International Journal of Food and Agricultural Economics
ISSN 2147-8988, E-ISSN: 2149-3766
Vol. 9, No. 2, 2021, pp. 109-123

PRICING THE REVENUE INSURANCE OF SUGAR APPLE AND BANANA IN TAIWAN: AN ACTUARIAL APPROACH

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Abstract:

This paper uses Fratini's (2017) actuarial approach and the traditional risk neutral approach to price the revenue insurance under the framework of Black Scholes put option pricing. Then, we extend the pricing to include a coverage limit. Further, we also decompose the revenue into price and yield and develop another pricing formula using the price and yield information. The risk neutral approach assumes that the underlying revenue is tradable while the actuarial approach assumes the revenue risk is diversifiable. We compute the premiums of the revenue insurance of sugar apple and banana in Taiwan. The results show that the premiums of the actuarial approach are generally lower than the risk neutral approach. Because revenue is often non-tradable, thus, if the individual revenue risk can be fully diversified, then the actuarial approach should be suggested for premiums calculation.
Keywords: *Actuarial approach, Black Scholes model, option pricing, put options, revenue insurance.*
JEL Codes: *G12, G22, Q14*

1. Introduction

The revenue risk of agricultural products can be decomposed into yield risk and price risk. While yield insurance and forward contracts can be used for coping the yield risk and price risk separately, revenue insurance considers both risks together. In addition, as yield and price are usually negatively correlated, it is a natural hedge for revenue risk. Thus, all else equal, the cost of using one revenue insurance contract tends to be lower than the total costs of yield insurance and forward contracts. Although revenue insurance does not completely replace yield insurance and forward contracts, revenue insurance has been widely used in many countries since its introduction in 1990s. In Taiwan, the program for revenue insurance of sugar was initiated in 2016, and that of banana was implemented in 2019. Accompanying the revenue insurance is the pricing issue which is the purpose of this paper.

Revenue insurance provides coverage to protect the buyer against loss of *revenue* caused by low price, low yield, or a combination of both. It guarantees the insured a minimum level of gross revenue from operations. The revenue can be based on a single commodity, multiply commodities, a farm, or certain area. No matter what the revenue is based, it can be regarded as a kind of European put option with the future uncertain revenue as the underlier and the guaranteed minimum as the strike price. Thus, the Black-Sholes option pricing model (Black & Sholes, 1973) for put options was used for the pricing (Turvey, 1992).

Although revenue insurance is conceptually like put options, applying the model for the revenue insurance pricing could be problematic. To price in the model one needs to replicate the option payoff by dynamically trading the underlying asset and a risk-free one. Then, under the no-arbitrage condition, the option pricing formula can be derived by solving a partial differential equation. The option price is the cost of replicating the option's payoff. This approach is known as a PDE approach. Under the same framework, risk neutral valuation can also be used for option pricing (Cox & Ross, 1976; Harrison & Kreps, 1978; Harrison & Pliska, 1981). The equivalence of the two methods is supported by the Feynman-Kac theorem. The key assumption to price options is that the options' payoff can be replicated. Thus, there should be some liquid market for trading the underlying assets. In revenue insurance, because the revenue is not a traded asset, theoretically it is not possible to use the PDE approach or the risk neutral approach to price the insurance (Stokes, Nayda, & English, 1997).

Fratini (2017) develops another way for option pricing by using an actuarial approach. He called this approach the expected value approach. That is, the value of an option is the discounted expected payoff of the option. Thus, it is like the way of calculating premiums for traditional insurance. This approach is not popularly used in financial option pricing because it requires that the underlying risk is a diversifiable risk which are not true for most financial risks. However, the actuarial approach, unlike the risk neutral approach, does not require the assumption of a liquid market nor tradability. In this paper, we would use both approaches to compute the revenue insurance premiums for comparison.

In pricing revenue insurance, we can assume the revenue as the underlying variable. By assuming that the revenue process follows a geometric Brownian motion (GBM), or the revenue is lognormal distributed, we can derive a pricing formula similar to the Black-Sholes option pricing model. This is one way we calculate the premium of revenue insurance. In addition, as revenue is the product of yield and price, if the yield and price are both lognormal distributed, then the revenue will also follow a lognormal distribution. Thus, the pricing formula can still be applied for computing the premium of revenue insurance by using the parameter information of both yield and price.

Based on which revenue is referred, there are two types of revenue insurance: farm-level revenue insurance, and area-based revenue insurance. One advantage of area-based revenue insurance is to reduce the problem of moral hazard in farm-level revenue insurance. However, there will be disadvantages for the area-based revenue insurance if the farm-level revenue and the area-based one are not highly correlated. In Taiwan, due to lacking accounting information of the farms, area-based revenue insurance is adopted. More often, the average revenue of a county or a town per hectare is used as the underlier. The premium can be calculated on a county basis. However, if the revenue of all counties is underwritten together, for actuarially fair pricing, they should be calculated on an aggregate basis. Thus, in this paper, we also develop a multi-area model for pricing area-based revenue insurance.

