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Multi-Criteria Optimization Methods in Transport Project Evaluation: The Case of Rural Roads in Developing Countries

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ABSTRACT

THERE IS INCREASING awareness that the evaluation of transport projects cannot be meaningfully carried out according to the single criterion of economic efficiency. Despite this recognition, important methodological advances in multi-criteria decision making and multi-criteria optimization which have appeared in the literature in recent years have yet to be applied to the evaluation of transport projects. This paper discusses the potential of usefulness of multi-criteria optimization methods in the evaluation of transport projects, emphasizing the strengths and weaknesses of the two primary schools of thought concerning the solution of multi-criteria optimization problems: generating methods and preference incorporation methods. A hypothetical transport project in a developing country where both efficiency and distribution are important objectives is analyzed in terms of the so-called weighting method originally suggested by Zadeh and Marglin. The results of applying this standard generating method are compared to those obtained from applying one of the more widely discussed preference incorporation methods, namely the iterative procedure due to Geoffrion, Dyer and Feinberg. Conclusions are drawn concerning the relative attractiveness of these two solution methods for the evaluation of transport projects.

INTRODUCTION

That the evaluation of transport projects involves the consideration of multiple criteria or objectives which are noncommensurable is increasingly recognized by planners and engineers. The multiple criteria necessary to the evaluation of urban transportation plans have been discussed by Hill (1973) who provides a list of 14 relevant objectives. Manheim (1974) has discussed the need for multiple criteria in the evaluation of more general transport projects. Although the importance of multiple criteria has been recognized, agreement as to an appropriate methodological approach for handling non-commensurable criteria in transport project evaluation has not been reached.

A useful vehicle for examining the role of multiple criteria in transport project evaluation is provided by rural roads in developing countries. The problem of investment in rural roads may be utilized to illustrate the application of standard multi-criteria evaluation tools and to compare the attractiveness of solution methods.

Multi-criteria evaluation problems are frequently most naturally articulated as vector mathematical programming problems — as the rural roads example of this paper will illustrate. Methods for the solution of vector mathematical programming problems can be divided into two categories (Cohon, 1978): (1) generating methods which identify all efficient solutions, and (2) preference incorporation methods which utilize decision maker preferences to examine only a subset of all efficient solutions. We illustrate in subsequent sections the characteristics of the so-called weighting method, perhaps the most widely known generating technique, generally attributed to Marglin (1967) and Zadeh (1963) and the iterative preference incorporation technique due to Geoffrion et. al. (1972). The Geoffrion et. al. technique is representative of the state of the art in preference incorporation methods. Before discussing the details of these methodological approaches, it is first necessary to describe the rural road investment problem in developing countries to which these methods are to be applied.

IDENTIFICATION OF THE RURAL ROAD PROBLEM

The lack of adequate transportation facilities has been a major determining factor of rural underdevelopment in developing countries. Thus, even when investment and technological assistance to

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the rural sector have been made available, in principle, by the government or by international agencies, they have often met with great difficulties in actually finding their way to the individual producers due to the non-existence or inadequacy of physical channels of distribution; and in the cases where they have reached their final destination, substantial delays and deterioration have reduced their effectiveness in boosting agricultural production. The fact that rural transportation improvement programs are a necessary condition for agricultural development has been amply demonstrated by past experience in less developed countries, and by drastic overestimations of expected benefits of rural development programs which failed to take this factor into account and which obtained results which were rather modest when compared with the "a priori" benefit analysis.

On the other hand, the dangers of excessive concern with the rural transportation sector and the consequent neglect of other complementary investment areas should be emphasized. Improvements in the rural transportation network will not "per se" necessarily boost agricultural production and economic development, if they are not complemented by additional investment in areas such as fertilizer and machinery production, improved seed development programs, irrigation, technical and credit assistance, training and education.

Lack of adequate rural transportation facilities has two main effects on the agricultural sector.

