PROCEEDINGS —
Twenty-first Annual Meeting

Theme:

"Transportation Challenges in
A Decade of Change"

October 27-28-29, 1980
Fairmont Hotel
Philadelphia, Pennsylvania

Volume XXI • Number 1 1980

TRF

TRANSPORTATION RESEARCH FORUM

A Study of Inter-fareclass Competition in Airline Markets†

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I. INTRODUCTION

THE DE-REGULATION MOVEMENT, which began officially in 1975 through the aggressive promotion of the legislative proposals for airline regulatory reform both by the U.S. Senate's Kennedy Subcommittee and by the Ford Administration, has continuously increased market pressures on the U.S. scheduled airlines. On the other hand, soaring fuel price coupled with double digit inflation made it impossible for the airlines to decrease their standard economy fares. In order to cope with the increasing market pressures for lower fares, therefore, the scheduled airlines competitively introduced and/or expanded availability of various forms of discount and promotional fare programs, and gradually relaxed the conditions under which the low fares can be applied. In the mean time, the airline industry hoped to keep the majority of business travellers in the first class or the standard economy fare class, while the discount fares generate additional traffic to help relieve the chronic situation of capacity underutilization.

Virtually on all long-haul routes, the majority of passengers now fly on discount fares. This is especially so on routes to and from resort cities. Because of the sharp increase in low fare passengers since the beginning of the de-regulation movement, airlines have already reached a point where major capacity expansion is necessary to accommodate the growing low fare passengers. Low fare passengers, initially served to increase load factor by filling in empty seats, are no longer marginal to the system. They now begin to impose capacity costs to the system.

As role of the low fare passengers in airline finance changes, it is natural to ask questions concerning the airlines' long-run profitability and economic efficiency of the current inter-fareclass pricing, which has evolved as airlines responded to the rising market pressure probably with short-term calculation. As the first step toward such a comprehensive analysis, this study shall attempt to estimate a system of derived demand functions for the three major air fare classes from a set of selected route-specific cross-sectional data of the U.S. intercity airline markets in 1978. The three fare-classes are first class, standard economy fare-class (includes first class discount fares), and discount fareclass (includes, among others, excursion fares, charter class fares, commuter fares, night flight fares, holiday and weekend fares and many other promotional fares).

The recent attempts to investigate the demands for fare classes include Mutti and Murai [1977] and Straszheim [1979] on the North Atlantic routes. Although their discussions on inter-fareclass competition are instructive, their demand functions suffer from the following weaknesses:

1. The yearly data aggregated over all North Atlantic routes (or by country in the case of Mutti and Murai's work), were used to estimate demand functions.
2. Cobb-Douglas demand function used in their studies is not appropriate for studying inter-fareclass competition because it restricts the elasticities of substitution between any pair of fareclasses on all routes to "unity."
3. Univariate multiple regression employed in their studies gives "biased test statistics," possibly leading to a choice of wrong model.

To avoid the above weaknesses and to provide reliable estimates for various measures indicating the degree of inter-fareclass competition, the following approach is taken in this study:

1. A system of demand functions for the three fareclasses are derived, consistently with theory of demand, from a 'translog' form of reciprocal indirect utility function. A translog form allows for free variation of the elasticities of substitution between any two fareclasses.
2. The demand functions for the three fare-classes are estimated jointly by a multivariate nonlinear least squares method using the route-specific cross-sectional yearly data (1978) of the selected intra-U.S. routes.
3. Within the translog context, we test several alternative structures of traveller

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† This work is supported in part by the Social Sciences and Humanities Research Council of Canada (SSHRCC) under a grant No. 410-78-0456 to Tae H. Oum, and in part by the Institute of Research on Public Policy (IRPP).

preferences such as non-homothetic general model, homotheticity and log-linear preferences which have been commonly assumed in most other studies including Mutti and Murai [1977] and Straszheim [1979].

This procedure allows us to choose the demand model which describes traveller choice behavior properly, and thus to obtain more reliable estimates of various measures indicating the degree of inter-fareclass competition on various routes.

This paper is organized as follows:

Section II discusses the derivation of the non-homothetic general form of demand system, and the formulae for computing elasticities of inter-fareclass substitution and price and route budget elasticities of fareclass demands. The conditions under which one can estimate our particular form of route-specific fareclass demands are also discussed in this section.

Section III presents data construction and sources of the data. Estimation and tests of the hypotheses concerning structure of the preferences are presented in Section IV.

Finally, empirical results are presented in Section V along with their implications on inter-fareclass competition.