For single area revenue insurance, the actuarial fair pricing literally implies that the revenue risk is spread over time. That is, sometimes the revenue is high, and no claim is paid; while during other period the revenue is low, and claims are paid. Thus, the insurance premium is the average claim payment over the time. This is a kind of time diversification albeit that time diversification effect is not accepted by some. In the actuarial fair pricing of multi-area revenue insurance, it is implied that the revenue risk is spread over both time and area, thus the revenue risk can be further diversified if the correlation among the areas is significantly low.

Finally, we also develop the actuarial fair pricing under multiple agricultural products considerations. That is, the revenue risk can be further diversified if the correlation among the products is not high.

The rest of this paper is organized as follows. The two approaches of revenue insurance pricing are compared in Section Two. Revenue insurance pricing models are presented in Section Three. In Section Four, the data for pricing sugar and banana insurance is presented, and the result is analyzed and discussed. The final section is the conclusion.

2. Risk Neutral Approach vs. Actuarial Approach

The advantage of using the risk neutral approach for pricing option is that we can get rid of the problem of estimating the option's risk premium. Though the risk of an option comes from its underlying variable, the riskiness of the option varies depending on the moneyness of the option. Generally, out-of-the-money options have higher risk than those of in-the-money options. Thus, in valuating options, we cannot use the discounted cash flow approach in which the discount rate consists of a constant risk premium. However, because the option's risk can vary, it is impossible to find an appropriate risk premium for discounting. The risk neutral approach solves this problem by changing a real probability measure into a risk neutral probability measure. Thus, under the risk neutral world, we can discount expected cash flows by using a risk-free rate because there is no required risk premium under the risk neutral world.

Similarly, as revenue insurance is a kind of options, it is not suitable to use the discounted cash flow approach for valuation. Thus, using the risk neutral approach to price revenue is considered. However, insurance as pointed out by Stokes, Nayda, & English (1997), pricing revenue insurance is also not proper because of the non-tradable property of the underlying, the revenue. Stokes, Nayda, & English (1997) propose not to use the risk neutral approach by assuming that equilibrium expected rate of return on revenue insurance is observable. However, it is really not. In addition, because of no price formula, their approach is complicate for application.

Fratini (2017) propose an actuarial approach for option pricing. The actuarial approach, like the risk neutral approach, can derive a pricing formula for options and revenue insurance. However, it might have the same problem of the discounted cash flow approach, because the actuarial approach uses the real probability measure for taking expectation. Thus, in pricing revenue insurance, we have to consider the risk premium the insurer requires for compensating the risk taking. However, if the underwriting risk can be fully diversified, then there will be no required risk premium. The risk-free rate can be used as a proper discount rate. Thus, the actuarial approach may not be good for pricing options in which the underlying risk is undiversifiable financial risk. It may be good for revenue insurance when the underlying risk can be diversified. Also, this approach is still good without the assumption that the underlying is tradable.

3. The Pricing Models

3.1 Revenue Insurance as a Put Option

As revenue insurance guarantees a minimum revenue for the insured, it can be treated as a European put option on the revenue thus calls for the application of the well-known Black-Sholes option pricing model. It can be derived by taking the expected payoff of the put option under the risk-neutral world and by discounting the expected payoff with the risk-free rate.

However, by the actuarial approach, the put price is the discounted expected payoff under the real world. Thus, by assuming that the revenue follows a Geometric Brownian Motion, the real revenue process can be written as:

$$dR_t = \mu R_t dt + \sigma R_t dw_t \tag{1}$$

where R_t is the revenue at time *t*, w_t is a Wiener process, and the parameters μ and σ are the drift and volatility, respectively. The integral version of the process becomes

$$R_T = R_t e^{\left(\mu - \frac{\sigma^2}{2}\right)(T-t) + \sigma w_{T-t}} \tag{2}$$

where *t* is the current time, and *T* is some future time. Assuming that the payoff of the revenue insurance at time *T* is $Max(K - R_T, 0)$, where *K* is the strike revenue, or guaranteed minimum revenue. Then the actuarial fair premium is $e^{-r(T-t)}E(Max(K - R_T, 0))$. This is the same as the value of a put option except that the expectation is taken under the real process, not the risk neutral process. Like the Black Scholes put pricing model, we can derive the discounted expected payoff under the real process as

$$PutPrice_t(R_t, K, T) = e^{-r(T-t)}KN(-d2) - R_t e^{(\mu-r)(T-t)}N(-d1) \tag{3}$$
$$d1 = (\ln\left(\frac{R_t}{K}\right) + (\mu + \sigma^2/2)(T-t))/\sigma\sqrt{T-t}$$
$$d2 = d1 - \sigma\sqrt{T-t}$$

where $PutPrice_t$ is the put price at *t*, *T* is the maturity time and *r* is the risk-free rate.

