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# **Evaluating the Efficiency of Transportation Services on Intermodal Commuter Networks**

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# Evaluating the Efficiency of Transportation Services on Intermodal Commuter Networks

M. P. Boile<sup>1</sup>

## Abstract

The Intermodal Surface Transportation Efficiency Act (ISTEA) of 1991 and its successor the Transportation Equity Act for the 21<sup>st</sup> Century (TEA-21) changed the scope of the transportation planning process from evaluating new regional transportation facilities to developing strategies which promote more efficient use of the existing transportation infrastructure, and created a need for new and improved analytical tools to be used in the analysis and evaluation of intermodal networks. Transportation plans that are developed must consider a range of transportation options designed to meet the transportation needs of a state including all modes and their connections. Transportation planners need to investigate programs aimed at reducing our reliance on single-occupant vehicles and making alternatives such as transit, high-occupancy vehicle lanes, bicycle and pedestrian facilities a more important part of the transportation program. This paper presents an efficient method for analyzing and evaluating intermodal commuter networks, modeling interactions between modes, making predictions regarding future network activity in terms of traffic volumes and travel costs, and aiding the decision making process in terms of future transportation plans by evaluating alternative policies for improving the efficiency of high occupancy modes, mitigating congestion, reducing energy consumption and air pollution.

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## Introduction

The transportation-related provisions of the Clean Air Act Amendments (CAAA) of 1990, the Intermodal Surface Transportation Efficiency Act (ISTEA) of 1991 and its successor the Transportation Equity Act for the 21<sup>st</sup> Century (TEA-21) create a need for new and improved analytical tools to be used in the analysis and evaluation of intermodal networks. An intermodal network can be defined as an integrated transportation system consisting of two or more modes, which are connected through facilities that allow travelers and freight to transfer from one mode to another during a trip from an origin to a destination. Intermodal networks aim to provide efficient, seamless transport of people and goods from one place to another.

The new legislation changes the scope of the transportation planning process from evaluating new regional transportation facilities to developing strategies which promote more efficient use of the existing transportation infrastructure while enhancing air quality. Transportation plans that are developed must consider a range of transportation options designed to meet the transportation needs of a state including all modes and their connections. Transportation planners need to investigate programs aimed at reducing our reliance on single-occupant vehicles and making alternatives such as transit, high-occupancy vehicle lanes, bicycle and pedestrian facilities a more important part of the transportation program.

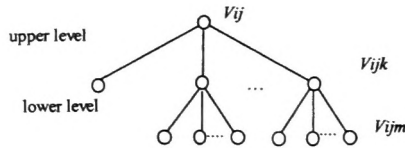
Most of the widely used applications, which are based on a sequential set of analyses, were developed to analyze capacity expansions. These applications do not have the ability to determine the interactions between different transportation modes serving the same network and evaluate the effects of a change in the service provided by one mode, to the performance of the other, competing modes. Existing applications are not suitable for meeting the new legislative requirements or performing tasks such as evaluating congestion pricing, transportation control measures, alternative development patterns or motor vehicle emissions (Shunk, 1992).

This paper presents an efficient method for analyzing and evaluating intermodal commuter networks, modeling interactions between modes, making predictions regarding future network activity in terms of traffic volumes and travel costs, and aiding the decision making process in terms of future transportation plans by evaluating alternative policies for improving the efficiency of high occupancy modes, mitigating congestion, reducing energy consumption and air pollution.

## Theoretical Background and Previous Developments

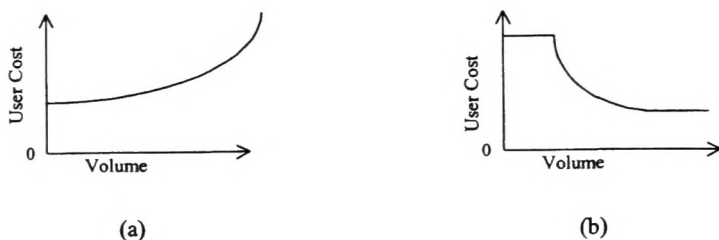
The specific problem of concern in this paper is that of network equilibrium modeling for intermodal transportation planning applications. *Network equilibrium* is defined (Friesz, 1985) as a nonnegative flow pattern occurring on a given network which is consistent with market clearing (i.e. with supply equals demand) and with postulated behavioral principles describing decision makers active on the network, such as the *user equilibrium* principle (Wardrop, 1952). The user equilibrium principle states that for every origin-destination pair on a network, the journey times of all utilized routes are equal, and less than those which would be experienced on any unused route. A more general expression of this statement considers a generalized cost, disutility, or negative utility function including monetary, qualitative and time costs as the journey impedance. *Discrete choice models*, also known as *random utility models*, describe the choices of individuals between competing alternatives (Domencich and McFadden, 1975; Oppenheim, 1995). *Nested logit discrete choice models* may be used to formulate the mode choice using various levels and groups of similar characteristics. For example, the upper level of the general model shown in Figure 1 may be used to model traveler's choices ( $V_{ij}$  being the number of travelers between origin  $i$  and destination  $j$ ) of using rail, auto, bus, or any other mode

group  $k$ . Assuming that a traveler uses auto, the second level would determine the choice of driving alone, using a 2-, 3-, or more-person carpool or any other alternative  $m$  within the same group. For travelers using bus, the lower level would model the choice of a specific bus route among the available alternatives  $m$  within the group.