1) **Input effect**
The physical inaccessibility to farm and agricultural cooperatives caused by deficiencies in the rural road network generally results in the impossibility of providing the rural sector with the necessary inputs for agricultural production.
2) **Output effect**
The more costly accessibility of the rural sector to agricultural markets has a marked negative-incentive effect on its production patterns. Higher transport costs will result in lower perceived prices for agricultural products and, therefore, in lower levels of output.

In order to analyze alternative rural transportation project evaluation techniques we will work with a simplified model based on the following assumptions:

(1) Transport cost savings caused by a road investment project are fully transferred to agricultural producers in the form of correspondingly higher ex-farm prices.
(2) Total cultivable land area is fixed.
(3) Agricultural producers are price-takers.
(4) Agricultural producers are profit maximizers.
(5) The total amount of the homogeneous agricultural product of concern is marketed only after transport over the road system.

Let us further suppose that we have two agricultural regions 1 and 2 which follow assumptions (1) – (5) and which are connected through roads R_1 and R_2 respectively to market A where their production is sold (see Figure 1). Both regions draw on common fixed resources (irrigation water, government credits, etc.). Improving road R_i will decrease the market transport cost and will thus increase the product price perceived by the producer. As a result, agricultural production in i will expand. On the other hand a production increase in one region will, "ceteris paribus," lead to a production decrease, of smaller magnitude in the other region since the common resource's share to the latter will diminish as a consequence of the production expansion in the former. A transport investment program is being considered which will improve both roads R_1 and R_2.

Figure 2 describes the agricultural production situation before and after the road investment project for region 1. Before the implementation of the R_1 improvement the perceived price is p_o and total output is determined by $p_o = MC_o$ (price equals marginal cost of production) at q_o which is the profit maximizing output. After the implementation

THE TWO-REGION RURAL ROAD INVESTMENT PROBLEM WITH PRODUCTION EXTERNALITIES

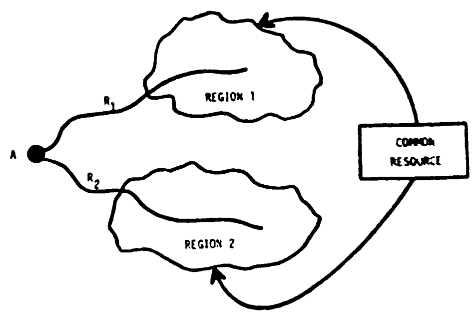

FIGURE 1

AGRICULTURAL PRODUCTION IN REGION 1 BEFORE AND AFTER IMPROVEMENTS IN R_1 AND R_2

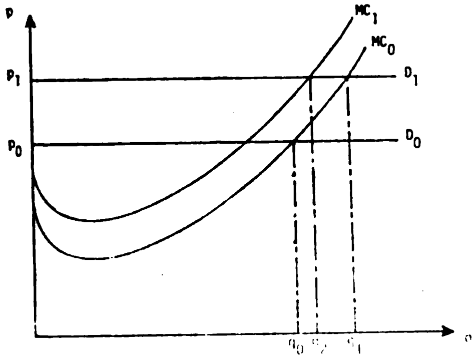

FIGURE 2

of the R_1 improvement the product demand curve (as perceived by the farmers) shifts upward to $p = p_1$, due to transportation cost reduction. If the improvement of R_2 is in addition implemented, production in region 2 will increase and, as a result, the marginal cost curve of region 1 will shift upwards from MC_0 to MC_1. Final equilibrium output in region 1 will be q_2 which is larger than q_0 but less than q_1, the level that could have been achieved had no R_2 investment been made.