II. FORMULATION OF THE DEMAND MODEL

(A) Theoretical Issues

Virtually all previous studies on cross-sectional demand models have estimated various forms of statistical relationships between demands and their potential determinants such as prices and quality variables of the alternative choices, and socio-economic and demographic characteristics of the origin and the destination, without directly relating to the theoretical framework in which consumers optimize their purchase decisions. This is understandable considering that when consumer theory is applied to an empirical demand study involving only a small sub-sector such as route specific travel mode choice and fareclass choice, it requires to impose many restrictive assumptions about the structure of preferences which may not be justified empirically. Nonetheless, these assumptions are implicitly imposed in their statistical demand equations.

Since a derived demand system will be used in this study, it seems essential to describe briefly about the assumptions required to single out the fareclass demand system from the over-all framework of consumer demand system. Given our interest in investigating inter-fareclass competition, it is necessary to express demands for the fareclasses as functions of only the available route-specific data such as prices and quality variables of alternative fareclasses, and route-specific total air travel cost (or budget) and market characteristics. Purely statistical demand studies normally include, among other variables, populations and incomes of the origin and destination cities as the explanatory variables in the demand equations. This practice is theoretically equivalent to the assumption that all passengers flown on a route were residents of the two cities. To avoid this problem, we use the route-specific total cost of air travel instead of the population and income.

This assumes that the consumer preferences satisfy the so-called **budgetability** condition[1] such that it is possible for the consumer to allocate the discretionary income **optimally** among, say, **m** budget categories (in our case, including the air travel budgets on various routes) knowing only the **m** aggregate category prices and without information on the prices of individual items in each category. This condition may not hold in reality because the first stage allocation of the travel budget to each route is likely to be influenced by the prices of individual fareclasses as well as by the route specific aggregate fare indexes on all routes. However, degree and nature of the inter-fareclass competition which are of our major interest can be fully measured by constructing the demand model **conditional** on the amount of budget allocated to a specific route regardless of optimality or non-optimality of the first stage budget allocation.

What we are interested in is essentially the representative behaviour of the second stage allocation of the predetermined route-specific air travel budget (or category expenditure in general terms) to the individual fareclasses (or items). The consumer preferences are said to **be strongly decentralizable** if it is possible for the consumer to **optimally** allocate the category expenditures knowing only the intra-category prices. Gorman [1971] has shown that weak separability of the utility functions into the categories is both necessary and sufficient for strong decentralizability.[2] In the context of our study, the weak separability implies that marginal rates of substitution between every pair of fareclasses are uninfluenced by prices other than those of all the fareclasses on that route. Empirically, there is no reason to believe that marginal rates of substitution and thus, choice of a fareclass should depend on prices of other goods or services and fares on other routes. Therefore, the assumption of strong decentralizability, does not seem to be too restrictive to undermine the results of our study. In sum, given the major interest in investigating structure of inter-fareclass competition, it is theoretically justifiable to estimate the system of demand equations for the alternative fareclasses as functions of only the route specific data on prices and qualities of all fare classes, route's market characteristics and the total air travel expenditures spent on each route.

Under the assumption of strong decentralizability in allocating the route specific air travel budget (Y^r) to **n** individual fareclasses, it is possible to write the route-wise indirect utility function in terms of only the route specific data as in equation (1):

$$(1)\quad H^r = H(P_1^r, \ldots, P_n^r, Y^r, Z_1^r, \ldots, Z_k^r)$$

$$r = 1, 2, \ldots, m,$$

where

H^r: the maximum utility that can be obtained from route r with given budget Y^r, price vector $(P_1^r, \ldots, P_n^r)$ and market characteristics of the route $(Z_1^r, \ldots, Z_k^r)$.

Y^r: total air travel expenditure on route r.

P_i^r: observed price of fareclass i on route r.

Z_k^r: kth market characteristics of route r.

(for convenience, the super-script r for P_i^r, Y^r, and Z_k^r are omitted hereafter.)

Note that the effects of variations in the route characteristics (Z_k) on fareclass choice are parameterized in the indirect utility function (1). Due to the duality relation between indirect and direct utility functions,[3] the indirect utility function H satisfying certain regularity conditions[4] can completely characterize the direct utility function satisfying the similar regularity conditions. Therefore, the system of utility maximizing demand functions X_i (V, Y, Z), i = 1, 2, —, n, where $V_i = P_i/Y$, can be derived by applying **Roy's Identity** directly to the indirect utility function H as follows [Diewert, 1974 p. 126]:

$$(2)\quad X_i(V, Y, Z) = \frac{\partial H(\cdot)}{\partial V_i} \Big/ \sum_j V_j \frac{\partial H(\cdot)}{\partial V_j}$$

$$i = 1, 2, \ldots, n$$

A brief discussion seems necessary as to why we choose to estimate the derived demand system from the indirect utility function rather than from the direct utility function. When the direct utility function is used, the normalized prices (P/Y) are expressed as functions of the quantities consumed (X), which is referred to as the system of indirect demand functions. This implies that quantities are taken to be exogenous, while normalized prices are endogenous. If we were estimating the preferences of a single consumer, it would be indifferent, at least theoretically, whether we used a direct or an indirect utility function. However, when the data are aggregated over a number of consumers, it would be improper to assume that the quantities consumed are approximately constant across individuals. But, it is reasonable to assume that the prices facing each consumer are approximately constant while incomes and quantities consumed differ.[5]

