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A Solution Method for the Traveling Salesman n person M town Problem (TSP(n/M)) using the Genetic Algorithm

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abstract

In this paper, we give a solution for traveling salesman n person M town problem (TSP(n/M)) which is an extension of traveling salesman problem (TSP) by using the genetic algorithm (GA). TSP(n/M) is a generalized problem of the ordinary traveling salesman to a problem when n person exists. Here, we propose a new algorithm named multi-parents exchange method which generates one offspring from several parents. Moreover, we propose a new method by combining this algorithm with the surface dividing method which is a method in the area of Operations Research.

1.Introduction

Ordinary traveling salesman problem is a problem which looks for the shortest Hamiltonian path when one salesman visits every one city. This is a typical NP problem considered to be solved only by calculating all the possible paths to find the optimal path. The number of possible paths increase exponentially according to the number of cities to visit. For example, as the number of cities increases from 10 to 20, the possible paths increase 10 billion times. To find the optimal combination among all the possible combinations is called the combinatorial optimization problem. The investigation for this optimization problem is done almost for TSP and is extended to the other problems. This is due to the fact that the definition of TSP is simple and TSP contains many practical applications.

However, it is necessary to extend the definition of TSP for solving the practical problem, since TSP is too simple to apply to the practical problem. Here, we extend the ordinary TSP to the problem concerning the several salesmen, since it is often happened that several salesmen share the cities to visit.

In this case, we usually assign each salesman a range to visit and try to find the shortest path for each salesman in his range. Though this method finds the shortest way for each salesman, we do not always minimize the distance for the salesman who has the longest path.

So, we consider the traveling salesman problem n person M town problem (TSP(n/M)) which does not concern the range to visit. TSP(n/M) is defined to be a problem how we

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minimize the distance for the salesman who has the longest path. This problem happens in the following case: When n salesmen have to visit M towns, how can we find the shortest path that all the salesmen come back to their company as soon as possible ? This contains many practical applications such as to the conveying plan by several trucks. The various solution methods for TSP have been proposed, such as neural network, genetic algorithm (GA) and etc. However, these methods are not for TSP(n/M) but for ordinary TSP. [1] shows that the method by neural network is not good for more than twenty towns. And GA which uses subtour exchange crossover has a serious problem that too much computation is needed to find the same subtour between both gene. In this paper, we propose multi-parents exchange method instead of the crossover used in ordinary GA. And we give a solution for TSP(n/M) by combining GA using multi-parents exchange method with surface dividing method which is often used in the area of OR. In section 2, we introduce the traditional encode/decode-crossover problem for TSP. In section 3, we propose multi-parents exchange method. Here, we introduce link matrix used in the multi-parents exchange algorithm. In order to show the effectiveness of this method, we compare GA using multi-parents exchange method with GA using subtour exchange crossover. In section 4, we propose to apply the surface dividing method to TSP(n/M) and examine goodness of this new method.

2.The Solution for TSP by the Ordinary GA

Here, we introduce the typical encode/decode-crossover in GA for TSP and give some consideration.

Let the name of city be gene. The encoding by the order of the city name is called path representation. The crossover which replaces the rest behind a point with that of the other parent entirely is called one point crossover. The problem is that GA using this path representation and one point crossover does not generally yield offspring which are legal tours.

In order to avoid this difficulty, GA using ordinal representation and one point crossover and GA using path representation and partially mapped crossover have been proposed.

Though these methods are guaranteed not to yield illegal tour, character-preservingness are neglected and these methods have almost the same ability as random search performed by mutation only.

Yamamura etal. [3] proposed GA using path representation which made much of character-preservingness and subtour exchange crossover. Subtour exchange crossover method does not destroy the subtour which has useful property and leave this useful property to the next generation. This is a kind of two point crossover and the range between two points on the chromosome is exchanged only when the set of cities involved in the subtour corresponds to that of the other parent. Since the distance for each path is not changed when we go through the path conversely, two parents generate four offsprings. This causes difficulty that much computation is needed.

In order to avoid these difficulties mentioned above, we propose GA using path representation and multi-parents method. Since several parents yield only one child in this method, we use the term "exchange method" in stead of crossover.

3.Multi-Parents Exchange Method

Multi-parents exchange method is made of two parts. One part is to prepare a link matrix in order to make much of character presevingness. And the other part is to generate one offspring by using this link matrix. In the following, we discuss on the link matrix and multi-parents exchange algorithm concretely.

3.1 link matrix

In multi-parents exchange algorithm, we make much use of the information where the salesman comes from and where the salesman goes for. This information obtained from each parent is called a link information and we record this information in the link matrix. For example, the link matrix of the parent who has the following chromosome is as follows.

Table 1. Link Information

Parent : 5 2 4 1 3 6

	1	2	3	4	5	6
1	0	0	1	1	0	0
2	0	0	0	1	1	0
3	1	0	0	0	0	1
4	1	1	0	0	0	0
5	0	1	0	0	0	1
6	0	0	1	0	1	0

The link matrix is made out by adding 1 to the corresponding column of the cities visited before and after for each row of the city. For example, for the city 2, (2,5) and (2,4) components are added to 1, since the cities before and after the city 2 are cities 5 and 4.