3.2 Revenue Insurance with Coverage Limits

Like other types of insurance, there are often coverage limits in revenue insurance. With the coverage limit, the payoff becomes

$$Min(Max(K - R_T, 0), L)$$

where *L* is the coverage limit. This payoff is the same as the sum of longing a put with the strike price *K* and shorting a put with the strike price *K-L,* where *L*<*K*. That is, the payoff can be written as

$$Max(K - R_T, 0) - Max(K - L - R_T, 0)$$

As one can expect, the put price with a coverage limit is lower than the put without one.

3.3 Pricing with Yield and Price Information

Revenue equals the product of yield and price. By assuming that the yield and the price are lognormal distributed the revenue also follows a lognormal distribution; thus, the pricing formula of the revenue insurance can be derived. Assume the yield process

$$Y_T = Y_t e^{\left(\mu_Y - \frac{\sigma_Y^2}{2}\right)(T-t) + \sigma_Y w_{T-t}^Y}$$

and the price process

$$P_T = P_t e^{\left(\mu_P - \frac{\sigma_P^2}{2}\right)(T-t) + \sigma_P w_{T-t}^P}$$

where Y_t is the yield, P_t is the price at time t, w_t^Y and w_t^P are Wiener processes, with correlation coefficient ρ, μ_Y and σ_Y are the drift and volatility for the yield process, and μ_P and σ_P are the drift and volatility for the price process. Thus, the revenue process can be written as

$$R_T = P_T Y_T = P_t Y_t e^{\left(\mu_P+\mu_Y-\frac{\sigma_P^2}{2}-\frac{\sigma_Y^2}{2}\right)(T-t)+\sigma_P w_{T-t}^P+\sigma_Y w_{T-t}^Y} \quad (4)$$

Comparing Equation (2) with Equation (4), we can derive

$$\mu = \mu_P + \mu_Y + \rho\sigma_P\sigma_Y$$
$$\sigma^2 = \sigma_P^2 + \sigma_Y^2 + 2\rho\sigma_P\sigma_Y$$

Thus, we can still use Equation (2) for pricing the revenue insurance with the yield and price parameters.

3.4 Pricing across Areas or Products

In this subsection, we develop a two-area model of area-based revenue insurance. The two areas can produce one or two products. The one-product model is for pricing across areas while the two-product model is for pricing across products, regardless of area. This model also applies to a two-price case. The two prices can be referred to as two local market prices for one product or the prices of two products, regardless of area.

Assume the prices P_{1t} and P_{2t}, the yields Y_{1t} and Y_{2t}, and the guaranteed minimum revenues K_1 and K_2 are for the two areas or the two products, respectively. Thus, the revenues for the two areas or the two products can be defined as $R_{1t} = P_{1t}Y_{1t}$ and $R_{2t} = P_{2t}Y_{2t}$. Then, the total actuarial fair premium for the two areas or the two products is

$$e^{-r(T-t)}(E(Max(K_1 - R_{1T}, 0) + Max(K_2 - R_{2T}, 0))) =$$
$$e^{-r(T-t)}E(Max(K_1 - R_{1T}, 0)) + e^{-r(T-t)}E(Max(K_2 - R_{2T}, 0)) \quad (5)$$

Thus, the total actuarial fair premium for the two areas or the two products is the sum of the individual actuarial fair premiums which can be computed by using Equation (3). However, if we use standard deviation as a risk measure, the risk of underwriting the revenue insurance for the two areas or the two products together will be less than the sum of underwriting each revenue insurance because of subadditivity

$$STD(e^{-r(T-t)}\left(\left(Max(K_1 - R_{1T}, 0) + Max(K_2 - R_{2T}, 0)\right)\right)) \leq$$
$$STD(e^{-r(T-t)}\left(Max(K_1 - R_{1T}, 0)\right)) + STD(e^{-r(T-t)}(Max(K_2 - R_{2T}, 0))) \quad (6)$$

where STD(.) is the standard deviation function. Thus, there is risk diversification effect across areas or products if there is no perfect correlation between areas or products. Furthermore, like the above inequality, underwriting across different guaranteed minimum revenues can also bring in diversification effect.

4. Data and Results

To facilitate referencing, a summary of the tables is presented here.

Table 1. The Current and Strike Revenue of Banana (unit: NT dollar)

Panel A: Kaohsiung Areas

Area Name	Qishan	Mino	Neimen	Tianliao	Shanlin	Yanchao	Daliao
StrikeRevenue	749,928	744,440	647,498	640,060	717,004	742,612	714,748
CurrentRevenue	558,028	562,014	518,169	520,162	554,042	617,816	547,797

Panel B: Pingtung Areas (I)

Area Name	Gaoshu	Ligang	City	Wandan	Neipu
Strike Revenue	739,014	772,263	736,250	717,939	756,462
Current Revenue	602,692	602,670	615,491	568,059	597,798

Panel C: Pingtung Areas (II)

Area Name	Zhutian	Kanding	Xinpi	Jiadong	Fangliao
Strike Revenue	745,233	720,012	742,733	755,252	713,732
Current Revenue	600,057	582,740	591,487	581,988	587,922

Panel D: Chiayi and Tainan Areas (I)

