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**Proceedings of the 36th Annual Meeting  
Transportation Research Forum**

*Volumes  
1 and 2*

**November 3-5, 1994**

**Daytona Beach, Florida**

**Published and Distributed by:**

**Transportation Research Forum  
1730 North Lynn Street, Suite 502  
Arlington, VA 22209**

# The Effects of Subsidies on Public Transit Long-Run Costs

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## ABSTRACT

This paper investigates sources of public transit long-run cost increases attributable to transit subsidies. The sources include wage, vehicle capital price and service increases. Service expansion is found to be the major source. Transit cost increases related to subsidies are classified as input price, output and finance effects of these subsidies. Transit costs are more responsive to federal operating subsidies, followed in declining order by local operating, state operating and capital subsidies, respectively.

## INTRODUCTION

Critics of transit subsidies complain that they inflate transit costs. Previous studies in general support this argument.<sup>1</sup> Their hypothesis that transit costs and subsidies are positively related has been used as a rationale for curtailing transit subsidies at the federal level in the United States. The Reagan administration opposed federal operating subsidies given its belief that such subsidies resulted in increased labor costs rather than in improving or expanding transit service. In 1980, federal operating and capital subsidies were \$1,120.7 million and \$2,787.1 million, respectively; by 1985, federal support had declined to \$881.1 million and \$2,510.3 million, respectively (American Public Transit Association, 1987).

Surprisingly, the literature provides little empirical evidence of the source (or sources) of the cost increases attributable to subsidies. The one exception is support for the hypothesis that increases in operating subsidies lead to increases in transit wages, thus increasing transit costs.<sup>2</sup> However, right not increases in operating subsidies also lead to increases in service levels, thus increasing transit costs?<sup>3</sup> Further, it is feasible that increases in capital subsidies will result in increases in capital (or vehicle) prices, thus increasing transit costs.<sup>4</sup>

This paper investigates sources of transit cost increases attributable to transit subsidies. The results of the add to our understanding of the impact of subsidies on transit costs. Also, they provide public policymakers with information for determining subsidy allocations to transit firms for the purpose of achieving certain subsidy impacts. Further, the investigation offers a more definitive interpretation (than that previously appearing in the literature) of the impact of subsidies on transit costs via the transit cost function.

We investigate the long-run cost relationship between U.S. transit costs and operating and capital subsidies.<sup>5</sup> We develop a theoretical model relating long-run transit costs to transit subsidies. To investigate whether increases in wages, capital prices, and levels of service are sources (attributable to subsidies) of transit cost increases, we hypothesize that: wages are a positive function of operating subsidies; capital prices are a positive function of capital subsidies; and levels of service (or vehicle miles) are a positive function of operating and capital subsidies.

The empirical findings support the above hypotheses. Indeed, estimates suggest that increases in service levels represent the major source of subsidy-related cost increases. Among the four types of subsidies (federal operating, state operating, local operating and capital subsidies), a given percentage increase in federal operating subsidy results in the largest percentage increase in transit costs, followed in declining order by increases in local operating, state operating and capital subsidies, respectively.

## THE THEORETICAL MODEL

Assume that a public (i.e., government-owned) transit firm seeks to minimize costs subject to both a minimum service level constraint and a budget constraint.<sup>6</sup> The minimum service level constraint is analogous to the output constraint in the traditional minimization problem in the theory of the firm. The minimum level of service (or output) to be provided is determined by the firm. The implicit production function contains the traditional arguments of output and inputs. The budget constraint is analogous to the cost constraint in the traditional output maximization problem in the theory of the firm. Here, revenue (fare and non-fare) plus operating and capital subsidies must at least equal to total costs incurred by the firm. In addition, fund balances may be carried forward into the next budget period or periods to minimize cost, as for example in the purchase of equipment.

A formal derivation of the model defines a Lagrangean as:

$$L = \sum_{i=1}^n W_i X_i - \lambda_1 [\bar{Q} - h(X_1, X_2, \dots, X_n)] \quad (1)$$

$$- \lambda_2 \left[ \sum_{i=1}^n W_i X_i - PQ - \sum_{j=1}^m A_{oj} - \sum_{j=1}^m A_{cj} - R - E \right] \quad j=1,2,3$$

Where,  $W_i$  is the  $i$ th input price;  $X_i$  is the amount of the  $i$ th input;  $\bar{Q}$  is the minimum level of transit service to be provided;  $h(X_1, X_2, \dots, X_n)$  is the firm's production function;  $P$  is the fare (or price) per unit of transit service;  $A_{oj}$  is the amount of operating subsidy received from the  $j$ th level of government;  $A_{cj}$  is the amount of capital subsidy received from the  $j$ th level of government;  $R$  is the firm's amount of non-fare revenue; and  $E$  is the transit firm's ending budget balance. Both  $R$  and  $E$  are endogenous to the model. The Lagrangean multipliers associated with the output and budget constraints are  $\lambda_1$  and  $\lambda_2$ , respectively. The choice variables are the inputs ( $X_i$ 's), fare ( $P$ ),  $R$ ,  $E$  and the  $\lambda$ 's. Solving the first order conditions of our model yields the long-run cost function.<sup>7</sup>

In addition to affecting the cost of providing transit services, operating subsidies may also have an impact on transit wages. If so, a transit firm's wage rate  $W_1$  (or labor price) is affected by the amounts of operating subsidies received from various levels of government. Specifically,

$$C = C(W_1, W_n, A_{ol}, A_{om}, A_{cl}, A_{cm}, Q) \quad (2)$$

we may state the following wage rate function:

$$W_1 = W_1(A_{ol}, \dots, A_{om}) \quad (3)$$

Further, a transit firm's capital price ( $W_3$ ) is expected to be affected by the amounts of capital subsidies received from various levels of government. If so, we may state the following capital price function:

$$W_3 = W_3(A_{cl}, \dots, A_{cm}) \quad (4)$$

In addition to their effects on transit wage rates and capital prices, subsidies can also increase the level of transit service. Increases in operating subsidies provide financial support for financing the cost of service expansion. Further, increases in capital subsidies provide financial support for acquiring vehicles to be used in service expansions. We may represent these impacts in the following vehicle mile (or service level) function:

$$Q = Q(A_{ol}, A_{om}, A_{cl}, A_{cm}) \quad (5)$$

The total effect of an operating subsidy ( $A_o$ ) on a transit firm's long-run costs ( $dC/dA_o$ ) is found by taking the total differential of cost function (2) and then dividing by  $dA_o$ , i.e.,

$$dC/dA_o = (\partial C/\partial W_1)(\partial W_1/\partial A_o) + (\partial C/\partial Q)(\partial Q/\partial A_o) + \partial C/\partial A_o \quad (6)$$