**Figure 1** Nested Logit Model Structure

Performance, or supply functions describe the relationship between flow, capacity and level of service-price. Typically, *average user cost-volume relationships* are used to describe the performance of transport systems. Factors that need to be considered in a motorist's average user cost function include travel time, comfort and safety, which can collectively be referred to as level of service, and tolls, parking fees and some of the operating and maintenance costs of the vehicle which comprise the out of pocket, monetary costs. A transit user's cost function would consist of similar factors, including travel time, comfort and safety, and fares as the out of pocket cost. Depending on the assumed behavior of management of transportation facilities in modifying characteristics under its control, several types of user cost-volume functions have been developed (Morlok, 1978 and 1979). The two types that are used in this paper are: *Type I* which presumes only the volume of traffic varies with all characteristics of the facility or carrier service under the control of management fixed and *Type II* which includes managerial responses to volume variations. Figure 2 shows examples of Type I and Type II functions.



**Figure 2.** User Cost-Volume Relationships for Users of Transportation Facilities

Part (a) of Figure 2 shows a Type I function which represents a system or facility for which the user cost is an increasing function of volume. This function can be representative of a congested highway segment. Examples of this function can be found in Levinson et al. (1975) and the US Bureau of Public Roads (1964) congestion curves. Part (b) shows a Type II function which represents a system or facility in which changes in the operating plan can be introduced based on considerations of volume. This function can be representative of a public transit system. As an example, the management of a bus or train service can schedule departure frequencies and other characteristics such as vehicle size and accelerated operating regimes on a route primarily on the basis of demand for transport.

The user equilibrium principle was formulated as a mathematical programming problem by Beckmann, McGuire, and Winsten (1956), who used the Kuhn-Tucker conditions to show that the solution to this problem is equivalent to the user equilibrium conditions. A computational algorithm which may be used to solve this problem was developed the same year by Frank and Wolfe (1956). Numerous mathematical formulations and efficient algorithms have been developed to model transportation networks (Florian and Nguyen, 1974; Evans, 1976; Florian, 1977; Abdulaal and LeBlanc, 1979; Boyce, 1980; LeBlanc and Farhangian, 1981; Fisk and



Nguyen, 1981; LeBlanc and Abdulaal, 1982; Dafermos, 1982; Boyce et al., 1982; Florian and Spiess, 1983; Boyce, 1984; Boyce and Zhang, 1996). These papers present models which consider either one- or two-mode networks. When two modes are considered, traffic is assigned over modal networks. None of the models considers connections between modes and as a result they do not apply to intermodal networks. The first network equilibrium models which explicitly consider and analyze intermodal networks are presented in Fernandez et al. (1994), and Boile et al. (1995). Fernandez et al. (1994) presents model formulations which consider two alternative modes available at each origin of the network. The alternatives are either auto and metro or auto and combined (auto-to-metro) modes. Combined modes are considered only at those origins where metro is not available. Boile et al. (1995) considers intermodal trips as an option at every origin of the network. This complicates the mathematical formulation and solution of the problem, however, it captures a common fact in most US urban areas, that even when a traveler has an option to walk to a nearby train station, he/she may prefer to drive to or be dropped off at another station along the route.

The performance of a transportation system is typically described through Type I functions. Type II functions were developed with a view toward their inclusion in network equilibrium models (Morlok, 1978). The need to use these functions in network equilibrium models is also evident in a conjecture by Mogridge (1985) who indicated that the only way to increase the road speed within and around a central conurbation is to increase the speed of the rail (or other high capacity) system. It seems appropriate to develop models that can predict such an equilibrium, if rail frequencies are adjusted in relation to transit demand. The only attempt to develop such models by including Type II functions in a passenger network equilibrium context is presented in Boile (1995) and Boile and Spasovic (1999).

Nested logit models have been tested and used in the estimation of travel volumes by mode, transit station, or both (Fan et al., 1993; Miller, 1993; Forinash and Koppleman, 1993; and Ortuzar, 1983). These models, however, only formulate the demand side and have not been implemented within a demand-supply network equilibrium context. The properties of these models are discussed in Hartley and Ortuzar (1980) and McFadden (1979).

This paper presents a network equilibrium model for intermodal transportation network planning. The supply side of the model uses Type I functions for systems that are subject to congestion, such as highways, and Type II functions for rail transit, to capture the effect that transit operators can adjust the rail service to better meet the expected demand. The demand side of the model uses a nested logit function the upper level of which determines traveler's preference between auto and transit and the lower level determines the choice between walking to a train station (pure rail trip) or driving to a station (intermodal trip). Walk is the only access to auto considered in this model (auto trip). The model may be used to analyze intermodal commuter corridors and evaluate operating and pricing policies aimed at improving the efficiency of the transportation service provided in these corridors.

