[\sum_{j=1}^n A_{j,f,t+1} (\bar{X}_{j,f,t+1})^{\rho_f} \right]^{\frac{1}{\rho_f}} \quad (72)$$

$$\text{s.t.} \quad \sum_{j=1}^n \mathbb{E}_{f,t} [P_{j,t+1}] \Delta \bar{X}_{j,f,t+1} = D_{f,t+1} \quad (73)$$

$$\bar{X}_{j,f,t+1} = \Delta \bar{X}_{j,f,t+1} + (1-\delta_j) X_{j,f,t} \quad (74)$$

$$\Delta \bar{X}_{j,f,t+1} \geq 0. \quad (75)$$

Solving the constrained optimization problem leads to the following demand for input:

$$\Delta \bar{X}_{j,f,t+1}^d = v_{f,t+1} \Delta X_{j,f,t+1}^d - (1-v_{f,t+1})(1-\delta_j) X_{j,f,t} \quad \forall j = 1, \dots, n \quad (76)$$

where

$$v_{f,t+1} = \frac{D_{f,t} + \sum_{j=1}^n \mathbb{E}_{f,t} [P_{j,t+1}] (1-\delta_j) X_{j,f,t}}{\psi_{f,t+1} Q_{f,t+1}} \quad (77)$$

is the ratio between available liquid resources and expected total costs.

A.2.3 Profit maximization and steady state

Assuming no constraints on input demands (i.e., $\lambda_{j,f,t+1} = 0 \forall j = 1, \dots, n$) and that the available liquidity $D_{f,t+1}$ is sufficient to produce the desired quantity $Q_{f,t+1}$ (i.e., $v_{f,t+1} = 1$), the expected profits of the firm f are:

$$\mathbb{E}_{f,t} [\Pi_{f,t+1}] = P_{f,t+1} Q_{f,t+1} - \psi_{f,t+1} Q_{f,t+1} + \sum_{j=1}^n (1-\delta_j) \mathbb{E}_{f,t} [P_{j,t+1}] X_{j,f,t} \quad (78)$$

Under the assumption of monopolistic competition, the price $P_{f,t+1}$ resulting from profits maximization is a mark-up $\mu_{f,t+1}$ on marginal costs:

$$\max_{Q_{f,t+1}} \mathbb{E}_{f,t} [\Pi_{f,t+1}] \Rightarrow P_{f,t+1} = (1 + \mu_{f,t+1}) \psi_{f,t+1} \quad (79)$$

which implies:

$$\mathbb{E}_{f,t} [\Pi_{f,t+1}] = \mu_{f,t+1} \psi_{f,t+1} Q_{f,t+1} + \sum_{j=1}^n (1-\delta_j) \mathbb{E}_{f,t} [P_{j,t+1}] X_{j,f,t}. \quad (80)$$

In steady state, the produced quantity, (relative) prices, and the mark-up are constant through time ($Q_{f,t} = Q_f$, $\mu_{f,t} = \mu_f$, and $\mathbb{E}_{f,t}[P_{j,t+1}] = P_j \forall t = 1, \dots, \infty$ and $\forall j = 1, \dots, n$). Accordingly, the demand for inputs in the previous period is the same as in the current one, namely:

$$\Delta X_{j,f,t} = \Delta X_{j,f,t+1} = \Delta X_{j,f} \quad \forall t = 1, \dots, \infty \text{ and } \forall j = 1, \dots, n. \quad (81)$$

As a result, the long-term profits of the firm f are:

$$\Pi_f = (1 + \mu_f - \Delta_f) \psi_f Q_f \quad (82)$$

where

$$\Delta_f = \frac{\sum_{j=1}^n \delta_j (P_j)^{1-\sigma_f} (A_{j,f})^{\sigma_f}}{\sum_{j=1}^n (P_j)^{1-\sigma_f} (A_{j,f})^{\sigma_f}} \quad (83)$$

is the weighted average depreciation rate.