For the purpose of estimation it is necessary to postulate a specific functional form for the indirect utility function H(•). To study inter-fareclass competition, the functional form should be general enough to allow for free variation of the elasticities of substitution, and be sufficiently 'flexible' to provide a valid quadratic approximation to the unknown true preferences at a point of approximation as well as to allow for a wide range of hypotheses tests concerning the structure of preferences. Recently, there has been considerable work developing general functional forms known as "flexible" functions. Generalized Leontief function [Diewert, 1971], translog function [Christensen/Jorgenson/Lau, 1975], generalized Cobb-Douglas function and square root quadratic function are such examples. For the following reasons, we have chosen to use a translog function[6] to approximate the **reciprocal**[7] of our indirect utility function (1) around the mean values of our data.

1. Berndt/Darrough/Diewert's experiment [1977] using the time-series data of Canadian consumer demands have shown that the translog model was preferred to Generalized Leontief and Generalized Cobb-Douglas models.
2. Starting with a general system of consumer demand functions representable as ratios, Jorgenson and Lau [1979] have shown that the only such systems which satisfy the integrability conditions and, at the same time, are capable of modelling arbitrary own- and cross-substitution effects are the systems generated by translog direct or indirect utility functions.

(B) General non-homothetic Model

The non-homothetic translog reciprocal indirect utility function can be written as:

$$(3)\quad -\ln H(V,Z) = a_0 + \sum_i a_i \ln V_i + \sum_k b_k \ln Z_k + \frac{1}{2}\sum_i\sum_j a_{ij} \ln V_i \ln V_j + \frac{1}{2}\sum_l\sum_k b_{lk} \ln Z_l \ln Z_k + \sum_i\sum_k c_{ik} \ln V_i \ln Z_k$$

where

$V_i = \frac{P_i}{Y}$: normalized price of fareclass i.

P_i: observed price of fareclass i.

Y: route-specific total air travel expenditure.

Z_k: kth route characteristics.

a_0, a_i, b_k, a_{ij}, b_{lk}, c_{ik}'s are parameters of the translog function.

Applying Roy's identity (2) to the translog function (3) and multiplying each of the resultant demand equations by the corresponding normalized price V_i, we obtain the following system of expenditure share functions for the n fareclasses:

$$(4)\quad S_i = \frac{a_i + \sum_j a_{ij} \ln V_j + \sum_k c_{ik} \ln Z_k}{a_m + \sum_j a_{mj} \ln V_j + \sum_k c_{mk} \ln Z_k}$$

$$i = 1, 2, \ldots, n$$

where

$S_i = \frac{P_i X_i}{Y}$: expenditure share on fareclass i.

$a_{ij} = a_{ji}$ for all $i \neq j$ (symmetry condition).

$a_m = \sum_i a_i \quad a_{mj} = \sum_i a_{ij},\ c_{mk} = \sum_i c_{ik}$ (adding-up condition)

Note that the homogenity of expenditure shares in prices (P) and total expenditures (Y) is implied by Roy's identity, and the conditions for symmetry of preferences and adding up of shares are imposed in equations (4). Since the expenditure shares in (4) are homogeneous of degree zero in parameters, we impose the following additional restriction for identifiability of the parameters:

$$(5)\quad \sum_{ij} a_{ij} = 0$$

The elasticity of demand for fareclass i with respect to price of fareclass j can be computed[8] as:

$$(6a)\quad E_{ij} = a_{ij}/(a_i + \sum_j a_{ij} \ln V_j + \sum_k c_{ik} \ln Z_k)$$

$$- a_{mj}/(a_m + \sum_t a_{mt} \ln V_t + \sum_k c_{mk} \ln Z_k)$$

when $i \neq j$

$$(6b)\quad E_{ii} = -1 + a_{ii}/(a_i + \sum_j a_{ij} \ln V_j + \sum_k c_{ik} \ln Z_k)$$

$$- a_{mi}/(a_m + \sum_t a_{mt} \ln P_t + \sum_k c_{mk} \ln Z_k)$$

when $i = j$

Due to the homogeneity condition imposed on the share functions, the elasticity of demand with respect to total route expenditure (Y) can be written as:

$$(7a)\quad E_{iy} = -\sum_j E_{ij}$$

Note that E_{iy} is, in general, different from income elasticity of the demand for fare class i. However, the income elasticity can be obtained by:

$$(7b)\quad E_{iI} = \frac{\partial \ln X_i}{\partial \ln Y} \frac{\partial \ln Y}{\partial \ln I} = E_{iy} \cdot E_{yI}$$

where

E_{iI}: income elasticity of demand for fareclass i.