The first term to the right of the equality sign in (6) is the "wage rate effect" of an operating subsidy on transit costs. If  $\partial W_1/\partial A_o > 0$  and  $\partial C/\partial W_1 > 0$ , an increase in  $A_o$  via wage rate function (3) will result in an increase in  $W_1$  which, in turn, via cost function (2) will result in an increase in transit costs. The second term to the right of the equality sign in (6) is the "output effect" of an operating subsidy on transit costs. If  $\partial Q/\partial A_o > 0$  and  $\partial C/\partial Q > 0$ , an increase in  $A_o$  via vehicle mile function (5) will result in an increase in  $Q$  which, in turn, via cost function (2) will result in an increase in transit costs.

The final term in (6) is the "finance effect" of an operating subsidy on transit costs. The "wage rate effect" represents the increase in transit costs from an operating subsidy via an increase in wage rates, holding service level constant. However, it does not necessarily follow that the increase in  $A_o$  resulting in the increase in  $W_1$  will be sufficient to finance the wage increase, holding the service level constant. The "output effect" represents the increase in transit costs from an operating subsidy via an increase in level of service (or vehicle miles), holding

input prices constant. However, it does not necessarily follow that the increase in  $A_0$  resulting in the increase in  $Q$  and the increase in fare revenue (following the increase in  $Q$  with fare held constant) will be sufficient to finance the service expansion, holding input prices constant. Thus, we interpret  $\partial C/\partial A_0$  in (6) as the increase in the firm's budget, i.e., the budget constraint in (1), from an increase in operating subsidy necessary to finance the remainder of the budget, i.e., in providing the given level of service at the given input prices. The remainder of the budget is that amount not accounted for by the wage rate and output effects. In the literature (see Pucher, Markstedt and Hirschman, 1983),  $\partial C/\partial A_0$  has been interpreted as the direct effect of subsidy on transit costs. Interpreted as the finance effect, we have provided a more definitive interpretation.

Similarly, the total effect of a capital subsidy ( $A_c$ ) on the transit firm's long-run costs ( $dC/dA_c$ ) is the total differential of cost function (2) divided by  $dA_c$ , i.e.,

$$dC/dA_c = (\partial C/\partial W_j)(\partial W_j/\partial A_c) + (\partial C/\partial Q)(\partial Q/\partial A_c) + \partial C/\partial A_c \quad (7)$$

As with operating subsidies, the first term to the right of the equality sign in (7) represents an input price effect from a change in the capital subsidy, i.e., the "capital price effect." The second term represents the "output effect" and the final term represents the "finance effect" of capital subsidy on transit costs. For further discussion of these effects, see the Appendix.

## THE EMPIRICAL MODEL

To estimate (2), the cost function is specified as a translog function (see Christensen, Jorgenson, and Lau, 1973).<sup>8</sup> Firm cost ( $C$ ) is the sum of operating and vehicle capital costs. Transit service ( $Q$ ) is measured in vehicle miles. Inputs include labor, fuel and vehicle capital. Operating subsidies are from federal, state and local governments. Since capital subsidies may not be provided by all three levels of government to a transit firm in a given time period, we

use the sum of these capital subsidies, i.e.,  $A_c = \sum_{j=1}^3 A_{cj}$ , to avoid the problem of missing observations. The translog cost function with one output, three input prices, three operating subsidies and one capital subsidy variable is written as:

$$\begin{aligned}
\ln C = & \alpha_o + \sum_{i=1}^3 \alpha_i \ln W_i + \gamma_q \ln Q + \sum_{i=1}^3 \beta_i \ln A_{oi} + \beta_a \ln A_c \\
& + 1/2 \gamma_{qq} (\ln Q)^2 + \beta_{aa} (\ln A_c)^2 + \sum_{i=1}^3 \sum_{j=1}^3 \alpha_{ij} \ln W_i \ln W_j + \\
& \sum_{i=1}^3 \sum_{j=1}^3 \beta_{ij} \ln A_{oi} \ln A_{oj} + \gamma_{qa} \ln Q \ln A_c + \sum_{i=1}^3 \alpha_{qi} \ln Q \ln W_i \\
& + \sum_{i=1}^3 \alpha_{ai} \ln A_c \ln W_i + \sum_{i=1}^3 \beta_{qi} \ln Q \ln A_{oi} + \sum_{i=1}^3 \beta_{ai} \ln A_c \ln A_{oi} \\
& + \sum_{i=1}^3 \sum_{j=1}^3 \theta_{ij} \ln W_i \ln A_{oj}
\end{aligned} \tag{8}$$

Where, the cross product matrix is symmetric (i.e.,  $\alpha_{ij} = \alpha_{ji}$ ,  $\beta_{ij} = \beta_{ji}$  and  $\theta_{ij} = \theta_{ji}$ ).