## **Model Formulation**

### **General Statement and Equilibrium Conditions**

The general expression of the network equilibrium model presented in this section is:

$$\min z, \text{ s.t.: } a_i x = b_i, \quad x \geq 0$$

where  $z$  is a non-linear objective function and  $a_i x = b_i$  is a set of linear constraints. A solution to this model is obtained using the Lagrangian method. The Lagrangian of the model is formulated

by multiplying the constraints with Lagrangian multipliers  $u_i$ , and introducing them in the objective function. The mathematical program then becomes equivalent to:

$$\min L = z + \sum_i u_i (b_i - a_i x), \quad x \geq 0$$

A solution is obtained by estimating the derivatives of the Lagrangian with respect to the decision variables, setting them equal to zero and solving the resulting equations. In mathematical terms, the Lagrangian multiplier  $u_i$  represents the shadow price for constraint  $i$  the value of which indicates the marginal change in the value of the objective function as a result of a marginal change in the right-hand-side of constraint  $i$ . The resulting solution must satisfy the following three equilibrium conditions:

First, for each trip type, no traveler has an incentive to unilaterally change routes for s/he cannot reduce her/his travel cost. This condition takes the mathematical form:

$$GC_{pk}^{ij} - GC_k^{ij} \begin{cases} = 0 & \text{if } f_{pk}^{ij} \geq 0 \\ \geq 0 & \text{if } f_{pk}^{ij} = 0 \end{cases} \quad \forall k, i, j$$

This condition indicates that a type  $k$  path  $p$  from origin  $i$  to destination  $j$  is utilized (i.e., has a non negative flow, or  $f_{pk}^{ij} \geq 0$ ) only if the generalized cost on this path  $GC_{pk}^{ij}$  is equal to the minimum generalized cost for type  $k$  trips for that O-D pair  $GC_k^{ij}$ .

Second, no transit user has an incentive to change trip type within each mode (i.e., no traveler has an incentive to change access type (walk or drive) to transit) for s/he cannot further reduce his/her travel cost. In this case, for each O-D pair, the difference between the generalized cost for intermodal and rail trips is given as:

$$GC_M^{ij} - GC_R^{ij} = -\frac{1}{\beta_1} \left( \ln \frac{T_M^{ij}}{T_R^{ij}} + \alpha_{MR} \right)$$

where:  $T_M^{ij}$  and  $T_R^{ij}$  represent the number of travelers between  $i$  and  $j$  using intermodal ( $M$ ) or rail ( $R$ ), respectively, and  $\alpha_{MR}$ ,  $\beta_1$  are model parameters.

Finally, no traveler has an incentive to change mode for s/he cannot reduce his/her travel cost. The difference between the generalized cost for transit ( $T$ ) and auto ( $A$ ) trips is given as:

$$GC_T^{ij} - GC_A^{ij} = -\frac{1}{\beta_2} \left( \ln \frac{T_T^{ij}}{T_A^{ij}} + \alpha_{TA} \right)$$

### Demand Function

The upper level of the nested logit function (D1) used in this model performs the choice between auto and transit modes and has the form:

$$T_A^{ij} = T^{ij} \frac{1}{1 + \exp(U_T^{ij} - U_A^{ij})}$$

Where the number of auto users between  $i$  and  $j$  ( $T_A^{ij}$ ) is given as a function of the total number of travelers between  $i$  and  $j$  ( $T^{ij}$ ) and the utilities for transit and auto. The transit and auto utilities are given as functions of the generalized cost, as follows:  $U_T^{ij} = -\beta_2 GC_T^{ij} - \alpha_{TA}$ , and  $U_A^{ij} = -\beta_2 GC_A^{ij}$ .

The lower level (D2) performs the choice, within transit, between rail and intermodal trips and has the form:

$$T_R^{ij} = T_T^{ij} \frac{1}{1 + \exp(U_M^{ij} - U_R^{ij})}$$

Where the utilities for intermodal and rail are given as:  $U_M^{ij} = -\beta_1 GC_M^{ij} - \alpha_{MR}$ , and  $U_R^{ij} = -\beta_1 GC_R^{ij}$ , respectively.

## Supply Function

A typical Type I generalized cost function of a link  $z$  has the form:

$$c(x_z) = OOP_z + VOTT * ff_z * (1 + 0.15 \frac{x_z^4}{cap_z^4})$$

where:  $OOP_z$  is the out of pocket cost on link  $z$ ,  $VOTT$  is the value of travel time, used to express time in monetary terms,  $ff_z$  is the free flow travel time,  $x_z$  is the flow, and  $cap_z$  is capacity on link  $z$ . A typical Type II generalized cost function of a link  $r$  has the form:

$$c(x_r, h_r) = OOP_r + VOTT * (ff_r + U_r * h_r)$$

where  $h_r$  is rail headway. An average rail user wait time is a fraction ( $U$ ) of headway. To ensure that the model counts the wait time only once for each user,  $U_r$  is set to be equal to zero for rail links other than the critical rail link ( $cr$ ). Critical rail link is the most heavily utilized rail link. On a commuter corridor, critical rail link is typically the last rail link before the trip destination.

According to Morlok (1978), the rail headway has the form:

$$h_r = \begin{cases} \min h, & \text{for } x_r \geq m \\ \frac{\lambda * S}{x_r}, & \text{for } n \leq x_r \leq m \\ \max h, & \text{for } x_r \leq n \end{cases}$$

where  $S$  is train capacity (in seats) and  $\lambda$  is a pre-specified load factor. For a small number of rail users ( $x_r \leq n$ ) at least a minimum service (maximum headway) must be provided. As the number of rail users increases ( $n \leq x_r \leq m$ ) the service becomes more frequent, until, due to safety considerations, it reaches a maximum frequency (or minimum headway) at  $x_r \geq m$ .