E_{iY}: elasticity of demand for fareclass i with respect to the total route expenditure Y.

E_{YI}: elasticity of total route expenditure Y with respect to income I.

Finally, using the result of Allen [1938, p. 512], the partial elasticities of substitution between fareclasses i and j can be computed as:

$$(8)\quad \sigma_{ij} = \frac{E_{ij}}{S_j} + E_{iy} = \sigma_{ji} \quad i \neq j$$

(C) Alternative Structures of Preferences

Given our primary interest on empirical results, in this paper we admittedly avoid any exhaustive tests concerning the structure of preferences.[9] However, we shall test the following hypotheses corresponding to the selected restrictive preference structures of our interest, most of which have been frequently assumed in the past empirical studies.

1. Route characteristics:

The route characteristics (Z_K, K = 1, 2, 3, 4) included in the general model were chosen purely on the basis of our intuition. Therefore, it is necessary to test whether or not the preference structure is strictly independent[10] of one or more of these route variables. Restrictions for the strict independence from the route characteristic Z_K are:

(9) $c_{iK} = 0$ for all i = 1, 2, 3.

2. Weak separability:

It is interesting to see if the ratio of demand functions for the economy and discount fareclasses, X_2/X_3, is independent of the normalized value of first class fare V_1. Restrictions for this weak separability [see Denny and Fuss, 1977] are:[11]

(10) $a_{12} = \rho_1 \bullet a_2 \quad a_{13} = \rho_1 \bullet a_3$.

3. Strong separability:

A stronger condition is strong separability,[12] implying that the demand functions for the economy and discount fareclasses, X_2 and X_3, are independent of the normalized value of first class fare V_1. Restrictions for this strong separability are:

(11) $a_{12} = a_{13} = 0$.

4. Homotheticity:

Under homotheticity, the budget shares of all fareclasses are independent of the amount of the total route expenditure Y. Equivalently, the ratios of the demand functions are homogeneous of degree zero in (V_1, V_2, V_3). The conditions for homotheticity are:

(12) $$\sum_j a_{ij} = \sigma a_i \quad \text{for all } i = 1, 2, 3.$$

However, the normalization of parameters (5) forces $\sigma = 0$ implying "linear homogeneity," a stronger condition than homotheticity. This condition implies that all expenditure elasticity (E_{iy}) are one.

5. Log-linearity:

Under the log-linearity of reciprocal indirect utility function, the budget shares of all fareclasses are independent of all prices and total expenditure, and depend only on route characteristics. The conditions for log-linearity are:

(13) $$\sum_j a_{ij} = \sigma a_i \quad i = 1, 2, 3,$$
$$a_{ij} = \rho_j \bullet a_i \quad i,j = 1, 2, 3.$$

The plan for testing hypotheses concerning these restrictive structures are summarized later in Tables 1 and 2, along with the test results.

III. SOURCES OF DATA AND CONSTRUCTION OF VARIABLES

The 1978 yearly cross-sectional data for the selected 100 intra U.S. routes are used in this study. These routes were chosen carefully so as to represent general pattern of the entire intra-U.S. route network by including routes with various distance and market density.

For computational reasons, we decided to aggregate all the scheduled airlines' passenger services into the following three fareclasses:

(i) the regular first class

(ii) the standard economy class; for the similarity of fares, all the first class discount services are combined in this fareclass.

(iii) the discount fareclass; all other services including excursion fares, holiday or weekend fares, night flight fares, charter class fares, commuter fares and all other promotional fares.

To account for the effects of route characteristics on fareclass choice, we decided to use the following variables: average weekly departure frequency (Z_1), route distance in miles (Z_2), number of competitors serving the route (Z_3) and average seat capacity of the aircrafts used on the route (Z_4).

Therefore, the route-specific data required to estimate the system of expenditure share functions in equations (4) are as follows:

P_i: average observed price of fareclass **i**, i = 1 (first class), 2 (economy), 3 (discount),

S_i: share of expenditure of fareclass **i**, i = 1 (first class), 2 (economy), 3 (discount),

Y: total air travel expenditures on the route,

Z_1: average weekly departure frequency (includes the flights with 0 or 1 stop only),

Z_2: distance of the route in miles,

Z_3: number of airlines serving the route,

Z_4: average seat capacity for the aircrafts being used on the route.