If we assume that the objective of the investment program is to maximize the vector of regional agricultural productions we can set up the problem mathematically as:

$$\max \overline{Z} = [Z_1, Z_2] \qquad (1)$$

s.t.

$$Z_1 = \alpha X_1 - \beta Z_2 \qquad (2)$$

$$Z_2 = -\gamma Z_1 + \delta X_2 \qquad (3)$$

$$X_1 + X_2 \leqslant b_1 \qquad (4)$$

$$-X_1 + X_2 \leqslant b_2 \qquad (5)$$

$$X_1 \leqslant b_3 \qquad (6)$$

$$X_2 \leqslant b_4 \qquad (7)$$

$$X_1, X_2 \geqslant 0 \qquad (8)$$

where:

Z_i is the agricultural production level of region i

X_i is the amount of investment in road R_i (10^6\$)

$$\alpha, \beta, \gamma, \delta > 0 \qquad (9)$$

$$\beta, \gamma < 1 \qquad (10)$$

b_1 is the total national (both regions) budget devoted to rural road improvements (10^6\$)

b_2 is a maximal allowable excess of region 2's transport investment share over region 1's (a non-negative number) (10^6\$)

b_3 is the rural road improvement budget of region 1 (10^6\$)

b_4 is the rural road improvement budget of region 2 (10^6\$).

The objective function (1), which denotes the vector of regional agricultural production levels, is to be maximized subject to constraints (2) — (8). Constraints (2) and (3) are the regional production functions as discussed above; production in one region is positively proportional to transport investment in that region and negatively proportional to production in the other region. Constraint (4) is an overall budget constraint. Constraint (5) is an interregional equity constraint. Constraints (6) and (7) are regional budget constraints. Constraint (8) is a non-negativity constraint.

In addition we will assume that a bicriterion welfare function is defined and given by

$$U = U[Z_1(\overline{X}), Z_2(X)] \qquad (11)$$

which is a monotonically increasing function of the objectives with convex isoquants in objective space.

SOLUTION OF AN EXAMPLE PROBLEM

In this section, we show that the original problem associated with the hypothetical transport project can be reformulated as a (linear) multiobjective programming problem. We then assign some numerical values to the coefficients and solve the example problem by a standard generating method, the weighting method [Zadeh (1963), Marglin (1967)], and one of the more widely discussed preference incorporation methods, namely the iteractive procedure due to Geoffrion, Dyer and Feinberg. [Geoffrion et. al (1972).]

As stated before, each of the two conflicting objectives of the hypothetical transport project, represents the gains (agricultural benefits) of a zone and are given by expressions (2) and (3). We reformulate the above equations in such a way that Z_1 and Z_2 are expressed as linear functions of the decision variables,

X_1, X_2. Some algebraic manipulations leads to:

$$Z_1 = \frac{1}{1-\beta\gamma} (\alpha X_1 - \beta\delta X_2)$$

$$Z_2 = \frac{1}{1-\beta\gamma} (-\alpha\gamma X_1 + \delta X_2). \quad (12)$$

Since all the coefficients shown in the above equations are positive and the parameters β and γ are less than one, expressions (12) may be rewritter as:

$$Z_1 = aX_1 - bX_2$$

$$Z_2 = -cX_1 + dx_2 \quad (13)$$

where the coefficients are all positive. To illustrate the solution process, we assign some arbitrary positive integers as the coefficients in expressions (13) and the constraint set stated before in equations (4), (5), (6), and (7). The example problem with two objectives and two decision variables is thus as follows:

$$\text{Maximize } Z(X_1,X_2) = [Z_1(X_1X_2), Z_2(X_1,X_2)]$$

where

$$Z_1(X_1,X_2) = 5X_1 - 2X_2$$

$$Z_2(X_1,X_2) = -X_1 + 4X_2 \quad (14)$$

s.t.

$$-X_1 + X_2 \leqslant 3 \qquad X_1 + X_2 \leqslant 8$$

$$X_1 \leqslant 6 \qquad X_2 \leqslant 4 \quad (15)$$

$$X_1, X_2 \geqslant 0.$$

Two different methods are used in the solution of the above described example problem. Throughout the discussion which follows the general solution of a multi-criteria or vector optimization problem is taken to be the set of all efficient or noninferior alternatives. An alternative is noninferior if it is impossible to increase the value of one objective without decreasing the value of one or more other objectives. The noninferior alternative which maximizes some aggregate social welfare measure (in our two objective case the welfare function [11]) is termed the best compromise solution.