In order to insure that (8) is positive-linearly homogeneous, the following restrictions are imposed:

$$\begin{aligned}
\sum_{i=1}^3 \alpha_i = 1, \sum_{i=1}^3 \alpha_{ij} = 0, \sum_{i=1}^3 \alpha_{qi} = 0, \sum_{i=1}^3 \alpha_{ai} = 0, \\
\text{and } \sum_{i=1}^3 \theta_{ij} = 0 \quad \text{where, } j = 1, 2, 3
\end{aligned}$$

Additional information can be introduced into the estimation by employing Shephard's lemma. In logarithmic form, Shephard's lemma for the  $i$ th input can be written:

$$\partial \ln C / \partial \ln W_i = (\partial C / \partial W_i) / (C / W_i) = X_i^* W_i / C = S_i \tag{9}$$

Where,  $X_i^*$  is the cost minimizing quantity of the  $i$ th input and  $S_i$  is the share of the  $i$ th input in total cost ( $C$ ). Applying Shephard's lemma directly to (8) yields the factor share equation for the  $i$ th input:

$$S_i = \alpha_i + \sum_{j=1}^3 \alpha_{ij} \ln W_j + \alpha_{qi} \ln Q + \alpha_{ai} \ln A_c + \sum_{j=1}^3 \theta_{ij} \ln A_{oj} \quad (10)$$

Since the factor share equations do not add additional parameters, it is useful to estimate (8) and the share equations (10) together, thereby increasing the degrees of freedom. However, to avoid the problem of a singular variance-covariance matrix of random error terms (since there are only two independent share equations), one share equation must be deleted. Since it does not matter which one is deleted, we arbitrarily delete the vehicle capital share equation.

Thus, we have six equations to be estimated—translog cost function (8), labor and fuel share equations (10), wage rate function (3), vehicle capital price function (4) and vehicle mile function (5). Functions (3), (4) and (5) are assumed to be log-linear functions. Since it is reasonable to expect that error terms across the six equations will be correlated, the six equations are estimated utilizing Zellner's (1962) seemingly-unrelated technique.

## DATA

Cross-section data are utilized. The primary data source is 1985 Section 15 data (Urban Mass Transportation Administration, 1986). Section 15 data are quite extensive and provide information on cost, performance, service characteristics and subsidies (operating and capital) received for all reporting transit firms. Transit firms that operate only bus transit service (the service most frequently provided by transit firms) are considered in this paper. A sample of 73 transit firms was selected. However, with six equations to be estimated jointly, the sample provides 388 degrees of freedom for estimation.

The sampled transit systems consist mainly of those with yearly revenues between one million and twenty million dollars. There are 54 such transit systems or 74 percent of the sample, while nine systems (12 percent) have revenues of less than one million dollars. Comparatively, 14 percent or ten transit systems have revenues exceeding \$50 million. The largest system in the sample earned \$124 million in revenue in 1985, while the smallest system earned \$300,000.

Since a few large firms may bias the estimation results, a number of larger firms were excluded from the full sample to obtain a sub sample of 50 firms. Specifically, all firms in the full sample with a "ratio of peak to mid-day buses in service" of more than 1.82 (which is equal to the arithmetic mean plus two times the standard deviation of this variable) were excluded to obtain the sub sample. This selection process eliminated transit systems from the full sample with fleet sizes exceeding 561 vehicles. A transit firm's wage rate is based on total labor compensation—wages and benefits. Labor hours are obtained by multiplying the equivalent labor data in the Section 15 report by 2080 hours (i.e., one equivalent labor unit represents 2080 hours).<sup>9</sup> This product is then divided into total labor compensation to obtain the hourly compensation rate, i.e., the firm's wage rate ( $W_i$ ).



The same data source also provides transit fuel prices. These data include diesel fuel, liquefied nitrogen gas and liquified petroleum gas consumed. Gallons of these fuels consumed by a transit firm are summed and then divided into the system's fuel cost to obtain the weighted arithmetic mean price per gallon, i.e., fuel price ( $W_2$ ).

Information on new bus prices is taken from actual contracts awarded to bus manufacturers by transit firms (Metro Magazine, 1985). The weighted arithmetic mean of new bus prices ( $W$ ) for 1985 was utilized in the following equation to obtain the vehicle capital price ( $W_3$ ) for a given transit firm:<sup>10</sup>

$$W_3 = W * (D + r) * \exp(-DA) \quad (11)$$

Where,  $D$  represents the straight line depreciation rate for a bus (where a 12 year replacement cycle is assumed);  $A$  is the average age of the transit system's bus fleet; and  $r$  represents the interest rate on high grade Standard and Poor municipal bonds which was 10 percent in 1985 (Economic Report of the President, p. 332). The average age of a transit system's bus fleet ( $A$ ) was computed utilizing the following equation:

$$A = \frac{\sum_b B_b A_b}{\sum_b B_b} \quad (12)$$

Where,  $B_b$  is the number of buses of type  $b$  and  $A_b$  is the average age for buses of type  $b$  for the transit system.

By multiplying the vehicle capital price (in equation 11) by the firm's number of buses ( $\sum B_b$ ), we obtain the total vehicle capital cost ( $C_3$ ) for the firm, i.e.,

$$C_3 = \left( \sum_b B_b \right) W * (D + r) * \exp(-DA) \quad (13)$$

Adding  $C_3$  to the transit firm's operating cost, yields the total cost ( $C$ ) that appears in cost function (2).

A problem with cost analyses of transportation firms is the absence of a unique measure of output. Transportation output includes produced output measures such as vehicle miles and seat miles and consumed output measures such as passengers and passenger miles. Vehicle miles have been utilized extensively in the estimation of cost functions of transportation firms; vehicle miles is used as our firm output measure.

Federal, state and local operating subsidies are symbolized by  $A_{o1}$ ,  $A_{o2}$  and  $A_{o3}$ , respectively. The federal operating subsidy program is a matching grant program for financing transit operating deficits, requiring a dollar of local or state operating subsidy for every dollar of federal operating subsidy, up to a specified maximum total federal contribution. This maximum amount varies by locality and is determined by a formula based on non-incentive tiers such as local population and population density and to a lesser extent on incentive tiers such as passenger miles squared and cost.<sup>11</sup> Since state and local operating subsidies generally do not entail explicit matching provisions, they may be classified as lump-sum grants.<sup>12</sup> Similarly, the

federal capital subsidy program is a matching grant program for financing transit capital purchases, requiring a dollar of local or state capital subsidy for every four dollars of federal capital subsidy up to a specified maximum total federal contribution. State and local capital subsidies are lump-sum grants. To avoid the problem of missing observations (as stated previously), we use the sum of federal, state and local capital subsidies (i.e.,  $A_c$ ) in our estimations.<sup>13</sup>

## EMPIRICAL RESULTS

Since a few large firms may bias the estimation results, a number of large firms were excluded from the full sample to obtain a sub sample of 50 firms. Specifically, all firms in the full sample with a "ratio of peak to mid-day buses in service" of more than 1.82 (which is equal to the arithmetic mean plus two times the standard deviation of this variable) were excluded to obtain the sub sample. This selection process eliminated transit systems from the full sample with fleet sizes exceeding 561 vehicles. Descriptive statistics for our variables related to the two samples are found in Table 1. The parameter estimates from joint estimation of translog cost function (8), labor and fuel share equations (10), wage rate function (3), vehicle capital price function (4) and vehicle mile function (5) are presented in Table 2 through 5. Cost-function parameter estimates for the full sample (73 transit firms) and the sub sample of (50 transit firms) are in Table 2A and 2B. Since the parameters of the labor and fuel share equations (10) are also parameters of translog cost function (8), their parameter estimates are also found in Table 2A and 2B.