Area Name	Meishan	Zhuqi	Fanlu	Zhongpu	Liujiao
Strike Revenue	682,662	620,695	627,669	625,438	580,252
Current Revenue	566,734	512,225	480,211	435,391	466,125

Panel E: Chiayi and Tainan Areas (II)

Area Name	Lucao	Dalin	Minxiong	Shuishang	Zuozhen
Strike Revenue	608,216	623,020	571,876	535,611	564,902
Current Revenue	443,928	469,540	480,211	512,225	496,218

Panel F: Yunlin Areas

Area Name	Linnei	Citong	Douliu	Huwei
Strike Revenue	959,451	476,099	641,060	611,759
Current Revenue	853,708	691,504	491,736	424,506

Panel G: Hualien Areas

Area Name	Shoufeng	Fenglin	Guangfu	Ruisui
Strike Revenue	502,135	613,721	476,410	535,611
Current Revenue	352,155	528,232	466,722	546,373

Panel H: Taitung Areas

Area Name	City	Luye	Guanshan	Chenggong
Strike Revenue	627,669	89,841	624,564	314,672
Current Revenue	640,281	420,067	433,385	242,026

Note: The current revenue is the actual revenue in 2019. The strike revenue is the average revenue based on the past five years (from 2015 to 2019) by excluding the maximum and minimum of the five years.

Table 2. The Current and Strike Revenue of Sugar Apple (unit: NT dollar)

Panel A: Type I Sugar Apple in Taitung Areas

Area Name	Taitung City	Penan	Taima	Luye	Tonho
Strike Revenue	892,742	849,405	1,101,742	801,157	691,283
Current Revenue	981,546	774,904	1,120,598	619,924	711,190

Panel B: Type II Sugar Apple in Taitung Areas

Area Name	Taitung City	Penan	Taima	Luye	Tonho
Strike Revenue	606,464	758,080	641,032	781,429	549,911
Current Revenue	981,546	774,904	1,120,598	619,924	711,190

Note: The current revenue is the actual revenue in 2019. The strike revenue is the average revenue based on the past five years (from 2015 to 2019) by excluding the maximum and minimum of the five years.

Table 3. The Basic Statistics of Log Revenue Difference of Banana

Panel A: Kaohsiung Areas

Statistics	Qishan	Mino	Neimen	Tianliao	Shanlin	Yanchao	Daliao
Mean	0.0812	0.0840	0.0913	0.1027	0.0994	0.1225	0.0852
Standard Deviation	0.3678	0.3648	0.4136	0.4145	0.4178	0.4440	0.4121
Skewness	-0.0545	-0.5721	-0.6425	-0.7673	-0.3741	-0.5545	-0.1640
Excess Kurtosis	-0.1826	-0.1006	-0.3102	-0.1518	0.1909	-0.4424	-0.2514

Panel B: Pingtung Areas (I)

Statistics	Gaoshu	Ligang	City	Wandan	Neipu
Mean	0.0732	0.0682	0.0704	0.0537	0.0594
Standard Deviation	0.4117	0.4061	0.3956	0.3860	0.3724
Skewness	-0.0519	-0.4010	0.7478	-0.0853	0.1809
Excess Kurtosis	-0.4426	-0.3621	-0.7449	-0.8105	-0.6928

Panel C: Pingtung Areas (II)

Statistics	Zhutian	Kanding	Xinpi	Jiadong	Fangliao
Mean	0.0554	0.0727	0.0631	0.0464	0.1237
Standard Deviation	0.3783	0.3632	0.3953	0.3408	0.4076
Skewness	0.1661	-0.3359	-0.4146	-1.0475	0.3086
Excess Kurtosis	-0.8698	-0.4828	-0.4863	-0.4364	-0.8098

Panel D: Chiayi and Tainan Areas (I)

Statistics	Meishan	Zhuqi	Fanlu	Zhongpu	Liujiao
Mean	0.1339	0.1121	0.1097	0.1168	0.1280
Standard Deviation	0.4492	0.4024	0.3963	0.4855	0.4551
Skewness	0.4825	0.7361	0.4255	0.1792	-0.4620
Excess Kurtosis	0.0417	-0.3785	-0.6444	-0.3299	-0.3449

Panel E: Chiayi and Tainan Areas (II)

Statistics	Lucao	Dalin	Minxiong	Shuishang	Zuozhen
Mean	0.0885	0.1123	0.1430	0.1627	0.1554
Standard Deviation	0.4016	0.4664	0.4827	0.5377	0.4702
Skewness	1.0627	2.4237	-0.5379	0.2519	-0.8421
Excess Kurtosis	-0.3449	-0.8070	0.8803	0.0493	0.5377

Panel F: Yunlin Areas

Statistics	Linnei	Citong	Douliu	Huwei
Mean	0.1665	0.1153	0.0654	0.0683
Standard Deviation	0.4887	0.3989	0.4068	0.3847
Skewness	-0.3295	-0.3102	0.9007	-0.2559
Excess Kurtosis	-0.1596	-0.0198	0.9573	-0.7039