The average observed prices of the three fareclasses, number of weekly departure frequency, and number of airlines serving were compiled from the Official Airline Guides, North American Edition for each of the selected routes. The traffic volumes for each fareclass and distance of each route were compiled from Domestic Origin-Destination Survey of Airline Passenger Traffic, Table 13 (Domestic Passenger-Stage Movement Between Cities, by Carrier and Fare Bases), which is supplied by the Air Transport Association of America. The total air travel expenditures and the expenditure shares on each route were computed using the appropriate price and volume information. The average seat capacity per aircraft on each route was approximated as the number of departures performed for each aircraft type multiplied by the average number of seats per aircraft for the aircraft type and then divided by the total departure frequency of all aircraft types. All the variables used for estimation were normalized around their respective means such that mean values of all the variables become one. Therefore, all the second-order approximation to unknown true preferences are made around mean values of the data.

IV. ESTIMATION AND HYPOTHESES TESTING

Each of the expenditure share equations derived in Chapter III can be interpreted as expected value of the consumers' utility-maximizing share of expenditures on that fareclass. In practice, there are errors in adjustment to the utility-maximizing shares due to, among others, inertia created by habit formation and imperfect information on prices and qualities of fareclasses. Therefore, empirical implementation requires that the share equations be imbedded in a stochastic framework. Thus, we define the column vector of disturbances at route **r** as $E^*(r) = [E_1(r), E_2(r), E_3(r) \mid r = 1, 2, \text{—}, R$, and the associated constant cross equation disturbance covariance matrix as Ω^*.

Since the expenditure shares always sum to one at each observation, Ω^* is singular and non-diagonal. If the estimators of parameters are to be efficient, this disturbance covariance matrix must be taken into account. As the result of singularity, however, the determinant of the disturbance covariance matrix is zero, and consequently the likelihood function is undefined. To avoid this problem, any one of the three share equations can be dropped. Since the maximum likelihood estimates are invariant to the equation deleted, we arbitrarily drop the first class share equation and jointly estimate the share equations for the economy and discount fareclasses. The computation is carried out by the nonlinear least squares routine available in the Harvard Version TSP package.[13]

Asymptotic likelihood ratio (λ) criterion is used to test the hypotheses concerning structure of preferences. Theil [1971, p. 396] has shown that, asymptotically, $-2\ell n\lambda$ has a Chi-square distribution with the degree of freedom being equal to the number of independent restrictions imposed in the restricted model.

V. EMPIRICAL RESULTS

Maintaining the general non-homothetic model as in equations (3) and (4), we first tested to see if each of the four variables indicating route specific market characteristics contributes significantly to the explanatory power of the model. The results of these first stage tests reported in Table 1 indicate clearly that consumer behaviour on the fareclass choice is strongly influenced by the weekly departure frequencies (Z_1, a market density indicator) and distance of the route (Z_2) but is not significantly affected by number of competing air carriers operating on the route (Z_3). The choice behavior appears to be affected only marginally by the average seat capacity per airplane being used on the route (Z_4). The statistical significance of the variables Z_1 and Z_2 may be justified on the grounds that the first class services and discount fare programs are more likely to be readily available on long-haul and/or dense routes than on short-haul and/or light density routes. The statistical insignificance of Z_3 and only a marginal significance of Z_4 are probably caused by the strong correlations between these variables and the variables Z_1 and Z_2. Using the results of these tests, we decided to delete the variables Z_3 (number of competitors) and Z_4 (average seat capacity per aircraft) from

TABLE 1

TESTS OF HYPOTHESES ABOUT THE ROUTE CHARACTERISTICS

Alternative hypothesis (H_a)	Null hypothesis (H_o)	restrictions	$-2\ell n\lambda$	Results at α = 1%
General model in (4)	1. Independence from Z_1 (departure freq.)	$c_{i1}=0$ i=1,2,3	50.2**	Reject H_o
"	2. Independence from Z_2 (distance)	$c_{i2}=0$ i=1,2,3	32.5**	Reject H_o
"	3. Independence from Z_3 (Number of Competitors)	$c_{i3}=0$ i=1,2,3	3.6	Cannot reject H_o
"	4. Independence from Z_4 (average aircraft size)	$c_{i4}=0$ i=1,2,3	8.6*	Cannot reject H_o

**Statistically significant at α = 1%.

*Statistically significant at α = 5%.