The variables in our cost function have been normalized by removing their sample means and are presented as logarithms. Hence, the cost function's first-order coefficients are interpretable as long-run cost elasticities evaluated at sample means. For the full sample (see Table 2A), the cost function's first-order coefficients for vehicles miles and input prices have the expected signs and are highly significant. Among the first-order coefficients for subsidies, only the coefficient for federal operating subsidy is significant (but at ten percent level). Similar findings follow for the sub sample of transit firms (see Table 2B) except the federal operating subsidy coefficient is now highly significant.

Estimation of the wage rate function reveals that the parameter estimates for state operating and local operating subsidies are significant and positive for both the full and sub samples (see Table 3). Since the first-order coefficient of the wage rate variable is positive and highly significant in the cost function (see Table 2A and 2B), our results indicate that state and local operating subsidies have positive wage rate effects on transit costs. The parameter estimate for federal operating subsidy in the wage rate function is insignificant (for both samples).<sup>14</sup> This result is in contrast to Pucher, Markstedt and Hirschman (1983), which reports a significant positive relationship between transit wage rate and federal operating subsidies per bus hour (utilizing 1979 and 1980 data). In addition to the operating subsidies, the unemployment rate ( $M$ ) in the firm's local service area was also included as an explanatory variable to account for differences among labor markets in the local service areas.<sup>15</sup> For both samples, its parameter estimate is insignificant.

TABLE 1

**Descriptive Statistics: Arithmetic Mean and Standard Deviation**

<b>Variable</b>	<b>Full Sample (73 Transit Firms)</b>	<b>Sub Sample (50 Transit Firms)</b>
C	11,645,815 (22,027,999)	8,461,419 (16,729,901)
W <sub>1</sub>	8.82 (2.18)	8.82 (2.36)
W <sub>2</sub>	0.97 (0.22)	0.99 (0.25)
W <sub>3</sub>	36,214 (8,662)	37,821 (9,014)
Q	3,585,521 (6,210,473)	2,633,141 (4,316,601)
A <sub>01</sub>	1,704,417 (2,199,410)	1,422,664 (1,691,846)
A <sub>02</sub>	1,488,752 (3,226,258)	1,239,288 (3,258,272)
A <sub>03</sub>	3,401,512 (9,017,479)	2,431,149 (5,275,691)
A <sub>c</sub>	4,012,800 (11,325,430)	2,479,870 (7,276,860)
M	0.07 (0.02)	0.07 (0.02)

\*Standard deviations are in parentheses. All variables except "Q" and "M" are expressed in monetary terms (i.e., in dollars).

TABLE 2A

## Estimated Coefficients: Transit Translog Cost Function (Full Sample)

Coefficient	Estimate	Coefficient	Estimate	Coefficient	Estimate
$\alpha_0$	-0.0464 (0.0334)	$\alpha_{23}$	-0.0292*** (0.0063)	$\beta_{q1}$	0.0920 (0.1149)
$\alpha_1$	0.5859*** (0.0061)	$\alpha_{33}$	-0.1272*** (0.0291)	$\beta_{q2}$	0.0401 (0.0446)
$\alpha_2$	0.0855*** (0.0515)	$\beta_{11}$	-0.0016 (0.0726)	$\beta_{q3}$	0.0632 (0.0549)
$\alpha_3$	0.3287*** (0.0057)	$\beta_{12}$	-0.0265 (0.0346)	$\beta_{a1}$	-0.0418* (0.0247)
$\gamma_q$	0.8606*** (0.0718)	$\beta_{13}$	-0.0323 (0.0509)	$\beta_{a2}$	-0.0150 (0.0105)
$\beta_1$	0.1001* (0.0515)	$\beta_{22}$	0.0254 (0.0228)	$\beta_{a3}$	-0.0237 (0.0179)
$\beta_2$	0.0307 (0.0188)	$\beta_{23}$	-0.0002 (0.0133)	$\Theta_{11}$	0.0100 (0.0104)
$\beta_3$	-0.0015 (0.0230)	$\beta_{33}$	0.0223 (0.0221)	$\Theta_{12}$	0.0126*** (0.0051)
$\beta_a$	0.0018 (0.0121)	$\gamma_{qa}$	0.0963** (0.0397)	$\Theta_{13}$	-0.0022 (0.0061)
$\gamma_{qq}$	-0.3328* (0.1807)	$\alpha_{q1}$	0.0159 (0.0141)	$\Theta_{21}$	-0.0049* (0.0026)
$\beta_{aa}$	-0.0018 (0.0056)	$\alpha_{q2}$	0.0092** (0.0035)	$\Theta_{22}$	-0.0026*** (0.0012)
$\alpha_{11}$	0.1350*** (0.0296)	$\alpha_{q3}$	-0.0251* (0.0134)	$\Theta_{23}$	0.1237*** (0.0549)
$\alpha_{12}$	-0.0369*** (0.0058)	$\alpha_{a1}$	-0.0044 (0.0033)	$\Theta_{31}$	-0.0051 (0.0099)
$\alpha_{13}$	-0.0980*** (0.0286)	$\alpha_{a2}$	0.0004 (0.0008)	$\Theta_{32}$	-0.0100*** (0.0048)
$\alpha_{22}$	0.0661*** (0.0068)	$\alpha_{a3}$	0.0041 (0.0032)	$\Theta_{33}$	-0.1215*** (0.0552)

\*(\*\*, \*\*\*)Significant at the ten (5, 1) percent level.