Panel G: Hualien Areas

Statistics	Shoufeng	Fenglin	Guangfu	Ruisui
Mean	0.1360	0.2381	0.1437	0.1771
Standard Deviation	0.5130	0.6070	0.4902	0.5122
Skewness	-0.6815	-1.0221	0.7083	2.7832
Excess Kurtosis	-0.4689	0.1408	-0.0417	0.5787

Panel H: Taitung Areas

Statistics	City	Luye	Guanshan	Chenggong
Mean	0.0734	0.0723	0.0472	0.2068
Standard Deviation	0.4410	0.4856	0.4506	0.6852
Skewness	0.0290	-0.3768	0.5354	2.5848
Excess Kurtosis	-0.2803	0.1297	-0.6104	0.4934

Table 4. The Basic Statistics of Log Revenue Difference of Sugar Apple
Panel A: Type I Sugar Apple in Taitung Area

Statistics	City	Penan	Taima	Luye	Tonho
Mean	0.0629	0.0640	0.1214	0.0181	0.0211
Standard Deviation	0.2593	0.3309	0.4051	0.1403	0.2046
Skewness	3.7325	3.1654	3.5314	0.5847	0.3668
Excess Kurtosis	1.2350	1.0423	-0.5429	-0.6651	-0.5294

Panel B: Type II Sugar Apple in Taitung Areas

Statistics	City	Penan	Taima	Luye	Tonho
Mean	0.0229	0.0637	0.0864	0.0499	0.0308
Standard Deviation	0.2306	0.3305	0.4136	0.2176	0.3086
Skewness	1.3866	1.1274	2.8093	2.3031	0.6933
Excess Kurtosis	0.5498	0.325	-0.4165	-0.8193	-0.6288

Table 5.1 The Pricing Results of Banana Insurance (Unit: NT dollar)
Panel A: Kaohsiung Areas (Actuarial Approach)

% of K	Qishan	Mino	Neimen	Tianliao	Shanlin	Yanchao	Daliao
100%	180,826	173,001	141,930	132,778	164,368	143,835	170,148
95%	155,392	147,860	120,954	112,608	141,813	123,470	147,068
90%	130,385	123,364	101,004	93,580	119,757	103,749	124,417
85%	106,564	100,228	82,482	76,040	98,779	85,141	102,805
80%	84,585	79,052	65,685	60,237	79,386	68,065	82,771

Panel B: Kaohsiung Areas (Risk Neutral Approach)

% of K	Qishan	Mino	Neimen	Tianliao	Shanlin	Yanchao	Daliao
100%	207,612	200,836	167,409	161,141	194,175	179,210	195,621
95%	181,212	174,491	144,695	138,871	170,265	156,873	171,442
90%	154,548	148,104	122,623	117,359	146,227	134,530	147,139
85%	128,496	122,529	101,721	97,089	122,751	112,771	123,421
80%	103,882	98,542	82,400	78,431	100,505	92186	100,964

Panel C: Kaohsiung Areas (At the Money)

% of K	Qishan	Mino	Neimen	Tianliao	Shanlin	Yanchao	Daliao
100%	62,797	62,012	65,768	64,069	69,617	78,781	70,411
95%	50,127	49,378	53,692	25,518	56,874	64,976	57,512
90%	39,008	38,315	42,906	41,653	45,483	52,490	45,976
85%	29,468	28,849	33,446	32,407	35,489	41,427	35,852
80%	21,500	20,968	25,326	24,489	26,907	31,835	27,156

Note: The Prices in Panel C are calculated by assuming that the current revenue is equal to the strike revenue.

Table 5.2 The Pricing Results of Banana Insurance (Unit: NT dollar)

Panel A: Pingtung Areas (I) (Actuarial Approach)

% of K	Gaoshu	Ligang	City	Wandan	Neipu
100%	158,041	178,989	146,988	165,910	168,905
95%	135,639	155,256	124,765	142,103	144,289
90%	113,848	131,670	103,473	118,983	120,320
85%	93,240	108,908	83,644	97,174	97,695
80%	74,312	87,617	65,711	77,195	77,002

Panel B: Pingtung Areas (I) (Risk Neutral Approach)

% of K	Gaoshu	Ligang	City	Wandan	Neipu
100%	179,590	199,696	167,902	181,637	187,537
95%	156,029	175,231	144,312	156,988	161,995
90%	132,644	150,422	121,258	132,694	136,666
85%	110,091	126,003	99,379	109,457	112,334
80%	88,991	102,730	79,228	87,888	89,709

Panel C: Pingtung Areas (I) (At the Money)

% of K	Gaoshu	Ligang	City	Wandan	Neipu
100%	79,988	79,793	78,409	73,911	73,234
95%	65,520	65,245	63,762	59,970	58,926
90%	52,510	52,181	50,667	47,543	46,256
85%	41,049	40,690	39,212	36,692	35,286
80%	31,176	30,811	29,429	27,440	26,031

Note: The Prices in Panel C are calculated by assuming that the current revenue is equal to the strike revenue.