Critical values of χ^2 distribution with 3 df:

α = 1%: 11.34
α = 5%: 7.81

the model before proceeding to the second stage tests of the hypotheses concerning the structure of preferences as described in Section II.

Table 2 presents the results of the second stage hypotheses tests concerning the preference structures as well as the interrelationships between the tests. The results in Table 2 can be summarized as follows:

1. The general non-homothetic model is preferred to the homothetic and the log-linear models. This implies that Cobb-Douglas form of demand models would give unreliable results.
2. The model with weak or strong separability of the economy and the discount fareclasses from the first class fares is pre-

TABLE 2

TESTS OF HYPOTHESES ABOUT THE STRUCTURE OF PREFERENCES

Alternative hypothesis (H_a)	Null hypothesis (H_o)	Restrictions	Number of independent restrictions	$-2\ln\lambda$	Results at α = 5%
General model in (4)	weak separability (10)	$a_{12}=\rho_1 \cdot a_2$ $a_{13}=\rho_1 \cdot a_3$	1	0.225	cannot reject H_o
"	strong separability (11)	$a_{12}=a_{13}=0$	2	0.308	cannot reject H_o
"	Homotheticity (12)	$\sum_j a_{ij}=0$ $i=1,2,3$	2	8.26*	reject H_o
"	Log-linearity (13)	$\sum_j a_{ij}=0$ $i=1,2,3$ $a_{ij}=\rho_j \cdot a_i$ $i=1,2,3$	3	9.01	reject H_o
Weak separability (10)	strong separability (11)	$\rho_1=0$	1	0.083	cannot reject H_o
"	Log-linearity (13)	$\sum_j a_{ij}=0$ $a_{ij}=\rho_j \cdot a_i$ $i,j=1,2,3$	2	8.785*	reject H_o at
Strong separability (11)	Log-linearity (13)		1	8.702*	reject H_o
Homotheticity (12)	Log-linearity (13)	$a_{ij}=\rho_j \cdot a_i$ $i,j=1,2,3$	1	0.75	cannot reject H_o

Critical values of χ^2-distribution:

significance \ df	1	2	3
α = 5%	3.84	5.99	7.8[illegible]
α = 1%	6.63	9.21	11.34

*Significant at α = 5%.

ferred to the general model while the strongly separable model is preferred to the weakly separable one. These results mean that the demand functions for the economy and the discount fareclasses, X_2 and X_3, are independent of the normalized price of first class service V_1.

3. The model with the weak or strong separability or homotheticity is preferred to the log-linear one. This reinforces the above result that Cobb-Douglas form of demand functions is inappropriate to use for the study of inter-fareclass competition.

The overall result of the two-stage hypotheses tests in Tables 1 and 2 indicates that the strongly separable model without the route specific market variables Z_3 and Z_4 is the most appropriate one to use in this study. The parameter estimates of this model are reported in Table 3 along with absolute values of their asymptotic t-ratios. The parameter estimates c_{if}'s and c_{id}'s indicate that the first class and the discount fareclass are being used relatively more on the long-haul and/or dense routes.

Using the formulae presented in equations (6), (7a) and (8), we evaluated the own and cross price elasticities E_{ij}, the total route expenditure elasticities E_{iy} and the elasticities of substitution between every pair of fareclasses. These elasticities are reported in Table 4 only for the selected routes to serve as the representative examples. Note that cross price elastici-

TABLE 3

ML PARAMETER ESTIMATES FOR THE STRONGLY SEPARABLE MODEL (asymptotic t-ratios are in parentheses)

Parameter	Estimate	Parameter	Estimate
a_1	.031864*	c_{1f}	.009108*
a_2	.189343 (1.31)	c_{2f}	-.020379 (.53)
a_3	.109595 (1.35)	c_{3f}	.033822 (1.14)
a_m	.330802 (2.33)	c_{1d}	.010040*
a_{11}	.002130*	c_{2d}	-.033559 (.71)
a_{22}	-.084936 (.92)	c_{3d}	.017339 (2.83)
a_{32}	.075156*	c_{mf}	.022552 (3.41)
a_{33}	-.067506*	c_{md}	-.006180 (.14)
a_{m1}	.002130*		
a_{m2}	-.009780 (1.02)		
a_{m3}	.007650 (2.87)		

Subscripts: 1 = first class, 2 = economy, 3 = discount,
f = departure frequencies, d = distance
$m = \Sigma_i$

*denotes the parameter estimates computed from the adding-up conditions in equation (4) and the normalization in equation (5).

ties between the first class and the other two classes are not reported in the table because they are all close to zero due to the strongly separable model. This implies that there is no price competition between the first class and any other class.

The results in Table 4 may be summarized as the following.