APPLICATION OF THE WEIGHTING METHOD

The weighting method is historically the first technique developed for generating or approximating the set of efficient or noninferior solutions without incorporating preference into the solution process. It follows directly from the necessary conditions for noninferiority developed by Kuhn and Tucker (1951). Application of the weighting method as a generating technique has been discussed by Gaas and Saaty (1955), Zadeh (1963), Marglin (1967) and Major (1969).

Given a vector mathematical optimization problem, of the form

$$\underset{(\text{Min})}{\text{Max}}\ Z(x) = [Z_1(X), Z_2(X) \ldots Z_k(X)] \quad (16)$$

s.t. constraints

we can define an associated scalar optimization problem by means of a vector of non-negative weights $(w_1, w_2, \ldots, w_k)$; that scalar optimization problem is:

$$\underset{(\text{Min})}{\text{Max}}\ Z(X) = w_1Z_1(X) + w_2Z_2(X) + \ldots + w_kZ_k(X). \quad (17)$$

This optimization problem forms an alternative scalar problem which can be solved, the solution of which, for each vector $(w_1, w_2, \ldots, w_k)$, will generate a point of the noninferior set for the original vector optimization problem. This will be shown in detail later.

By using weights to scalarize the vector objective function $[Z_1, Z_2]$, the example problem may be put into the following scalar form:

$$\text{Max } Z(X_1,X_2,w_1,w_2) = w_1 (5X_1 - 2X_2) + w_2(-X_1 + 4X_2) \quad (18)$$

s.t. $(X_1,X_2) \in F_d$

where F_d is the feasible region in decision space which satisfies the constraint set given by (15). Note F_d is also drawn in Figure 3.

The values of w_1, w_2 are non-negative. The non-negativity of the w_i's is required by the optimality conditions for noninferiority. The w_i's are also not allowed to be all zero to assure the objective is non-trivial.[1] For simplicity, the weight on objective Z_1 is set to be one, i.e. $w_1 = 1$. This amounts to selecting objective Z_1 as the numeraire and does not in any way affect the generality of the procedure.

The weighted problem in (18) becomes:

THE FEASIBLE REGION IN DECISION SPACE FOR THE EXAMPLE PROBLEM

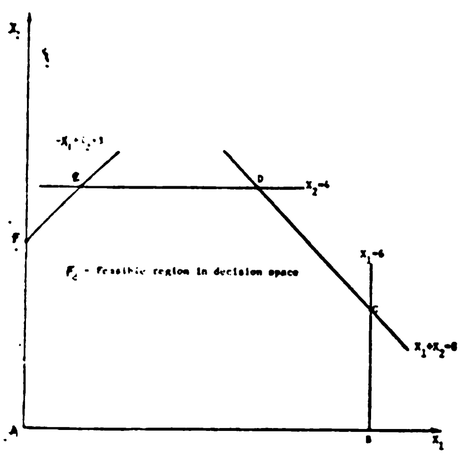

FIGURE 3

$$\text{Max } Z(X_1,X_2,w_2) = 5X_1 - 2X_2 + w_2(-X_1 + 4X_2) \quad (19)$$

s.t. $(X_1,X_2) \in F_d$.

The constraints are unchanged, and so for a given value of w_2, the solution can be found graphically in the decision space. For example, for $w_2 = 1$ the objective function in (19) becomes

$$\text{Max } Z(X_1,X_2,1) = 4X_1 + 2X_2 \quad (20)$$

The solution is that point for which a straight line with slope determined by Z is tangent to F_d in Figure 4. This occurs at point C where $X_1 = 6$, $X_2 = 2$, $Z_1 = 26$ and $Z_2 = 2$. We can confirm this result by drawing a linear indifference curve with slope $-w_1 \div w_2 = -1$ in Figure 5 and observing the point at which it is tangent to F_o, the feasible region in objective space. By definitions of Z_1 and Z_2, there is a one-to-one correspondence between F_d and F_o. A noninferior point generated in Figure 4, e.g. point C = (6,2), will give rise to a similarly labeled point in Figure 5, i.e. point C = (26,2).