Standard errors are in parentheses. The system weighted  $R^2$  equals 0.9551 and degrees of freedom are 388.

TABLE 2B

## Estimated Coefficients: Transit Translog Cost Function (Sub Sample)

Coefficient	Estimate	Coefficient	Estimate	Coefficient	Estimate
$\alpha_0$	-0.0098 (0.0215)	$\alpha_{23}$	-0.0318** (0.0072)	$\beta_{q1}$	0.1496 (0.0997)
$\alpha_1$	0.5724*** (0.0082)	$\alpha_{33}$	0.1406** (0.0360)	$\beta_{q2}$	0.1256** (0.0380)
$\alpha_2$	0.0874*** (0.0020)	$\beta_{11}$	-0.0401 (0.0479)	$\beta_{q3}$	-0.1109* (0.0534)
$\alpha_3$	0.3403*** (0.0076)	$\beta_{12}$	-0.0797** (0.0263)	$\beta_{a1}$	-0.0442* (0.0186)
$\gamma_q$	0.8267*** (0.0900)	$\beta_{13}$	-0.0450 (0.0464)	$\beta_{a2}$	-0.0153 (0.0080)
$\beta_1$	0.1467* (0.0347)	$\beta_{22}$	-0.0082 (0.0145)	$\beta_{a3}$	-0.0243* (0.0120)
$\beta_2$	0.0019 (0.0119)	$\beta_{23}$	-0.0085 (0.0122)	$\theta_{11}$	0.0191 (0.0142)
$\beta_3$	-0.0422 (0.0150)	$\beta_{33}$	0.0274 (0.0163)	$\theta_{12}$	0.0105 (0.0075)
$\beta_a$	-0.0077 (0.0077)	$\gamma_{qa}$	0.1359*** (0.0323)	$\theta_{13}$	0.0002 (0.0093)
$\gamma_{qq}$	-0.6099* (0.1642)	$\alpha_{q1}$	0.0064 (0.0207)	$\theta_{21}$	-0.0062 (0.0035)
$\beta_{aa}$	-0.0031 (0.0082)	$\alpha_{q2}$	0.0115* (0.0051)	$\theta_{22}$	0.0028 (0.0017)
$\alpha_{11}$	0.1449*** (0.0366)	$\alpha_{q3}$	-0.0178 (0.0197)	$\theta_{23}$	-0.1520*** (0.0345)
$\alpha_{12}$	-0.0361*** (0.0072)	$\alpha_{a1}$	0.0002 (0.0093)	$\theta_{31}$	-0.0128 (0.0135)
$\alpha_{13}$	-0.1088*** (0.0350)	$\alpha_{a2}$	0.0016 (0.0013)	$\theta_{32}$	-0.0076 (0.0072)
$\alpha_{22}$	0.0679*** (0.0076)	$\alpha_{a3}$	0.0069 (0.0050)	$\theta_{33}$	-0.1523*** (0.0351)

\*(\*\*, \*\*\*)Significant at the ten (5, 1) percent level.

Standard errors are in parentheses. The system weighted  $R^2$  equals 0.9789 and degrees of freedom are 250.

TABLE 3

**Wage Rate Function: Dependent Variable  $\ln W_1$** 

Variable	Parameter Estimate	
	Full Sample	Sub Sample
Intercept	-0.0003 (0.0244)	-0.0002 (0.0333)
$\ln A_{01}$	-0.0254 (0.0298)	-0.0070 (0.0433)
$\ln A_{02}$	0.0538*** (0.0173)	0.0482** (0.0232)
$\ln A_{03}$	0.0512*** (0.0164)	0.0450* (0.0229)
$\ln M$	0.3642 (1.1419)	0.7095 (1.5124)

\*(\*\*, \*\*\*)Significant at the ten (5, 1) percent level.  
Standard errors are in parentheses.

TABLE 4

**Vehicle Capital Price Function: Dependent Variable  $\ln W_3$** 

Variable	Parameter Estimate	
	Full Sample	Sub Sample
Intercept	0.0002 (0.0252)	-0.0000 (0.0298)
$\ln A_c$	0.0275*** (0.0103)	0.0310** (0.0139)

\*\*(\*\*\*)Significant at the ten (5, 1) percent level.  
Standard errors are in parentheses.

TABLE 5

**Vehicle Mile Function: Dependent Variable  $\ln Q$** 

Variable	Parameter Estimate	
	Full Sample	Sub Sample
Intercept	$-2.6173 \times 10^{-5}$ (0.0528)	0.0004 (0.0599)
$\ln A_{01}$	0.5054*** (0.0643)	0.4316*** (0.0784)
$\ln A_{02}$	0.1371*** (0.0375)	0.1680*** (0.0441)
$\ln A_{03}$	0.2664*** (0.0382)	0.3258*** (0.0431)
$\ln A_c$	0.0584** (0.0272)	0.0201 (0.0373)

\*\*(\*\*\*).Significant at the five (1) percent level.  
Standard errors are in parentheses.

Estimation of the vehicle capital price function reveals that the parameter estimate for capital subsidy is significant and positive in both samples (see Table 4). Since the first-order coefficient for vehicle capital price is positive and highly significant in the cost function (see Table 2A and 2B), the estimation results indicate that the capital subsidy has a positive capital price effect on transit costs.

Estimation of the vehicle mile function reveals that the parameter estimates for all three operating subsidies are highly significant and positive for both samples (see Table 5). For the full sample, the parameter estimate for capital subsidies is significant at the five percent level and positive; for the sub sample, it is insignificant. The first-order coefficient for vehicle miles is positive and highly significant in the cost function (see Table 2A and 2B). Thus, the estimation results indicate that operating subsidies have positive output effects on transit costs. For the full sample, the estimation indicates that capital subsidy has a positive output effect on transit costs.