**Model Statement**

The objective function of the mathematical model is:

$$\min z = \sum_{i \in L \times cr} x_i \int_0^{x_i} c(\psi) d\psi + \sum_{cr} x_i \int_0^{x_i} c(h(\psi)) d\psi - \sum_{ij} \int_0^{T_{ij}^u} D1_{ij}^{-1}(\omega) d\omega - \sum_{ij} \int_0^{T_{ij}^u} D2_{ij}^{-1}(\varphi) d\varphi$$

The first two components are the mathematical expression of the user equilibrium principle (Sheffi, 1985) while the last two components are the integrals of the inverted demand functions, D1 and D2, which account for traveler preference between auto and transit, and between rail and intermodal. The total demand conservation constraint indicates that the total demand between each origin-destination (O-D) pair is equal to the sum of the auto and transit trip rates for this O-D pair:

$$T^{ij} = T_A^{ij} + T_T^{ij} \quad \forall ij$$

The auto demand conservation constraint indicates that the auto trip rate for an O-D pair is equal to the sum of flows on all auto paths of this O-D pair:

$$T_A^{ij} = \sum_{PA} f_{PA}^{ij} \quad \forall ij$$

The same constraint is written for the rail and intermodal trip rates.

$$T_R^{ij} = \sum_{PR} f_{PR}^{ij} \quad \forall ij$$

$$T_M^{ij} = \sum_{PM} f_{PM}^{ij} \quad \forall ij$$

The demand for transit conservation constraint indicates that the transit trip rate between each O-D pair is equal to the sum of rail and intermodal trip rates between this O-D pair.

$$T_T^{ij} = T_R^{ij} + T_M^{ij} \quad \forall ij$$

## Evaluation of Network Efficiency

The mathematical model may be used within the framework shown in Figure 3 to analyze potential improvements and evaluate their effects on traveler's costs and choice of mode, on the performance of the transportation systems, and on the overall performance of the network.

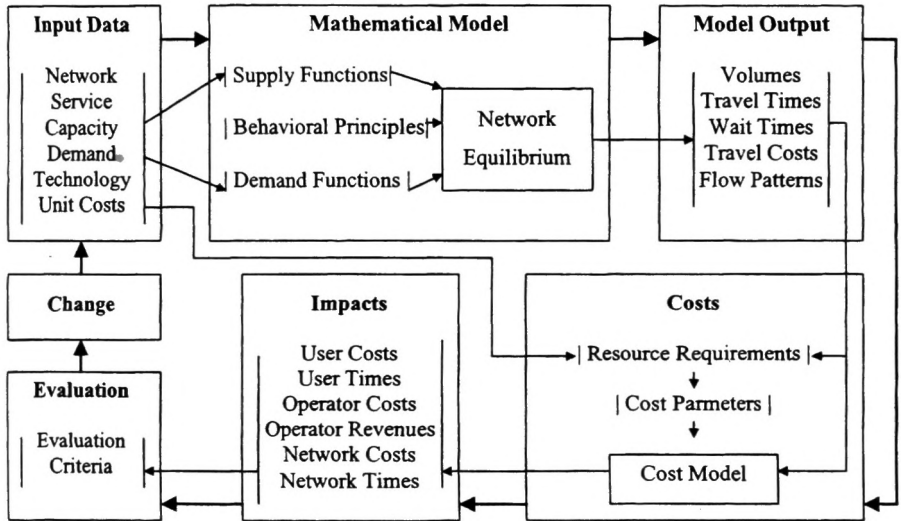


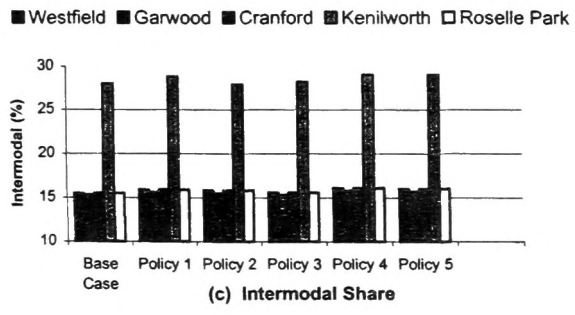
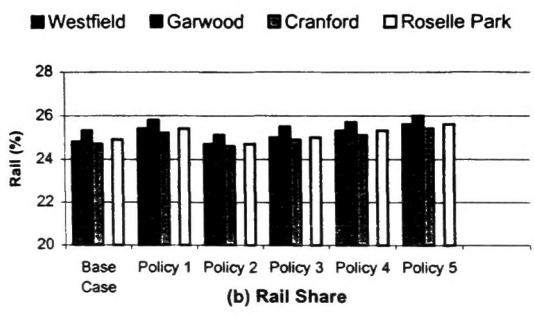
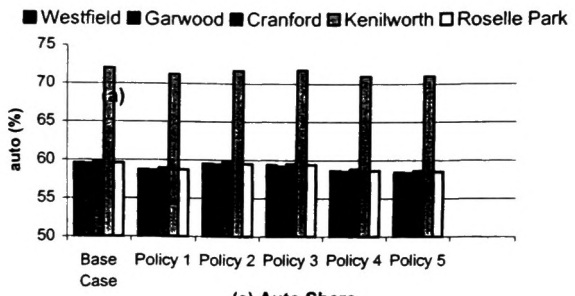
Figure 3 Methodological Framework