Table 5.3 The Pricing Results of Banana Insurance (Unit: NT dollar)

Panel A: Pingtung Areas (II) (Actuarial Approach)

% of K	Zhutian	Kanding	Xinpi	Jiadong	Fangliao
100%	162,806	147,385	166,750	177,364	133,768
95%	138,825	123,997	143,205	150,878	113,041
90%	115,595	101,813	120,210	125,117	93,395
85%	93,762	81,375	98,399	100,872	75,268
80%	73,861	63,085	78,330	78,789	58,997

Panel B: Pingtung Areas (II) (Risk Neutral Approach)

% of K	Zhutian	Kanding	Xinpi	Jiadong	Fangliao
100%	179,593	170,277	185,711	192,197	170,837
95%	154,679	145,312	161,232	165,004	147,559
90%	130,152	121,117	136,886	138,159	124,728
85%	106,740	98,365	113,391	112,538	102,962
80%	85,084	77,595	91,412	88,888	82,803

Panel C: Pingtung Areas (II) (At the Money)

% of K	Zhutian	Kanding	Xinpi	Jiadong	Fangliao
100%	75,826	66,327	76,951	66,999	66,471
95%	61,269	52,905	62,631	53,033	53,796
90%	48,328	41,122	49,828	40,827	42,559
85%	37,074	31,017	38,619	30,423	32,802
80%	27,528	22,584	29,031	21,818	24,525

Note: The Prices in Panel C are calculated by assuming that the current revenue is equal to the strike revenue.

Table 5.4 The Pricing Results of Banana Insurance (Unit: NT dollar)

Panel A: Chiayi and Tainan Areas (I) (Actuarial Approach)

% of K	Meishan	Zhuqi	Fanlu	Zhongpu	Liujiao
100%	132,907	119,492	142,459	182,814	125,691
95%	113,529	100,339	121,091	161,422	107,428
90%	95,022	82,530	100,865	140,092	90,222
85%	77,779	66,325	82,152	119,355	74,317
80%	62,113	51,896	65,212	99,691	59,876

Panel B: Chiayi and Tainan Areas (I) (Risk Neutral Approach)

% of K	Meishan	Zhuqi	Fanlu	Zhongpu	Liujiao
100%	169,757	149,952	174,052	212,986	157,382
95%	147,955	128,233	150,556	190,653	136,740
90%	126,450	107,536	127,746	167,790	116,840
85%	105,786	88,253	106,138	145,021	98,048
80%	86,458	70,667	86,122	122,970	80,621

Panel C: Chiayi and Tainan Areas (I) (At the Money)

% of K	Meishan	Zhuqi	Fanlu	Zhongpu	Liujiao
100%	71,367	58,951	54,527	63,691	60,785
95%	58,807	47,667	43,968	53,325	50,232
90%	47,505	37,679	34,642	43,889	40,730
85%	37,517	29,011	26,570	35,414	32,308
80%	28,861	21,660	19,747	27,927	24,980

Note: The Prices in Panel C are calculated by assuming that the current revenue is equal to the strike revenue.

Table 5.5 The Pricing Results of Banana Insurance (Unit: NT dollar)

Panel A: Chiayi and Tainan Areas (II) (Actuarial Approach)

% of K	Lucao	Dalin	Minxiong	Shuishang	Zuozhen
100%	159,716	158,280	115,213	89,066	98,216
95%	137,403	137,667	98,449	75,857	82,924
90%	116,003	117,614	82,711	63,611	68,786
85%	95,925	98,564	68,204	52,427	55,939
80%	77,464	80,874	55,060	42,362	44,465

Panel B: Chiayi and Tainan Areas (II) (Risk Neutral Approach)

% of K	Lucao	Dalin	Minxiong	Shuishang	Zuozhen
100%	184,425	187,707	148,439	120,111	133,096
95%	160,700	165,590	129,030	104,019	114,625
90%	137,510	143,547	110,367	88,775	97,106
85%	115,377	122,141	92,776	74,565	80,787
80%	94,685	101,862	76,485	61,505	65,836

Panel C: Chiayi and Tainan Areas (II) (At the Money)

% of K	Lucao	Dalin	Minxiong	Shuishang	Zuozhen
100%	54,693	65,902	65,385	77,480	63,075
95%	44,410	54,833	54,496	65,624	52,231
90%	35,266	44,803	44,629	54,728	42,460
85%	27,287	35,850	35,814	44,847	33,790
80%	20,482	27,995	28,071	36,013	26,234

Note: The Prices in Panel C are calculated by assuming that the current revenue is equal to the strike revenue.