One can generate an approximation of the noninferior solution set by joining the noninferior points generated by each weighted problem considered with straight line segments. A possible result is summarized in Table 1. The approximate noninferior set generated in Table 1 happens to be the exact noninferior set as is shown in Figure 5 where we see that every noninferior extreme point in F_o has been found by some weighted problem. Suppose, however, that we generate the non-inferior points by using the sets of weights (1,0), (1,2), (1,4) and (0,1); the point C will be skipped in this approximation. Due to this property, it is difficult to know just how good the current approximation is. There are no efficient rules that can be applied to assessing the sufficiency of an approximation. Generally, if there are no inordinately large gaps and the generated solutions give a reasonable account of the range of choice, the approximation is adequate.

In short, despite the fact that there is no guarantee of obtaining the exact noninferior set, the weighting method provides a convenient and effective way for the analyst to solve the multi-criteria problem when the analyst cannot acquire prior statements of the decision maker's preferences or value judgements about the objectives, and the number of objectives is small.[2]

APPLICATION OF THE GEOFFRION ET. AL METHOD

The preference incorporation methods require the explicit articulation of preferences either prior to solution or in an iterative manner. We will now concen-

APPLICATION OF THE WEIGHTING METHOD TO THE EXAMPLE PROBLEM: DECISION SPACE

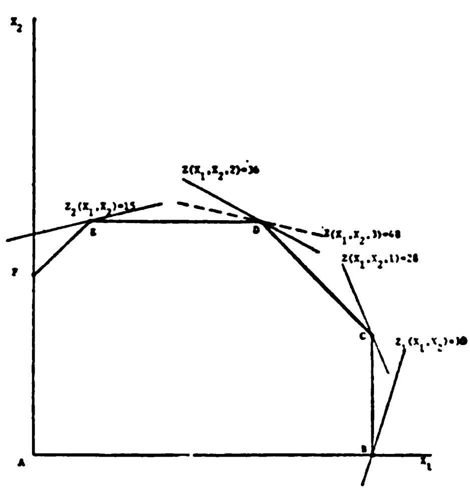

FIGURE 4

APPLICATION OF THE WEIGHTING METHOD TO THE EXAMPLE PROBLEM: OBJECTIVE SPACE

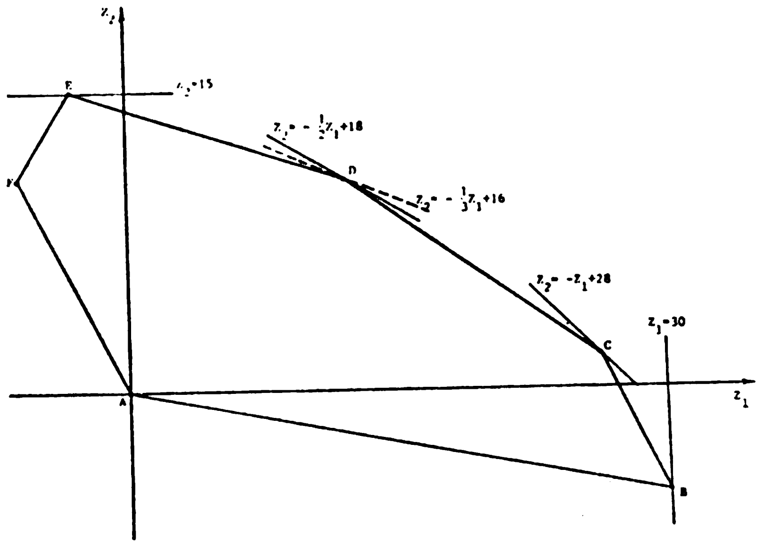

FIGURE 5

trate our attention on the method due to Geoffrion et. al (1972) which provides an interative solution technique leading to an approximation of the best compromise solution by assuming an underlying welfare or utility function which is approximated locally as the algorithm proceeds. The mechanism of this method is based on the well-known Frank-Wolfe algorithm [Frank and Wolfe (1956)]. To implement the algorithm we also need to assume, as usual, a compact, convex feasible region and a concave, differentiable welfare or utility function.