A.3. Initialization

We initialize the system at the steady state under the following assumptions:

1. All markets clear:²⁴

$$\text{labour market: } n^W = n^E \Delta X_{N,E}^d + n^C \Delta X_{N,C}^d + n^K \Delta X_{N,K}^d \quad (85)$$

$$\text{energy market: } n^E Q_E = n^C \Delta X_{E,C}^d + n^K \Delta X_{E,K}^d \quad (86)$$

$$\text{capital market: } n^K Q_K = n^E \Delta X_{K,E}^d + n^C \Delta X_{K,C}^d \quad (87)$$

$$\text{natural resource market: } Q_O = n^E \Delta X_{O,E}^d \quad (88)$$

2. All firms set their price to maximize profits and operate under perfect competition:

$$P_f = (1 + \mu_f) \psi_f \quad (89)$$

$$\Pi_f = 0 \Rightarrow 1 + \mu_f - \Delta_f = 0 \quad (90)$$

3. Oil expenditure over total final consumption is equal to $\nu^O = 0.03$:

$$P_O Q_O = \nu^O n_C P_C Q_C \quad (91)$$

4. Firms' deposits and loans are equal to total revenues:

$$D_f = L_f = P_f Q_f \quad (92)$$

5. The initial wage is set $W_0 = 10$ as a numerator.

²⁴We remove the equilibrium condition on the consumption market:

$$n^E \Pi_E + n^C \Pi^C + n^K \Pi_K + W n^W + P_O Q_O = n^C P^C Q_C \quad (84)$$

since it is redundant by Walras's law.

A.4. Interest rate rule

Assume that each bank b maximize the expected profit ($\mathbb{E}[\Pi_{b,f,t}]$) of a new loan issuance to firm f at the time t with respect to the required interest rate $i_{b,f,t}$ and the size of the credit line $L_{b,f,t}$, namely:

$$\max_{\{L_{b,f,t}, i_{b,f,t}\}} \mathbb{E}[\Pi_{b,f,t}] = (1 - PD_{b,f,t})i_{b,f,t}L_{b,f,t} + PD_{b,f,t}(-LGD_{b,f,t})L_{b,f,t} \quad (93)$$

where $PD_{b,f,t}$ is the probability of default, and $LGD_{b,f,t}$ is the loss given default. Under the assumption that the probability of default increases with the interest rate and the size of the loan (i.e., with the leverage), the optimization problem implies the following first-order conditions:

$$\frac{\partial \mathbb{E}[\Pi_{b,f,t}]}{\partial L_{b,f,t}} = \frac{\partial PD_{b,f,t}}{\partial L_{b,f,t}}(LGD_{b,f,t} + i_{b,f,t})L_{b,f,t} + PD_{b,f,t}(-LGD_{b,f,t}) + (1 - PD_{b,f,t})i_{b,f,t} = 0 \quad (94)$$

$$\frac{\partial \mathbb{E}[\Pi_{b,f,t}]}{\partial i_{b,f,t}} = \frac{\partial PD_{b,f,t}}{\partial i_{b,f,t}}(LGD_{b,f,t} + i_{b,f,t})L_{b,f,t} + (1 - PD_{b,f,t})L_{b,f,t} = 0 \quad (95)$$

from which we obtain the (approximate) equality:

$$\frac{\partial i_{b,f,t}}{\partial L_{b,f,t}} \approx \frac{\frac{\partial PD_{b,f,t}}{\partial L_{b,f,t}}}{\frac{\partial PD_{b,f,t}}{\partial i_{b,f,t}}} = \frac{(1 - PD_{b,f,t})i_{b,f,t}L_{b,f,t} + PD_{b,f,t}(-LGD_{b,f,t})L_{b,f,t}}{(1 - PD_{b,f,t})L_{b,f,t}^2} = \frac{\mathbb{E}[\Pi_{b,f,t}]}{(1 - PD_{b,f,t})L_{b,f,t}^2}. \quad (96)$$

In other words, when the expected profit of a new loan issuance is positive (i.e., when the bank does not ration credit), the interest rate increases with the leverage of the firm (i.e., with the size of the loan).