1. The own price elasticity for the first class services E_{11} varies between -.85 and -.95. This price inelastic demand conforms with the fact that majority of the first class users are business travellers.
2. The demands for discount fare services and for standard economy fareclass are highly price elastic with respect to their respective own prices, with the discount fare services being substantially more price elastic. The own price elasticity for the economy fareclass E_{22} ranges from -1.3 to -1.5 with a tendency for the long-haul route being relatively more price elastic. These figures are substantially higher than the own price elasticities for North Atlantic routes in the Mutti and Murai's work (-1.01) and in the Straszheim's work (-1.116) estimated by using Cobb-Douglas model. Our estimates of the own price elasticity for the demand for the discount fare services lie between -1.5 and -1.9. Mutti and Murai reported -.51 as the own price elasticity for charter class services on the North Atlantic routes, while Straszheim's figure was -2.735 for the "high discount" fare services.
3. The cross price elasticity of demand for the economy class with respect to discount fares ranges from 0.28 to 0.45. This figure is generally above or below 0.4 for the routes longer or shorter than 1000 miles, respectively. This implies that 1% increase in the discount fares would result in more than 0.4% increase in the economy class passengers on most of the long-haul routes.
4. The cross price elasticity of demand for the discount fare services is between 0.6 to 1.0, fluctuating significantly from one route to another depending on market conditions. This strong cross price elasticity modifies the airline industry's claim 'the discount fares have eroded the standard economy class market' by showing that the threat is bi-directional rather than unidirectional. A substantial proportion of the customers now flying on the discount fare programs may end up switching to regular economy fare if the standard economy fares are adjusted downward by a substantial margin. This may in fact improve the airlines' combined revenue from the two classes.
5. The high degree of competition between the economy and discount fare classes discussed above is confirmed again by the extremely high elasticities of substitution between the two fareclasses: σ_{23} is between 2.0 and 2.4 depending on the route's market condition. This implies that one percent change in price ratio of the two classes would result in 2 to 2.4 percent change in the demand ratio.

TABLE 4
ELASTICITY ESTIMATES FOR THE SELECTED ROUTES

Route	Distance (in miles)	1978 passenger volume	E_{11}	E_{22}	E_{23}	E_{32}	E_{33}	E_{1Y}	E_{2Y}	E_{3Y}	σ_{12}	σ_{13}	σ_{23}
L.A. - San Diego	101	27,594	-.85	-1.31	.28	.84	-1.75	.85	1.04	.92	.89	.77	2.12
N.Y. - Kansas City	102	14,292	-.93	-1.43	.38	.76	-1.68	.93	1.05	.92	.98	.85	2.25
N.Y. - Washington, D.C.	199	180,252	-.90	-1.34	.30	.76	-1.68	.90	1.04	.92	.94	.82	2.10
L.A. - San Francisco	335	103,552	-.93	-1.36	.32	.66	-1.59	.93	1.04	.93	.97	.86	2.05
Boston - Cleveland	558	12,085	-.90	-1.36	.33	.96	-1.86	.89	1.04	.90	.94	.79	2.34
Philadelphia - Atlanta	672	18,620	-.92	-1.38	.33	.85	-1.76	.92	1.04	.91	.97	.83	2.26
N.Y. - Atlanta	756	66,833	-.94	-1.40	.36	.67	-1.60	.93	1.05	.93	.98	.87	2.12
Chicago - Dallas	800	67,979	-.93	-1.39	.35	.76	-1.68	.92	1.04	.92	.97	.85	2.19
Washington, D.C. - Miami	820	19,607	-.93	-1.41	.36	.80	-1.72	.92	1.05	.92	.97	.84	2.25
N.Y. - Miami	1,091	154,991	-.94	-1.44	.39	.63	-1.56	.94	1.05	.93	.99	.87	2.13
L.A. - Dallas	1,240	38,017	-.94	-1.45	.40	.65	-1.58	.94	1.05	.93	.99	.87	2.18
N.Y. - Houston	1,432	36,892	-.94	-1.44	.39	.75	-1.67	.93	1.05	.92	.98	.86	2.24
Seattle - Chicago	1,731	10,465	-.94	-1.47	.42	.74	-1.66	.93	1.05	.92	.99	.86	2.29
L.A. - Chicago	1,740	73,020	-.95	-1.49	.43	.61	-1.54	.94	1.06	.94	1.00	.88	2.19
San Francisco - Detroit	2,086	13,514	-.94	-1.46	.41	.82	-1.73	.93	1.05	.91	.99	.85	2.35
L.A. - Washington, D.C.	2,288	34,572	-.94	-1.48	.43	.70	-1.63	.94	1.05	.93	1.00	.87	2.27
L.A. - N.Y.	2,453	130,716	-.95	-1.50	.45	.60	-1.54	.94	1.06	.94	1.00	.89	2.21
L.A. - Boston	2,600	28,576	-.94	-1.46	.41	.84	-1.75	.93	1.05	.91	.99	.85	2.37

*E_{ij} = the elasticity of demand for ith fareclass with respect to variable j.