Let us now investigate the total effect of each operating subsidy on transit costs utilizing function (6) and the parameter estimates found in the estimated cost, wage rate and vehicle mile functions. Since the variables in the functions are in logarithms, we rewrite function (6) in terms of logarithms, i.e.,

Equation (14) measures the degree of cost elasticity with respect to each subsidy variable. This elasticity is the sum of "wage rate effect", "output effect," and "finance effect" operating

$$d\ln C/d\ln A_o = (\partial \ln C/\partial \ln W_1)(\partial \ln W_1/\partial \ln A_o) + (\partial \ln C/\partial \ln Q)(\partial \ln Q/\partial \ln A_o) + \partial \ln C/\partial \ln A_o \quad (14)$$

subsidy cost elasticities. The operating subsidy cost elasticity for the "wage rate effect" is the product of the wage rate cost elasticity (from the cost function) and the operating subsidy wage rate elasticity (from the wage rate function). The operating subsidy cost elasticity for the "output effect" is the product of the vehicle mile cost elasticity from the cost function and the operating subsidy vehicle mile elasticity from the vehicle mile function. The operating subsidy elasticity for the "finance effect" is simply the operating subsidy cost elasticity from the cost function.

In evaluating the variables in (14) at their sample means, the elasticities from the estimated cost function are the respective first-order coefficients in the function. This follows, since the variables in the estimated cost function were normalized prior to estimation by removing their sample means. The estimated total operating subsidy cost elasticity for each operating subsidy and their component elasticities taken from the various estimated functions (evaluated at sample means) are found in Table 6 for both the full and sub samples. These elasticities suggest that transit costs are more responsive to federal than to state and local operating subsidies and are more responsive to local than to state operating subsidies. Further, for each total operating subsidy cost elasticity, the "output effect" operating subsidy cost elasticity is the largest elasticity, thus suggesting that service expansion (and not rising wage rates) attributable to rising operating subsidies is the major source of the increase in transit costs attributable to operating subsidies.

Based upon function (7), the total effect of capital subsidy on transit costs expressed in logarithms may be written as:

$$d\ln C/d\ln A_c = (\partial \ln C/\partial \ln W_3)(\partial \ln W_3/\partial \ln A_c) + (\partial \ln C/\partial \ln Q)(\partial \ln Q/\partial \ln A_c) + \partial \ln C/\partial \ln A_c \quad (15)$$

Thus, the total capital subsidy cost elasticity ( $d\ln C/d\ln A_c$ ) is the sum of "capital price effect," "output effect," and "finance effect" elasticities. The capital subsidy cost elasticity for the "capital price effect" is the product of the capital price cost elasticity (from the cost function) and the capital subsidy capital price elasticity (from the capital price function). The capital subsidy cost elasticity for the "output effect" is the product of the vehicle mile cost elasticity (from the cost function) and the capital subsidy vehicle mile elasticity (from the vehicle mile function). The capital subsidy cost elasticity for the "finance effect" is the capital subsidy cost elasticity (from the cost function).

Our estimate of the total capital subsidy elasticity is found by substituting into equation (15) the various elasticities (evaluated at sample means) found in the estimated functions and solving. Estimates for both samples appear in Table 6. Estimates of the total capital subsidy cost elasticity suggest that transit costs are less responsive to capital subsidies than to operating subsidies. Here too, the "output effect" elasticity is larger than the two other subsidy effect



TABLE 6

## Total Subsidy Cost Elasticity

$$d\ln C/d\ln A = \epsilon_{CW}\epsilon_{WA} + \epsilon_{CQ}\epsilon_{QA} + \epsilon_{CA}$$

Subsidy	Full Sample		Sub Sample	
Federal Operating	0.5201 =	(0.5859)(-0.0254) + (0.8605)(0.5054) + 0.1001	0.4995 =	(0.5724)(-0.0070) + (0.8267)(0.4316) + 0.1467
	=	-0.0149 + 0.4349 + 0.1001	=	-0.0040 + 0.3568 + 0.1467
State Operating	0.1802 =	(0.5859)(0.538) + (0.8605)(0.1371) + 0.0307	0.1684 =	(0.5724)(0.0482) + (0.8267)(0.1680) + 0.0019
	=	0.0315 + 0.1180 + 0.0307	=	0.0276 + 0.1389 + 0.0019
Local Operating	0.2577 =	(0.5859)(0.0512) + (0.8605)(0.2664) - 0.0015	0.2529 =	(0.5724)(0.0450) + (0.8267)(0.3258) - 0.0422
	=	0.0300 + 0.2292 - 0.0015	=	0.0258 + 0.2693 - 0.0422
Capital	0.0611 =	(0.3287)(0.0275) + (0.8605)(0.0584) + 0.0018	0.0194 =	(0.3403)(0.0310) + (0.8267)(0.0201) - 0.0077
	=	0.0090 + 0.0503 + 0.0018	=	0.0105 + 0.0166 - 0.0077

Note: The symbol "ε" represents an elasticity coefficient.

elasticities, thus suggesting that service expansion attributable to rising capital subsidies is the major source of the increase in transit costs attributable to capital subsidies.

Utilizing factor share equation (10) and estimated parameters found in Table 2A (i.e., the full sample), we also investigate the effects of subsidies on factor (or input) shares. If the estimates for parameters  $\alpha_{ai}$  and  $\theta_{ij}$  are negative (positive) and statistically significant, the capital subsidy and the jth operating subsidy, respectively, are said to be ith factor saving (using)

since the cost share of the  $i$ th factor is lowered (raised).<sup>16</sup> Otherwise, the effect is neutral, since factor cost shares are not affected. Our estimation results (summarized in Table 7) suggest that federal operating subsidy lowered fuel's cost share; state operating subsidy also lowered fuel's cost share as well as vehicle capital's cost share but raised labor's cost share; local operating subsidy also lowered vehicle capital's cost share but raised fuel's cost share; and capital subsidy had no effect on factor cost shares.