Note that the city before the starting city is the last visiting city and the city after the last visiting city is the starting city. For example, the link matrix of the four parents who have the following chromosome is as follows.

Table 2. Link Matrix

Parent1 : 5 2 4 1 3 6
Parent2 : 5 2 3 1 6 4
Parent3 : 6 5 4 2 3 1
Parent4 : 1 2 5 3 6 4

	1	2	3	4	5	6
1	0	0	1	1	0	0
2	0	0	0	1	1	0
3	1	0	0	0	0	1
4	1	1	0	0	0	0
5	0	1	0	0	0	1
6	0	0	1	0	1	0

3.2 Multi-Parents Exchange Algorithm

Next, we show an algorithm which generates one offspring by using this link matrix. As the component of the matrix is larger and larger, the corresponding city of the row and the corresponding city of the column are connected more closely.

Multi-Parents Exchange Algorithm

1. Let the row of the component which has the largest value among all the components in the link matrix be the starting city and add the city at the beginning of the list. And let the column of the component be the present city. (If several components which have the same value exist, choose one of these at random.)
2. Remove the coresponding row and column involved in the list from the link matrix.
3. Add the present city to the list and let the column which has the largest value among the row of the present city be the new present city. (If several components which have the same value exist, choose the city which has the smallest sum of the corresponding row.)
4. If the unvisited cities exist, go to Step 2. Otherwise, we can obtain the list as the new offspring.

Note that, in step 3, we let the character preservngness to be meaningful. The offspring generated from the link matrix in Table 2 is as follows.

Table 3.

Parent : 5 2 4 1 3 6

However, the same offspring as that in Figure 3 is not always generated, since this algorithm is a kind of random algorithm.

3.3 Simulation Results

In order to examine the usefullness of this multi-parents exchange algorithm. we compare GA using this multi-parents exchange algorithm with GA using subtour exchange crossover by simulation studies. We evaluate the goodness of these method by the distance of the obtained shortest path and their searching time. We locate cities at random in the range of a 1×1 square and let the center of the square be a starting point. We have done simulation studies for the following cases:

- The number of cities is 30 and searching number is 100 thousand times.
- The number of cities is 50 and searching number is 200 thousand times.

We repeat the calculation three times and show the average of the results. In Table 3, we show the results of our simulation studies.

Table 4. Simulation Results

	GA(Subtour exchange crossover)	GA(Multi-parents exchange Method)
TSP(2/30)	2.777	2.426
	329	163
TSP(4/30)	1.767	1.696
	754	209
TSP(2/50)	4.785	3.693
	9786	1217
TSP(4/50)	2.67	2.353
	9145	1398

We have the following observations. There is not so much difference between the distance obtained by GA with subtour exchange crossover and that obtained by GA with multi-parents exchange method in the case of 30 cities. On the other hand, the searching time for finding its shortest path by GA with multi-parents exchange method is about half of that by GA with subtour exchange crossover. Moreover, the number of salesmen does not affect not so much the searching time for GA with multi-parents exchange method. On the other hand, for GA with subtour exchange crossover, the searching time is longer and longer as number of the salesman increase.

In the case of 60 cities. GA with multi-parents exchange method gives a better solution than GA with subtour exchange crossover does. And the searching time of the former is about 1/6 of that of the latter. By the above observations, we find that GA with multi-parents exchange method generally gives a better solution and finds the solution for a shorter time than GA with subtour exchange crossover does. And the number of salesman does not affect so much the seraching time for GA with multi-parents exchange method.

4. GA combined with surface dividing method

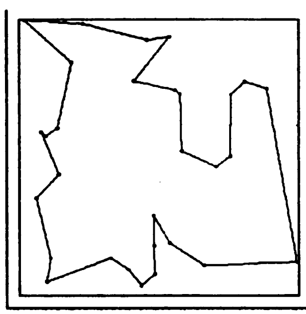
In this section. we discuss on the surface dividing method for TSP simply first. and apply this method to TSP(n/M).

4.1 The Surface Dividing Method

In the surface dividing method, we divide the given cities into parts called backett and get an approximate solution quickly by patrolling the backett in the appropriately order. Here, a triangular backett is used, since we can find in which backett the city exists quickly. And let the objective area be a square. The square can be divided into the congruent right-angled isosceles triangle called triangular backett by a diagonal line. We can divide the right-angled isosceles triangle into the the right-angled isosceles triangles recursively. We show the 2^t (t=4) triangular backetts in Figure 1. In Figure 1, we also show the path through all the backetts. This curve when t tends to infinity is called Sierpinski curve.