As the figure indicates, the mathematical model estimates the traffic volumes, travel patterns, travel times and costs. This information is used to determine the impacts to users and operators of the transportation systems and to the overall network performance. The user costs are estimated in terms of out of pocket expenditures and time per mode, for a trip between an origin and a destination. Transit operator costs are based on the frequency of the service provided and the number of transit vehicles necessary to operate at this frequency, while operator revenues are estimated based on the number of transit users and the fare each user is paying for a trip between

The network consists of five origins: Westfield, Garwood, Cranford, Kenilworth and Roselle Park and one destination: Newark. The street network consists of major highways, including Interstate 78, U.S. Route 22, and the Garden State Parkway, as well as a number of local state routes. The rail corridor, Raritan Valley line, is operated by NJ Transit. Peak hour travel demand from the five origins to Newark was obtained from Bureau of Census information. The network consists of 120 links and 141 paths. Network characteristics such as link length and number of lanes were obtained from the National Transportation Atlas Database (1998). Other data such as highway tolls, transit fees and commuter parking availability were obtained through the Internet and from site visits. A detailed description of network characteristics is presented in Boile and Spasovic (1999). The objective of this section of the paper is to present an example of the type of output files generated by the model and how this information can be used to evaluate alternative policies. Six cases were analyzed to demonstrate model results. The first one, base case, represents the current situation on the network during peak period. Policy 1 increased parking fees in the downtown area by 25%. Policy 2 decreased parking fees at suburban train stations by 25%. Policy 3 doubled highway tolls on Garden State Parkway. Policy 4 is a combination of policies 1 and 3. Policy 5 is a combination of policies 2 and 3.

Figure 5 shows the modal shares by network origin and policy. Part (a) of the figure shows the auto share, part (b) shows the rail share and part (c) the intermodal share. In terms of increasing transit ridership and reducing the number of auto users, Policies 1, 4 and 5 seem to be the most promising ones. The percent reduction in auto use is in the magnitude of 0.9% for Policy 1, 1% for Policy 4 and 1.2% for Policy 5 compared to the base case. These results are along the line with nationwide surveys, which predict that transit incentives may divert a maximum of about 2% of the auto users to transit (Manheim, 1978).





**Figure 5** Modal Shares by Origin and Policy

**Table 1** Average Generalized User Costs

Policy	O-D Pair	Auto	Rail	Intermodal
		(\$/passenger)	(\$/passenger)	(\$/passenger)
Base case	Westfield - Newark	15.5	16.5	16.9
	Garwood - Newark	14.2	14.9	15.8
	Cranford - Newark	13.0	14.4	14.6
	Kenilworth - Newark	12.2	-	16.5
	Roselle Park - Newark	12.4	13.5	13.9
Policy 1	Westfield - Newark	16.7	16.5	16.8
	Garwood - Newark	15.5	14.9	15.8
	Cranford - Newark	14.3	14.4	14.6
	Kenilworth - Newark	13.4	-	16.5
	Roselle Park - Newark	13.7	13.5	13.9
Policy 2	Westfield - Newark	15.5	16.5	16.5
	Garwood - Newark	14.2	14.9	15.5
	Cranford - Newark	13.1	14.4	14.3
	Kenilworth - Newark	12.2	-	16.1
	Roselle Park - Newark	12.5	13.6	13.5
Policy 3	Westfield - Newark	15.8	16.6	16.9
	Garwood - Newark	14.6	14.9	15.8
	Cranford - Newark	13.4	14.4	14.6
	Kenilworth - Newark	12.6	-	16.5
	Roselle Park - Newark	12.8	13.5	13.9
Policy 4	Westfield - Newark	16.7	16.5	16.5
	Garwood - Newark	15.5	14.9	15.5
	Cranford - Newark	14.3	14.4	14.3
	Kenilworth - Newark	13.4	-	16.1
	Roselle Park - Newark	13.7	13.5	13.5
Policy 5	Westfield - Newark	17.1	16.5	16.8
	Garwood - Newark	15.8	14.9	15.8
	Cranford - Newark	14.7	14.4	14.6
	Kenilworth - Newark	13.8	-	16.5
	Roselle Park - Newark	14.0	13.5	13.9

Table 1 reports the estimated average generalized user cost by mode. Policy 1 increased the average cost for auto users, since it increased the parking fee in the downtown area. Even though, as a result of the parking fee increase, this policy resulted in some travelers switching from auto to transit, it did not have any major effect on the rail and intermodal average user costs. Policy 2, which decreased parking fees at suburban train stations, resulted in reduced

average user cost for intermodal users, while it did not substantially affect auto and rail costs. Policy 3, which increased highway tolls resulted in an increase in auto cost without substantially affecting rail and intermodal costs. Policy 4 increased auto and decreased intermodal costs without affecting rail costs. Finally, Policy 5 substantially increased auto costs but did not have a major effect on rail and intermodal costs.

**Table 2** Network Performance

Policy	Total Auto Time (minutes)	Total Rail Time (minutes)	Total Intermodal Time (minutes)
Base Case	18638	16745	11319
Policy 1	18303	17092	11561
Policy 2	18599	16651	11472
Policy 3	18544	16842	11387
Policy 4	18265	16996	11716
Policy 5	18210	17190	11629

Policy	Total Auto Cost (\$)	Total Rail Cost (\$)	Total Intermodal Cost (\$)
Base Case	14336	5833	4457
Policy 1	15407	5954	4553
Policy 2	14308	5800	4416
Policy 3	14640	5867	4484
Policy 4	15377	5920	4512
Policy 5	15700	5988	4581

Table 2 shows estimates of network performance, in terms of modal time and generalized cost. The values in the table indicate that, in general, the total auto travel time decreased for each of the policies compared to the base case. This is due to the decrease in the number of auto users. The increased number of transit users resulted in an increase in rail and intermodal travel time for each of the policies compared to the base case. The second part of the table indicates that, in general, the total auto generalized cost increased for each of the policies with the exception of

Policy 2. This is due to the increase in parking fees in the downtown area (Policies 1 and 4), or in highway tolls (Policy 3), or both (Policy 5). Policy 2, which decreased parking fees at suburban stations, resulted in some auto and rail users switching to intermodal paths and reduced the generalized cost for all modes.