In order to investigate the effect of subsidies on firm scale economies, we may utilize the following equation (obtained from cost equation 8) representing the cost elasticity of output:

$$\begin{aligned} \partial \ln C / \partial \ln Q = & \gamma_q + \gamma_{qq} \ln Q + \gamma_{qa} \ln A_c + \sum \alpha_{qi} \ln W_i \\ & + \sum \beta_{qi} \ln A_{oi} \end{aligned} \quad (16)$$

Evaluating the variables in (16) at their sample means, the point estimate of the cost elasticity of output is  $\gamma_q$ . In Table 2A, its parameter estimate is 0.8605 and 0.8267 in Table 2B. These parameter estimates are significant from zero at the one percent level and significant from one at the ten percent level, thus indicating firm increasing returns to scale. If the estimates for parameters,  $\gamma_{qa}$  and  $\beta_{qi}$ , are negative (positive) and statistically significant, the capital subsidy and the  $i$ th operating subsidy, respectively, increase (lower) scale economies. In both Table 2A and 2B, the parameter estimate for  $\gamma_{qa}$  is positive and significant from zero, suggesting that capital subsidy lowers scale economies. From Table 2B, the parameter estimates for  $\beta_{q2}$  and  $\beta_{q3}$  are positive and negative, respectively as well as being significant—suggesting that state operating subsidy lowers scale economies and local operating subsidy increases scale economies.

## CONCLUSION

The literature suggests that transit subsidies inflate transit costs. However, there has been little investigation of the source (or sources) of these cost increases. The one exception being that increases in operating subsidies lead to increases in transit wages. The purpose of this paper has been to extend the literature by investigating sources of public transit long-run cost increases attributable to transit subsidies. In addition to wage increases, these sources include increases in vehicle capital prices and service.

From estimation of a multi-equation model consisting of a transit (bus) firm's cost, wage rate, capital price and vehicle mile functions, transit costs in general were found to be positively related to transit subsidies. These findings are in agreement with the previous literature. Transit cost increases related to transit subsidies were classified as input price, output and finance effects of these subsidies. The input price effect is the increase in cost following an increase in an input price that is attributable to a subsidy increase. The output effect is the increase in cost following service expansion that is attributable to a subsidy increase. The finance effect is the increase in a firm's budget (and thus cost) from increasing a subsidy in order to finance the remainder of the firm's budget not financed as an outcome of the input price and output effects of the given subsidy.

TABLE 7

**Biased Subsidy Effects**

<b>Input</b>	<b>Federal Operating Subsidy</b>	<b>State Operating Subsidy</b>	<b>Local Operating Subsidy</b>	<b>Capital Subsidy</b>
Labor	Neutral	Using	Neutral	Neutral
Fuel	Saving	Saving	Using	Neutral
Vehicle Capital	Neutral	Saving	Saving	Neutral

The total effect of a given subsidy on transit costs was measured in terms of its total subsidy cost elasticity. The total subsidy cost elasticity is the sum of the input price effect, the output effect and the finance effect subsidy cost elasticities. For the three operating subsidies as well as capital subsidy, the output effect elasticity was the largest elasticity among the three subsidy cost elasticities for each subsidy--thus suggesting that service expansion (and not rising wage and vehicle capital prices) is the major source of the increase in transit costs attributable to transit subsidies. Further, based upon total subsidy cost elasticities, transit costs are more responsive to federal operating subsidies, followed in declining order by local operating, state operating and capital subsidies, respectively.

The estimation results suggest that operating subsidies but not capital subsidy affect the cost shares of transit inputs. Capital subsidy was found to lower scale economies, but less evidence was found to suggest that operating subsidies affect scale economies.

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## ENDNOTES

- \* The authors are, respectively, Professor, Department of Economics, North Carolina A&T University; Professor, Department of Economics, Old Dominion University; and Professor, Department of Economics, Old Dominion University. The authors acknowledge R.L. Basmann's helpful comments.
- 1. For transit subsidy studies utilizing U.S. transit firms, see Anderson (1983), Cervero (1984a, 1984b), Pickrell (1985), Pucher, Markstedt and Hirschman (1983), Pucher and Markstedt (1983). For a study where transit firms across countries are considered, see Bly, Webster and Pounds (1980).
- 2. Pucher, Markstedt and Hirschman (1983) found that federal and state operating subsidies per bus hour of service have a positive and significant impact on transit hourly wage rates. The estimated parameter in their linear hourly wage rate equation for both federal and state operating subsidies was 0.06.
- 3. This hypothesis has been suggested by Frankena (1981).
- 4. Although Frankena (1987) does not specifically suggest this hypothesis, his empirical results support the hypothesis that capital subsidies increase the scrapping probability for buses. By deduction, the probability of purchasing new buses is increased, thereby increasing the likelihood that new bus prices will rise as a result of capital subsidies.

5. In 1982, total U. S. transit operating subsidies from all three levels of government were \$3.6 billion; of which 25.6%, 30.5% and 43.9% were from federal, state and local governments or \$0.92, \$1.10 and \$1.58 billion, respectively. In 1986, the total transit operating subsidies were \$6.2 billion; of which 14.2%, 35.3% and 50.4% were from federal, state and local government or \$0.88, \$2.19 and \$3.13 billion, respectively. These data are found in an Urban Mass Transportation Administration (1990) report.
6. Such assumptions for a transit firm have also been made by Kim and Spiegel (1987). Also see Deller, Chicoine and Walzer (1988). Our transit firm, however, does differ from the one considered by Kim and Spiegel (KS). The U.S. transit firm is generally owned and regulated by local government. Hence, it is not a profit seeking firm but may seek to maximize ridership subject to a deficit constraint subsidized by various levels of government. The KS firm is a profit seeking firm and a lump-sum subsidy recipient operating under rate of return regulation. It "will adjust its reported profits by choosing the desired level of output and factor inputs to the size of a given level of subsidy in such a way that the reported rate of return per unit of capital employed does not exceed the reasonable amount" (Kim and Spiegel, 1987, pp. 105-106). The behavior of the firm under this rate of return regulatory constraint is assumed to follow that of the Averch and Johnson (1962) model. The U.S. transit firm generally does not operate under a rate of return regulatory constraint, since it is non-profit seeking and since capital purchases are generally financed by capital subsidies from various levels of government. Hence, we do not incorporate KS's rate of return constraint in our model nor utilize the Averch and Johnson (1962) firm behavior model.
7. We assume subsidies to be exogenously determined. This is a reasonable assumption. The bases for allocating federal operating subsidies among transit firms, for example, are incentive and non-incentive tiers. However, only 9.2 percent of the total allocation is based upon incentive tiers such as passenger miles squared and cost, with the remaining 90.8 percent based upon non-incentive tiers such as population size and density of urban areas.
8. For applications of the translog cost function to transportation firms, see Talley (1988).
9. The 2080 hours represent annual hours per employee, i.e., a 40 hour week per employee times 52 weeks in a year.
10. This equation is based upon an equation found in Berechman and Giuliano (1984) for determining transit vehicle capital prices. However, unlike Berechman and Giuliano, we do not adjust our equation for the federal government's subsidy share of a transit firm's capital expenditure. The rationale is that our budget constraint includes capital subsidies and hence our vehicle capital price should reflect the firm's use of capital subsidies.
11. See footnote #7.