σ_{ij} = the elasticity of substitution between fareclasses i and j.
i,j = 1 (first class), 2 (economy), 3 (discount), y (total route expenditure).

6. The elasticity of demand with respect to the route specific total expenditure, E_{iy}, is 0.85 ~ 0.95, 1.04 ~ 1.05 and 0.89-1.0 for the first, the economy and the discount fare-classes, respectively. It is important to know that these elasticities are different from the fareclass specific income elasticities (E_{iI}) as explained in Section II. The income elasticity (E_{yI}) for the aggregate air travel markets was estimated to be about 1.7 in our other study[14] [Oum and Gillen, 1979]. Therefore, the income elasticity for demand for each fareclass E_{iI} (see equation (7b)) can be computed by multiplying 1.7 to the corresponding route specific expenditure elasticity E_{iy}. Therefore, the income elasticity of demand is 1.4 ~ 1.6, 1.75, and 1.5 ~ 1.7, for the first, the economy and the discount fare classes, respectively.

In sum, our study finds that: (i) There is strong competition between the economy and the discount fareclasses and no significant competition between the first class and the other two classes. (ii) The demand is highly own price elastic for the discount fareclass, slightly less price elastic for the economy fareclass, and price inelastic for the first class services. (iii) Income elasticities are very high for all the three fareclasses. (iv) The strongly separable model is chosen as being the most appropriate model to use in the study of inter-fareclass competition.

Owing to the symmetric nature of inter-fareclass competition between the economy and the discount fareclasses, the issue of optimal inter-fareclass pricing is crucial for the airline industry's future financial performance, as well as for the efficiency of resource allocation.

FOOTNOTES

1. Lau [1969] has shown that the consumer preference is **budgetable** if the utility function is continuous, strictly quasi-concave, non increasing, and strictly increasing in at least one coordinate.
2. Blackorby/Primont/Russell [1978, pp. 103-259 or 1975] presents a most comprehensive and systematic discussion on budgeting, aggregation and decentralization of consumer's budget allocation decisions.
3. For various versions of the duality theorem, see Lau [1969], Shepard [1970], Afriat [1972] and especially Diewert [1974].
4. The required regularity conditions are: the indirect utility function H(•) be continuous, non increasing, and quasi convex in the normalized price vector ($V_i = P_i/Y$, i = 1, 2, —, n) over the positive orthant.
5. Pollak and Wales [1978] and Berndt/Darrough/Diewert [1977] are examples, in which indirect utility function is used to derive demand system. Jorgenson and Lau [1975], Christensen and Manser [1976], and Christensen/Jorgenson/Lau [1975] have used both direct and indirect utility functions. It is not surprising that they obtained very different test results by estimating direct and indirect translog utility functions, although the two are supposed to approximate an identical preference.
6. In this study, the translog function is used as a quadratic approximation to the true unknown preferences rather than as an exact form of reciprocal indirect utility function. See Simmons and Weiserbs [1979] for an elegant discussion about the inability to distinguish between the hypothesis that translog preferences hold globally and the hypothesis that true unknown preferences satisfy integrability conditions at the point of approximation. Friedlaender and Spady [1977], Oum [1979a, 1979b], Oum and Gillen [1979], Spady and Friedlaender [1978] are a few examples of transport studies, in which translog functions were used.
7. Translog function is not suitable for approximating any convex function such as indirect utility function. Therefore, we have chosen to approximate the reciprocal function rather than the indirect utility function itself.
8. Derivations of these elasticities are available in Oum and Gillen [1979].
9. For nearly complete tests on structures of the preferences, see Jorgenson and Lau [1975] and Oum and Gillen [1979].
10. See Blackorby/Primont/Russell [1978, pp. 165-168] for a formal description of strict independence.
11. Weak separability is a necessary but not sufficient condition for the two stage optimization procedure.
12. Strong separability is a sufficient condition for the two stage optimization procedure.
13. The convergence criterion used is that (a) the largest change in parameter estimates from one iteration to the next should be no greater than 0.5 percent, and (b) the largest absolute deviation of the elements of the transformed residual covariance matrix from the identity matrix of same size should be no greater than .005.
14. Although income elasticity of the combined demand varies from route to route, the findings reported in the study by Verledger [1972] seem to conform that the average overall routes is about 1.7.

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