Table 5.6 The Pricing Results of Banana Insurance (Unit: NT dollar)

Panel A: Yunlin Areas (Actuarial Approach)

% of K	Linnei	Citong	Douliu	Huwei
100%	145,322	12,652	160,119	180,072
95%	126,800	9,388	137,685	156,060
90%	108,432	6,761	116,069	132,735
85%	90,595	4,706	95,750	110,600
80%	73,693	3,148	77,091	90,015

Panel B: Yunlin Areas (Risk Neutral Approach)

% of K	Linnei	Citong	Douliu	Huwei
100%	193,195	20,392	178,106	199,251
95%	172,877	15,521	154,627	174,372
90%	151,880	11,490	131,661	149,839
85%	130,597	8,237	109,776	126,247
80%	109,519	5,689	89,418	104,031

Panel C: Yunlin Areas (At the Money)

% of K	Linnei	Citong	Douliu	Huwei
100%	104,734	77,436	65,832	52,649
95%	88,960	62,656	53,830	42,552
90%	73,957	49,462	43,081	33,595
85%	60,012	37,980	33,628	25,804
80%	47,384	28,253	25,493	19,202

Note: The Prices in Panel C are calculated by assuming that the current revenue is equal to the strike revenue.

Table 5.7 The Pricing Results of Banana Insurance (Unit: NT dollar)

Panel A: Hualien Areas (Actuarial Approach)

% of K	Shoufeng	Fenglin	Guangfu	Ruisui
100%	147,706	122,857	69,526	69,670
95%	129,457	107,584	58,257	58,301
90%	111,983	92,739	47,994	47,964
85%	95,456	78,593	38,772	38,705
80%	79,994	65,395	30,617	30,547

Panel B: Hualien Areas (Risk Neutral Approach)

% of K	Shoufeng	Fenglin	Guangfu	Ruisui
100%	177,134	173,755	93,902	101,431
95%	157,130	155,592	80,044	86,645
90%	137,678	137,224	67,181	72,864
85%	119,022	119,040	55,385	60,211
80%	101,308	101,465	44,717	48,764

Panel C: Hualien Areas (At the Money)

% of K	Shoufeng	Fenglin	Guangfu	Ruisui
100%	53,022	81,550	64,824	74,511
95%	44,693	69,937	54,171	62,524
90%	37,074	59,125	44,496	51,580
85%	30,191	49,207	35,831	41,745
80%	24,066	40,235	28,197	33,050

Note: The Prices in Panel C are calculated by assuming that the current revenue is equal to the strike revenue.

Table 5.8 The Pricing Results of Banana Insurance (Unit: NT dollar)

Panel A: Taitung Areas (Actuarial Approach)

% of K	City	Luye	Guanshan	Chenggong
100%	85,718	247,473	198,524	90,133
95%	70,982	225,473	175,552	79,981
90%	57,570	201,919	152,519	70,278
85%	45,621	177,363	130,042	61,062
80%	35,206	152,541	108,661	52,372

Panel B: Taitung Areas (Risk Neutral Approach)

% of K	City	Luye	Guanshan	Chenggong
100%	101,123	264,733	209,937	114,198
95%	84,660	243,159	186,635	102,480
90%	69,463	219,616	163,032	91,153
85%	55,732	194,607	139,792	80,258
80%	43,592	168,878	117,512	69,840

Panel C: Taitung Areas (At the Money)

% of K	City	Luye	Guanshan	Chenggong
100%	85,716	68,210	68,525	47,288
95%	70,982	57,447	57,227	42,168
90%	57,570	47,585	46,930	35,616
85%	45,621	38,666	37,683	30,350
80%	35,206	30,724	29,522	25,486

Note: The Prices in Panel C are calculated by assuming that the current revenue is equal to the strike revenue.

Table 6. The Pricing Results of Sugar Apple Insurance (Unit: NT dollar)

Panel A: Type I Taitung Areas (Actuarial Approach)

% of K	City	Penan	Taima	Luye	Tonho
100%	40,150	109,652	92,055	168,737	41,533
95%	28,262	90,680	77,646	132,573	27,977
90%	18,867	72,567	63,804	97,655	17,639
85%	11,853	55,917	50,853	66,249	10,277
80%	6,942	41,258	39,107	40,297	5,447

Panel B: Type I Taitung Areas (Risk Neutral Approach)

% of K	City	Penan	Taima	Luye	Tonho
100%	53,999	127,791	122,340	174,433	45,062
95%	39,193	107,623	106,036	138,182	30,664
90%	27,033	87,822	89,733	102,800	19,552
85%	17,589	69,098	73,824	70,612	11,536
80%	10,697	52,128	58,746	43,628	6,201

Panel C: Type I Taitung Areas (At the Money)

% of K	City	Penan	Taima	Luye	Tonho
100%	70,665	76,906	97,061	29,378	50,806
95%	52,759	61,273	82,285	16,905	34,990
90%	37,455	47,162	67,988	8,590	22,606
85%	25,095	34,908	54,510	3,747	13,533
80%	15,727	24,711	42,187	1,355	7,393

Note: The Prices in Panel C are calculated by assuming that the current revenue is equal to the strike revenue.