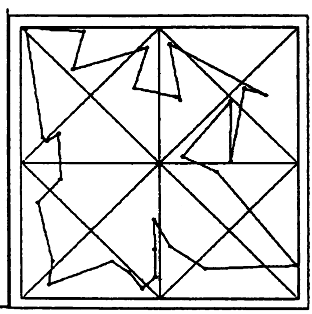
4.2 Comparison between GA and Surface Dividing Method

Next, we give a solution for TSP with 30 cities both by GA and by the surface dividing method to examine the effectiveness of the surface dividing method. We show the results in Figures 2 and 3.



The distance = 4.737

Fig 1. The solution for TSP by GA



The distance = 5.363

Fig 2. The solution for Surface dividing method

From Figures 2 and 3, we find that the solution by GA is better than the surface dividing method. But it takes only a second for the surface dividing method to obtain such a solution, while it takes about one minute for GA. We can say that the surface dividing method finds a good solution in some degree very quickly.

4.3 the Method Combining the Surface Dividing Method

We are able to obtain a good solution in some degree quickly by combining the surface dividing method. However, the surface dividing method is not for TSP(n/M) but for TSP and we can not apply this directly to TSP(n/M). Therefore we use the surface dividing method for the one salesman in TSP(n/M). As we showed in 4.2, by GA we can obtain a better solution than by the surface dividing method and the better solution obtained by GA may be destroyed by combining the surface dividing method with GA. Therefore, we use the surface dividing method as a mutation in the operation of GA and let the ratio of the mutation decrease as the generation proceeds.

4.4 Simulation Results

In order to show the effectiveness of GA combined with the surface dividing method, we compare this method with GA using the subtour exchange crossover and using the multi-parents exchange method. We use the multi-parents exchange method as an exchange method for GA combined with the surface dividing method. We have done simulation studies as in 3.3. We show the results in Table 5.

Table 5. Simulation Results

	GA(Subtour exchange crossover)	GA(Multi-parents exchange Method)	GA(Multi-parents exchange Method)+Surface Dividing Method
TSP(2/30)	2.777	2.426	2.379
	329	163	185
TSP(4/30)	1.767	1.696	1.555
	754	209	195
TSP(2/50)	4.785	3.693	3.318
	9786	1217	1320
TSP(4/50)	2.67	2.353	1.949
	9145	1398	1375

We find from the Table that by GA combined with the surface dividing method, we can obtain a shorter path than by GA using subtour exchange crossover and using the multi-parents exchange method in every case. And it takes almost the same time to finish searching for a solution. However, we do not show as yet whether or not we can obtain a good solution at the beginning of the searching. To examine this, we show a change of a distance against a searching number and a change of a distance against a searching time in Figures 4 and 5.

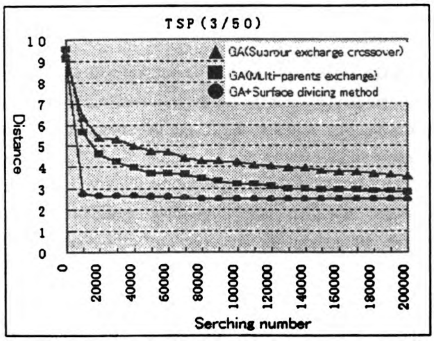


Fig 4. The change of distance against the searching number

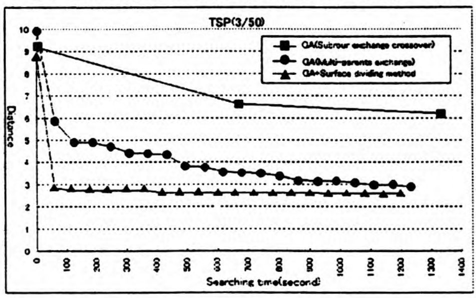


Fig 5. The change of distance against the searching time

We find from Figure 4 that in 1/10 of all the searching number GA combined with the surface dividing method generates an offspring having almost the same distance with an offspring after finishing all the search. Even if we let the searching number to be 1/10. we can get the solution almost as good as the solution after all the searching and this means we can decrease the searching time to be 1/10. On the other hand. by the other two methods. we can not obtain a solution as good as a solution given after all the searching in a short time. It takes a much longer time for GA using subtour exchange crossover to obtain a solution as good as a solution after searching than it does for GA using multi-parents exchange method. So. we can attain the object that we get a good solution at the beginning of searching by GA combined with the surface dividing method.

In ordinary GA, offspring which has the same chromosome can exist at the same time. In this paper, we do not admit the existence of this duplication, since duplication causes an initial convergence problem that there becomes to exist only the same offsprings which are not even a local solution.

5.concluding remarks

In this paper, we propose GA combined with the surface dividingmethod for TSP(n/M) and propose to use multi-parents exchange method in stead of subtour exchange crossover used in ordinary GA. We can get a better solution by our proposed method

than by GA using the subtour exchange crossover, but it remains a problem in the ability of local search. From now on, we need to be able to perform local search by using heuristic method in the last stage of GA with multi parents exchange method and the surface dividing method.

References

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