Table 5: Model parameters

#	Description	Name	Value
1	Number of workers	n^W	1000
2	Discount rate households	β^C	0.993
3	Memory parameter	ε	β^C
4	Marginal propensity to consume out of wealth	χ	$1 - \beta^C$
5	Discount rate firms	β^F	0.985
6	Dividend payout ratio firms	μ^F	$1 - \beta^F$
7	Insider-outsider bargaining power	θ^W	0.75
8	Labour depreciation	δ_N	1
9	Number of C-firms	n^C	100
10	Factor share capital (C-firms)	$A_{N,C}$	0.60
11	Factor share labour (C-firms)	$A_{K,C}$	0.38
12	Factor share energy (C-firms)	$A_{E,C}$	0.02
13	Depreciation rate of consumption goods	δ_C	1
14	Elasticity of substitution (C-firms)	σ_C	0.50
15	Number of E-firms	n^E	100
16	Factor share capital (E-firms)	$A_{N,E}$	0.60
17	Factor share labour (E-firms)	$A_{K,E}$	0.38
18	Factor share natural resource (E-firms)	$A_{E,E}$	0.02
19	Depreciation rate of energy services	δ_E	1
20	Elasticity of substitution (E-firms)	σ_E	0.50
21	Number of K-firms	n^K	500
22	Factor share labour (K-firms)	$A_{N,K}$	0.95
23	Factor share energy (K-firms)	$A_{E,K}$	0.05
24	Depreciation rate of physical capital	δ_K	0.0125
25	Elasticity of substitution (K-firms)	σ_K	0.50
26	Size of exploration: price	γ^P	0.15
27	Speed of adjustment: quantity	ζ^Q	0.75
28	Speed of adjustment: price	ζ^P	0.75
29	Intensity of choice	ω	15
30	Foreign natural resource expenditure over GDP	ν^O	0.03
31	Depreciation rate of foreign natural resource	δ_O	1
32	Number of banks	n^B	10
33	Capital adequacy ratio	γ^B	0.105
34	Risk weighting	ω^B	1
35	Equity to deposits ratio	ν^B	$\gamma^B \omega^B$
36	Maximum single exposure to borrowers	κ^B	0.25
37	Interest rate setting parameter	ϱ^B	0.010/4
38	Interest rate setting parameter	ρ^B	0.015/4
39	Interest rate setting parameter	ι^B	0.005/4
40	Share of loans repaid at each time-step	θ^B	0.0125
41	Number of entrepreneurs	n^F	$n^C + n^E + n^K + n^B$
42	Number of consumers	n^H	$n^W + n^F$
43	Inflation target	p^*	0.02/4
44	Target unemployment rate	u^*	0.06
45	Steady state real interest rate	r^*	$1/\beta^C - 1$
46	Monetary policy rule weights: inflation	λ^P	1.3
47	Monetary policy rule weights: unemployment	λ^u	1.3
48	Speed of adjustment of the monetary policy rule	ρ^{CB}	0.80
49	Target debt-GDP ratio	b^*	0.80
50	Speed of adjustment to target debt-GDP ratio	ρ^G	0.01
51	Share of social expenditures	ψ^G	0.066
52	Maximum number of new partners (C-market)	\mathcal{Z}^C	0.50
53	Maximum number of new partners (E-market)	\mathcal{Z}^E	4
54	Maximum number of new partners (K-market)	\mathcal{Z}^K	4
55	Maximum number of new partners (labour market)	\mathcal{Z}^N	10
56	Maximum number of new partners (credit market)	\mathcal{Z}^B	0.50
	Initial wage rate	W_0	10
	Time length	T	5000
Total number of non-composite parameters			49

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Fondazione Eni Enrico Mattei

Corso Magenta 63, Milano – Italia

Tel. +39 02.520.36934

Fax. +39.02.520.36946

E-mail: letter@feem.it

www.feem.it