According to the pricing formula, we need the information of current revenue R_t, the strike revenue *K*, and the related parameters for pricing. Table 1 and Table 2 report the current revenue and strike revenue for banana and sugar apple, respectively. The current revenue is the actual revenue in 2019. The strike revenue is the average revenue based on the past five years (from 2015 to 2019) by excluding the maximum and minimum of the five years. For banana, the current revenues are much lower than the strike revenues in most areas. Thus, as the revenue insurance is like a put option, the premiums of revenue insurance tend to be high under this situation. For both types of sugar apple, the current revenue is much lower than the strike revenue only in one area, Luye. Thus, other things equal, the premium in Luye should be higher than those of others.

Table 6. The Pricing Results of Sugar Apple Insurance (Unit: NT dollar) (Continued)

Panel A: Type II Taitung Areas (Actuarial Approach)

% of K	City	Penan	Taima	Luye	Tonho
100%	112,238	111,662	131,953	94,159	137,005
95%	88,041	92,509	114,881	70,334	115,259
90%	66,040	74,176	97,706	49,651	93,989
85%	47,035	57,282	80,879	32,832	74,000
80%	31,501	42,370	64,892	20,103	55,993

Panel B: Type II Taitung Areas (Risk Neutral Approach)

% of K	City	Penan	Taima	Luye	Tonho
100%	118,232	129,768	153,113	112,486	144,646
95%	93,449	109,465	135,493	86,354	122,563
90%	70,681	89,484	117,273	62,798	100,708
85%	50,807	70,543	98,912	42,896	79,936
80%	34,382	53,337	80,956	27,227	61,015

Panel C: Type II Taitung Areas (At the Money)

% of K	City	Penan	Taima	Luye	Tonho
100%	49,144	75,618	79,245	50,527	59,560
95%	35,255	60,143	65,724	35,001	46,369
90%	24,015	46,219	53,163	22,810	34,905
85%	15,382	34,161	41,795	13,839	25,292
80%	9,151	24,153	31,822	7,712	17,524

Note: The Prices in Panel C are calculated by assuming that the current revenue is equal to the strike revenue.

Table 3 and Table 4 list the statistics for banana and sugar apple, respectively. The volatility parameter σ is estimated by the standard deviation of log revenue differences and the mean parameter μ by the mean of log revenue differences plus one half of the standard deviation squared. These two parameters indicate that both the growth rate and the volatility vary across areas and products. In general, the growth rates are positive, and the revenues are highly volatile. In addition, the skewness and the excess kurtosis are also in the tables. In general, they don't deviate a lot from zero. Thus, the lognormal distribution could be reasonably assumed in the pricing model.

By using the put option pricing formula of the actuarial approach and the risk neutral approach, the result of banana and sugar apple revenue insurance is reported in Table 5.1 to 5.8 and 6. In each table, Panel A and Panel B report the pricing result of the actuarial approach and the risk neutral approach, respectively. Because the growth rates are often higher than the risk-free rate, the premiums calculated from the actuarial approach are generally lower than those from risk neutral approach. However, it is important to know that the premiums of the actuarial approach do not consider risk premium. Except that the underwriting risk of revenue insurance could be fully diversified, the risk premium should be added up. Without considering the risk premium, the premiums of the actuarial approach are already much higher than the premiums currently charged in practice in general. It is because the current revenues used for calculation are generally much lower than the strike revenues.

The current revenue of 2019 was relatively low, making the put option in-the-money which is rarely bought in practice as an insurance. To solve this, we calculate another premium by assuming that the current revenue equals the strike revenue and using the actuarial approach. Under this situation, the revenue insurance is an at-the-money put option.

The pricing result is presented in Panel C of Table 5.1 to 5.8 and 6. The premiums are closer than before, but still higher than the premiums currently charged. It could be that we use the annual revenues data dating back to fifteen (15) years and the revenue is more volatile during this period.

5. Conclusion

In this paper, we employ Fratini's (2017) actuarial approach to calculate the premium of revenue insurance which can be regarded as a put option. This approach is different from the popular risk neutral approach to option pricing, in which the underlying has to be a tradable asset. Because the underlying of the revenue insurance is the revenue of some product and the revenue is not traded in any market, the actuarial approach is more appropriate than the risk neutral. The actuarial approach is the same as the traditional insurance pricing. In computing option prices, we take the discounted expected value under real-world probability. This is different from the risk neutral approach which computes the option prices by taking the discounted expected value under risk-neutral probabilities.

By using the actuarial approach to price revenue insurance, we derive a price formula which is like the Black Scholes option pricing formula. In addition, we also derive the pricing formula of revenue insurance with a coverage limit. The revenue insurance with a coverage limit can be decomposed as the difference of two put options. Thus, the pricing formula can be still applied. Furthermore, we decompose the revenue process into a price process and a yield process and develop another pricing formula using the parameters of price and yield.

To apply the pricing formula empirically, we use the data of sugar apple and banana revenue insurance in Taiwan. We calculate the premiums for several areas which have already implemented revenue insurance. The result shows that the premiums are generally lower than the risk neutral approach. Because revenue is often non-tradable, thus, if the individual revenue risk can be fully diversified, then the actuarial approach should be suggested for premiums calculation.

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