**Table 3** Operating Characteristics

Policy	Operating Schedule (headway in min.)	Operator Revenues (\$/peak period)
Base Case	17.3	1158
Policy 1	17.1	1168
Policy 2	17.22	1159
Policy 3	17.22	1161
Policy 4	17.1	1170
Policy 5	17.1	1171

Table 3 shows the operating characteristics estimated by the model for each of the alternative policies. The operating schedule is determined based on the optimal headway. For each of the policies, the estimated optimal headway is approximately equal to the base case headway. The suggested headway is equal to 17 minutes, which is equivalent to a frequency of approximately 3.5 trains per peak hour. Based on the service frequency, the hourly operating expenses for transit may be estimated. The model estimates the operator farebox revenue by multiplying the appropriate fare price by the number of transit riders.

The model results may be used to evaluate the alternative policies based on selected criteria. If for example the objective of the analysis is to determine the most efficient method (among a given set of alternative policies) to reduce highway congestion, the suggestion would be to implement Policy 5, since it resulted in the largest decrease in auto share and increase in transit share, as shown in Figure 5. The same policy should be implemented according to results

in Table 3 if the objective is to increase the operator's revenue or, according to Table 2, if the objective is to decrease the total network time spend by auto users.

**Table 4** Peak vs. Off Peak Period Travel Demand, Network and Operating Characteristics

	Modal Shares					
	Auto (%)		Rail (%)		Intermodal (%)	
	peak	off peak	peak	off peak	peak	off peak
Westfield - Newark	59.6	65.7	24.8	20.9	15.5	13.4
Garwood - Newark	59.5	65.3	25.3	21.5	15.3	13.2
Cranford - Newark	59.8	65.6	24.7	21.1	15.5	13.3
Kenilworth - Newark	72.2	76.9	-	-	27.8	23.1
Roselle Park - Newark	59.6	65.5	24.9	21.3	15.5	13.3
	Average Generalized User Costs					
	Auto (\$/passenger)		Rail (\$/passenger)		Intermodal (\$/passenger)	
	peak	off peak	peak	off peak	peak	off peak
Westfield - Newark	15.5	14.0	16.5	23.8	16.9	23.8
Garwood - Newark	14.2	15.6	14.9	22.2	15.8	22.8
Cranford - Newark	13.0	12.0	14.4	21.7	14.6	21.9
Kenilworth - Newark	12.2	11.2	-	-	16.5	23.7
Roselle Park - Newark	12.4	11.4	13.5	20.8	13.9	21.2
	Total Network Time (minutes)					
	Auto		Rail		Intermodal	
	peak	off peak	peak	off peak	peak	off peak
	18638	4975	16745	6469	11319	4503
	Total Network Cost (\$)					
	Auto		Rail		Intermodal	
	peak	off peak	peak	off peak	peak	off peak
	14336	4308	5833	2219	4457	1675
Operating Schedule (headway in minutes)	peak			off peak		
	17.3			61		
Operator Revenues (\$/peak period)	1158			328		

A similar type of analysis was performed to compare the performance of the network during off peak, to that of the peak period. Results of the analysis are shown in Table 4. Due to reduced

congestion on highways, over 65% of the travelers use auto during the off peak period. As a result, the average travel cost on rail and intermodal paths increases substantially, due to the larger rail headway. The optimal headway is 61 minutes, which is equivalent to a train frequency of about one train per hour. The reduced frequency substantially increases the average wait time at the train station. The modal network time and generalized cost are much lower during the off peak compared to peak period, due to the substantial decrease in the number of travelers.

## Conclusions

A mathematical model which may be used by transportation planners to analyze and evaluate alternative improvement policies on intermodal commuter networks was developed. In addition to estimating travel times, cost and flow patterns, the model can be used to determine the transit operating characteristics that better satisfy travel demand. The model determines the choice of mode, type of access to a mode, and actual path, using a simultaneous approach, thus overcoming some of the problems of the widely used sequential travel demand forecasting models. Furthermore, the model has the ability analyze intermodal networks and predict the effects of operating and pricing changes in one mode to the performance of other, competing modes.

Having the ability to analyze intermodal commuter networks is of great importance due to the increasing number of commuters living in transit poor suburbs and working in congested urban areas, who are favoring intermodal trips. Intermodal network planning has the potential to further improve transit attractiveness and increase the number of travelers using facilities such as park and ride as an intermediate point in their trip to work.