There are two major parts of the algorithm: (1) the determination of the best direction from an existing solution toward the optimal solution and (2) the step size along that direction. The basic mathematical foundation of this algorithm is summarized below. Given that the objective is to maximize a multiattribute welfare or utility function which is defined over the p objectives

$$\text{Max } U[Z_1(X), Z_2(X), \ldots, Z_p(X)] \qquad (21)$$

Geoffrion et.al (1972, p. 358) showed that the direction subproblem is to find a direction $d^i = y^i - X^i$, where y^i is the optimal solution to

$$\text{Max } \{\nabla_X U [Z_1(X^i), Z_2(X^i), \ldots, Z_p(X^i)]y\} \qquad (22)$$

$$\text{s.t. } y \in F_d \qquad (23)$$

where X^i is an existing feasible solution and F_d is the feasible region in decision space. By application of the chain rule for differentiation, (22) can be written as:

$$\text{Max } \sum_{k=1}^{p} [\partial U \div \partial Z_k \mid Z(X^i)] \; \nabla_X Z_k(X^i)y. \qquad (24)$$

Also by the definition of the MRS (marginal rate of substitution)

$$\partial U/\partial Z_k \div \partial U/\partial Z_r \mid Z(X^i) =$$

$$- \partial Z_r \div \partial Z_k \mid Z(X^i) =$$

$$MRS_{kr} (X^i) \; k = 1,2, \ldots p \qquad (25)$$

where Z_r is a reference objective used to compute all MRS_{kr}'s.

The direction subproblem is therefore given by:

$$\text{Max } \sum_{k=1}^{p} MRS_{kr}(X^i)\nabla_X Z_k(X^i)y \qquad (26)$$

TABLE 1

EXAMPLE APPLICATION OF THE WEIGHTING METHOD

Weights		Optimal Value of Objective Function	Optimal Solution		Non-inferior Point Generated in Fig. 4 and Fig. 5.
			in decision space	in objective space	
w_1	w_2	Z^*	(X_1^*, X_2^*)	(Z_1^*, Z_2^*)	
1	0	30	(6, 0)	(30, -6)	B
1	1	28	(6, 2)	(26, 2)	C
1	2	36	(4, 4)	(12, 12)	D
1	3	48	(4, 4)	(12, 12)	D
0	1	15	(1, 4)	(-3, 15)	E

s.t. $y \in F_d$.

It follows that the step size subproblem may be expressed as

$$\text{Max } U\,[Z_1(X^i + td^i),\; Z_2(X^i + td^i), \ldots,\; Z_p(X^i + td^i)] \qquad (27)$$

$$\text{s.t. } 0 \leqslant t \leqslant 1 \qquad (28)$$

where $d^i = y^i - X^i$ is the optimal direction obtained from the direction subproblem. When the utility function is not specified, Geoffrion suggests plotting each objective over the range of interest $Z_k(X^i + td^i)$, $0 \leqslant t \leqslant 1$; then letting decision-maker choose a solution in this range of choice.

The algorithm may clearly be executed in an iterative fashion involving the decision maker and analyst as follows:

Step 0:

The dicision-maker (or the analyst) selects a feasible solution $X^o \in F_d$, $i = 0$

Step 1:

(a) The decision-maker articulates the local MRS_{kr}'s at X^i.

(b) Solve the direction subproblem (26). Set $d^i = y^i - X^i$.

(c) If $d^i = 0$ or $|\,d^i\,| \leqslant \epsilon$, where ϵ is preset small positive number, stop. The BCS (best compromised solution) has been found. Otherwise go to next step.

Step 2:

(a) If the utility function can be specified, solve (27) subject to (28) to obtain the optimal step t^*. Go to Step 2(c). Otherwise go to Step 2(b).

(b) Plot the functions $Z_k(X^i + td^i)$ over the region where $0 \leqslant t \leqslant 1$, and let the decision-maker determine a t^* by inspection.