12. Transit operating subsidy programs are specific-grant (as opposed to general-grant) programs, since the subsidies must be spent on transit operating deficits as stipulated by the grantor. The specific-grant is a lump-sum grant if the transit firm receives a fixed sum of money. The specific-grant is a matching grant if the latter sum received is variable and depends on how much of the grantee's own revenue (as from state or local operating lump-sum grants) it spends in financing the operating deficit. For further discussion of lump-sum and matching grants, see King (1984, Ch. 3).
13. For further discussion of federal operating and capital subsidy programs, see Talley (1983, pp. 304-307).
14. Shughart and Kimenyi (1991) found a significant but negative relationship between the transit hourly wage rate and the percent of transit revenue that is federal operating subsidy. Our sign between the hourly wage rate and the amount of federal operating subsidy is negative but insignificant. Shughart and Kimenyi (1991, p. 28) suggest that one possible explanation for their result is that "factors entering into calculating the federal operating subsidy amount cause transit systems in smaller cities to receive a proportionately greater share of operating revenues from federal sources than their counterparts in larger cities who consequently rely more heavily on state and local subsidies." Obeng (1991) found a negative relationship between federal operating subsidy and cost and, like Shughart and Kimenyi (1991), attributes the result to overallocation of the subsidy to small transit systems.
15. Data for unemployment rates (M) are those reported for October 1985 by the United States Department of Labor (1985). Unemployment rates (if available) of the relevant cities in which transit systems provided service were utilized. If these rates were not available, unemployment rates of the state of the relevant cities were utilized as proxies.
16. For further discussion, see Kim and Spiegel (1987).

## APPENDIX

This appendix develops a graphical analysis of the effect on transit costs from a change in the operating and/or capital subsidies. Recall, as demonstrated by equation (6) that there are three effects on firm costs from a subsidy change: (1) the input price effect, (2) the output effect, and (3) the finance effect, or:

$$\frac{dC}{dA} = \frac{\partial C}{\partial W} \frac{\partial W}{\partial A} + \frac{\partial C}{\partial Q} \frac{\partial Q}{\partial A} + \frac{\partial C}{\partial A} \quad (1A)$$

Further recall that transit firm costs are minimized subject to two different constraints: the production function  $\bar{Q} = h(X_1, X_2, \dots, X_n)$  and the budget constraint

$$PQ + R + E = \sum_{i=1}^n W_i X_i - \sum_{j=1}^m A_{oj} - \sum_{j=1}^m A_{cj}.$$

The constraints are binding when the firm is in long-run equilibrium.

Without loss of generality assume an increase in an operating subsidy increases the relative price of labor, holding constant the price of capital and the price of fuel. Assume, for the graphical analysis, no change in the use of capital due to the introduction of the subsidy and that the levels of all other subsidies are equal to zero. This essentially assumes strong separability of labor and fuel from capital. Hence the graph will allow us to concentrate on how changes in the operating subsidy affect the use of labor, fuel, firm costs, and total output, ceteris paribus. Also, so that the vertical axis of the diagram may be considered as expenditures on fuel and labor (sub-group costs), assume the price of fuel is fixed at \$1. Finally assume that  $R = E = 0$ , i.e., non-fare revenue is equal to zero and the budget is balanced. The simplifying assumptions imply that the budget constraint may be written as:

$$PQ = W_1 X_1 + X_2 - A_{01} \quad (2A)$$

In Figure 1, the transit firm minimizes total cost for the output level  $Q_0$  along the isocost line  $M_0-N_0$  as indicated by point  $E_0$ . Total expenditures for labor and fuel are equal to  $M_0$ . Assume that the level of transit operating subsidy increases resulting in an increase in the price of a unit of labor. Since we are holding output constant, this pure substitution effect will always cause the quantity of labor to decrease following the increase in total input costs. The increase in the price of labor rotates the isocost curve to  $M_1-N_1$ , where equilibrium is attained at  $E_1$ . Hence the distance  $M_1-M_0$  measures the cost increase due to the input price effect.

The output effect from an increase in operating subsidy is the increase in cost following the expansion to a new level of service while holding input prices constant. This is indicated by a movement from the  $Q_0$  isoquant tangency point  $E_0$  to the  $Q_1$  isoquant tangency point  $E_2$ . Hence the distance  $M_2-M_0$  measures the output effect.



The finance effect from an increase in operating subsidy is the increase in cost that is necessary to finance the remainder of the budget (in providing the new level of service at the new labor price attributable to the input price and output effects), where the remainder of the budget is that amount left unfinanced by these two effects. The cost increase incurred in moving from  $Q_0$  isoquant tangency point  $E_0$  to  $Q_1$  isoquant tangency point  $E_3$  (i.e., at the new service level and labor price) is measured by the distance  $M_3-M_0$ . Hence the distance  $(M_3-M_0) - (M_1-M_0) - (M_2-M_0)$  measures the finance effect.

In summary the total effect on cost  $(M_3-M_0)$  due to the introduction of the subsidy is the combination of the input price, output, and finance effects and is illustrated in Figure 1 as the sum of the distances  $M_1-M_0$  (the input price effect),  $M_2-M_0$  (the output effect), and  $(M_3-M_0) - (M_1-M_0) - (M_2-M_0)$  (the finance effect).