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## Appendix

$$\text{Consider } \frac{\partial x_l}{\partial f_{pk}^{ij}} = \frac{1}{occ} \delta_{lpk}^{ij} \quad \forall l \in LZ, LT \quad (1)$$

$$\frac{\partial x_l}{\partial f_{pk}^{ij}} = \delta_{lpk}^{ij} \quad \forall l \in LR, LW \quad (2)$$

The Lagrangian of the model is:

$$\begin{aligned} L(f, T, u) = & \sum_l \int_0^{x_l} c(x_l) dx_l + \sum_{ij} \int_0^{T_T^{ij}} \frac{1}{\beta_2} \left( \ln \frac{T_T^{ij}}{T_T^{ij} - T_T^{ij}} + \alpha_{TA} \right) dT_T^{ij} + \sum_{ij} \int_0^{T_M^{ij}} \frac{1}{\beta_1} \left( \ln \frac{T_M^{ij}}{T_T^{ij} - T_M^{ij}} + \alpha_{MR} \right) dT_M^{ij} + \\ & \sum_{ij} u^{ij} (T_T^{ij} - T_A^{ij} - T_T^{ij}) + \sum_{ij} u^{ij} (T_T^{ij} - T_R^{ij} - T_M^{ij}) + \\ & \sum_{ij} \left( u_A^{ij} (T_A^{ij} - \sum_{P_A} f_{P_A}^{ij}) + u_R^{ij} (T_R^{ij} - \sum_{P_R} f_{P_R}^{ij}) + u_M^{ij} (T_M^{ij} - \sum_{P_M} f_{P_M}^{ij}) \right) \end{aligned} \quad (3)$$

with  $f_{P_k}^{ij} \geq 0, \quad \forall P \in P_A, P_R, P_M$

The derivatives with respect to path flow and demand are:

$$\begin{aligned} \frac{\partial L}{\partial f_{P_A}^{ij}} = & \frac{\partial \sum_l \int_0^{x_l} c(x_l) dx_l}{\partial x_l} \frac{\partial x_l}{\partial f_{P_A}^{ij}} + \frac{\partial \sum_{ij} u_A^{ij} (T_A^{ij} - \sum_{P_A} f_{P_A}^{ij})}{\partial f_{P_A}^{ij}} = \\ & \sum_{l \in LZ, LT} \frac{1}{occ} \delta_{lp_A}^{ij} c(x_l) + \sum_{l \in LR, LW} \delta_{lp_A}^{ij} c(x_l) - u_A^{ij} = GC_{P_A}^{ij} - u_A^{ij} \end{aligned} \quad (4)$$

$$u_A^{ij} = \min GC_{P_A}^{ij} \quad (5)$$

$$\text{Similarly, } \frac{\partial L}{\partial f_{P_R}^{ij}} = GC_{P_R}^{ij} - u_R^{ij} \quad (6)$$

$$u_R^{ij} = \min GC_{P_R}^{ij} \quad (7)$$

$$\text{and } \frac{\partial L}{\partial f_{P_M}^{ij}} = GC_{P_M}^{ij} - u_M^{ij} \quad (8)$$

$$u_M^j = \min GC_{P_M}^j \tag{9}$$

$$\frac{\partial L}{\partial T_A^j} = \frac{\partial \sum_{ij} u^j (T^j - T_A^j - T_T^j)}{\partial T_A^j} + \frac{\partial \sum_{ij} u_A^j (T_A^j - \sum_{P_A} f_{P_A}^j)}{\partial T_A^j} = -u^j + u_A^j \tag{10}$$

$$\frac{\partial L}{\partial T_T^j} = \frac{\partial \sum_{ij} \int_0^{T_T^j} \frac{1}{\beta_2} (\ln \frac{T_T^j}{T^j - T_T^j} + \alpha_{TA}) dT_T^j}{\partial T_T^j} + \frac{\partial \sum_{ij} u^j (T^j - T_A^j - T_T^j)}{\partial T_T^j} + \frac{\sum_{ij} \int_0^{T_M^j} \frac{1}{\beta_1} (\ln \frac{T_M^j}{T_T^j - T_M^j} + \alpha_{MR}) dT_M^j}{\partial T_T^j} + \frac{\partial \sum_{ij} u_T^j (T_T^j - T_R^j - T_M^j)}{\partial T_T^j} \tag{11}$$

$$\frac{\sum_{ij} \int_0^{T_M^j} \frac{1}{\beta_1} (\ln \frac{T_M^j}{T_T^j - T_M^j} + \alpha_{MR}) dT_M^j}{\partial T_T^j} = \frac{\partial \sum_{ij} \frac{1}{\beta_1} [\int_0^{T_M^j} \ln T_M^j dT_M^j + \int_0^{T_M^j} \ln (T_T^j - T_M^j) d(T_T^j - T_M^j) + \int_0^{T_M^j} \alpha_{MR} dT_M^j]}{\partial T_T^j} = \frac{1}{\beta_1} [\ln (T_T^j - T_M^j) + (T_T^j - T_M^j) \frac{1}{(T_T^j - T_M^j)} - 1 - \ln T_T^j - T_T^j \frac{1}{T_T^j} + 1] = \frac{1}{\beta_1} \ln \frac{T_T^j - T_M^j}{T_T^j} \tag{12}$$