(c) Set $X^{i+1} = X^i + t^* d^i$, $i = i+1$ and go to Step 1.

To apply the above algorithm to the example problem stated in (14) and (15), we first assume the multivariate utility in (21) is expressible in the product form given by:

$$U\,[Z_1(X), Z_2(X)] = Z_1 Z_2, \qquad (29)$$

The MRS_{21} evaluated at a current solution point X^i can be thus given by:

$$MRS_{21}(X^i) = -\,dZ_1 \div dZ_2 \mid Z(X^i) = Z_1(X^i) \div Z_2(X^i) \qquad (30)$$

To get an initial feasible solution, we may solve a weighted problem with appropriate weights. Suppose that weights are $w_1 = w_2 = 1$; the solution associated with this weighted problem has been solved and presented in Table 1. Thus we obtain the initial solution,

$$X^o = (6,2) \text{ and } Z_1(X^o) = 26,\quad Z_2(X^o) = 2.$$

Therefore, the algorithm be described for the example problem as follows:

First Iteration

Step 1:

(a) We calculate

$MRS_{21}(X^o = 26 \div 2 = 13.$

(b) Since

$\nabla Z_1 = (5,-2),\ \nabla Z_2 = (-1,4)$

and

$MRS_{11} = 1\ ,\ MRS_{21} = 13,$

the direction subproblem (26) becomes:

$$\begin{aligned}&Max(5,-2)(y_1,y_2)^T+\\&\quad 13(-1,4)(y_1,y_2)^T =\\&\quad -8y_1+50y_2\end{aligned}$$

s.t. $(y^1,y^2) \in F_d$.

The optimal solution of this subproblem is $y^o = (1,4)$

Set $d^o = (1,4)-(6,2) = (-5,2) \neq 0$. Go to Step. 2.

Step 2:

(a) Using (22), the stepsize subproblem (27) subject to (28) becomes:

$$\begin{aligned}&Max\ U\ [Z_1(X^o+td^o),\\&Z_2(X^o+td^o)]\\&= Z_1(X^o+td^o)\ Z_2(X^o+td^o)\\&= [5(6-5t)-2(2+2t)]\\&\quad [-(6-5t)+4(2+2t)]\\&= (-277t^2+280t+52)\end{aligned}$$

s.t. $0 \leqslant t \leqslant 1$

Solving this problem we get $t^* =$ 0 37.

(b) Note $X' = X^o+t^*d^o =$ (4.2,2.7), $Z(X') = (15.6,\ 6.6)$.

Second iteration

Step 1:

(a) Note $MRS^{21}(X') = 15.6/6.6 =$ 2.4. Use this in (27) get a new direction subproblem.

(b) Solve the new direction subproblem

$Max\ 2.6y_1+7.6y_2$

s.t. $(y_1,y_2) \in F_d$.

The result is $y' = (4,4)$. Set $d' = y'-X' = (-0.2,1.3) \neq 0$. Go to Step 2.

Step 2:

(a) Solve the new stepsize problem

$Max\ (-19.4t^2+60.5t + 103)$

s.t. $0 \leqslant t \leqslant 1$

The result is $t^* = 1$.

(b) Note $X^2 = X'+d' =$ (4,4), $Z(X^2) = (12,12)$.

Repeating the procedure, we obtain $y^2 = (6,2)$, $d^2 = (2,-2)$, $x^3 = (4.3, 3.7)$, and $Z(X^3) = (14.4,\ 10.3)$ in the 3rd iteration. Finally convergence is achieved at the 4th iteration, where we get $d^3 = (0,0)$; thus the algorithm stops with the best-compromize solution (4.3, 3.7), at which $(Z_1,Z_2) = (14.4,\ 10.3)$. The convergence to the BCS is also shown in the objective space in Fig. 6.

The above procedure has been completely computerized (Dyer, 1973) allowing the decision maker to interact directly with the algorithm through a computer terminal. However, the overall performance of the Geoffrion method, as Wallenius (1975) pointed out, is not as good as might be expected, mainly due to the difficulties experienced by the subjects in estimating the MRS.