Substituting (12) into (11) and deriving, we obtain:

$$\frac{\partial L}{\partial T_T^j} = \frac{1}{\beta_2} (\ln \frac{T_T^j}{T^j - T_T^j} + \alpha_{TA}) - u^j + \frac{1}{\beta_1} \ln \frac{T_T^j - T_M^j}{T_T^j} + u_T^j$$

$$\frac{\partial L}{\partial T_R^j} = \frac{\partial \sum_{ij} u_T^j (T_T^j - T_R^j - T_M^j)}{\partial T_R^j} + \frac{\partial \sum_{ij} u_R^j (T_R^j - \sum_{P_R} f_{P_R}^j)}{\partial T_R^j} = -u_T^j + u_R^j \tag{13}$$

$$\frac{\partial L}{\partial T_M^j} = \frac{\partial \sum_{ij} \int_0^{T_M^j} \frac{1}{\beta_1} (\ln \frac{T_M^j}{T_T^j - T_M^j} + \alpha_{MR}) dT_M^j}{\partial T_M^j} + \frac{\partial \sum_{ij} u_T^j (T_T^j - T_R^j - T_M^j)}{\partial T_M^j} +$$

$$\frac{\partial \sum_{ij} u_M^{ij} (T_M^{ij} - \sum_{P_M} f_{P_M}^{ij})}{\partial T_M^{ij}} = \frac{1}{\beta_1} \left( \ln \frac{T_M^{ij}}{T_T^{ij} - T_M^{ij}} + \alpha_{MR} \right) - u_T^{ij} + u_M^{ij} \quad (14)$$

Setting the above derivatives equal to zero and considering that the Lagrangian multipliers,  $u_k^{ij}$ , represent the minimum average cost for mode  $k$  and for O-D pair  $ij$  ( $GC_k^{ij}$ ), equations (7), (8), and (9) become:

$$GC_{P_A}^{ij} - GC_A^{ij} \begin{cases} = 0 & \text{if } f_{P_A}^{ij} > 0 \\ \geq 0 & \text{if } f_{P_A}^{ij} = 0 \end{cases} \quad (15)$$

$$GC_{P_R}^{ij} - GC_R^{ij} \begin{cases} = 0 & \text{if } f_{P_R}^{ij} > 0 \\ \geq 0 & \text{if } f_{P_R}^{ij} = 0 \end{cases} \quad (16)$$

$$GC_{P_M}^{ij} - GC_M^{ij} \begin{cases} = 0 & \text{if } f_{P_M}^{ij} > 0 \\ \geq 0 & \text{if } f_{P_M}^{ij} = 0 \end{cases} \quad (17)$$

Equations (15), (16), and (17) are the expressions of the first equilibrium condition.

From equation (13)  $u_T^{ij} = u_R^{ij}$ , so equation (14) becomes:

$$u_R^{ij} - u_M^{ij} = \frac{1}{\beta_1} \left( \ln \frac{T_M^{ij}}{T_T^{ij} - T_M^{ij}} + \alpha_{MR} \right) \quad (18)$$

which for positive rail and intermodal path flows is the expression of the third equilibrium condition:

$$GC_{P_M}^{ij} - GC_{P_A}^{ij} = -\frac{1}{\beta_1} \left( \ln \frac{T_M^{ij}}{T_R^{ij}} + \alpha_{MR} \right) \quad (19)$$

Assuming a positive transit demand,  $T_T^{ij}$ , and considering  $u^{ij} = u_A^{ij}$  from equation (10) and  $u_T^{ij} = u_R^{ij}$ , equation (13) becomes:

$$GC_{P_A}^{ij} - GC_{P_A}^{ij} + \frac{1}{\beta_2} \left( \ln \frac{T_T^{ij}}{T^{ij} - T_T^{ij}} + \alpha_{TA} \right) + \frac{1}{\beta_1} \ln \frac{T_T^{ij} - T_M^{ij}}{T_T^{ij}} = 0, \text{ or} \quad (20)$$

$$\left( GC^{ij} + \frac{1}{\beta_1} \ln \frac{T_R^{ij}}{T_T^{ij}} \right) - GC_A^{ij} + \frac{1}{\beta_2} \left( \ln \frac{T_T^{ij}}{T^{ij} - T_T^{ij}} + \alpha_{TA} \right) = 0 \quad (21)$$

By definition:

$$T_R^{ij} = T_T^{ij} \frac{e^{U_R^{ij}}}{e^{U_R^{ij}} + e^{U_M^{ij}}}, \text{ or} \quad (22)$$

$$\ln \frac{T_R^{ij}}{T_T^{ij}} = U_R^{ij} - \ln(e^{U_R^{ij}} + e^{U_M^{ij}}), \text{ or} \quad (23)$$

$$\ln \frac{T_R^{ij}}{T_T^{ij}} = -\beta_1 * GC_R^{ij} - \ln(e^{U_R^{ij}} + e^{U_M^{ij}}) \quad (24)$$

Substituting (24) in (21):

$$-\frac{1}{\beta_1} \ln(e^{U_R^{ij}} + e^{U_M^{ij}}) - GC_A^{ij} + \frac{1}{\beta_2} \left( \ln \frac{T_T^{ij}}{T^{ij} - T_T^{ij}} + \alpha_{TA} \right) = 0 \quad (25)$$

Considering that the generalized cost for transit can be expressed as follows:

$$GC_T^{ij} = -\frac{1}{\beta_1} \ln(e^{U_R^{ij}} + e^{U_M^{ij}}) \quad (26)$$

equation (25) becomes:

$$GC_T^{ij} - GC_A^{ij} = -\frac{1}{\beta_2} \left( \ln \frac{T_T^{ij}}{T^{ij} - T_T^{ij}} + \alpha_{TA} \right)$$

which is the expression of the second equilibrium condition.