CONCLUSION

We have shown that the problem of investment in rural roads may be formulated as a multi-criteria optimization problem. Moreover, we have illustrated that the form of this problem is such that two widely known techniques for "solving" multi-criteria mathematical programming problems may be applied. These techniques, the weighting method and the iterative preference incorporation method due to Geoffrion et.al, differ dramatically in the types of information required by the decision-maker and the degree of interaction between the decision-maker and the analyst.

The weighting method, prototypical of the methods generally classified as generating methods, strive to approximate the noninferior set (the set of all efficient alternatives), with the implicit assumption that knowledge of this set will allow the decision-maker to select a best compromise solution. The iterative method of Geoffrion et.al, like all preference incorporation approaches of which it is but one example, seeks to identify the best compromise solution without generating the entire noninferior set; this is

APPLICATION OF GEOFFRION'S METHOD TO THE EXAMPLE PROBLEM

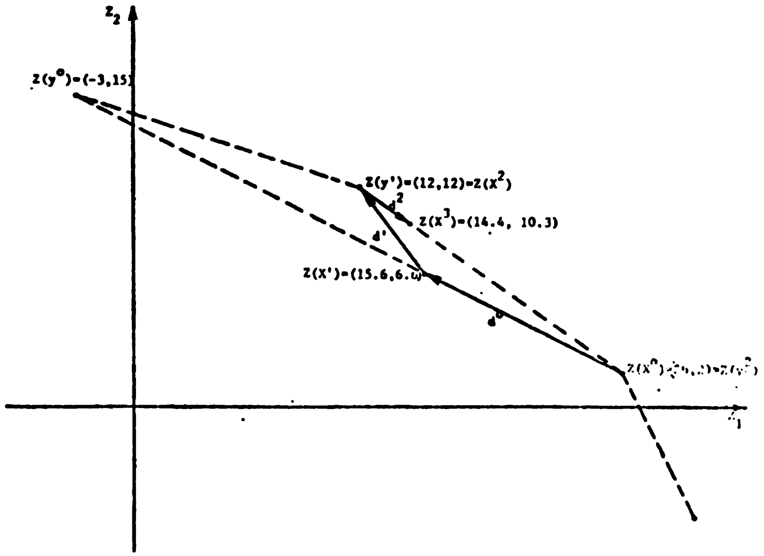

FIGURE 6

accomplished by soliciting preference information from the decision maker which leads to the best compromise solution through the generation of fewer noninferior alternatives.

The two methods discussed here illustrate the significant difficulties associated with the multi-criteria evaluation of transport investment projects—such as rural roads in developing countries—which involve conflicting objectives. Chief among these difficulties is the fact that generating methods (such as the weighting method) do not systematically guide the decision maker to a final decision and may, for problems with large numbers of objectives, require the consideration and assimilation of more information (principally information in the form of "trade-off" curves describing noninferior alternatives) than can be reasonably handled. Preferences incorporation methods, the alternative to generating methods, particularly more sophisticated methods like the Geoffrion et.al method, require preference information (e.g. marginal rates of substitution) which may be beyond the ability of the decision maker to supply due either to problem complexity or lack of technical understanding of the problem at hand.

In light of the above observations and the current state of the art, it can be concluded that the multi-criteria optimization method selected to analyze projects in which the evaluation problem has the form of a vector mathematical program must be selected in light of the prevailing decision making environment. Where problems are small and a desire to know all noninferior alternatives prevails, methods such as the weighting method are appropriate. When adequate decision maker technical sophistication and time for interaction with the analyst exists, iterative preference incorporation methods such as that due to Geoffrion et.al are appropriate.

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FOOTNOTES

1 The MRS (marginal rate of substitution) of two objectives, as we shall point out, is the negative reciprocal of the ratio of w_i's. When all w_i's are zero, it implies that all the MRS's of objectives are undetermined. In other words, there exists no utility function for the decision-maker in this case.

2 The computational cost for applying a generating technique increases exponentially with the number of objectives.

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